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Greek Musical Writings II

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Greek Musical Writings

Volume II
Harmonic and Acoustic Theory

Edited by
Andrew Barker
Senior Lecturer in Philosophy
University of Warwick

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In this volume even more than in its predecessor, I owe a particular debt to Professor Winnington-Ingram, especially for his generosity in allowing me to see his manuscript translation of Aristides Quintilianus, *De Musica*, together with detailed notes on some of its early chapters. My work on that difficult treatise would otherwise have been even more imperfect than it is: he is not to be blamed for my errors and confusions.

Writing a book is an absorbing activity, but one that gets inextricably entangled with others. I should like to dedicate this volume compendiously to my wife Jill, my family and my friends, without whom it would all have been very different. It is also a little offering in memory of Derek Macnutt, at whose feet I first learned the rudiments of the translator's art, and whose shade even now, between exertions on the Elysian fairways, still notes our mistakes with an unerring eye, and smiles benignly on our small successes.

Abbreviations, texts and typographic conventions

In citing ancient and modern works I have used only a few esoteric abbreviations:

DK	H. Diels and W. Kranz, *Die Fragmente der Vorsokratiker*, 8th edition, Berlin, 1956–9
GMW vol. 1	A. Barker, *Greek Musical Writings*, vol. 1, Cambridge, 1984
MSG	C. von Jan, *Musici scriptores graeci*, Leipzig, 1895
SVF	H. von Arnim, *Stoicorum veterum fragmenta*, 4 vols., Leipzig, 1905–24.

Most other references to modern writings are by author's name and date. Full details of works indicated in this way will be found in the Bibliography.

The translations are based on the following editions (cases where I have preferred a different reading of the texts are mentioned in the notes):

Chapter 1 The text of the second passage is from vol. 4 of the edition by R. G. Bury, London, 1961 (Loeb Classical Library). Extracts from authors translated more fully elsewhere in the volume are taken from the editions mentioned below. For the remainder I have used the texts printed in DK

Chapter 2 Plato: texts from editions printed in the Oxford Classical Texts

Chapter 3 Aristotle: texts from editions printed in the Oxford Classical Texts

Chapter 4 Texts from *Aristotle, Problems I–XXI*, ed. W. S. Hett, London, 1970 (Aristotle, vol. 15, Loeb Classical Library)

Chapter 5 The Peripatetic *De Audibilibus*: from Porphyry, *Commentary on Ptolemy's Harmonics*, ed. I. Düring, Göteborg, 1932

Chapter 6 Theophrastus, frag. 89: from Düring (1932), as above

Chapter 7 Aristoxenus, *Elementa Harmonica*, ed. R. da Rios, Rome, 1954. Appendix to ch. 7, Aristoxenus, *Elementa Rhythmica* Book II, from *Aristoxenus von Tarent: Melik und Rhythmik des Classischen Hellenenthums*, ed. R. Westphal, vol. 2, Leipzig, 1893 (reprinted Hildesheim, 1965); see also Pighi (1969).

Chapter 8 The Euclidean *Sectio Canonis*: text from *MSG*

Chapter 9 Passages of Theon Smyrnaeus, *Expositio rerum mathematicarum ad legendum Platonem utilium*: from the text of E. Hiller, Leipzig, 1878; passages of Porphyry from Düring (1932)

Chapter 10 Nicomachus, *Enchiridion* : text from *MSG*

Chapter 11 Ptolemy, *Harmonics*, ed. I. Düring, Göteborg, 1930

Chapter 12 Aristides Quintilianus, *De Musica*, ed. R. P. Winnington-Ingram, Leipzig, 1963

Other musicological works frequently mentioned include:

(a) The three treatises each entitled *Eisagoge Harmonike* by Bacchius, Cleonides and Gaudentius: texts are in *MSG*, as are the selections entitled *Excerpta ex Nicomacho*; Cleonides is translated in Strunk (1952), Bacchius in Steinmayer (1985)

(b) The group of short works whose familiar title I abbreviate as *Anon. Bell.*, the *Anonyma de musica scripta Bellermanniana*, ed. D. Najock, Leipzig, 1975; see also Najock (1972)

(c) Philodemus, *De Musica*, ed. J. Kemke, Leipzig, 1884; see also Rispoli (1969)

(d) The Plutarchian *De Musica*: text and translation in *Plutarch's Moralia*, vol.14, ed. B. Einarson and P. H. De Lacy, London, 1967 (Loeb Classical Library); other important editions, both with French translation and substantial commentary, are *Plutarque: de la musique*, ed. H. Weil and T. Reinach, Paris, 1900, and *Plutarque de la musique*, ed. F. Lasserre, Olten and Lausanne, 1954; English translation also in *GMW* vol. 1

In referring to passages translated in this volume, I have generally used the systems of numbering that stem from canonical printed editions of the texts: these are indicated in the left-hand margin beside the translations. The names of author and work are preceded by bold numerals indicating where they occur in this book. In cases where a chapter contains a single passage or treatise, the numeral is the chapter number; where it contains several passages, the first numeral is the chapter number, and the second identifies the position of the passage in its chapter. Thus a reference to line 8 of Meibom's page 16 of Aristoxenus' *Elementa Harmonica*, which occupies the whole of chapter 7, will appear as 7 Aristox. *El. Harm.* 16.8. A reference to Plato, *Republic* 531a, which occurs in the first passage of chapter 2, will appear as 2.1 Plato *Rep.* 531a.

Introduction

Preliminaries

The roots of the Greek sciences of harmonics and acoustics go back to the fifth century B.C., perhaps even the sixth. No treatises survive from this period, and only one or two short quotations: even these are of questionable authenticity. Later writers, reflecting on the past, offer tantalising hints about pioneering efforts in the field. Some of these, referring to just one of several traditions of enquiry, are collected in my first chapter, and others appear in texts translated elsewhere in the book. It seems likely that these beginnings were fairly unsystematic, and were usually embedded in writings of wider scope. The classification of sciences into distinct domains and their pursuit as autonomous intellectual enterprises are things that were only beginning in the later fifth century, and came into their own in the fourth.

Even for the first three quarters of the fourth century we have very little from the pens of specialists in the musical sciences: our small collection of quotations and paraphrases of the work of Archytas, important though they are, give a pretty thin representation of the work of seventy-five years. Some of them are also included in chapter 1. But for these years we have another significant source of information in the writings of the philosophers, especially Plato and Aristotle (chapters 2 and 3). Though neither made the scientific study of music a central part of his own investigations, both found that their reflections in other areas required them to pay these subjects careful attention, and each made contributions to them which exercised a powerful influence on later theorists. Both also give us valuable information about the work of their contemporaries and predecessors.

Among the sciences which Aristotelian methods of classification identified as independent enquiries was physical acoustics; and from the years after Aristotle's death there survives a compilation, made within his 'school', the Lyceum, of problems that arise in that field, together with suggested solutions. In the same collection is a comparable set of puzzles relating to music more generally, some of which bear on harmonics. Selections from both are given in chapter 4. Something approaching the status of a complete treatise is translated in chapter 5: parts of it are certainly missing, but even as it stands it is a substantial essay in acoustics, still within the Aristotelian or 'Peripatetic' tradition. A long fragment by another philosopher in this school, Aristotle's successor Theophrastus, occupies chapter 6. It reviews, very critically, the assumptions and procedures of all theorists up to that time who had conceived

the study of music, from any point of view, as a quantitative or mathematical discipline.

So far, all that we know of specialist writings in harmonics comes either indirectly, in the reports and comments of others, or in the form of fragmentary quotations, torn from their context. But from the end of the fourth century we have two impressive works, the *Elementa Harmonica* of Aristoxenus (chapter 7), and the *Sectio Canonis* attributed to Euclid (chapter 8). The former is incomplete, and as we have it is probably the remains of more than one treatise, edited into a continuous piece at a later date, but its importance for the history of the subject can hardly be exaggerated. Subsequent theorists treated Aristoxenus' writings both as the foundation stone of one major tradition in Greek harmonics, and as the pinnacle of its achievements. The *Sectio Canonis* is dated only uncertainly to this period, but it represents neatly and systematically, and perhaps in almost its complete and original form, a rounded exposition of harmonic theory in a quite different style from that of Aristoxenus, one based in mathematics and physics rather than in musical experience. It is an illuminating specimen of the tradition that may roughly be called 'Pythagorean' or 'Platonist', in the more scientific and rigorous of its various guises.

The enterprises of theoretical harmonics, under the banners of two principal schools of thought (loose confederations of interests and approaches, each internally divided in significant ways), were now well under way. Disappointingly, very little remains of writings in either tradition over the next three hundred years. From the time when our direct evidence begins to resurface, in the first century A.D., we can put together a tolerable amount of material representing contemporary ideas, and giving some notion of their relation to their fourth-century precursors: relevant passages from two major sources are assembled in chapter 9. My remaining chapters contain three complete treatises, those of Nicomachus, from about the end of the first century (chapter 10), Ptolemy in the second century (chapter 11), and Aristides Quintilianus, whose date is uncertain, but perhaps belongs to the third century or fourth (chapter 12). Of the three, those of Ptolemy and Aristides are of great intrinsic value: Ptolemy's for his intellectual rigour, his compelling and original method, his detailed critiques of previous theories and his independent development of new ones; Aristides' for its impressive (if only fitfully successful) attempt at the extraordinary project to which its author set himself with such enthusiasm, that of bringing everything that could be said about music into a compendious scheme embracing all human life, the cosmic order and God. Both convey a mass of information about musical practice, and about the ideas of earlier writers, much of it unknown elsewhere. The work of Nicomachus, though slighter than the others, is significant as our earliest complete specimen of the genre from which Aristides' much wider vision developed, the 'Pythagorean' essay in musical metaphysics. Like the other two, it must also be treated as a landmark in musicological history for the influence it came to exert on theorists of post-classical times.

In selecting the texts to translate I have been guided by the conviction that certain major works should be presented complete, or in as complete a form as the surviving documents permit. By applying this policy to the *De Audibilibus*, Aristoxenus, Euclid, Nicomachus, Ptolemy and Aristides Quintilianus, I have inevitably squeezed out many writings with a good claim on our attention. Those who know the field will no doubt deplore the absence, in particular, of the 'Aristoxenian handbooks', essays put together in the first few centuries A.D. as compilations of the doctrines of that school. (They include the treatises of Cleonides, Bacchius and Gaudentius, and those collectively known as 'Bellermann's Anonymous'.) In mitigation I can plead that the bulk of the information they offer can be found either in Aristoxenus' *El. Harm.* itself, or in the first book of Aristides Quintilianus. On points of detail, admittedly, these authors often differ among themselves, and I have tried to make comparisons where they are relevant, especially in the notes to chapter 12. The most important of them, Cleonides, is translated in Strunk (1952). I might add that they make dreary reading: Aristides Quintilianus, for all his faults, at least has some fire in his belly.

Two other kinds of material are obtrusively missing from this volume. One relates to notation, which I have dealt with only by brief comments on the sources who mention or use it (principally Aristoxenus and Aristides Quintilianus). The treatise on which we must rely for most of our knowledge of Greek notation, that of Alypius (printed in *MSG*), is a work whose translation would serve little purpose: it is principally valuable for its tables of notational signs. These and the systems of ideas underlying them have been ably discussed elsewhere: see especially Gombosi (1939), chapters 3 and 5, Winnington-Ingram (1956), Henderson (1957), pp. 358ff., Barbour (1960), Pöhlmann (1970), pp. 141ff. Similarly, I have not reproduced any of the surviving scores of Greek melodies, thought I have referred to them occasionally in the commentary. Most of them are printed and discussed in Pöhlmann (1970), and there is also a useful brief analysis in Chailley (1979). They are not documents in the 'literature' of music, and would therefore be out of place in this book, since no ancient theorist discusses them. The exercise of applying the theorists' constructions to the analysis of the scores is one that I must leave to the reader.

Traditions of enquiry in harmonic and acoustic science

Greek harmonics, broadly conceived, is the study of the elements out of which melody is built, of the relations in which they can legitimately stand to one another, of the organised structures (e.g., scalar systems) formed by complexes of these relations and of the ways in which different structures are generated by combinations or transformations of others. As I have already hinted, there was no single, homogeneous Greek approach to this study. For most of its history, Greek harmonic writing can be classified under two fairly distinct traditions, the 'Aristoxenian' and the 'Pythagorean', each with its own

characteristic presuppositions, methods and goals. It must be said at once that neither school is monolithic – there are important internal distinctions to be drawn – and that the work of each did not flow onwards quite independently of the other. There were occasional attempts to bring the two approaches together in a coherent synthesis, as well as more frequent polemical interactions across the doctrinal divide.

As Aristoxenus conceived it, harmonics is a science whose data and explanatory principles are independent of those in any other domain of enquiry. Its subject is music as we hear it, the perceptual data offered to the discerning musical ear. Its task is to exhibit the order that lies within the perceived phenomena; to analyse the systematic patterns into which it is organised; to show how the requirement that notes must fall into certain patterns of organisation, if they are to be grasped as melodic, explains why some possible sequences of pitches form a melody while others do not; and ultimately to display all the rules governing melodic form as flowing from a coordinated group of principles that describe a single, determinate essence, that of 'the melodic' or 'the well-attuned' itself.

Three points are of special importance in this programme. First, the science begins from the data presented to perception and grasped by it as musical. It is these that must be brought into a comprehensible order, in precisely that guise under which they strike the ear as melodic, or as exhibiting specific kinds of melodic relation: they must not be redescribed, for scientific purposes, as (for instance) physical movements of the air. The order Aristoxenus seeks is a set of relations between items grasped in their character as notes, and there is no need, and no reason, to suppose that the same relations hold between the physical movements that are their material causes. (To suggest a modern analogy, the note A is the dominant of D major, but 440 cycles per second cannot be the dominant of anything.) Secondly, and as a consequence, harmonics must *describe* the phenomena in terms that reflect the way in which they are grasped by the ear. Various special conclusions flow from this, as well as a general tendency to write in a language developed out of the terminology of practising musicians: the most important is that notes should be treated as located at points lying on a continuum of pitch, and the relations between them as 'distances' or 'intervals', *diastēmata*. Intervals must themselves be described in autonomously musical terms, as distances of various sizes in the dimension of pitch (as tones, half-tones, and the like). An interval is not to be defined by reference to something non-musical, something that the musical ear does not grasp as such (for instance, as the ratio between the speed of the movements by which the relevant pitches are produced). Thirdly, the coordinating principles of the science must themselves be found by abstraction from the perceived musical data. We explain why this sequence is melodic while that is not, by identifying a general pattern of order, essential to all melody, to which the former, and not the latter, conforms. The principle stating that this order is essential is extracted, along with all other principles in the domain, from the experience of the trained musical ear. They are not sought outside it,

for instance in mathematics: it is not because a sequence can be described by a neat mathematical formula that it is melodically coherent, nor is it because two structures differ in ways significant for mathematics that they fall into aesthetically distinct harmonic categories. (Boundaries between structures that differ significantly from a mathematical point of view may not coincide with ones that are musically important.)

Aristoxenus' own approach involves a number of subtleties that this sketch ignores. It is also true that he had predecessors and successors, discernibly of the same musicological tendency, whose methods do not tally with his in all respects. Those whom he acknowledges as his precursors seem to have been 'empiricists' of a cruder sort, content to tabulate the perceptual data without seeking to discover their coordinating principles. By contrast, many later 'Aristoxenian' writers sought only to give a scholastic exposition of the master's 'doctrines', and to reduce them to an academic system, neglecting the need for harmonic understanding to be grounded in real musical experience, and ironing out many of the penetrating ideas that Aristoxenus had derived from that source himself. Where that experience was lacking, or not applied in the proper way, no suitable yardstick was left by which the relative importance of different aspects of Aristoxenus' analyses could be assessed. Throughout the tradition, however, two central features persist. The data are described in autonomously musical terms (conceived as representing the phenomena in the way that perception grasps them, and developed in large part from the vocabulary of musicians themselves): they are neither described nor explained by reference to concepts drawn from mathematics and physics. Secondly, intervals are consistently treated as linear distances on a continuum of pitch, measured only by the units through which musical experience is articulated, and not transformed to fit into some extra-musical metric: notes are located in (sometimes, identified with) the 'breadthless' points that are these intervals' boundaries.

On the other side of the fence are those theorists who can be bundled together, for convenience, under the description 'Pythagorean'. The word is a slippery one. Not all of those embraced by it here would have professed any allegiance to Pythagoras himself. Few were genuine members of the Pythagorean brotherhood, dedicated to its rituals and its way of life as well as to its intellectual ideas: in its original form the brotherhood had evaporated by the early fourth century. Some, in particular, were specialist mathematicians whose wider philosophical commitments, if any, had little bearing on their work in this field. Even if we restricted the term to people who would have given it to themselves, there is a world of difference between the immediate followers of Pythagoras in the sixth and fifth centuries B.C., the Pythagorising successors of Plato in the fourth and third, and the enthusiasts of the neo-Pythagorean revival in the first centuries A.D. But in harmonics they had certain views and attitudes in common, and if their work is not a single enterprise it is at least a network of interconnected strands; the crude classification is not altogether arbitrary.

We have an enormous amount of information about Pythagoreans, but so far as it bears on the sixth and fifth centuries B.C., the bulk of it is misinformation, a mixture of speculative reconstruction, anachronistic interpretation and plain forgery. From Plato onwards the ideas allegedly developed by early Pythagoreans were refurbished, embroidered and newly tailored to suit each passing intellectual or religious fashion; and such was the veneration inspired by the long-dead founder of the sect that these novel, often bizarre and gaudy philosophical garments were seldom labelled with their own makers' names, but with that of Pythagoras himself, or those of his direct disciples. As a result, our evidence is in confusion. I can do little to untangle it here: the reader should consult especially the pioneering work of Burkert (1962, translated 1972), along with that of Thesleff (1961, 1965) and Philip (1966).

From the beginning, Pythagoreans were not typically interested in the study of music for its own sake. Their researches in harmonics arose out of a conviction that the universe is orderly, that the perfection of a human soul depends on its grasping, and assimilating itself to that order, and that the key to an understanding of its nature lies in number. Music enters the matter with the discovery that the relations between notes framing an organised melodic structure can themselves be expressed in very simple and neat numerical formulae. The lengths of two sections of a string giving notes an octave apart are in the ratio 2:1, while the ratio 3:2 gives a fifth and 4:3 a fourth. These fundamental harmonic relations thus correspond to what are evidently elegant and fundamental mathematical relations, and encourage the idea that all properly harmonic intervals gain their musical status *because* of their mathematical properties. So we arrive at a position that conflicts sharply with that of the Aristoxenians. The order found in music is a mathematical order; the principles of the coherence of a coordinated harmonic system are mathematical principles. And since these are principles that generate a perceptibly beautiful and satisfying system of organisation, perhaps it is these same mathematical relations, or some extension of them, that underlie the admirable order of the cosmos, and the order to which the human soul can aspire.

These ideas fuelled enthusiasm for investigations in mathematics, together with less rationalistic speculations about the symbolic import of individual numbers. Particular attention was focussed on the mathematics of ratio and proportion, with constant reference back to the paradigmatic ratios governing basic musical structures. By the end of the fifth century, an analysis in terms of ratios had been extended to at least one pattern of attunement spanning a complete octave, where all the important relations between its notes are described as ratios of numbers; others, representing different systems of attunement, appear during the early fourth century in the work of Archytas. Questions of a more abstract order also come into sharp focus in the writings of Archytas, and insistently in Plato. Most importantly, given that such and such a set of ratios combines to form a coherent scalar system, why? What are

the mathematical principles to which a set of ratios must conform if its structure is to be harmoniously coordinated? What is the special feature of those principles that gives them their role as the source of order and coherence, whether musical, psychic or cosmic? Again, what is it about some mathematical relations that is reflected in the concordance of their musical counterparts, while other equally intelligible ratios correspond to discords?

Such questions looked for answers in mathematics, but not only there. Pythagorean ideas about music probably began, as I have said, from observations about lengths of string. But musical sounds can be produced by other means, and in any case the audible sounds are not themselves lengths. Then when two notes stand in the relation of the octave, what are the items that stand to one another in the ratio 2:1? Here mathematical harmonics needs a supplement from speculations in physics, enquiries into the nature and causes of sound as a physical phenomenon. We know little about their origins: the earliest substantial account is in the first fragment of Archytas, which most later researches in physical acoustics took as their point of departure. A few of the details will be mentioned below: more will appear in the translations. The crucial fact is that these theorists were able to identify a quantitative physical variable (most commonly, the speed of a sound's transmission through the air) as that which determines its pitch. Ratios between different values of this variable could then be identified as the ratios with which harmonics concerns itself, and it became possible to offer physical, as well as mathematical explications of such phenomena as concordance.

In some writings, notably the Euclidean *Sectio Canonis*, the task undertaken by the author is to show how the propositions of harmonics can be demonstrated as theorems within mathematics itself, given certain assumptions about the physical nature of the musical phenomena. In others, the metaphysical and mystical sides of Pythagoreanism come to the fore, in attempts to relate the order of music to the harmony of the heavens and the intelligible organisation of the universe at large. Sometimes, as in Plato and the writers who follow him, harmonic analysis focusses only on those forms of attunement which exhibit the mathematical and metaphysical principles of coordination in their purest form – other kinds of system are dismissed as mere human aberrations. Elsewhere in the tradition, however, a mismatch between theory and practice will more probably be taken as a sign of defective theory. Archytas, for instance, and most notably Ptolemy (though his concepts and methods derive as much from his original genius as from a Pythagorean tradition), direct their analyses at least in part to musical systems in actual use, in an attempt to show that they too exhibit coherent patterns of mathematical order. But in all 'Pythagorean' harmonics there are common features that distinguish their enterprises from those of the Aristoxenians. The most important are these.

First, notes are treated as entities one of whose attributes, that of pitch, varies quantitatively, and can be expressed in numbers. Intervals between notes

are to be expressed as ratios of numbers. Notes, then, are items possessing magnitudes of some sort. They are not points on a line, with intervals as the linear 'distances' between them.

Secondly, the principles on which the structure of harmonic systems is to be analysed and by which their coherence is to be explained are mathematical. More generally, the proper language for the rigorous discussion of harmonic issues is that of mathematics, of which harmonics is a branch: it is not an autonomous discipline to be discussed in an independent terminology developed out of the professional patois of practising musicians.

Thirdly, the application of mathematical concepts to musical phenomena is mediated by a physical theory that re-identifies the entities under discussion, perceived in the guise of notes, as movements in a material medium. It is to these movements that the quantitative characteristics can be attached directly.

Finally, in the majority of 'Pythagorean' writers, the study of harmonics is part of a much larger enterprise, designed to show how the same principles govern 'harmonious' relations between the elements of all significant structures in the cosmos. The universe and its parts are all subject to the same perfect patterns of intelligible mathematical order. This programme was pursued in many different ways and with different preconceptions about the nature and source of that order, but the projects undertaken in their various ways by Philolaus, Archytas, Plato, Theon, Nicomachus, Ptolemy and Aristides Quintilianus all stem from a similar aspiration. In mathematics, and especially in mathematical harmonics, lies the key to the rational organisation of the universe.

Some writers, particularly during and after the first century A.D., made heroic attempts to combine elements drawn from both harmonic traditions. From the point of view of musical analysis, the Aristoxenians had a far richer and more flexible repertoire of concepts to draw on, and were able to give systematic accounts of a much greater variety of musical structures. The Pythagoreans offered nothing comparable, but their procedures appealed to a demand for intellectual rigour and demonstrative argument, and fed, as we have seen, into scientific and metaphysical enquiries of far wider scope. Few theorists found persuasive ways of reconciling the two approaches, which were indeed often held to conflict in their concrete conclusions as well as their conceptual apparatus. In Theon, Nicomachus and Aristides Quintilianus, Pythagorean and Aristoxenian analyses sit uneasily side by side. Only Ptolemy, whose methodological and mathematical ingenuity far exceeded theirs, offered an intellectually convincing way of coordinating a mathematically rigorous form of analysis, close to that of the Pythagoreans, with a realistic sensitivity to the complexity and variability of actual musical structures, preserving some of the musical richness of Aristoxenian accounts while wholly rejecting their framework of concepts and methods.

Ideas about the physical nature and attributes of sound were first developed in detail in the context sketched above, as part of the explanatory paraphernalia

of mathematical harmonics. Certainly there were speculations in this area before Archytas, some within the Pythagorean school, others offered by Presocratic cosmologists in the course of their researches into the constitution of the material world, and the way its perceptible phenomena arise. But Archytas frag. 1 is our first surviving sustained essay on the subject, and its enquiries set the agenda for most later researches, which refined and modified its hypotheses many times over. After Archytas and Plato, it was the school of Aristotle that conducted the most important work in physical acoustics. Here it was to some extent pursued as a science in its own right, detached from harmonics, though inheriting some of the problems and most of the presuppositions bequeathed by Pythagorean students of music and mathematics. The two disciplines are recombined in some of the Aristotelian *Problems* and in the *Sectio Canonis*, and later in the neo-Pythagorean writings of the first centuries A.D. Their authors were able to draw on physical theories from a wide range of Pythagorean, Platonist, Aristotelian and Stoic sources to underpin their programmes for harmonics.

All Greek acoustic theories begin from the proposition that sound is a form of movement in the air, caused by an impact made on the air by a solid body, or by an emission of breath. One major question for debate was how it is transmitted from place to place. The earliest authorities suppose that portions or currents of air actually travel from the source to the hearer's ear. Others (particularly in some of the *Problems*), reflect on differences between the behaviour of a sound and that of a solid missile, and hint at a different view, expressed most clearly in the *De Audibilibus*. No parcel of air travels when a sound moves: its transmission is rather a spreading pulsation in a stationary elastic medium. The second issue of prime importance concerned pitch. What is it that distinguishes a movement perceived as a high-pitched sound from that perceived as a deep one? The question is obviously crucial to mathematical harmonics, and to the interpretation of the ratios by which musical intervals are described. It is essential to identify the variable whose 'magnitudes' the terms of the ratios quantify. Several approaches can be disentangled. The earliest (that of Archytas and Plato) and much the most popular in later writings identified this variable with the speed of a sound's transmission from place to place: theories of this sort continually resurface throughout antiquity. But they posed obvious problems, most notably that of explaining how two sounds of different pitches, simultaneously produced, could be heard as simultaneous at some distance from their origin. A second theory is suggested in a number of sources, but explicitly stated only in one, the *Sectio Canonis*. It begins from the observation that a plucked string, in generating an apparently continuous sound, oscillates back and forth. The string is therefore conceived as making not one impact on the air, but a sequence of detached blows, though these follow one another so rapidly that our hearing grasps the resulting movements as a single, sustained and uninterrupted sound. This conception is next generalised to cover all forms of sound production. It is allied to a second observation, that a string generating a higher pitch oscillates

more rapidly. The conclusion of the writer of the *Sectio Canonis* is that it is precisely this relative frequency of impacts striking the ear that is responsible for or constitutes the perceived phenomenon of higher or lower pitch. Other writers (for example the author of the *De Audibilibus*) subscribed to the theory of discrete impacts, and remarked on the greater frequency of those of higher pitched sounds, but saw this as a secondary characteristic, associated with higher pitch, but not its cause; they continued to identify that cause with the velocity of a sound's transmission. There were other theories too, not all of which treated a sound's pitch as rooted in something directly quantifiable: some linked it, for example, with the 'shape' of the sound's movement. But the two I have sketched were much the most influential.

A third fundamental question, straddling the divide between acoustics and mathematical harmonics, concerned the phenomenon of concordance. Pairs of sounds heard as concordant were held to 'blend' in a characteristic way, whereas discordant pairs did not. In mathematics the question was: 'What is it that marks out the classes of ratio corresponding to concords as ones that are peculiarly well coordinated and unified?' In physical theory it was rather: 'What sort of physical interaction is there between movements with certain relative velocities, or with certain relative frequencies of impact, which causes or constitutes this blending, and which is lacking in other cases?' 'Velocity' theorists found the problem particularly troublesome: Plato offered a solution, but its details are barely comprehensible. Under the aegis of 'relative frequency' theories an ingenious hypothesis was developed of which there are traces in reports of early Pythagorean speculations, in the *Problems*, and in the *De Audibilibus*, but the issue cannot be said to have been satisfactorily resolved.

These three questions attracted the most persistent and painstaking attention, though many others were discussed, especially by Aristotle and in the *De Audibilibus*. The science never advanced to a very high level of theoretical rigour or methodological sophistication. Speculations about the quantitative determination of pitch gained support from observations of the properties of sounding strings or pipes, and we have reports of 'experiments' with other sound-generating devices too, but few inspire any confidence, and many are plainly mere 'thought-experiments' which could not have worked in practice. None, in any case, was adequate to decide between the rival theories. They were supported, if at all, by abstract arguments of modest persuasiveness: satisfactory empirical tests were never devised. Hypotheses about the physical causes of other qualifications of sound, such as volume, clarity, harshness and so forth, were similarly mounted on a basis of plausible argument combined with informal observation: in the *De Audibilibus* such hypotheses are loosely unified under an impressionistic theory about the qualitative resemblance of physical cause to acoustic effect. The central conclusions of the science, though problematic and inadequately explored, were sufficient to encourage the programmes of Pythagorean musical metaphysics, and they could stand as presuppositions that underpinned, for example, Ptolemy's detailed pre-

scriptions for the construction of instruments through whose use his theoretically based conclusions in harmonics could be assessed by the ear. In other parts of the field, there are isolated examples of acute observation and inspired guess-work. But while the harmonic sciences exhibit some of the Greeks' most impressive intellectual achievements, it would be disingenuous to make a similar claim for their acoustics.

The organisation of harmonic space

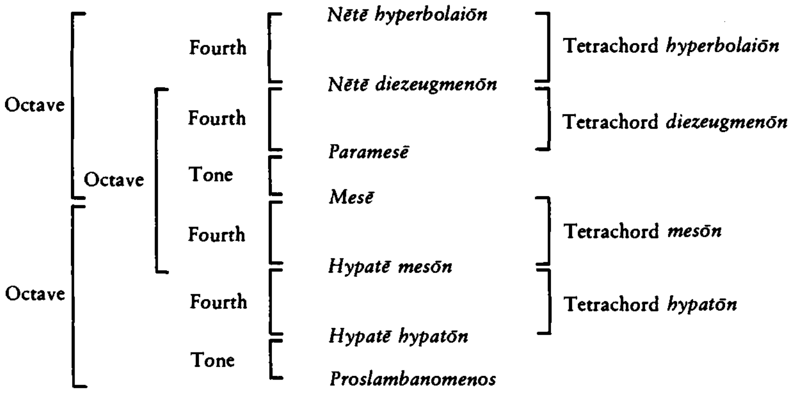
A complete account of the concepts and structures analysed by the harmonic theorists would be out of place here. A schematic sketch of some of the most important of them may be helpful as a preliminary, but I must emphasise in advance that my account will be oversimplified both conceptually and historically, particularly in its first two sections. It attempts to set out what is broadly common ground between a variety of writers and periods. It will say little about their areas of disagreement except in the section on *tonoi* below (p. 17), and less about the ways in which musical structures changed over time. Those of the period before about 400 B.C., in particular, raise problems which I shall not try to address.

Tetrachords and fixed notes

The Greeks conceived their scalar systems and patterns of attunement as expressions of the divisions and organisations imposed by melody on the tonal 'space' or range which it inhabits. The range analysed did not normally exceed two octaves, and it was of 'abstract' pitch: that is, it was not identified as any particular pitch-range, but simply as the range occupied by any melody, or by the structure implied by a melody as its foundation.

The structure was standardly envisaged as being framed by a set of notes forming the boundaries of tetrachords. In the most basic form of organisation, these tetrachords were themselves ordered in a regular way, and the notes bounding them were regarded as 'fixed', invariable in their relations to one another. Thus the central octave of the most fundamental system was divided into two principal parts, each spanning a fourth, and separated ('disjoined') by a tone. Above this octave, and sharing its highest note, lay a further tetrachord; below it was another, also in 'conjunction' with the lower of the two central tetrachords. The double octave range was completed by the addition of one more note at the bottom, at the interval of a tone below the lowest note of the lowest tetrachord. Names were attached to these fixed notes, and to the tetrachords, as in the table below.

An alternative structure, sometimes treated as a variant of the first, sometimes as an independent parallel system, retained the two lowest tetrachords, but proceeded upwards from *mesē* to a third tetrachord in conjunction with the second. This tetrachord was called *synēmmenōn* ('of conjoined notes'), as its counterpart was *diezeugmenōn* ('of disjoined notes').



The structure was not usually conceived as extending to a further tetrachord above the highest note of this one, *nētē synēmmenōn*.

Moveable notes and the genera

Each tetrachord spans a fourth: between the boundaries of each two further notes remain to be inserted. These, however, were not invariable in their relations to their neighbours. Different systems were available in which these internal notes might be higher or lower with respect to the tetrachord's boundaries. According to Aristoxenus, for example, the higher 'moveable' note in a tetrachord might lie at any distance from a tone to a ditone below the upper boundary, and the lower moveable note at any distance from a quarter-tone to a half-tone above the lower boundary. Though an indefinite number of variations of position within these ranges were in principle permissible, certain sets of tetrachordal divisions were the most familiar, and these fell into three aesthetically distinguishable groups or 'genera', the enharmonic, the chromatic and the diatonic. Theorists differed considerably in their quantifications of these divisions (especially those theorists who described the intervals as ratios of numbers, rather than as 'quarter-tones', 'semitones', etc.). They differed also over the question of how many distinct and legitimate kinds of division there were in each genus. They typically agreed, however, that what determined difference of genus was primarily the distance between the higher moveable note and the upper boundary of the tetrachord. This distance was greatest in the enharmonic genus, smallest in diatonic, intermediate in chromatic. Let us pass over the complications, and record only the simplest and commonest of the tetrachordal divisions, as expressed in the terms used by Aristoxenus and his successors. Here an enharmonic tetrachord divides the span of a fourth between fixed notes, from the bottom upwards, into quarter-tone, quarter-tone and ditone; a chromatic tetrachord into semitone, semitone, tone-and-a-half; and a diatonic tetrachord into semitone, tone, tone.

The notes lying between fixed notes were therefore given names which

qualified them as enharmonic, chromatic or diatonic. We can now fill in the two-octave framework with the moveable notes placed between the boundaries of each tetrachord, to complete what is known as the Greater Perfect System (hereafter GPS). I have shown the details of the commonest tetrachordal divisions only in the highest tetrachord; the others are identically formed. Fixed notes are capitalised.

		diatonic	chromatic	enharmonic	
Tetrachord *hyperbolaiōn*	*NĒTĒ HYPERBOLAIŌN*				
	Paranētē hyperbolaiōn (diatonic)	tone	three semitones	ditone	fourth
	(chromatic)				
	(enharmonic)				
	Tritē hyperbolaiōn (diatonic or chromatic)	tone	semitone	quarter-tone	
	(enharmonic)				
		semitone	semitone	quarter-tone	
	NĒTĒ DIEZEUGMENŌN				
Tetrachord *diezeugmenōn*	*Paranētē diezeugmenōn* (diatonic or chromatic or enharmonic)				
	Tritē diezeugmenōn (diatonic or chromatic or enharmonic)				
	PARAMESĒ				
Tone					
	MESĒ				
Tetrachord *mesōn*	*Lichanos mesōn* (diatonic or chromatic or enharmonic)				
	Parhypatē mesōn (diatonic or chromatic or enharmonic)				
	HYPATĒ MESŌN				
Tetrachord *hypatōn*	*Lichanos hypatōn* (diatonic or chromatic or enharmonic)				
	Parhypatē hypatōn (diatonic or chromatic or enharmonic)				
	HYPATĒ HYPATŌN				
Tone					
	PROSLAMBANOMENOS				

The two lower tetrachords, if conjoined at *mesē* with the tetrachord *synemmenōn*, formed the Lesser Perfect System (LPS). The moveable notes between *nētē synēmmenōn* and *mesē* were named as *paranētē synēmmenōn* and *tritē synēmmenōn*. Notes in this tetrachord will sometimes stand in the same pitch-relations to *mesē* as do some of those in the parallel system between *paranētē diezeugmenōn* and *paramesē* (just which coincidences occur will depend on genus): the analyst has to inspect the whole structural context, not merely the size of the interval between, for example, *mesē* and some other note, to determine that other note's identity. Something similar is true of other coincidences of pitch in the system, for instance that between an enharmonic *lichanos* and a diatonic *parhypatē*.

The *harmoniai*

The systems so far described provide the foundation for all others, but they are not the end of the story. Nor indeed, are they properly speaking its beginning, but represent a convenient intermediate stage in the history of harmonic investigations. It will be appropriate to move next to a form of analysis that is relatively early, predating Aristoxenus, one that concerns itself with structures spanning only an octave.

From the seventh century, if not before, the Greeks were familiar with a number of distinct melodic styles, associated with different regions or peoples of the Aegean area. Although one such style, called 'Dorian' after the Dorian race of Greeks, came to be thought of as peculiarly and nobly Greek, interaction between Greeks from different places, and contact with non-Hellenic cultures, led to the adoption of several other such styles into the music of the major centres of civilisation. By the sixth century this process was well advanced, and the literature of the sixth and fifth centuries gives hints of the ways in which the styles were distinguished and employed. They were not assimilated into a single, undifferentiated cosmopolitan mélange: Ionian, Phrygian, Lydian and Dorian music seem to have retained distinct characters, credited with distinct emotional, aesthetic and moral effects, and found their places in different religious or cultural niches. Large-scale works by sophisticated fifth-century composers might shift from one style to another in the course of a single piece, but this was a way of generating changes of feeling and mood, not merely an exhibition of complicated technique. Poets like Aristophanes in the late fifth century could still indicate distinguishable musical characters simply by means of the regional names, and the differences were still sufficiently marked in the fourth century for philosophers, notably Plato, to use them as the foundation for their theories about the distinct moral characters and influences of music of different sorts.

Even if our authorities are right in attributing theoretical musical writings to authors as early as Lasus of Hermione (late sixth century), all such works are lost, and we have little solid information about the ways in which distinctions between regional styles might, at that period, have been described from a technical point of view. By the later fifth century, however, a fairly clear general conception begins to emerge: the main differences between regional types are identified as differences of what is called *harmonia*. The word has many uses, but here its primary significance is 'attunement', specifically 'pattern of attunement over the span of an octave'. Its principal application is to the organisation of intervals between notes sounded by the strings of a *lyra* or a *kithara*. Whatever may have been the case earlier, it seems that at this stage differences of pitch-range had little to do with the matter. One *kithara* probably differed little from another in usable range of pitch, but by retuning the intervals between the strings a performer could prepare his instrument for a piece in a different *harmonia*, Dorian, Phrygian or whatever.

There is no reason to suppose that these patterns of tuning originally stood

in any straightforward and organised relations to one another. Our information is sparse: just one late source sets out what purport to be a collection of scales 'which the ancients used for their *harmoniai*', that is, to prepare the attunements of their instruments (12 Arist. Quint. *De Mus.* 18.5ff.), and even that does not claim to refer to anything before the time of Plato. If the scales it describes are genuine (and it is a big 'if'), they reveal attunements that differ in the sizes of intervals used, in their order, in the number of notes brought into play, and in their overall ranges, and they cannot all be easily treated as modifications or transformations of one another.

Attempts to reduce such *harmoniai* to a system, and in particular to express them as orderly transformations of a single structure, probably originated in the later fifth century, partly as a consequence of the growing use of modulations between *harmoniai* in practical music, partly under the influence of detached, abstract musical theory, which was just beginning to appear as a serious technical discipline. We cannot assign a definite date to Eratocles, but somewhere within a decade or two of the year 400 is a reasonable guess, and it is to him and his school of harmonic theorists that Aristoxenus attributes a representation of 'the seven octachords which they call *harmoniai*', as cyclic reorderings of a given series of intervals within the octave. That is, if we begin from a series of intervals spanning an octave, constituting some one *harmonia*, we can generate another by removing the extreme interval at one end and replacing it at the other end of the series, shifting the pitches of the inner strings so that the whole always remains within the same overall compass. In this way we produce the seven 'species' of the octave, each constituting one *harmonia*.

We also hear from Aristoxenus that his predecessors' analyses were confined to the enharmonic genus, whose tetrachords were divided into two steps of a quarter-tone each, plus one of a ditone. Drawing on later sources (especially Cleonides and Aristides Quintilianus), as well as on Aristoxenus, we can ascribe to the school of Eratocles the following system of *harmoniai*, represented on a diagram that divided the space of an octave into twenty-four quarter-tones.

Mixolydian $\frac{1}{4}$, $\frac{1}{4}$, 2; $\frac{1}{4}$, $\frac{1}{4}$, 2; 1
Lydian $\frac{1}{4}$, 2; $\frac{1}{4}$, $\frac{1}{4}$, 2; 1; $\frac{1}{4}$
Phrygian 2; $\frac{1}{4}$, $\frac{1}{4}$, 2; 1; $\frac{1}{4}$, $\frac{1}{4}$
Dorian $\frac{1}{4}$, $\frac{1}{4}$, 2; 1; $\frac{1}{4}$, $\frac{1}{4}$, 2
Hypolydian $\frac{1}{4}$, 2; 1; $\frac{1}{4}$, $\frac{1}{4}$, 2; $\frac{1}{4}$
Hypophrygian 2; 1; $\frac{1}{4}$, $\frac{1}{4}$, 2; $\frac{1}{4}$, $\frac{1}{4}$
Hypodorian 1; $\frac{1}{4}$, $\frac{1}{4}$, 2; $\frac{1}{4}$, $\frac{1}{4}$, 2

The list retains a number of the old names. Others, such as 'Iastian' and 'Aeolian' have disappeared, and instead we find what are obviously specialists' terms, Hypolydian, Hypophrygian, Hypodorian. This is already evidence of a shift away from traditional practice towards systematic theorising: never-

theless, it is unlikely that the process of tidying up the *harmoniai* was completely divorced from the realities of performance. Comparison with the allegedly ancient and relatively disorganised scales mentioned by Aristides Quintilianus suggests that the Eratoclean *harmoniai* might fairly be construed as rationalised but recognisable versions of their older counterparts. Nor is there good reason to doubt that the rationalised versions represented systems of attunement which practical musicians did in fact adopt, and to which, perhaps, they had already begun to approximate before the theorists got to work.

If we now consider the table of *harmoniai* in relation to the notes and intervals of the GPS, it will be clear, first, that the central, Dorian species of the octave corresponds to the sequence, in the GPS, from *hypatē mesōn* to *nētē diezeugmenōn*, two tetrachords between fixed notes separated by a tone. This remains the case whether the analysis is set out in the enharmonic genus, as above, or in any version of the chromatic (e.g., $\frac{1}{2}$, $\frac{1}{2}$, $\frac{3}{2}$; 1; $\frac{1}{2}$, $\frac{1}{2}$, $\frac{3}{2}$) or diatonic (e.g., $\frac{1}{2}$, 1, 1; 1; $\frac{1}{2}$, 1, 1, as on the white notes from e to e′ on a modern keyboard). Each of the others is also represented in its own range of the GPS. Thus the Mixolydian structure is that which runs upwards for an octave from *hypatē hypatōn*; Lydian begins from *parhypatē hypatōn*; Phrygian from *lichanos hypatōn*; Dorian, as we have seen, from *hypatē mesōn*; Hypolydian from *parhypatē mesōn*; Hypophrygian from *lichanos mesōn*; Hypodorian from *mesē*. This sort of account is also found in our sources, for instance at 12 Arist. Quint. *De Mus.* 15.10ff.

One might therefore suppose that if these structures were to be described as standing to one another at various relative pitches, against the background of the GPS, Mixolydian would be treated as lowest, Hypodorian as highest. In fact the reverse is the case, consistently in all our sources, and it is important to see why. First, a given melody remains the same melody just so long as its pattern of movements through intervals is preserved. The range of pitch within which it is performed is not relevant. Correspondingly, the notes of the melody are identified and named by reference to the organisation of the series of intervals surrounding them, not by their absolute pitches: the note below the higher disjunction of the GPS, for instance, is always *mesē*, no matter whether, in absolute terms, it is performed at a high or low pitch. Then, if we think of an instrument with eight strings spanning an octave, two different melodies, each of the same genus and each within an octave range, may require the strings to be attuned in different arrangements of intervals. (Thus, both might be playable on the white notes of a keyboard, but one might demand an arrangement of intervals like that between e and e′, while the other required one like that between a and a′.) Each will thus reflect a different *harmonia*; and given that all *harmoniai* can be found, in one location or another, in octave sequences in the GPS, each will project onto the range used a different slice of that system.

The absolute pitch of the range used is irrelevant: *harmoniai* are of 'abstract pitch', and may be described as if the pitch-range inhabited by the melody was

in each case the same. (Concretely, we may imagine a performer retuning his instrument for each *harmonia* without altering the pitches of the highest and lowest of his eight strings, those bounding the octave.) Then Mixolydian, for instance, projects onto that range the interval series, and the corresponding notes, between *hypatē hypatōn* and *paramesē*, while Hypodorian projects onto it the series from *mesē* to *nētē hyperbolaiōn*. In that case any given note or interval of the GPS appears in Mixolydian higher in the range of the octave employed than it does in any other *harmonia*, and in Hypodorian it appears lower. In Mixolydian, the disjunction between *mesē* and *paramesē* is the highest interval of the octave; in Hypodorian it is the lowest.

This, then, is the sense in which Mixolydian is 'higher' than Lydian, Lydian than Phrygian, and so on. It follows that relations between the *harmoniai* were not conceived as displaying the relative locations of the octave-species on a diagram, an instrument or a group of instruments whose notes covered the whole span of the GPS. If they had been, the pitch-relations would have come in precisely the reverse order. The relations are those between the levels of the octave range onto which a particular note or segment of the GPS is in each case projected: it is as though the octave range were held constant, and the GPS moved up or down to bring different parts of it within the range. When it is moved up (bringing 'lower' notes into the range) the *harmonia* is 'higher', because *mesē*, for example, has travelled into a higher part of the octave. In that case the *harmoniai* are not distinguished by their location in different regions of pitch, and their differences have nothing to do with relative pitch of performance. Though the terminology is risky, we may say that a *harmonia* is a good deal more like a 'mode' than a 'key'.

The *tonoi*

Questions about the 'relative pitches' of different *harmoniai*, conceived in the way discussed above, become musically important in connection with issues concerning the possibility of modulation between them. Thus if a melody were to shift in mid-course, for instance, from a Dorian arrangement of the intervals in its octave range to a Hypodorian arrangement, this would require that a sequence of intervals corresponding to a certain stretch of the GPS should be available in two positions, the latter at the interval of a fourth below the former. Such a melody could obviously not be played on an instrument with just eight strings, unless their pitches were altered by some expedient during the performance. (This may sometimes have been done, and we also know that composers of the later fifth and early fourth centuries, who were notoriously addicted to the practice of modulation, often added extra strings to their instruments to make a total of eleven or twelve. See, for example, the passage of Pherecrates quoted at ps.-Plut. *De Mus.* 1,141d ff., *GMW* vol. 1, pp. 236–8.) If we number the strings from 1 to 8 (from highest to lowest), we can assign note-names to them in each *harmonia*, and identify the intervals between them (see table below): I have specified the relations according to

Aristoxenus' quantification of the enharmonic genus, with their diatonic counterparts in brackets.

Dorian	Hypodorian
1 *Nētē diezeugmenōn*	*Nētē hyperbolaiōn*
Ditone (tone)	Ditone (tone)
2 *Paranētē diezeugmenōn*	*Paranētē hyperbolaiōn*
Quarter-tone (tone)	Quarter-tone (tone)
3 *Tritē diezeugmenōn*	*Tritē hyperbolaiōn*
Quarter-tone (semitone)	Quarter-tone (semitone)
4 *Paramesē*	*Nētē diezeugmenōn*
Tone (tone)	Ditone (tone)
5 *Mesē*	*Paranētē diezeugmenōn*
Ditone (tone)	Quarter-tone (tone)
6 *Lichanos mesōn*	*Tritē diezeugmenōn*
Quarter-tone (tone)	Quarter-tone (semitone)
7 *Parhypatē mesōn*	*Paramesē*
Quarter-tone (semitone)	Tone (tone)
8 *Hypatē mesōn*	*Mesē*

Such a modulation in enharmonic would require adjustments of pitch, or alternative strings, in three cases, numbers 5, 6 and 7. In diatonic only number 7 is altered. The modulation is relatively straightforward (some others would involve more radical adjustments). It is clear that a systematisation of the relations between *harmoniai*, and an analysis of the shifts involved in modulating from one to another, would have been useful to performers as well as interesting for theorists.

No such treatment survives: Aristoxenus implies that none was given, or none of any value for an understanding of modulation. Even in his time, analysis in terms of the *harmoniai* was outmoded. He refers to them as elements in his predecessors' constructions, but makes no direct use of them in his own (though there are associated issues that he does discuss, and treats as relevant to his own articulation of harmonic structures). In their place, connected with similar problems about modulation, we find references to systems called *tonoi* (sometimes *tropoi* in later writers).

Aristoxenus' full-dress account of the *tonoi* is lost, though he mentions them in several surviving passages. We are left with the hints those passages give, the compressed and often confused reports of later Aristoxenians, and a beautifully clear, meticulously detailed exposition by Ptolemy. Unfortunately Ptolemy tells us little about the systems of Aristoxenus and his followers beyond what can be gleaned from other sources. His account describes his own novel construction, and he mentions others only to criticise them, without fully explaining their nature or their authors' intentions.

The incompleteness, internal confusion and mutual contradictions of our sources make the topic of the *tonoi* one of the thorniest in Greek musical

science; many of its details remain obscure, and I cannot pursue all of them here. But some sketch of their character and functions must be offered, and it is important at least to identify the principal source of difficulty. It lies, I think, in the fact that theorists conceived the *tonoi* in two quite different ways and used them for two different purposes, which they themselves did not always distinguish clearly, and indeed they are interconnected. Very broadly, some treatments envisage them in a manner comparable to our notion of 'key', others in a way closer to that of 'mode' (though both terms, as I have suggested above, are to some degree misleading). Perhaps these conceptions appeared at different times, reflecting real changes in musical practice, or perhaps both – or some fusion of the two – were originally worked out by a theorist involved in a period of transition. In the latter case the theorist in question must be Aristoxenus. If he was responsible for only one version of the theory of *tonoi*, the confusions in our sources will be due to an uncritical conflation of his account with those of later writers, who approached the topic from a different angle. Let us first try to clarify, in general terms, the nature of the two main uses to which the idea of *tonos* was put.

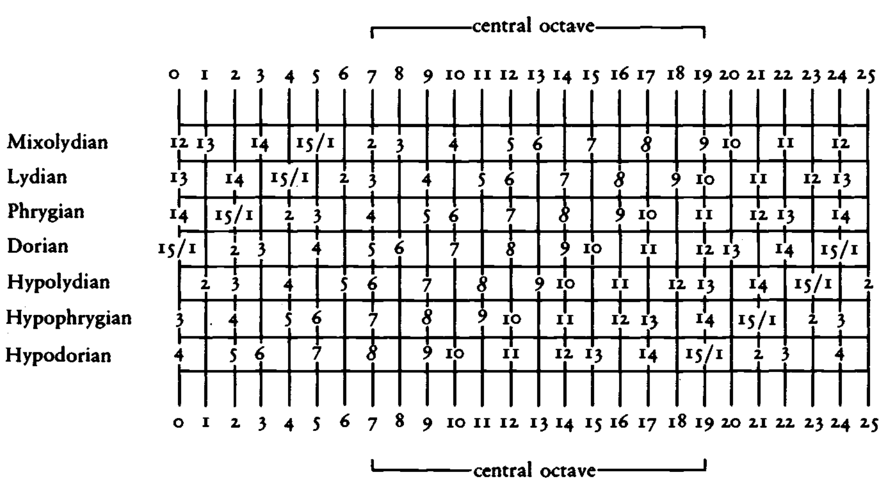
I shall simplify my exposition by ignoring chronology, and considering first, fairly briefly, the system of Ptolemy. Though it is late and perhaps idiosyncratic, it brings out clearly a connection between *tonoi* and species or rearrangements of the octave, which other accounts tend to obscure. For Ptolemy the connection is intimate and essential.

He does not directly follow the principle of the cyclic reordering of intervals which is found in the pre-Aristoxenian system of *harmoniai*. Nevertheless, his results are comparable to those that would arise from an application of this principle to the whole two-octave structure of the GPS, taken in a familiar version of the diatonic genus. As intervals are shifted from the top of the structure to the bottom, they carry with them the names of the notes bounding them: hence for these purposes the highest note of the GPS, *nētē hyperbolaiōn*, is treated as identical with the lowest, *proslambanomenos*. Ptolemy's main intention, like that of the exponents of the *harmoniai*, is that each *tonos* should project onto a specified octave range a different species of the octave: the range in question lies roughly in the centre of the system, running from its fifth degree to its twelfth. In each *tonos* the fifth degree is occupied by the note from which the corresponding *harmonia* was conceived as beginning – *hypatē hypatōn* in Mixolydian, *parhypatē hypatōn* in Lydian, and so on.

This organisation sounds neat and simple, but in practice involves two related sorts of awkwardness. First, if the intervals of the central, characteristic range are to be projected in each case onto an octave between the same pitches, the outer limits of one double-octave *tonos*, the Hypolydian, must be shifted upwards by a semitone. (Talk of 'semitones' is in fact inappropriate in discussions of Ptolemy, whose representation of intervals as ratios of numbers does not admit exact half-tones, but I shall ignore this complication here.) The alternative would be to locate the outer notes of its central octave a semitone below those of the other *tonoi*, but since the differences between *tonoi* are

conceived as those between their organisations of intervals within a single abstract octave of pitch, such a manoeuvre would be out of place. But secondly, as I have said, Ptolemy does not derive the pitch-relations between his *tonoi* (defined by the intervals between their *mesai*) directly from the principle of cyclic reordering. Such a method would obviously generate serious complications when applied to different generic divisions of the tetrachords of the system. The relations between the *mesai* of Ptolemy's *tonoi* do not alter with changes of genus, and correspond in fact to a form of the diatonic series. They are derived, however, from a principle that is independent of considerations of genus: roughly, it is that each *mesē* should be locatable by movements through concords (fourths, fifths, octaves) from every other. But given that the relative positions of the *mesai* are unaffected by genus, it is inevitable that in some genera, the boundaries of the 'central octave' will sometimes be moved upwards or downwards after all, since in some *tonoi* these boundaries are occupied by moveable notes.

The diagram below may be helpful, but in certain respects it simplifies the matter drastically. First, it represents its intervals in the way they would have been treated by an Aristoxenian, as tones, semitones, etc., not in ratios as Ptolemy does. Secondly, it presents them in only one genus, the most familiar form of Aristoxenus' diatonic (it corresponds roughly, but not exactly, to Ptolemy's ditonic diatonic). These expedients blur some of the complexities mentioned above. Thirdly, I have avoided the difficulties of expression introduced by Ptolemy's dual terminology for naming notes, by *thesis* and by *dynamis* (see 11 Ptol. *Harm.* Book II ch. 5). The vertical lines are the boundaries of semitones, numbered from 0 to 25. The notes of each *tonos* are located on this grid, and are also represented by numbers, corresponding to their ordinal positions in the GPS, so that 1 is *proslambanomenos*, 2 is *hypatē hypatōn*, and so on. The numeral 8, which indicates *mesē*, is emphasised in the diagram, since its changing locations determine what Ptolemy thinks of as the



pitch-relations between his *tonoi*. Because the systems are cyclic, the top of each *tonos* is conceived as joining on to the bottom: the highest and lowest notes are identical. For the same reason, *proslambanomenos* is always identical with *nētē hyperbolaiōn*: since the former is the first note in the GPS and the latter the fifteenth, I have represented their joint location by the symbol 15/1. The fifth degree of the scale stands at the same pitch in every *tonos*, seven semitones from the bottom, as does the twelfth, nineteen semitones from the bottom: these mark the boundaries of the central octave, in which the seven species appear.

Where *tonoi* are conceived in this way, the Dorian *tonos* is identical with the GPS in its usual form. All the others take the intervals of the GPS in the same sequence, but rotate them so as to begin from a different starting-point. All of them inhabit the same (abstract) range of pitch, but organise it differently. Relative pitch enters the matter not because the *tonoi* are transpositions of the same structure to a different pitch-level in the manner of keys (for they are not), but because issues to do with modulation require us to consider the size of the interval through which the GPS must be rotated in order to project the intended new organisation onto the same range of pitch. One might seek to define these pitch-relations by reference to the movements of *proslambanomenos*, but its travels between the bottom of the system and the top would confuse the scheme: it is simpler to track the peregrinations of *mesē*, which always remains within the central octave.

Despite the late date of Ptolemy's system, it seems likely that it reflects some of the preoccupations of much earlier theorists, writing at a time when the old *harmoniai* were still a live element in musical composition. This notion of *tonos* will have continued to be useful just so long as differences between patterns of attunement, within the same range of pitch, remained an important source of aesthetic distinctions in practical music. Music of this sort may fairly be called 'modal', where the modal character of a composition depends crucially on the order in which intervals are taken on a scale spanning its compass. (This of course does not exhaust the notion of 'mode', but we know virtually nothing that would help us to decide whether Greek music of any period incorporated other salient features of modal systems.)

While 'modal' conceptions remained influential, discussions of *tonoi* might be expected to display the following features.

(a) Though the note, *mesē*, by reference to whose position each *tonos* is located, moves up and down in the range (as does every other named note), the two-octave scales belonging to the various *tonoi* do not: their notes and intervals move round in the circle of the same range of pitch. Hence they can all be represented in their proper relations within the same span of two octaves (or two octaves and a semitone), as in our diagram. The *mesē* of each *tonos* is at a different pitch, but every *tonos* occupies the same pitch-range overall. To put it another way, a composer interested in modulations would find the whole compass of every *tonos* available within the boundaries of any single two-octave range he chose.

(b) Correspondingly, in a shift from one *tonos* to another the crucial change is in the arrangement of the intervals in the system, and especially in those of its central octave. It involves no necessary movement by the performer to a new range of pitch, no transposition of 'key', and the scheme is not designed to express relations between scales played on instruments of higher and lower compass.

(c) The number of *tonoi* will correspond to the number of species of the octave, as with the *harmoniai*. As Ptolemy argues, there can be only seven.

In some theoretical writings, however, these features become blurred or disappear. *Tonoi* are said to differ in the levels of pitch they occupy. They are all conceived as identical double-octave scales, following the standard or 'Dorian' form of the GPS, each being a progression in a straight line from low to high, not a cycle, and each beginning and ending above or below its neighbours. Since they form a set of overlapping systems, each spanning the full extent of the GPS, they jointly cover a range that substantially exceeds two octaves. I shall not represent them in a diagram here: the 'wing-shaped' figure printed in the translation of Aristides Quintilianus (see pp. 428–9 below) will serve the purpose. They are commonly described as providing for the needs of higher and lower voices or instruments: again, they are being treated as transpositions of identical interval-sequences to different levels of pitch. Their number is not restricted to seven, for we hear of one system incorporating thirteen, another of fifteen.

These changes in theoretical presentation, found mainly in the Aristoxenian compilers of the first few centuries A.D., must reflect what happened to musical theory in a period when the importance of distinctions between *harmoniai* or 'modes' had waned, leaving one species of the octave, the Dorian, in possession of the field. As a result, the concept of *tonos* came to be treated, by some theorists at least, as something very close to 'key' in its modern sense. To describe a composition as being 'in' a certain *tonos* was no longer an indication of the order of intervals forming its structure: such structures were treated as uniformly 'Dorian'. The *tonos* indicated merely the pitch at which the piece was performed, whether absolutely (in connection with roughly standardised ranges belonging to common types of instrument or voice), or relative to preceding sections of the same composition. There is good evidence of the divorce of *tonoi* from *harmoniai* in some of the surviving fragments of Greek musical scores. Their notation allows us to identify the *tonos* in which they were written: in most cases this bears no relation to the *harmonia* constructible out of the interval-sequence they use, and must indicate only 'key', in a sense related to pitch. Curiously enough, though the Dorian species of the octave retained its primacy, the Dorian *tonos* did not, at least for purposes of notation. At some stage in history (not clearly dateable) it became normal to write melodies of middling pitch in the notation of the Lydian *tonos*, this being conceived, apparently, as representing the two octaves most comfortably fitted to the commonest kind of voice.

Plainly this new conception of 'key', once detached from the modal

harmoniai and used to distinguish only the various transpositions of a single series of intervals, need no longer confine itself to seven *tonoi*. Indeed, it would be odd if it did, since even the seven diatonic versions of the 'modal' *tonoi*, set out as in our diagram, make use at one point or another of every semitonal step in the range of the central octave. Why should not a different *tonos*, in its role as 'key', be associated with each step? That is, why should not the *mesai* of the various possible *tonoi* be arranged at intervals of a semitone over the whole span of an octave?

We might expect a system of *tonoi* conceived in this manner to offer us twelve keys, just as there are in modern 'classical' theory if one ignores the differences between, for example, B flat and A sharp. In fact, however, our sources never speak of a collection of twelve, but typically attribute thirteen to Aristoxenus and fifteen to some later, unspecified authorities. I shall say little about the latter system. It was evidently a purely theoretical construction, three of whose 'keys' are merely repetitions of others at the octave: the additions were made only to yield a certain neatness of nomenclature (see 12 Arist. Quint. *De Mus*. 21.1–4).

The common ascription of thirteen *tonoi* to Aristoxenus raises more interesting issues. There is no doubt that in his time the modal systems were still alive, or at the least vividly remembered, and Aristoxenus' extant writings contain several hints that a discussion of the species of the octave would have substantial importance. The sketchy remarks about *tonoi* and related matters that survive in the *El. Harm.* do not allow us to conclude with certainty whether his treatment linked them to the *harmoniai*, or developed only the newer conception of 'key'. The likeliest hypothesis, I suggest, is that he made some attempt to accommodate both. If so, the confusion of ideas among the compilers would be the more understandable.

Where, then, does a system of thirteen *tonoi* fit into the picture? In particular, how could they have been related to the seven *harmoniai*? Even if they were conceived purely as keys, there should be only twelve. To insist on completing the octave with a thirteenth looks like the act of a theorist more concerned with tidiness than with musical realities, and that is not a description which fits Aristoxenus. Some modern scholars, as a result, have simply refused to believe the sources that ascribe this system to him.

Perhaps some scepticism is warranted, but I think we should avoid it if we can: the unanimity of the sources cannot be disregarded lightly. Let us reconsider some features of the set of 'modal' *tonoi*, each with its own focal note or *mesē*, represented in our previous diagram. Each *mesē* falls on a different degree of the central octave, but there are several semitonal steps in the octave on which no *mesē* falls. If we ignore for the present the repetition of the first note at the octave, there are five steps to which no *mesai* belong; and it is worth noticing that three of them are at pitches on which notes do fall in the fundamental Dorian scale (pitch-numbers 8, 10, 15). One can imagine a practical musician raising the question why he should not use these notes, already present in the Dorian attunement, as *mesai* to which he could

modulate, while generating attunements corresponding to other *harmoniai* as legitimately as he could by the modulations prescribed. Specifically, why should he not use pitch 8 (Dorian *parhypatē mesōn*) instead of pitch 9 as the *mesē* of his Hypophrygian structure; pitch 10 (Dorian *lichanos mesōn*) instead of pitch 11 as the *mesē* of his Hypolydian; and pitch 15 (Dorian *tritē diezeugmenōn*) instead of pitch 16 as the *mesē* of his Lydian? No doubt this involves shifting the boundaries of the central octave down by a semitone, but does that matter? For practical purposes it only means that the outer strings must be adjusted slightly as well as some of the inner ones, and the total number of strings that will need to be shifted from their 'Dorian' pitches is not always greater in these lowered versions of the three *tonoi* than it is when they have their original positions. (The lowered Hypophrygian requires five strings to be altered from their Dorian pitches, including the two outermost ones, by comparison with three in the original version; the lowered Hypolydian requires three alterations by comparison with five; both versions of Lydian require four.) We have already seen that the Hypolydian *harmonia* or species of the octave can only be kept exactly within the central octave by a slightly dubious expedient. The purities of theory are already compromised.

A musicologist who accepted the strength of this argument would find himself acknowledging the legitimacy of two alternative *tonoi*, a higher and a lower, associated with each of these three octave-species. The degree of the octave on which the *mesē* of each *tonos* stands is still the same – the second degree in Hypophrygian, the third in Hypolydian, the sixth in Lydian. What have changed are the distances between these *mesai* and those of the other *tonoi*, and one result of this is to allow more flexibility in modulation. Sometimes it will be more convenient or melodically acceptable to move to the Lydian structure in its higher position, sometimes in its lower one. It depends where we begin from, and what effect the modulation is designed to produce. But now, of course, there is every reason to assign *mesai* to the remaining empty semitonal steps, numbers 13 and 18. They do not, admittedly, appear as notes in the Dorian series, but they do in others, from which one might also wish to modulate. These give us, respectively, a lower *tonos* for the Phrygian octave-structure and a higher one for the Mixolydian.

This procedure has given us twelve *tonoi*: the thirteenth, the so called Hypermixolydian with its *mesē* on pitch 19, remains to be accounted for. The name means merely 'above Mixolydian', which is apt enough. The problem it raises is why one should posit a distinct *tonos* for a *harmonia* which is merely a repetition of the Hypodorian, and which the new *tonos* does not move into a different pitch-relation with the others. Unlike the cases of the variant Lydians, Phrygians, and so on, calling pitch 19 '*mesē*' instead of pitch 7 will make no difference to the pitch-levels of the strings that form the attunement. The distinction seems merely verbal, or at best 'abstract'. One might perhaps attempt to justify it, granted that *mesē* is in some sense a melodic focus, by a distinction between melodic forms with an impetus upwards and those focussing downwards towards the cadence (we know that the Greeks felt such

distinctions to have definite aesthetic significance). Such a distinction belongs, broadly, to a conception of modally differentiated melody rather than one based on the different keys of a single modal form; and hence, though the argument for the addition of this *tonos* is not strong, and is different in kind from those leading to the adoption of the others, some sort of case can be made. But there is no case for a thirteenth *tonos* at all, beyond a misguided notion of neatness, if the *tonoi* are merely keys.

The reconstruction of these arguments for thirteen *tonoi* in connection with the seven *harmoniai* has been largely hypothetical. According to the hypothesis, there are five pairs of *tonoi* of which each presents two alternative positions for one *harmonia*, a semitone apart, and the fact that these are named as pairs in our sources, as a higher and a lower Phrygian, for instance, may be an indication that the reconstruction is on the right lines. The first and thirteenth *tonoi* are not related in quite the same way, though their *harmoniai* are identical, and the latter could scarcely have been named on the same 'pairing' principle: the expression 'Hypermixolydian' is a pardonable makeshift. Only the Dorian pattern of organisation remains fixed to a single *tonos*, a fact again borne out by the nomenclature. This is only to be expected, since it is by their relation to this *tonos*, the one in which the Dorian structure appears, that the positions of all the others are calculated.

I am inclined to accept, then, that the system of thirteen *tonoi* is correctly attributed to Aristoxenus, and that he may well have developed it initially in connection with the seven *harmoniai*. But there is no doubt that the conception of *tonos* as pure 'key' is also present in what purport to be Aristoxenian sources, though the crucial distinctions are seldom made explicit: they have to be imported in order to make sense of what is otherwise mere confusion. Only in one source is the difference made perfectly plain, and that is in Ptolemy, who insists, as we have seen, on returning to a system of seven *tonoi* corresponding to the seven species of the octave, and who, in developing his ideas, mounts a vigorous attack on those who reduce change of *tonos* to nothing more than the transposition of a fixed sequence of intervals. He also argues that any number of *tonoi* beyond seven must be otiose for their proper purpose of locating the different octave-species in a given range, since additional *tonoi* will only produce duplicate species. If the argument I have offered carries any weight, this need not mean that the thirteen Aristoxenian *tonoi* were developed with no thought of their connection with octave-species or *harmoniai*. The reverse is perhaps more likely, since Ptolemy's counterarguments would otherwise merely miss the point.

Nevertheless, Ptolemy is clearly concerned to reinstate in theoretical analysis the modal conceptions which some accounts of the *tonoi* had obscured. Perhaps, as I suggested earlier, this reflects a renewed attention to distinctions of mode on the part of practical musicians, a 'modal revival' in Ptolemy's own time, at least in the parts of the Greek world with which he was acquainted. Since the mode-related notion of *tonos* and that of key are intertwined in the Aristoxenian writers, there are grounds for supposing that Aristoxenus' own

works included considerations of both, and that the period in which modal distinctions were temporarily eclipsed began – no doubt by gradual stages – at around the time at which he wrote.

There is one fairly well established fact about fourth-century music which may help to make this development understandable. A new focus on key, and an associated loosening of distinctions between *harmoniai*, may have been due, in part, to the increasing importance of the *aulos*, both as a performing instrument and as an adjunct of theory. Aristoxenus' *El. Harm.* mentions no stringed instruments at all, whereas the *aulos* is referred to in several places, and we are even told of a school of theorists who based their whole harmonic system on the properties of this instrument. It is mentioned again, furthermore, in connection with the disposition of the *tonoi*, which some people are said to have set out 'with an eye to the boring of *auloi*'. Now the fourth-century *aulos* was a sophisticated instrument of substantial range: unlike the *lyra* and *kithara* it came in a more or less standardised set of different sizes, corresponding to different ranges of pitch. For performers on such instruments it was natural to regard a shift of *mesē* as involving movement up or down in pitch, rather than as a reorganisation of intervals within a constant range – the conception natural to lyrists and kitharists. Typically, perhaps, an aulete would execute this sort of modulation simply by picking up a pipe pitched in a different key.

This suggestion should not be pressed too far. The predominance of the Dorian *harmonia* was already an established fact in the fifth century. Its foundational status in Aristoxenus' thought is obvious from his analysis of the genera, and of the conjunction and disjunction of tetrachords, where his attention is invariably focussed on the tetrachords lying between the fixed notes of the Dorian perfect systems, not on structures bounded by the outer notes of octave-species belonging to other *harmoniai*. If the modal distinctions between *harmoniai* were losing their importance, for whatever reason, nothing is more probable than that a system of *tonoi* originally designed to articulate relations between modalities might be pressed into service to operate as transposition keys for the triumphant Dorian. Aristoxenus seems to have been by instinct a musical conservative, and despite his emphasis on 'Dorian' structures there are clear indications in the *El. Harm.* of an interest in the other species (or 'forms', or 'arrangements') of systems spanning an octave, a fifth or a fourth. But if he was also a realist, he may well have added an account of the ways in which the mode-related *tonoi* could be adapted, with some differences of presentation, to serve the purposes of the new interest in key. Conceivably he hoped that an account of the relations between mode and key might persuade contemporary composers, seduced by the possibilities of key-variation, to rediscover connections with the music of the past, and to apply their skills to the revitalisation of mode.

There need have been no confusion about these matters in Aristoxenus' mind. It appears, however, that what he wrote was sufficiently obscure or involved to breed nonsense in the minds of some of his later followers. Given the use of the same term, *tonos*, in the two different contexts, and the use of the same names both for *harmoniai* and for *tonoi* in either of their roles, the

ambiguities were always likely to create confusion. Lapses of understanding are found exactly where these considerations would lead us to expect them, in writers more concerned to represent 'Aristoxenian doctrine' in academic summaries than to investigate the application of these ideas to actual musical practice. The fact that both conceptions of *tonos* appear in an author such as Aristides Quintilianus, for instance, may indicate his reliance on different sources, both descended from Aristoxenus, but selecting different parts of his discussions as the basis of their summaries. Aristides himself shows no sign of being aware that he has different ideas in play: the reader must disentangle them for himself. Even Ptolemy, who did at least set out to formulate the relevant distinctions, can hardly be said to have given a straightforward and unbiased account of the approaches and intentions of his rivals. If the subject remained an awkward one for an intellect as impressive and as ruthlessly logical as his, that is some measure of its difficulty, and it is no surprise that the Aristoxenian compilers' grasp on it was less than perfect.

A note on the introductions and commentary

Each chapter in this volume is prefaced by a short introduction. In writing these little essays I have not attempted to follow any fixed plan. I have tried to bring out features of the works translated which it would be helpful for the reader to meet in advance, whether these features are technical, methodological, concerned with the general plan of a treatise or with the biography and other interests of the author. The introductions are also uneven in length and in the amount of detail offered. I have said a good deal less about writers such as Plato, who are the subject of many excellent modern discussions, than about ones like Aristides Quintilianus, who are not. Most of the introductions include a few suggestions for further reading, but they give only the briefest sample of works available. More will be found in the Bibliography.

Something similar is true of the notes. Passages that are technically difficult, or which benefit from full cross-referencing, are heavily annotated, and sometimes the notes have grown quite long. (One or two issues seemed to require fuller treatment than a footnote could stand, and I have moved these into appendices.) Others seemed to call for little explanatory comment, and in some places where thickets of cross-references could have been provided, I have decided that they would serve no useful purpose. As in volume 1, I have entered into the controversies of recent commentators only when it seemed quite unavoidable, restricting myself wherever possible to direct comparisons and interpretations of the ancient texts. Where I know that my own views are heterodox, I have given references to other discussions, but I have used this strategy sparingly. As in the introductions, I have pursued no preconceived plan in deciding which kinds of topic called for discussion and which did not. The scope of my comments has of course been influenced by the direction of my personal interests, but I hope that matters which most readers will think important have not been wholly neglected, and that I am not asking them to ride mere hobby-horses of my own into territory that is not worth exploring.

CHAPTER I

Pythagoras and early Pythagoreanism

The 'Pythagoras' whose life and doctrines are profusely described in documents of late Hellenistic and Roman times is more a myth than a man. There was a real Pythagoras, but surprisingly few early writers mention him, and when the layers of legend, prejudice, philosophical reinterpretation and plain forgery have been peeled away from the stories that later sources tell, little historical fact remains. Of the points that seem relatively secure, the following have some relevance here.

Pythagoras was born in Samos around the middle of the sixth century B.C. He migrated, probably for political reasons, to the colonies of the Greek West in southern Italy, and settled in Croton. There his teachings attracted a group of followers, and it seems clear that what united them was a way of life defined primarily by a set of moral and ritual injunctions, rather than anything recognisable as philosophical 'doctrine'. The main aim of this way of life was the purification of the soul, whose fall from perfection and transmigration through a series of incarnations was asserted by Pythagoreans from the beginning of the tradition, and whose ultimate return to a condition of unblemished divinity might be achieved through these purifying practices. It also had political implications. Pythagoreans formed a governing elite at various times in several Italian cities, and were not always popular: a violent revolt against them in Croton is well attested.

Though early Pythagoreanism was more a set of practices than a body of doctrines, it was connected with speculations of a more or less metaphysical sort. The universe was conceived as an ordered unity, a system whose parts were held together in a pattern of divine 'harmony' or coordination. The key to the nature of this order lay in number, and though the notion that Pythagoreans were the founders of Greek mathematics is no longer accepted by serious scholars, it is certain that some mathematical investigations (often bizarrely mixed with mystical numerology) were undertaken by them from early times. A persistent tradition held that the Pythagoreans' interest in number and their deployment of numerical conceptions in cosmological contexts began from Pythagoras' discovery that certain fundamental musical relations can be expressed as ratios of numbers. Though the details of the later stories cannot be believed, there is no good reason to doubt the gist of this tradition (even if Pythagoras' originality lay in the theoretical attention he gave to these ratios, rather than in the discovery of them). The strangely unified relation of the musical octave corresponds to the simple ratio 2:1. Its unity conceals a structure of two lesser concords (themselves conceived acoustically as unified 'blendings' of sound), the fifth of 3:2 and the fourth of 4:3. The numbers involved in the articulation of this *harmonia* (a name attached by fifth-century musicians to the octave itself), 1, 2, 3, 4, form the *tetraktys* of the decad, a seminal conception in Pythagorean cosmology (see 1.2).

In the Pythagorean world-view, the 'harmony' of the universe (and sometimes those of the microcosms of state and soul) was rooted in mathematical relations of the sort that this musical structure displays. Often these ideas led to nothing but imaginative speculation on the symbolic significance of particular numbers. But by the later fifth century they had been elaborated into a metaphysical scheme which Aristotle thought worth analysing (see 1.5), and had generated a version of the famous astronomical

theory of the harmony of the spheres (1.6, cf. 1.7). Attempts were also made to investigate the musical phenomena themselves more closely (see particularly 1.8), and by the time of Philolaus (1.9–1.18) the analysis of ratios that combine to form an octave had been extended to give a mathematical description of a complete musical scale, a full articulation of the complex of relations internal to the structure of *harmonia* (1.12).

In all these cases the main motive was metaphysical or cosmological: these Pythagoreans were not engaged in 'musicology' for its own sake. But the work of Archytas is different. He was the last and greatest of the series of Pythagoreans who stood in direct succession to the founder of the school; after his time it faded into obscurity for several centuries. He was an eminent mathematician, a successful statesman, and a respected friend of Plato. Though there is no doubt that his musical and mathematical researches were linked with metaphysical ideas of a characteristically Pythagorean sort, his genius led him to pursue them almost as autonomous fields of enquiry, and at a high level of sophistication. Like that of his predecessors, most of Archytas' work is lost. Its power and originality can still be discerned, however, in the surviving fragments and reports of his writings on acoustics and harmonics (see 1.19–1.21): these writings, together with Plato's adaptations of them (2.1, 2.3–2.5) formed the basis for the whole tradition of musicology known to later writers as 'Pythagorean' (which figures largely in chapters 8–12), as well as for the studies in physical acoustics undertaken by Aristotle and his successors (see especially chapters 3–5).

Not all students of harmonics in the period before Plato were Pythagoreans. Aristoxenus refers to the sixth-century musicians Epigonus and Lasus as musical theorists, but we have little idea of the nature of their work. Rather more is known of the empirically minded musicologists whom Aristoxenus calls *harmonikoi*. They are probably the same as those mentioned by Plato in 2.1 and implicitly alluded to by Aristotle in 3.7. All three writers distinguish their methods and intentions sharply from those of the mathematical or Pythagorean theorists (see also 6 Theophrastus ap. Porph. *Comm.* 62.1–3): Aristoxenus grudgingly recognises them as his own precursors. Their work certainly originates in the fifth century, but as far as we know there was no significant interaction between them and the Pythagoreans. Aristotle, but no earlier writer, attempts to place the activities of the two traditions in an intelligible relation to one another. These 'empiricists' are mentioned here only to dispel the impression that musical theory in this early period was a Pythagorean monopoly. The principal passages that bear on their work appear in later chapters and will be discussed there: see especially 2.1, 3.7, and 7 Aristox. *El. Harm.* 2.7–3.1, 3.16–32, 5.6–27, 6.11–33, 7.3–5, 7.22–33, 27.34–28.29, 32.8–11, 36.15–38.17, 39.4–43.24, 53.2–15. A fifth-century theorist of a different sort was the Athenian Damon, well known from Plato's respectful remarks about him. It has often been assumed, plausibly enough, that he was influenced by Pythagoreanism, but there is no good evidence that he was an exponent of harmonic analysis in their mathematical style. His work falls for the most part outside the scope of this book, but see chapter 2, notes 42–3; chapter 6, note 44; 12 Arist. Quint. *De Mus.* 80.25–81.6, and compare *GMW* vol. 1, pp. 168–9.

The subject of early Pythagoreanism is tangled and difficult. Modern scholarship in the field was revolutionised by the appearance of Burkert, *Lore and Science* (in German, 1962, in English, 1972). Though it predates Burkert, Guthrie's *History*, vol. 1, chapter 4 is still worth consulting. See also Philip (1966), and Kirk, Raven, Schofield (1983) chapter 7.

1.1 Porphyry *Comm.* 30.1–5 (Xenocrates fr. 9, Heinze)

Heraclides writes as follows about these things in his *Introduction to Music*. 'Pythagoras, so Xenocrates says, discovered also that the intervals in music do not come into being apart from number; for they are an interrelation of quantity with quantity. So he set out to investigate under what conditions concordant intervals come about, and discordant ones, and everything well-attuned and ill-attuned.'[1]

1.2 Sextus Empiricus *Adv. Math.* vii.94–5

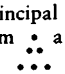
The Pythagoreans ... are in the habit of sometimes saying 'All things resemble number',[2] and of sometimes swearing this most fundamental oath: 'No, by him that gave to us the *tetraktys*, which contains the fount and root of ever-flowing nature.' By 'him that gave' they mean Pythagoras (for they deified him); and by '*tetraktys*' they mean a number which, being constituted out of the first four numbers, fits together the most perfect number, as for instance ten: for one and two and three and four becomes ten.[3] This number is the first *tetraktys*, and is described as the 'fount of ever-flowing nature' in as much as the whole universe is organised on the basis of these numbers according to *harmonia*; and *harmonia* is a *systēma* of three concords, the fourth, the fifth and the octave; and the proportions [*analogiai*] of these three concords are found in the four numbers previously mentioned, in one, two, three and four.[4]

1.3 Schol. to Plato *Phaedo* 108d4 (on the phrase '*Glaukou technē*', 'skill of Glaucus') (DK 18.12)

It is said either of things that are not accomplished easily, or of things that are

1 For the continuation of the quotation from Heraclides see 9.8. It is unlikely that numerical ratios were identified for intervals other than the basic concords and the tone before Philolaus in the late fifth century (see 1.12 Philolaus frag. 6). Thorough analysis of the ratios of scalar intervals seems to begin with Archytas in the early fourth century (1.21 Ptol. *Harm.* 30.9ff.).

2 See particularly 1.5 Aristotle *Metaph.* 985b23ff., 1.11 Philolaus frag. 4, and, for example, 12 Arist. Quint. *De Mus.* Book III ch. 6.

3 A *tetraktys* is any coordinated set of four items. This one, the *tetraktys* of the decad, was fundamental to Pythagorean metaphysics. It represents the basis of the number series (the perfect number 10) in the first four numbers, $1+2+3+4$, which themselves display the principal concordant ratios, as explained below. It is often represented in the diagram  a perfect triangle. Cf. 9.2 Adrastus ap. Theon Smyrn. 58–9; also Theon Smyrn. 87.5ff., 93.17ff., Lucian *Vit. Auct.* 4. For some other *tetraktyes* see 12 Arist. Quint. *De Mus.* 119.23, 127.18, 128.20.

4 On this use of *harmonia* see 1.12 Philolaus frag. 6. A *systēma* is something put together in a linked series, in musical usage roughly equivalent to 'scale' (see 7 Aristox. *El. Harm.* 1.21 with n. 3). The parallel with Philolaus suggests that it is here an octave 'system' composed of a fourth and a fifth, not the sequence of octave followed by fourth and fifth. Of the ratios, 2:1 is an octave, 3:2 a fifth, 4:3 a fourth: their terms are all found in the *tetraktys* of the decad.

made with great care and skill. For a certain Hippasus[5] made four bronze discs in such a way that while their diameters were equal, the thickness of the first disc was epitritic in relation to that of the second, hemiolic in relation to that of the third, and double that of the fourth, and when they were struck they produced a concord.[6] And it is said that when Glaucus noticed the notes made by the discs, he was the first to set himself to making music with them, and that it is as a result of this endeavour that people still speak of the 'skill of Glaucus', as it is called.[7]

1.4 Theon Smyrn. 59.4–21 (DK 18.13)

Some people thought it proper to derive these concords from weights, some from magnitudes, some from movements and numbers, and some from vessels. Lasus of Hermione, so they say, and the followers of Hippasus of Metapontum, a Pythagorean,[8] pursued the speeds and slownesses of the movements, through which the concords arise[9]... Thinking that... in numbers, he constructed ratios of these sorts in vessels.[10] All the vessels were equal and alike. Leaving one empty and filling the other up to halfway with liquid, he made a sound on each, and the concord of the octave was given out for him. Then, again leaving one of the vessels empty, he poured into the other one part out of the four, and when he struck it the concord of the fourth was given out for him, as was the fifth when he filled up one part out of the three.[11] The one empty space stood

5 A Pythagorean from Metapontum, probably of the early fifth century, important in the early development of the school. See the next passage, and cf. Burkert (1972), pp. 206–7, 377–8.

6 Epitritic ratio is 4:3, hemiolic 3:2. Compare the stories about Pythagoras at 10 Nicomachus *Ench.* ch. 6 and 12 Arist. Quint. *De Mus.* Book III ch. 1. In fact the procedures described in those passages give the wrong results, whereas that of Hippasus works: if the discs have equal diameters, the frequencies are proportional to their thicknesses. The present report may well be historically sound (cf. next note).

7 This Glaucus is Glaucus of Rhegium, a musical writer of the late fifth century from whom substantial passages of ps.-Plut. *De Musica* are derived (see *GMW* vol. I ch. 15). Here he is represented as a practising musician rather than a historian and critic: hence his proverbial skill. The whole passage is based on a report by Aristoxenus (frag. 90), whose testimony carries weight.

8 On Lasus see 7 Aristox. *El. Harm.* 3.21 with n. 11. It has been conjectured that he was a Pythagorean, but no source confirms it: he was roughly contemporary with Pythagoras himself. On Hippasus see n. 5 above. For the context and continuation of this passage see 9.2.

9 There is no good evidence that the early investigators associated pitch with velocity of movement. A theory of that sort appears first in 1.19 Archytas frag. 1: he attributes it to unnamed predecessors, but perhaps only by Pythagorean convention. For later developments (on which the present passage no doubt draws) see especially 2.4, 2.5 Plato *Tim.* 67a–c, 79e–80b; 3.14, 3.15, 3.17 Aristotle *De Sensu* 448a–b, *De Anima* 420a–b, *De Gen. An.* 786b ff.; 4.2, 4.4, 4.5 ps.-Ar. *Probs* XI.6, 13, 14 (and other passages in ch. 4); 5 *De Audib.* 801a, 803a, 803b–804a; 6 Theophrastus ap. Porph. *Comm.* 63.1ff.; 8 Eucl. *Sect. Can.* 148–9; 9.7, 9.8 Aelianus and Heraclides ap. Porph. *Comm.* 33.16ff., 30.1ff. The text of the present passage is in disrepair.

10 For other references to experiments with vessels of this sort see, for example, 4.28 ps.-Ar. *Probs* XIX.50, Porph. *Comm.* 121.11–12.

11 These locutions mean that in the first case the vessel was filled to one quarter of its capacity, in the second to one third. The relevant size of each sounding object is taken

to the other in the octave as 2 to 1, in the fifth as 3 to 2, and in the fourth as 4 to 3.

1.5 Aristotle *Metaphysics* 985b23ff. (DK 58 B4, B5)

At the same time as these people [Leucippus and Democritus] and before them, the people called 'Pythagoreans' took up mathematics, and were the first to advance this science; and having been reared on it, they thought that its principles [*archai*] were the principles of all things.[12] Since of these it is the numbers that are by nature first, and in the numbers they thought that they saw many likenesses to the things that are and that come into being, more so than in fire and earth and water (thinking that such and such a modification [*pathos*] of numbers is justice, that such and such another is soul and intellect, that another is opportunity,[13] and so on similarly for each one, more or less, of the others); since, again, they saw that the attributes [*pathē*] and ratios of the *harmoniai* are found in numbers;[14] since, finally, all other things seemed to have been framed, in the whole of their nature, in the likeness of the numbers, and of all nature the numbers seemed to be first, they supposed that the elements of the numbers are the elements of all things, and that the whole heaven is a *harmonia* and a number.[15] And all the things in the numbers and in the *harmoniai* that they could show to agree with the attributes and parts of the heaven and with its whole organisation, these they collected and fitted to them.[16] And if something was missing somewhere, they still clung to the objective that their whole theory [*pragmateia*] should be linked coherently together. I mean, for example, that since the number 10 seems to be perfect and to embrace the whole nature of the numbers,[17] they say that the things that

to be the volume of the empty space it contains. If the sounds are produced by striking the vessels, the results will not in fact be those listed in the next sentence. (If the vessels were cylindrical bottles and the sounds were made by blowing across the top, a version of the experiment might be made to work. Even here, the relevant variable is not the volume but the length of the air-column in the cylinder.)

[12] This is one of our most important documents on early Pythagorean metaphysics. It raises many difficult issues that cannot be pursued here. Burkert (1972) does not discuss the passage as a whole, but see his index of passages, pp. 512–13. See also Kirk, Raven, Schofield, ch. 11, particularly pp. 328–32.

[13] See, for example, 12 Arist. Quint. *De Mus.* 126.28ff. and cf. Book III ch. 6.

[14] Here a *harmonia* is a distinctive pattern of tuning, often associated with a specific ethos or character. (See, for example, Plato *Republic* 398c–400b, and *GMW* vol. I ch. 10, Appendix A.) The reference to attributes suggests a theory in which these characters were themselves held to inhere in the numerical relations (ratios) between notes.

[15] That is, an attunement expressible as a system of numerical relations. For theories of 'cosmic harmony' see especially 1.6 Aristotle *De Caelo* 290b; 2.2, 2.3 Plato *Rep.* 616b ff., *Tim.* 34b ff.; 10 Nicomachus *Ench.* ch. 3; 11 Ptol. *Harm.* Book III chs. 8–16; 12 Arist. Quint. *De Mus.* Book III chs. 20–24. Other references are given in Burkert (1972), ch. IV.4.

[16] A much later project of this sort is elaborately exemplified in 12 Arist. Quint. *De Mus.* Book III. Aristotle offers some examples in the last two books of the *Metaphysics*: see 3.10 *Metaph.* 1093a.

[17] See n. 3 above.

travel through the heaven are ten; but since those that are visible are only nine, they therefore make a tenth, the 'counter-earth'...[18]

986a15 These people, too, then, evidently believe that number is the principle, both as matter for things that exist and as attributes and states of them, and that the elements of number are the even and the odd, the former of these being unlimited, the latter limited; and that the one comes from both of these (for they say it is both even and odd), and that number comes from the one, and that the whole heaven, as we have said, is numbers.[19]

1.6 Aristotle *De Caelo* 290b12ff. (DK 58 B35)

It is clear from these points that the thesis that a *harmonia* occurs when the stars move,[20] on the grounds that the sounds arising are concordant, though it is elegantly and strikingly stated by those who enunciate it, is nevertheless not true. For it seems inevitable to some people that when bodies that are so large move, sound must occur, since it does with bodies in our region, which do not have bulk equal to theirs and do not move with so great a speed.[21] When the sun and the moon and the stars, so great in number and in magnitude, are moving with so swift a motion, it is impossible, they say, that there does not arise a sound extraordinary in magnitude. Taking these claims as assumptions, and assuming also that from the distances between them the speeds acquire the ratios of the concords,[22] they say that the sound produced by the stars as they travel in a circle is harmonic. And since it seems unaccountable that we do not hear this sound, they say that the reason for this is that the noise is there from the moment we are born, so that it is not apparent in relation to a contrasting silence: for, they say, sound and silence are distinguished in relation to one

[18] See 1.7 Aristotle *De Caelo* 293a. The visible bodies are the Sun and Moon, five planets (Mercury, Venus, Mars, Jupiter, Saturn), the Earth and the sphere of the fixed stars.

[19] Odd and even, limit and the unlimited, are fundamentals of Pythagorean cosmology. See 1.9, 1.10, 1.12 Philolaus frags. 1, 2, 6; 2.6 Plato *Philebus* 17a ff.; 12 Arist. Quint. *De Mus.* 102.10–12, 123.1–4, 126.5ff., and, for example, Burkert (1972), pp. 32ff., Kirk, Raven, Schofield, pp. 336–9. For the unit as both odd and even see Aristotle frags. 199 and 203 (Rose), Plutarch *De E* 388a–c.

[20] See n. 15 above.

[21] For comparable statements of the thesis see 10 Nicomachus *Ench.* ch. 3, 12 Arist. Quint. *De Mus.* Book III ch. 20. The underlying theory about the causation of sound is that of Archytas at 1.19 frag. 1.

[22] On the association of velocity with pitch see n. 9 above. The pitches sounded by the heavenly bodies were not always linked to their velocities or distances. (Nicomachus *Ench.* ch. 3 mentions several other views, and the account of Arist. Quint. is based on qualitative, not quantitative features of each divinity. See also notes to Plato's discussion at 2.2 *Republic* 616b ff.) The system intended was probably not just an extended sequence of concords, but a *harmonia* or scale based on upwards and downwards movements through octaves, fifths and fourths (the 'method of concordance': cf. 7 Aristox. *El. Harm.* 55.11ff.; 8 Eucl. *Sect. Can.* proposition 17; 12 Arist. Quint. *De Mus.* 21.8ff.). By these means a complete diatonic sequence can be generated (as in the context of cosmology at 2.3 Plato *Timaeus* 35b ff.). A few sources give arrangements using only fixed notes of the system, notably Plutarch *De Anim. Procr.* 1029a–b, but there are no early examples, and even these are not just sequences of concords.

another, so that just as to blacksmiths the noise makes no difference because they are accustomed to it, so the same thing happens to mankind.[23]

1.7 Aristotle *De Caelo* 293a18ff. (DK 58 B37)

Most people say that it [the earth] lies at the centre, ... but the people in Italy who are called 'Pythagoreans' speak in opposition to this. For they say that at the centre is fire, while the earth is one of the stars, and by travelling in a circle around the centre makes night and day. Again, they construct another earth, opposite to this one, which they call by the name 'counter-earth', not seeking theories and explanations to account for the observed phenomena, but forcibly dragging the phenomena towards certain theories and opinions of their own, and trying to organise them together.[24]

1.8 Porphyry *Comm.* 107.15ff. (DK 47 A17)

Some of the Pythagoreans, as Archytas and Didymus record, after the ratios of the concords had been established, comparing these ratios with one another and wishing to show which were the more concordant, proceeded as follows.[25] Taking the first numbers – which they called *pythmenes*[26] – of those that produce the ratios of the concords (that is, the smallest numbers in which concords are produced, as for instance the first numbers in which the octave is apprehended are 2 and 1, since two of the one is the first double, and the foundation of the other doubles; and the fourth is in the epitritics 4 and 3, for four related to three is the first and foundational epitritic), and giving these numbers to the concords, they enquired in the case of each ratio of the numbers comprising the boundaries, after taking away a unit from each of the boundaries, what numbers they were that were left after the subtraction. From 2 and 1, for instance, the numbers of the octave, they subtracted a unit each and considered the remainder, which was 1. From 4 and 3, which were the numbers of the fourth, they subtracted a unit each and had 3 as the remainder of the 4, and 2 as the remainder of the 3. Thus after the subtraction the remainder from both terms together was 5. From 3 and 2, which were the numbers of the fifth, they subtracted a unit each and had 2 as the remainder from the 3, and 1 as the

[23] For a different explanation of the sounds' inaudibility see 12 Arist. Quint. *De Mus.* Book III ch. 20.

[24] On the counter-earth cf. 1.5 Aristotle *Metaph.* 986a, and on the central fire see 1.13 Philolaus frag. 7. Aristotle's attitude to Pythagorean methodology was echoed by musicologists with reference to their own concerns, e.g., at 7 Aristox. *El. Harm.* 32.20–8, and in the passages from Ptolemaïs and Didymus at 9.11–9.14. Contrast 2.1 Plato *Republic* 531b–c.

[25] For the passage on which Porphyry is commenting see 11 Ptol. *Harm.* 14.1ff. Ptolemy does not mention his sources: this and other features make it fair to assume that Porphyry had independent evidence, very probably a report by Didymus quoting or paraphrasing Archytas. (Some scholars have taken the passage as a report about Archytas' own practices, but that is not what Porphyry says.) On Didymus see chapter 9.

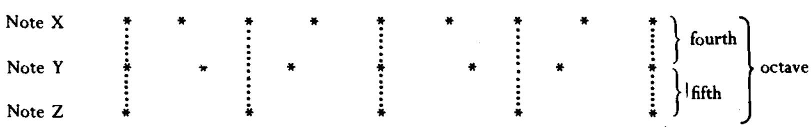
[26] The word means 'base' or 'foundation'. Porphyry explains its sense here in the parenthesis below. Cf. particularly Plato *Republic* 546c.

remainder from the 2, so that their combined remainder was 3. They called the units subtracted 'similars', and the remainders left after the subtraction 'dissimilars', for two reasons – because the subtraction from both the terms was similar and equal. For unit is similar to unit. When these are subtracted, the remainders are necessarily dissimilar and unequal. For if equals are subtracted from unequals, what is left will be unequal. The multiple and epimoric ratios,[27] in which the concords are found, are constituted from unequal terms; and when equals are subtracted from them, what is left is always unequal. The 'dissimilars' of the concords arise through 'blending': the Pythagoreans use the word 'blending' [*symmisgein*] for the construction of one number out of the two.[28] The 'dissimilars' put together in the case of each of the concords are these: for the octave, 1; for the fourth, 5; for the fifth, 3. They say that those whose 'dissimilars' are smaller are more concordant than the others. The octave is concordant, since its 'dissimilars' are 1; after it comes the fifth, since its 'dissimilars' are 3; and last is the fourth, since its 'dissimilars' are 5.[29]

27 Multiple ratios are of the form mn:n, epimoric or 'superparticular' ratios of the form n+1:n. Pythagorean theory always restricted concords to intervals with ratios of multiple or epimoric form: see especially 8 Eucl. *Sect. Can.* 149.11ff. and propositions 10–12; 11 Ptol. *Harm.* Book 1 ch. 5, and compare nn. 63–6 below.

28 The word is probably intended within the context of a theory of concord as the 'blending' of two sounds, as, for example, 8 Eucl. *Sect. Can.* 149.18–20.

29 The procedure is mathematically absurd, and is mercilessly pilloried by Ptolemy at 11 *Harm.* 14.6ff. (The fact that its mathematical credentials are shaky need not impugn the authenticity of the report: see n. 36 below.) But it is not pointless. First, it is based on the correct intuition that ratios of concords are ratios of small whole numbers: in the case of epimoric ratios (n. 27 above), the larger the numbers involved, the less concordant the interval. Secondly, it suggests a physical model, derived from the theory that sounds are generated by impacts on the air, and that higher sounds involve more frequent impacts. In sounds a fourth apart, for instance, the higher sound has four impacts to the lower one's three; in the fifth the numbers are three and two. Take three notes, X, Y, Z, such that X is a fourth above Y and Y a fifth above Z. Imagine that the first impact of each occurs at the same time. Then we can draw the following diagram, in which the first impact of each note is on the left: time proceeds from left to right, and each asterisk represents an impact. The vertical dotted lines represent temporal



coincidences between impacts in the different series. Impacts that coincide may be deemed to blend acoustically, and correspond to the 'similars' of the Pythagorean discussion. Impacts that do not coincide fail to blend, and these 'dissimilars' disrupt the perceived unity of the mixture of any two notes. Between any two coinciding impacts of notes a fourth apart there are 5 'dissimilars', in the case of the fifth there are 3, and in that of the octave 1. These are the numbers generated by the Pythagorean calculation. The greater number of disruptions in the case of the fourth explains why it is 'less concordant'.

Theories of pitch consistent with this model are at 5 *De Audib.* 803b–804a; 8 Eucl. *Sect. Can.* 148–9; 9.8 Heraclides ap. Porph. *Comm.* 30.1ff.; and a treatment close to the present one is at 4.24 ps.-Ar. *Probs* XIX.39. Compare also the language of 2.5 Plato *Timaeus* 79e–80b.

The procedure gives plausible results for intervals within the octave. If extended

1.9 Philolaus frag. 1 (Diog. Laert. viii.85 (DK 44 B1))

Nature in the universe was harmonised from unlimiteds and limiters, both the whole universe and all things in it.[30]

1.10 Philolaus frag. 2 (Stob. *Anth.* i.21, 7a (DK 44 B2))

It is necessary for the things that exist to be all either limiters or unlimiteds or both limiters and unlimiteds. But they could not be only unlimiteds. Since it is evident, then, that they are not from things that are all limiters or from things that are all unlimiteds, it is clear that the universe and the things in it are harmonised together from both limiters and unlimiteds. Things as they are in practice make this clear too. For some of them, being from limiters, limit, while others, being from limiters and unlimiteds, both limit and do not limit, and others, being from unlimiteds, are evidently unlimited.

1.11 Philolaus frag. 4 (Stob. *Anth.* i.21, 7b (DK 44 B4))

And all things, indeed, that are known have number: for it is not possible for anything to be thought of or known without this.[31]

1.12 Philolaus frag. 6 (Stob. *Anth.* i.21, 7d; the second paragraph also at 10 Nicomachus *Ench.* 252.17ff. (DK 44 B6))

Concerning nature and *harmonia*, it is like this. The being of the things, which is eternal, and nature itself, admit of divine and not human knowledge, except

further, oddities appear. For instance the octave plus fifth (3:1) turns out to be 'more concordant' than either the pure fifth (3:2) or the double octave (4:1) (cf. 11 Ptol. *Harm.* 14.16ff.). The octave plus fourth (8:3) is a particular embarrassment, as it always was in Pythagorean ratio theory (because, though apparently a concord, its ratio is neither multiple nor epimoric; compare n. 27 above). See especially 11 Ptol. *Harm.* 13.1ff., and note the absence of any reference to this interval in 8 Eucl. *Sect. Can.*

[30] Passages 1.9–1.18 derive from Philolaus. Fragments attributed to him include many that are certainly spurious; those quoted here are generally thought to be authentic. They make use of a concept of *harmonia* in cosmological and psychological contexts. The underlying idea is that the world and everything in it, including the soul, is constituted out of elements or kinds that belong to distinct categories (here the limiter and the unlimited), which cannot of their own nature come together to form coordinated wholes. The unity and coordination that things (and the whole cosmos) display arises from the presence of a third principle, *harmonia*, which reconciles the other two. Though the term *harmonia* has non-musical uses (meaning a 'fitting together'), it is clear that the musical sense was in Philolaus' mind, at least as a metaphor: see particularly 1.12 frag. 6. There are obvious affinities with the doctrine of the harmony of the spheres (1.6 Aristotle *De Caelo* 290b, with notes): ideas like those of Philolaus were greatly elaborated in the later Pythagorising tradition, e.g., 12 Arist. Quint. *De Mus.* Book III. For discussion of the cosmological fragments see Burkert (1972), pp. 250–77 (with pp. 386–400 on the musical passages), and more briefly Kirk, Raven, Schofield, pp. 324–8. I shall not attempt to expound their metaphysics further in these notes.

[31] That it is in their numerable aspects that things can be understood is a theme adopted by Plato (e.g., 2.1 *Republic* 531c, 2.6 *Philebus* 17d–e, cf. 56a ff.), and is characteristic of the Platonising Pythagoreanism of the later period.

that it was not possible for any of the things that exist and are known by us to have come into being, if it were not for the existence of the being of the things from which the universe is composed, both the limiters and the unlimiteds. And since there existed these principles, being neither alike nor of the same race, it would then have been impossible for them to be organised together, if *harmonia* had not come upon them, in whatever way it arose. The things that were alike and of the same race had no need of *harmonia* as well; but things that were unlike and not of the same race nor equal in rank, for such things it was necessary to have been locked together by *harmonia*, if they were to be held together in a cosmos.

The magnitude of *harmonia* is *syllaba* and *di'oxeian*.[32] The *di'oxeian* is greater than the *syllaba* in epogdoic ratio.[33] From *hypatē* to *mesē* is a *syllaba*, from *mesē* to *neatē* is a *di'oxeian*, from *neatē* to *tritē* is a *syllaba*, and from *tritē* to *hypatē* is a *di'oxeian*.[34] The interval between *tritē* and *mesē* is epogdoic, the

[32] Here *harmonia* acquires the sense '(attunement of) the octave'. *Syllaba* is the fourth and *di'oxeian* the fifth. The terminology is explained in 10 Nicomachus *Ench*. ch. 9, where this paragraph is quoted. A somewhat different account is given by Aelianus ap. Porph. *Comm*. 96.21–3, 96.29–97.8; cf. also 12, Arist. Quint. *De Mus*. 15.8–10. The use of *harmonia* to mean 'octave' is related to its sense 'system of attunement' through the treatment of such systems as exemplifying species of the octave: see, for example, 7 Aristox. *El. Harm*. 46.30–2, 12 Arist. Quint. *De Mus*. 15.9–20.

It seems unlikely that Philolaus wrote this paragraph as the immediate successor of the one printed before it, though that is how Stobaeus quotes them. But their uses of the notion of *harmonia* must be related. In the musical case, notes and intervals are coordinated by taking their place within the embracing framework of the octave, becoming articulations of its parts. Something similar holds for the components of the universe and its all-inclusive *harmonia*. Then concordant and melodic musical relations are not so in their own right, but only as entering into the octave structure. Contrast the standpoint of the Pythagoreans discussed in 1.8 Porph. *Comm*. 107.15ff.

[33] That is, the ratio 9:8, that of the tone. (The tone, whether conceived as a ratio or as an intervallic distance, is standardly defined by Greek writers as the difference between the fourth and the fifth, e.g., 7 Aristox. *El. Harm*. 21.22–3, 45.34–46.1; 8 Eucl. *Sect. Can*. proposition 13.)

[34] Here *hypatē* is *hypatē mesōn*, *neatē* (a variant form of *nētē*) is *nētē diezeugmenōn*. What Philolaus calls *tritē* corresponds to the position of the note which most writers call *paramesē*, *tritē diezeugmenōn* being the name normally given to the note a semitone or *leimma* above it (in the diatonic genus).

Standard terminology	*Philolaus*
Nētē (diezeugmenōn)	Neatē
Paranētē (diezeugmenōn)	
Tritē (diezeugmenōn)	
Paramesē	Tritē
Mesē	Mesē
Lichanos (mesōn)	
Parhypatē (mesōn)	
Hypatē (mesōn)	Hypatē

The oddity of Philolaus' terminology is important, because *tritē* means 'third (note)', and its use here implies that only one note lay between it and *neatē*. Then Philolaus' octave system contained at most seven notes. A persistent tradition held that early scale systems, whether they covered an octave or not, had only seven notes (the *lyra* often had only seven strings, even in the fifth century); an eighth was added later. See especially 4.27 ps.-Ar. *Probs* XIX.47, cf. 7 and 32 (and GMW vol. 1, p. 198, n. 62), 10 Nicomachus *Ench*. chs. 5 and 9, ps.-Plut. *De Mus*. 1140f (and GMW vol. 1, p. 233, n. 177), cf.

syllaba is epitritic, the *di'oxeian* hemiolic, and the *dia pasōn* is duple.[35] Thus *harmonia* consists of five epogdoics and two dieses; *di'oxeian* is three epogdoics and a diesis, and *syllaba* is two epogdoics and a diesis.[36]

1.13 Philolaus frag. 7 (Stob. *Anth.* i.21, 8 (DK 44 B7))

The first thing fitted together [*hērmosthen*], the one, in the middle of the sphere, is called the hearth.[37]

1.14 Philolaus frag. 10 (Nicomachus *Arithm.* ii.19, p. 115.2 (DK 44 B10))

Harmonia comes to be in all respects out of opposites: for *harmonia* is a unification of things multiply mixed, and an agreement of things that disagree.

1.15 Plato *Phaedo* 61d

So Cebes asked him: 'How can you say this, Socrates, that it is forbidden to do violence to oneself, and yet that the philosopher would want to follow the dying?'

1137b–e. These sources do not tell a single, consistent story. Their confusions may be due in part to the existence of two different seven-note systems, one formed by a conjunction of two tetrachords, spanning a seventh, the other a 'gapped' system, spanning an octave, formed by two fourths disjoined by a tone, but lacking a note in the upper tetrachord. See notes to the passages mentioned, and cf. Levin (1975), pp. 75–84, Burkert (1972), pp. 391–4, with the additional references given there.

35 These are the ratios standardly given for the tone and the concords, respectively 9:8, 4:3, 3:2, 2:1. On the names of the ratios see 8 Eucl. *Sect. Can.* 149.14–24. *Dia pasōn*, literally 'through all (the strings or notes)', is the usual term for 'octave'.

36 *Diesis* is used here for the interval by which a fourth exceeds two tones (Aristoxenian usage is different). Philolaus' form of expression suggests that he understood that this interval is not exactly half a tone (see, for example, 8 Eucl. *Sect. Can.* propositions 15 and 16), otherwise *harmonia* would be exactly six epogdoics (cf. *Sect. Can.* proposition 14). It is what Plato and others call the *leimma*, whose ratio is 256:243 (see particularly 2.3 Plato *Tim.* 35b). According to Boethius (*Inst. Mus.* 3.5 (DK 44 A26), translated in Burkert (1972), p. 395), Philolaus called this smaller part of the tone *diesis*, the larger remnant *apotomē*, and the difference between them *komma*; elsewhere (*Inst. Mus.* 3.8 (DK vol. 1, p. 410.4–10)) he adds the *schisma* and the *diaschisma*, the halves, respectively, of the *komma* and the *diesis*. Perhaps the halving of the *diesis* related to an analysis of the enharmonic tetrachord (in Aristoxenian theory two quarter-tones followed by a ditone). The *apotomē* might have been needed for an analysis of the chromatic tetrachord (two 'semitones' followed by an interval of a tone and a 'half', which could be represented as *diesis*, *apotomē*, *diesis*-plus-tone). The *schisma* was presumably a purely theoretical measure, expressing the difference between the precise mid-point of the tone and the point at which the conjunction of *diesis* and *apotomē* divides it. All this suggests a sophisticated form of analysis, but the halvings of *komma* and *diesis* are mathematically improper. So too are the ascriptions, in the earlier passage of Boethius, of numerical values as well as ratios to the musical intervals. If Boethius' reports are reliable (which is open to dispute), the former indicates that Philolaus was not acquainted with the theorem of 8 *Sect. Can.* proposition 3, which goes back to Archytas, while the latter might suggest a link with the practices reported in 1.18 Porph. *Comm.* 107.15ff. See Burkert's discussion, pp. 394–9. Note that the diatonic division of the octave given here corresponds to that of 2.3 Plato *Tim.* 35b ff., whereas that of Archytas (1.21 Ptol. *Harm.* 30.9ff.) does not.

37 See 1.7 Aristotle *De Caelo* 293a.

'Come now, Cebes: didn't you and Simmias hear about things of this sort when you were associating with Philolaus?'[38]

'Nothing clear, at any rate, Socrates.'

1.16 Plato *Phaedo* 86b

[Simmias speaking] 'For I think, Socrates, that you have realised yourself that we believe the soul to be something much like this: our body is as it were tensioned and held together by hot and cold and dry and wet and other things of this sort, and our soul is the blending and *harmonia* of these same things, when they have been finely and proportionately [*metriōs*] blended with one another. So if the soul turns out to be some sort of *harmonia*, it is clear that when our body is excessively [*ametrōs*] slackened or tautened by diseases and other evils, it is inevitable that the soul must perish at once, most divine though it be, just like the other *harmoniai*, those in the notes and in all the things that craftsmen make...'[39]

1.17 Aristotle *De Anima* 407b27 (DK 44 A23)

There is another opinion too, that has been handed down concerning the soul...for they say that it is a kind of *harmonia*: for *harmonia*, they say, is a blending and putting together of opposites, and the body is constituted out of opposites.[40]

1.18 Aristotle *Politics* 1340b18 (DK 58 B41)

Hence many of the wise say, some of them that the soul is a *harmonia*, others that it contains a *harmonia*.[41]

1.19 Archytas frag. 1 (Porph. *Comm.* 56.5–57.27; also quoted, in part, in other sources, for which see DK 47 B1)[42]

Those who are concerned with the sciences [*mathēmata*] seem to me to be men of excellent discernment, and it is not strange that they conceive particular

38 This lends colour to the common suggestion that the theory enunciated in the next passage is that of Philolaus.

39 This is not explicitly attributed to Philolaus, but see the previous note. It has some affinities with the ideas expressed in 1.9–1.12.

40 Aristotle probably bases his account on Plato's, and like him reports these views anonymously. Plato's context strongly suggests that they are Pythagorean, if not necessarily those of Philolaus.

41 According to the first view, the soul is an attunement that coordinates something else, presumably the bodily parts or elements (the doctrine of 1.16 and 1.17). According to the second, it is itself something coordinated by *harmonia*. That is, the soul itself has 'parts' which are brought into a harmonious relation with one another. This has a more Platonic ring (see, for example, *Republic* 443d–e).

42 Apart from Porphyry, the most important source for this fragment is Nicomachus, who quotes a version of the opening paragraph in his *Introduction to Arithmetic*. Burkert

things correctly, as they really are. For since they exercised good discrimination about the nature of the wholes, they were likely also to get a good view of the way things really are taken part by part. They have handed down to us a clear understanding of the speed of the heavenly bodies and their risings and settings, of geometry, of numbers, and not least of music [*mousikē*].[43] For these sciences seem to be sisters, since their concern is with the two primary forms of what is, which are sisters themselves.[44]

They noticed first that there can be no sound unless there has been an impact of things upon one another.[45] They said that an impact occurs when things in motion meet one another and collide. Those travelling in opposite directions and meeting make a sound as each slows down the other, while those travelling in the same direction but at different speeds make a sound when overtaken and struck by the ones coming after them. Many of these sounds are not capable of being discerned by our nature, some because of the weakness of the impact, some because of the extent of their distance from us, and some even because

((1972), pp. 379–80, n. 46), was inclined to think that the fragment is spurious, and he was especially dubious about the opening paragraph. His conclusions were challenged by Bowen (1982) and Huffman (1985), both of whom thought that though certain words in Nicomachus' version are due to Nicomachus' own rewriting and addition, the bulk as quoted by Porphyry is genuine. I believe that in general they are right, but that Huffman's arguments for excising the last clause of the first paragraph ('since... themselves') are inconclusive. See n. 44 below.

43 Here I follow Bowen and Huffman in accepting Porphyry's text, rather than that of Nicomachus or the composite version printed in DK.

44 The words from 'since...' appear only in Nicomachus' text. They are regarded with suspicion by both Burkert and Huffman, mainly because these scholars can find no interpretation for the 'two primary forms of what is' other than one depending on Nicomachus' own distinction between multiplicity (*plēthos*) and magnitude (*megethos*). But even if Nicomachus used these words to mean something that Archytas could not have meant, it does not follow that they are his own addition. Now Huffman accepts the preceding clause 'For... sisters', which is in Porphyry's text, despite the supposedly unlikely coincidence that a surviving passage of Archytas should be one plainly echoed by Plato (2.1 *Rep.* 530d) – Burkert saw this 'coincidence' as a reason for treating the fragment as a post-Platonic forgery. If 'For... sisters' is accepted, however, we can fairly hope to find other Archytan echoes in this part of the *Republic*, and to use Plato as an aid to interpreting Archytas (see also the Appendix to this chapter). I suggest that this strategy does indeed allow us to make sense of the suspect last clause without anachronism. In Plato the sister sciences are astronomy and harmonics, and he explains their kinship by remarking that the one is concerned with visible motion, the other with audible. Why should not the 'two primary forms of what is', in a writer who (unlike Plato) does not posit a realm of reality beyond the perceptible, be precisely these, the visible and the audible? Admittedly Archytas' scheme, embracing four sciences, not just two, will be less than perfectly neat: presumably astronomy and geometry are concerned with the visible, harmonics with the audible, and number-theory with both, since all knowables are linked by their capacity to be grasped in terms of number (see 1.5 Aristotle *Metaph.* 985b23ff., 1.11 Philolaus frag. 4: cf. Archytas frag. 4 and 1.20 frag. 2). It is worth noting in this connection the idea, treated as a commonplace by Ptolemy, for example, that sight and hearing are the best and most scientifically informative of the senses: see 11 Ptol. *Harm.* 5.23–4. The authenticity of the clause remains doubtful, but I do not think that the case against it has been proved.

45 This becomes a familiar theme in fourth-century and later acoustics. See, for example, 2.4 Plato *Tim.* 67a–c; 3.15 Aristotle *De Anima* 419b–421a; 4.8 ps.-Ar. *Probs* XI. 19; 5 *De Audib.* 800a; 8 *Sect. Can.* 148.3ff.; 9.2 Adrastus ap. Theon Smyrn. 50, etc.

of their excessively great magnitude.[46] For large sounds do not slip into the ear, just as nothing enters the narrow neck of a vessel when one pours a large quantity.

Now when things strike against our organ of perception, those that come swiftly and powerfully from the impacts appear high-pitched, while those that come slowly and weakly seem to be low-pitched.[47] Thus if someone moves a stick sluggishly and weakly, he will make a low-pitched sound with the impact, but a high-pitched one if he moves it swiftly and strongly. We can grasp the fact not only from this example, but also when we want to utter something loud and high-pitched, either in speaking or in singing, since we utter with a violent breath.[48] The following also happens, as it does with missiles: the ones that are thrown strongly travel to a distance, while those thrown weakly fall nearby.[49] For the air yields more to those travelling strongly, and less to those travelling weakly. The same thing will happen with voices too: those travelling under the agency of a strong breath are loud and high-pitched, while those travelling under the agency of a weak breath are quiet and low-pitched. Here is the most powerful evidence in which we may see this fact: we can hear a person shouting loudly from far away, but when the same man speaks quietly, we cannot hear him even from nearby.

In *auloi* too, if the breath travelling from the mouth goes into the finger-holes [*trypēmata*] near to the mouth it emits a higher-pitched sound, because of its vigorous force, but a lower one if it goes into the holes that are further away.[50] Hence it is clear that a swift movement makes a sound high-pitched, while a slow one makes it low-pitched. The same thing happens, once again, with the *rhomboi* that are whirled about in the mystery cults: moved gently they give out a low-pitched sound, moved powerfully a high-pitched one.[51] Similarly, again, with the *kalamos*: if one blocks up its lower end and blows into it, it will give out a low-pitched sound, but if one blows into its half-length or any other

[46] Possibly a reference to the harmony of the spheres: cf. 1.6 Aristotle *De Caelo* 290b.

[47] The remainder of the passage is concerned to give grounds for this thesis, which was extremely influential. It was modified by Aristotle (3.15 *De Anima* 420a–b, cf. 3.17 *De Gen. An.* 786b–787a) and rejected by 6 Theophrastus ap. Porph. *Comm.* 63.1ff.: an alternative is proposed in 8 Eucl. *Sect. Can.* 148–9. But it was accepted by Plato (2.4, 2.5, *Timaeus* 67a–c, 79e–80b), by the writers of most of the *Problems* (ch. 4), and in 5 *De Audib.* at 803a. It remained the dominant theory throughout antiquity.

[48] Archytas' theory makes no provision for distinguishing the loud from the high-pitched. The need to do so is one of the reasons for Aristotle's modification of Archytas' view: see 3.17 *De Gen. An.* 786b–787a, and cf. 4.1 ps.-Ar. *Probs* XI.3. Related difficulties associated with the theory appear in a number of the *Problems*: see 4.2, 4.5, 4.8, 4.9, 4.13, 4.22, 4.23, and cf. 3.14 Arist. *De Sensu* 448a20ff., 6 Theophrastus ap. Porph. *Comm.* 63.15ff. For a further distinction that the Archytan account fails to make see 5 *De Audib.* 803a with n. 28.

[49] For other discussions of the 'missile' analogy see 4.2, 4.14 ps.-Ar. *Probs* XI.6 and 52.

[50] Cf., for example, 4.19 ps.-Ar. *Probs* XIX.23, 6 Theophrastus ap. Porph. *Comm.* 63.6ff., 9.7 Aelianus ap. Porph. *Comm.* 33.31ff., 10 Nicomachus *Ench.* ch. 4. There are many other parallels.

[51] These *rhomboi* ('bull-roarers') are flat pieces of wood or metal whirled around on the end of a string, used in ritual in many cultures. The name is derived from a verb (*rhembein*) meaning to whirl or revolve. The geometrical use to designate a rhombus is

part of it, it will utter a high pitch.[52] For if the breath travels through a long passage it comes out weakly, but if the same breath travels through a shorter passage it comes out more vigorously[53]... Thus the fact that high-pitched notes move more quickly and low-pitched ones more slowly has become clear to us from many pieces of evidence.

1.20 Archytas frag. 2 (Porph. *Comm.* 93.6–17)

There are three means in music.[54] One is arithmetic, the second geometric, the third subcontrary [*hypenantia*], which they call 'harmonic'. There is an arithmetic mean when there are three terms, proportional in that they exceed one another in the following way: the second exceeds the third by the same amount as that by which the first exceeds the second. In this proportion it turns out that the interval between the greater terms is less, and that between the lesser terms is greater.[55] There is a geometric mean when they are such that as the first is to the second, so is the second to the third.[56] With these the interval made by the greater terms is equal to that made by the lesser.[57] There is a subcontrary mean, which we call 'harmonic', when they are such that the part of the third by which the middle term exceeds the third is the same as the part of the first by which the first exceeds the second.[58] In this proportion the interval between the greater terms is greater, and that between the lesser terms is less.[59]

secondary, meaning 'shaped like a bull-roarer'. For other references see, for example, Eurip. *Helen* 1362, Theocr. *Idylls* 2.30, Diogenes ap. Athenaeus *Deipn.* 636a, *Anth. Pal.* 6.309 (where it is a child's toy).

52 *Kalamos* means 'reed'. The procedure of progressively shortening a length of reed by partly filling it with wax was used in the tuning of the Pan-pipe *syrinx*, whose pipes were then sounded by blowing across the open end. Archytas is referring to 'experiments' with just one pipe. Cf. especially 4.19 ps.Ar. *Probs* XIX.23.

53 Comparable explanations of the properties of wind instruments are assumed by all those who adopt a 'velocity' theory of pitch: e.g., explicitly 10 Nicomachus *Ench.* ch. 4 (244.1ff.); cf. 11 Ptol. *Harm.* 8.25ff., and contrast 6 Theophrastus ap. Porph. *Comm.* 63.6ff.

Here Porphyry breaks his quotation, introducing the final sentence with the words: 'after saying other things too, concerning the fact that the movement of the voice is intervallic, he sums up his account as follows'. (On the 'intervallic' movement of the voice see 7 Aristox. *El. Harm.* 8.13ff.)

54 Uses of these three 'means' and the corresponding forms of proportion recur repeatedly in musicological texts of the Pythagorean and Platonist traditions. They lie behind the scalar divisions attributed to Archytas in 1.21 (see the Appendix to this chapter); see also especially 2.3 Plato *Tim.* 35c, 10 Nicomachus *Ench.* ch. 8, 12 Arist. Quint. *De Mus.* Book III chs. 5 and 24.

55 Thus the three numbers 12, 9, 6 are in arithmetical proportion. The ratio of 9 to 6 is 3:2, that of the musical fifth, while that of 12 to 9 is 4:3, that of the fourth.

56 That is, there are three terms x, y, z such that $x:y = y:z$ (or $\frac{x}{y} = \frac{y}{z}$, etc.).

57 The simplest examples are sequences of multiples, such as 6, 12, 24, where $24:12 = 12:6 = 2:1$, the ratio of the octave.

58 That is, there are three terms, x, y, z such that $y - z = \frac{z}{n}$ and $x - y = \frac{x}{n}$. Take for instance 12, 8, 6. Here $8 - 6 = \frac{6}{3}$ and $12 - 8 = \frac{12}{3}$. The difference in each case is a third part of the relevant extreme term. For a different approach to this relation see 12 Arist. Quint. *De Mus.* Book III ch. 5.

59 Thus in the case given in n. 58, the interval represented by 12:8 is 3:2, the fifth, and that represented by 8:6 is 4:3, the fourth. The various examples set out above also show

1.21 Ptolemy *Harm.* 30.9–31.18 (DK 47 A16)[60]

Archytas of Tarentum, of all the Pythagoreans the most dedicated to the study of music, tried to preserve what follows the principles of reason [*logos*][61] not only in the concords but also in the divisions of the tetrachords,[62] believing that a commensurable relation between the excesses is a characteristic of the nature of melodic intervals.[63] But though he sets off from this presupposition, at

how the three kinds of mean can be used, in an elementary way, to map out the basic framework of a standard musical system. The series 6, 12, 24 etc., in geometrical proportion, represents a sequence of notes an octave apart. If we take the first two numbers and insert the arithmetic mean, we get 6, 9, 12, the octave being divided into a fifth followed by a fourth. A harmonic mean inserted between the original terms gives 6, 8, 12, dividing the octave into a fourth followed by a fifth. When the two sequences are combined, 6, 8, 9, 12, they yield two fourths separated by the 'tone' of ratio 9:8, and can represent the fixed notes bounding a pair of disjoined tetrachords. These are the fundamental relations on which all the complex structures of Pythagorean and Platonist harmonics are built. For a more sophisticated use of the Archytan theory of proportions see the Appendix to this chapter.

60 This passage will also be found in its place in the translation of 11 Ptol. *Harm.*, but it seemed appropriate to make it available among the material concerning Archytas. Reference to the context in Ptolemy will clarify some points that may be obscure when the report is taken in isolation.

61 The reference to *logos* reflects the controversy, developed among Hellenistic commentators, as to whether a 'proper' scalar sequence is one conforming to rationally excogitated mathematical principles (Pythagoreans, Platonists), or one to be defined in terms of what musical perception accepts (Aristoxenus and his followers). See particularly 2.1 Plato *Rep.* 531a–c; 7 Aristox. *El. Harm.* 32, 18ff.; 9.11–9.14 Ptolemaïs and Didymus ap. Porph. *Comm.* 22.22ff.; 11 Ptol. *Harm.* Book I chs. 1–2, with chs. 5–6 on the Pythagoreans.

62 When the ratios of the fourth, fifth and octave have been identified, making it possible to express the relations between the 'fixed' notes of the scale, there remain the smaller intervals bounded by the two 'moveable' notes in the interior of each fourth. These, together with the boundaries of the fourth, form a tetrachord. The positions of the notes inside the tetrachord, and hence the ratios of the intervals in which they occur, vary to create the different 'genera', and these are therefore represented by different 'divisions of the tetrachord'. Various approaches to these divisions are described in 11 Ptol. *Harm.* Book I chs. 12–16. Aristoxenus, who offers his own divisions at 7 *El. Harm.* 21.31ff., 46.19–47.8, 50.15ff., also claims (2.7ff.) that none of his predecessors had analysed any genus but the enharmonic. The claim might cast doubt on the authenticity of the present passage, except that it also blatantly ignores the diatonic division incorporated into 2.3 Plato *Timaeus* 35b–36b, not to mention the diatonic implications of 1.12 Philolaus frag. 6. In Aristoxenus' view the Pythagorean and Platonist analyses in terms of ratio simply do not count as examples of the science he pursues (cf. 7 *El. Harm.* 32.18ff.). But Ptolemy is perhaps right to hint that Archytas was the first to work out ratios for the tetrachordal divisions on a 'rational' basis. Philolaus (1.12) and probably others before him (see 1.2–1.4) had identified the ratios of the basic concords and the tone, but may have gone no further, at least in a systematic way (compare n. 36 above).

63 That is, of the small intervals in a legitimate scalar series, contrasted with octaves and concords (11 Ptol. *Harm.* 15.3ff., etc.). The word 'commensurable' (*symmetros*) here means more than just 'expressible as a ratio between whole numbers'. It also indicates that the ratios must be of a mathematically suitable kind (cf. 3.11 Arist. *De Sensu* 439b, with notes 21 and 22). The major condition of suitability that Ptolemy attributes to Archytas (and uses himself) is one entailing that the ratio of any melodic interval (i.e., any incomposite scalar step) should be of the 'epimoric' form n+1:n. See 11 Ptol. *Harm.* 32.1–3, cf. 11.8ff., 16.12ff. and Book I chs. 15–16. An insistence on ratios of this or the multiple form in the analysis of concords appears at 8 Eucl. *Sect. Can.* 149.11ff. (cf. especially propositions 10–12). Plato too emphasises the difference between ratios

several points he seems to fall hopelessly short of it; and though in most cases he is well in control of this sort of thing, he is patently out of tune with what has already been straightforwardly accepted by the senses, as will be seen at once from the division of the tetrachords that he proposes. He posits three genera, the enharmonic, the chromatic and the diatonic, and he makes his division of each of them in the following way. He makes the last ratio the same in all three genera, 28:27; the middle one in the enharmonic 36:35 and in the diatonic 8:7, so that the first interval in the enharmonic turns out to be 5:4, in the diatonic 9:8.[64] In the chromatic genus he locates the note second from the highest by reference to that which has the same position in the diatonic. For he says that the second note from the highest in the chromatic stands to the equivalent note in diatonic in the ratio 256:243.[65] Such tetrachords, on the basis of the ratios set out, are constituted in their lowest terms by the following numbers. If we postulate that the highest note of each tetrachord is 1,512, and the lowest, in epitritic ratio with this, is 2,016, this latter term will make the ratio of 28:27 with 1,944, and this will be the quantity of the second note from the lowest in all three genera. As to the second note from the highest, that in the enharmonic genus will be 1,890, since that makes a ratio of 36:35 with 1,944, and a ratio of 5:4 with 1,512. The equivalent note in the diatonic genus will be 1,701, since that makes a ratio of 8:7 with 1,944, and a ratio of 9:8 with 1,512. The equivalent note in the chromatic genus will be 1,792, since that has a ratio to 1,701 as is 256 to 243. The table of these numbers is set out below.[66]

such as these and those that are 'number to number' (epimeric or superpartient), 2.3 *Timaeus* 36b. See also the Appendix to this chapter.

64 Here the 'first' interval is the highest in the tetrachord, the 'last' the lowest.

65 This is the ratio of the *leimma* of 2.3 Plato *Timaeus* 36b and many other sources. As the ratio of the interval left in the fourth after two tones of 9:8 each, it has a respectable role in harmonics, despite its deviation from the norm mentioned in n. 63 above. Since the highest interval of Archytas' diatonic is 9:8, his chromatic's extension of this by 256:243 ensures that the remainder of the chromatic tetrachord (the two lower intervals together) also spans an interval of sound pedigree, once again that of ratio 9:8. (The subdivisions of this interval are determined by Archytas' decision to have a lowest interval of 28:27 in each genus. See the Appendix to this chapter.) These points do something to explain the anomalies in Archytas' chromatic division, though not to Ptolemy's satisfaction. Note that it was not unusual, at least in informal terms, to envisage the chromatic as a 'colouring' or modification of the diatonic. See, for example, the interpolated passage at 12 Arist. Quint. *De Mus.* 92.19ff., and cf. *GMW* vol. 1, p. 143, n. 61.

66 Ptolemy's criticisms appear in 11 *Harm.* Book 1 ch. 14. As he has said, they accuse Archytas of offences against both reason and the ear, the former because he is unfaithful to the principle outlined in n. 63 above, the latter mainly because – unlike all other Greek theorists – he makes the lowest interval the same in all three genera (but he raises other issues too). In the Appendix to this chapter I offer some remarks on these matters, and on the principles on which Archytas' divisions are based.

Enharmonic		Chromatic		Diatonic	
1,512		1,512		1,512	
	5:4		(32:27)		9:8
1,890		1,792		1,701	
	36:35		(243:224)		8:7
1,944		1,944		1,944	
	28:27		28:27		28:27
2,016		2,016		2,016	

Appendix: the scalar divisions of Archytas

In 2.1 Plato *Rep.* 531c we find two related comments on Pythagorean musical theorists. First, they seek numbers – presumably ratios – in things that they hear, not in purely mathematical constructions. Secondly, they do not 'rise to problems, to investigate which numbers are concordant and which are not, and why each are so'. Now Plato and Archytas were friends (see especially Plato's seventh letter, 338–9, 350), and it is likely that by the time he wrote the *Republic* Plato was acquainted with Archytas' work in harmonics (cf. n. 44 above). In connection with Archytas' scalar divisions, reported in 1.21, two questions then arise. First, were they intended to describe musical systems actually in use at the time, or were they constructions of pure theory? Secondly, if they were analyses of 'things heard', were they just attempts to describe the phenomena as accurately as possible, or did Archytas design them to conform to certain theoretical principles, which he took to underlie the coherence of any well formed scalar system? Plato's comments might suggest answers to both these questions: that they were indeed intended as empirical descriptions of existing scalar systems; and that Archytas had not worked out any principle distinguishing well formed from ill formed systems at a mathematical level, a principle that would explain *why* one set of relations is harmonically proper while another is not.

Let us consider the mathematical issues first. It will be helpful to have in front of us the divisions set out not just over the range of a tetrachord, as in 1.21, but over a full octave, as in 11 Ptol. *Harm.* Book II ch. 14. They are presented in the table A1 to which I have added the standard Greek names of the notes between which the ratios hold.

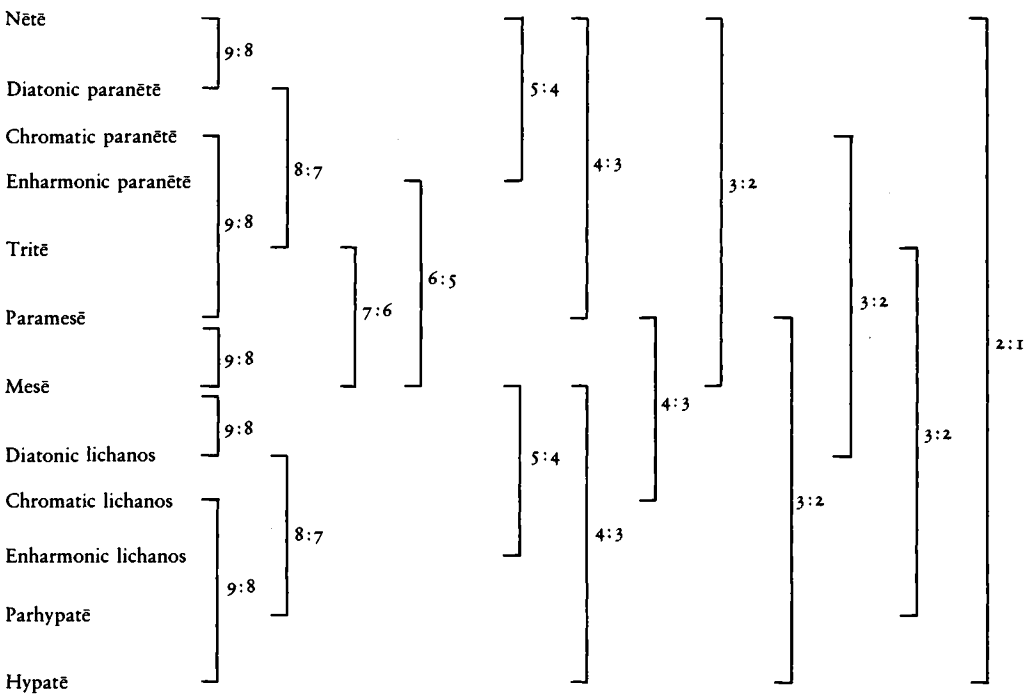
Table A 1

			Enharmonic	*Chromatic*	*Diatonic*
Nētē diezeugmenōn					
			5:4	32:27	9:8
Paranētē diezeugmenōn					
	4:3		36:35	243:224	8:7
Tritē diezeugmenōn					
			28:27	28:27	28:27
Paramesē					
	9:8		9:8	9:8	9:8
Mesē		2:1			
			5:4	32:27	9:8
Lichanos mesōn					
	4:3		36:35	243:224	8:7
Parhypatē mesōn					
			28:27	28:27	28:27
Hypatē mesōn					

If these divisions were based on a theory about the mathematical relations that constitute something as a 'harmonic' system, we should expect them to exhibit some clear pattern of mathematical order. At first sight such order is not apparent. The first clue to its character is provided by Ptolemy's remark that Archytas sought to give divisions based on ratios the 'excesses' between whose terms are 'commensurable'.

This implies, in Ptolemy's usage, that all the ratios used should be epimoric (n + 1 : n, see n. 63 above), and he complains that in the case of Archytas' chromatic they are not. But though not every step in Archytas' scales is of epimoric ratio, it is in fact possible to construct every note in all three genera, from a given starting point, by movements through intervals whose ratios are epimoric. Further, the ratios involved eliminate the need to use in the construction any terms other than small whole numbers, those in the series from 1 to 9. (Hence all the terms fall within the decad, a fact that may have been important to a Pythagorean: see 1.2.) The relations are represented in the table A2, which combines the three genera, and again assigns to each note the name by which it was usually known.

Table A 2



The table does not show all the relations that can be expressed in the relevant way. Notice in particular that every note of the upper tetrachord in enharmonic, chromatic or diatonic stands to its counterpart in the lower tetrachord in the ratio 3:2. The table shows only the relations that are most important in determining the positions of the 'fixed' notes from a given starting point (which might be *nētē*, *hypatē* or *mesē*), and in locating the positions of the moveable notes from those of the fixed ones.

This is satisfactory, but the requirement that every note be constructible through epimoric ratios is hardly enough to distinguish well formed harmonic systems from all others. In 2.3 *Timaeus* 35–6, Plato seems to accept as the real, primary principle of harmonic organisation the thesis that the ratios must divide the system in an orderly pattern of proportions, based on three kinds of mathematical mean, arithmetic, geometric and harmonic. Now we know from 1.20 that Archytas classified these means

and described them as 'used in music'. We should expect them to play some part in his divisions, but none of these is identical with Plato's. If the principle is at work here too, its application must be different.

If Plato's construction is adapted to fit the range of a musical octave, the system it generates is identical with that implied in 1.12 Philolaus frag. 6. It was later known as the 'Pythagorean diatonic', and is represented in the following schema.

Table A 3

Note	Interval	Tetrachord	Octave
Nētē			
	9:8		
Paranētē			
	9:8	4:3	
Tritē			
	256:243		
Paramesē			
	9:8		2:1
Mesē			
	9:8		
Lichanos			
	9:8	4:3	
Parhypatē			
	256:243		
Hypatē			

As in the divisions of Archytas, and all similar constructions, each note in the upper tetrachord stands to its counterpart in the lower tetrachord in the ratio 3:2.

The system can be built from the simplest possible application of Archytan proportion theory. Notes an octave apart are represented by terms in a geometrical progression by doubles (e.g., 6, 12). If the harmonic and arithmetic means are inserted between such terms (e.g., 6, 8, 9, 12), the new terms are the inner boundaries of the tetrachords, separated by a tone. (Terms bounding a tetrachord are in the ratio of the fourth, 4:3, and the ratio of the tone is 9:8.) So far, then, we have *nētē* (6), *paramesē* (8), *mesē* (9), *hypatē* (12). To find the remaining notes, construct an octave upwards from *mesē* and insert between the terms of this octave their harmonic mean: this gives *paranētē*. From here take an octave downwards, and again insert the harmonic mean: this gives *lichanos*. Repetitions of the procedure will give *tritē* and *parhypatē*. All the notes that are harmonic means between notes of the system and their octaves are also arithmetic means between others: the attunement procedure may use either form of proportion, and is most economical if it uses both.

Archytas' divisions cannot be reached in this way, but a simple change in the application of the theory of proportions gives the required results. The first steps, locating the boundaries of the octave and the tetrachords, are the same. But then, instead of inserting means only within further instances of the octave relation, we place them also between terms bounding the lesser concords of the fifth and the fourth (3:2 and 4:3). When harmonic and arithmetic means are placed between terms in the ratio 3:2, the ratios between means and extremes are 5:4 and 6:5. When they are placed

between terms in the ratio 4:3, the resulting ratios are 7:6 and 8:7. The ratio 9:8 has already appeared, as that between the two mean terms in the octave. Hence all the ratios underlying Archytas' divisions (table A2 above) can be constructed proportionally, by the location of means first in the octave, then in the concords generated by the first construction.

Archytas' theory of proportions can thus be found at work in his divisions. His approach may have originated in puzzles about epimoric ratios. Given that epimorics are fundamental to harmonic systems (as quite uncontroversially are the ratios 3:2, 4:3, 9:8, and the closely related 'multiple' ratio 2:1), the analyst of scales must ask how they are to be subdivided. Archytas is on record as having proved that they cannot be divided into equal parts (see 8 *Sect. Can.* prop. 3), and his original question may have been how, in that case, they can be divided in a mathematically intelligible way. His answer is that such division can be achieved through the insertion of harmonic and arithmetic means, and that the notes of a coordinated musical system must divide it in an integrated pattern of proportions.

These theoretical considerations do not by themselves determine the form of Archytas' divisions. His system of proportions can be applied so as to generate a good many others, notably the 'diatonic' of Philolaus and Plato. His divisions present several features that no purely mathematical principles will explain. The crucial questions are firstly whether these features are merely accidental, or reflect real aspects of current musical practice, and secondly why Archytas did not use Philolaus' simple division to represent the diatonic genus. There is no doubt that he knew of it.

Four peculiarities in the divisions stand out:

(i) The lowest interval of the tetrachord is the same in each genus. No later author accepts this equality: Ptolemy protests at it, and Aristoxenus would have done so too, had he discussed Archytas. In their view, the lowest enharmonic interval is much smaller than that of the diatonic, and that of the chromatic is sometimes smaller too (though not always). Correspondingly, in almost all later treatments, both the inner notes of a tetrachord are moveable, and alter in changes of genus, not just one of them.

(ii) The ratios of Archytas' chromatic are strange, as Ptolemy remarks, and their strangeness is not imposed by his mathematical principles, any more than is the equality of the lowest intervals in each kind of tetrachord. It is not difficult to construct suitable divisions, within the constraints of Archytan proportion theory, where this equality is avoided, and where all the chromatic ratios are epimoric. (See, for example, the tables recording the divisions of Ptolemy, Eratosthenes and Didymus in 11 Ptol. *Harm.* Book II ch. 14.)

(iii) The upper interval of Archytas' enharmonic is a major third, ratio 5:4. According to Aristoxenus the proper interval in this position is a ditone (see 7 *El. Harm.* 23.3ff.), whose ratio in Pythagorean terms would be 81:64.

(iv) Finally we should note again Archytas' refusal to accept the division indicated by Philolaus as a correct representation of the diatonic.

This last point is crucial. Ptolemy's statement of Archytas' own explanation for the ratios of the chromatic (1.21 Ptol. *Harm.* 31.4–6) makes no sense unless Archytas both knew of the Philolaic division (see Burkert (1972), p. 389) and thought it to be of genuine significance, even though he was not prepared to treat it as a scalar system in its own right.

Now the division of Philolaus and Plato is a mathematical representation of a system that arises musically from an attunement by pure fourths and fifths. The use of this 'method of concordance' as a practical way of attuning notes was well known in

antiquity: see, for example, 7 Aristox. *El. Harm.* 55.2ff., 8 Eucl. *Sect. Can.* proposition 17. Its relevance to Archytas' chromatic extends beyond the point made by Ptolemy, since the problematic *lichanos* and *paranētē* are readily found by this procedure. All the standard 'fixed notes' (*nētē, paramesē, mesē, hypatē*) can be located, taking any one of them as a starting point, by moves through fourths and fifths: Archytas' chromatic *lichanos* is a fourth below *paramesē*, and chromatic *paranētē* is of course a fifth above *lichanos*. It seems possible that Archytas knew of a method of tuning in which *lichanos* and *paranētē* were in practice found by this method. Then the strange ratio between them and the highest notes of their tetrachords is mathematically entailed. So is the ratio of the interval below each of them, if – for some other reason – the tetrachord's lowest interval has to be 28:27.

This conclusion suggests a more general hypothesis. Though the 'Pythagorean diatonic' does not appear as such in Archytas' divisions, perhaps it was, none the less, the practical basis of the tunings he sought to represent, and their peculiarities are partly explicable by reference to it, either as applications of it, as in the case considered above, or as deliberate deviations from it. In the latter kind of case the mathematical theorist's task will have been that of making the deviations intelligible, rather than merely arbitrary.

Consider the upper enharmonic interval, 5:4. The lower boundary of this interval cannot be found by movements of pure fourths and fifths from fixed notes; that would be possible only if the interval were the true ditone of 81:64. Here, as Winnington-Ingram has shown, the explanation lies in a passage of Aristoxenus (Winnington-Ingram (1932), p. 199: see 7 Aristox. *El. Harm.* 23.12–22). Aristoxenus prefers the ditone in this position, and complains that most musicians 'sweeten' the interval in practice by slightly reducing its magnitude. Now the major third of 5:4 is indeed perceptibly sweeter, more aesthetically agreeable, than the ditone. But though Aristoxenus could of course have detected this aesthetic difference between the two intervals, how could he have known that the sweeter of them was not a ditone, but fractionally less? And how could Archytas have known that the interval he wanted to represent was not, in fact, the one whose ratio is 81:64? Aristoxenus' method gives no way of measuring intervals with this degree of precision, and even that of the Pythagoreans, if at this period they had begun to make use of the monochord or similar devices (which is disputable), is most unlikely to have been sufficiently sensitive. If he were relying on experiments with the monochord, Archytas would apparently have had to do two things. He would have had to be able to distinguish these two intervals with precision on his experimental apparatus, and he would also have had to convince himself that the interval standardly and rightly used in musical practice was the smaller of them, not the larger. It seems improbable that the monochord was a sufficiently accurate device. It also seems unlikely that musical practice was itself so exact and unvarying that the conclusion could reliably be drawn in this way. How could one decide that any such slight apparent discrepancy between the ditone and the interval actually produced was not merely the result of a musician's error, or the technical limitations of his instrument? One wonders what suggested to Archytas, in the first place, the hypothesis that this discrepancy not only existed, but was in some sense meant to exist, and that a proper analysis of the scale would have to represent it, rather than 'correcting' it.

I suggest that only one hypothesis is plausible. It was not, after all, just the sound of the musicians' scales that alerted the theorists to this feature of them. It was rather that Archytas and Aristoxenus *saw* that musicians, in tuning the string corresponding to enharmonic *lichanos*, first tuned it at a true ditone below *mesē* by the method of concordance; and having done so, they then tightened it further, very slightly, to achieve the 'sweetness' they sought. Only evidence of this sort could have given these theorists

convincing grounds for the belief that the interval used in musical practice was not a genuine ditone, and was not intended to be one, but was deliberately formed as a ditone reduced by a very small amount.

What about Archytas' diatonic itself? (Note that it is identical with Ptolemy's tonic diatonic, analysed in 11 Ptol. *Harm.* Book 1 ch. 15.) Here I must deal very briefly with an extremely complex matter. The highest interval of the system is the same as that of Philolaus. Why are the others different? It is clear that this question is related to another which I mentioned above: why does Archytas, and no later writer, make the lowest interval of the tetrachord the same in all three genera, and assign it the ratio 28:27? For if this interval is fixed, the departure from Philolaus' system is inevitable.

Winnington-Ingram has given good reasons for believing two important propositions: I cannot go into the reasons here. The first is that in pre-Aristoxenian usage, the lowest interval of the tetrachord was indeed invariable. The other is that an interval comparable to a modern minor third was structurally important in these older systems: it lay immediately below the notes *tritē* and *parhypatē*. It was not an incomposite interval, of course: it is exemplified by the interval between *tritē* and *mesē*, where it is divided by the intervening note *paramesē*. Now in the Archytan system the *tritē* finds its place through its relation to a note that is lower in the ratio 7:6, and 7:6 is an acceptable version of the minor third. Winnington-Ingram concludes that it is the importance of this interval that was instrumental in determining the positions of the Archytan *tritai* and *parhypatai*, and if the ratio of the interval between *tritē* and *mesē* is 7:6, the interval between *tritē* and *paramesē* is thereby determined too, and must be 28:27. (The existence of this interval, and its importance independently of a system's genus, may be hinted at by occasional references in our sources to a note called *hyperhypatē*, a tone below *hypatē*. See notes 65 and 71 to 8 Eucl. *Sect. Can.* proposition 19.)

If this is correct, it adds substantially to our evidence for the hypothesis that Archytas was seriously concerned to represent systems belonging to real musical practice. But we must ask, once again, how a musician would have set about tuning the notes in question. Our previous reflections offer an obvious answer. The *tritē* string, for example, would first have been tuned, by the method of concordance, at a ditone below *nētē*. Then it would have been slightly relaxed, just as enharmonic *paranētē* was tightened, until the required relation of a 'minor third' with *mesē* was achieved. This practice will have given Archytas the empirical information that *tritē* was intended to be more than a 9:8 tone below diatonic *paranētē*. The difference is small; hence 8:7 is the obvious choice. This leaves 7:6 as the ratio between *tritē* and *mesē*, an interval that required a simple ratio, in view of its structural importance in contemporary music. And in that case the ratio between *tritē* and *paramesē* comes out at 28:27, as Archytas says.

I have been trying to show that Archytas was pursuing two goals. He attempted both to analyse the attunements underlying contemporary musical practice, and to reveal the principles of mathematical order on which they were based. I have also tried to explain the role, in his theory, of the structure described by Philolaus, and adopted as metaphysically fundamental by Plato. For Archytas, as for them, this structure was of primary importance. He was satisfied, however, that it did not appear anywhere as a system to which any instrument was actually tuned. Rather, it was a mathematical representation of the basic tuning procedure, the 'method of concordance', by which musicians established the outlines of their patterns of attunement, but which was then redeployed and modified to create the precise relationships they sought. For Plato, this fact might have served as an additional encouragement for his belief that only this system, based on pure concordances and the most economical application of proportion-theory, is truly 'harmonious' at a metaphysical level. Archytas could properly have

agreed that it is in all respects fundamental. But in his view, the additional complexities introduced by the tuning systems of current practice were not a sign of their mathematical or metaphysical incorrectness. On the contrary, these attunements provided a basis for the aesthetic variety of contemporary music, precisely through subtle applications of the proportional principles underlying the Platonic scale, and not through random departures from them. They achieved their specific musical characters by subtly modifying the role of the maximally concordant system, not by abandoning it. The theorist's task was to show how these modifications themselves created unified and intelligible systems that could be explicated in terms of the theory of proportions, so that the same mathematical principles that gave the Platonic system its metaphysical credentials were found to govern also the attunements of real musical performance. Seen in this light, Archytas' analysis is a triumphant fusion of attentive observation, metaphysical commitment and mathematical ingenuity.

What are we to say, then, of the relation between Archytas' work and the remarks made by Plato in 2.1 *Republic* 531c? If Plato had Archytas in mind, he was certainly right to claim that his attention was directed to the sounds that he heard, and not to pure mathematical abstractions. But the complaint that the Pythagoreans did not enquire 'which numbers are concordant and which are not, and in each case why' seems quite inappropriate as a criticism of Archytas. His theory of means and proportions is an answer to exactly this question, and Plato himself treated it as such, at least when he wrote the *Timaeus*. The truth, perhaps, is that for Plato it was indeed in Archytan proportion theory that the ideal science of harmonics would consist, and Plato's Socrates, in his dramatic context, was being allowed to predict its birth. But at the same time, Plato will not have been altogether happy with Archytas' application of his theory, since he had allowed himself to be diverted from the further pursuit of issues in pure mathematics, into the 'futile' task of describing and justifying the patterns of attunement that musicians actually employed.

CHAPTER 2

Plato

Plato's dialogues were written during the first fifty years of the fourth century. They are full of allusions to music, and several reflect at length on its aesthetic qualities, its effects on human character and emotion, and its uses as an instrument of educational and social policy. Some of the central passages concerned with these matters are collected in *GMW* vol. I ch. 10. His writings also contain discussions of harmonic theory, both actual and ideal, and apply musical ideas to the study of the structure of the universe. His investigations of the nature of the material world, in the *Timaeus*, include remarks about the physical basis of sound and our perception of it. The most important of these are translated in the present chapter.

In the *Republic*, after presenting his views about the proper organisation of the city-state, and introducing his famous conception of the 'philosopher-ruler', Plato proceeds in Book VII to consider the form that the intellectual education of these rulers should take (their physical and moral training has already been discussed). The studies he prescribes are mathematical, on the grounds that these are best suited to the task of elevating the mind from a concern with what is perceived to an investigation of more fundamental realities, those that are intelligible but not perceptible. Five branches of mathematics are distinguished: number-theory, plane geometry, solid geometry ('stereometry'), astronomy and harmonics. This curriculum is to be completed by the study of 'dialectic', philosophical argumentation, through which the aspiring philosophers will be led beyond mathematics (which, in Plato's view, still depends essentially on unexamined concepts derived from the manner in which things appear to our senses), and through which they will ascend to an understanding of the immaterial entities which Plato calls 'Forms', the realities that are the objects of that true knowledge which the mind can grasp quite independently of perceptual representations, and of which the perceived properties of everyday things are imperfect reflections in a misleading medium. The philosophers' final goal is understanding of the Form of the Good, whose nature determines that of all other Forms: for the reality on which the world is based is not only perfectly intelligible but perfectly good (in Platonic metaphysics the two conditions are virtually synonymous), and has the nature it does because that is the best possible.

Plato's accounts of the sciences of astronomy and harmonics are strikingly idiosyncratic. His astronomy is not a science that aims to describe accurately the observed movements of the heavenly bodies, and the task of his harmonics is not to analyse any musical systems in actual use. For the reasons given in the previous paragraph, both sciences, if they are to have more than a humble, mundane utility, must seek to reveal truths of a higher order, transcending the sphere of perception. Just as geometry has as its subject matter such intelligible entities as the square and the circle, which are not things that can be perceived, and is not concerned with the description of the individual, perceptible and approximate squares and circles that it uses in its diagrams, so astronomy and harmonics are concerned with an ideal mathematics of motion. The visible movements of the stars and the audible movements that constitute sounds are to be treated merely as 'diagrams' or perceptual aids, from which the mind can be led to a grasp on the intelligible mathematical principles that perceptible movements may imperfectly exhibit.

Plato's notion of harmonics is Pythagorean in inspiration, but his radical separation of the intelligible from the perceptible is not, and in 2.1, after distinguishing the Pythagoreans' approach from that of certain others (the passage is an important source of information about these schools), he chides them for engaging in researches at too low a level of abstraction. They are too concerned with the quantitative description of what they hear, too little with the mathematical principles which explain why some numerical relations are 'concordant' while others are not. (On the justice of this judgement as applied to Archytas in particular, see the Appendix to chapter 1.) Plato seeks principles that constitute harmonic order at a mathematical and metaphysical level: if the musical systems of actual human practice fail to exhibit this order, that merely shows their imperfection and the inadequacy of human perception to judge what is truly harmonious. (This attitude came to infect some later schools of harmonic theory that took the name 'Pythagorean': see, for example, 9.11–9.14. It is already criticised in 7 Aristox. *El. Harm.* 32.19–28.)

Pythagorean ideas are also prominent in 2.2, another extract from the *Republic*, and the earliest document we have that elaborates a version of the doctrine of the harmony of the spheres. They are pervasive throughout the *Timaeus*, the source of 2.3–2.5. Of these 2.3, which describes the quasi-musical structure of the soul of the world, was enormously influential in later antiquity. Here Plato adopts a set of mathematical principles from Archytas (1.20) as an answer to the question posed at the end of 2.1, and uses them to construct a metaphysically perfect scalar system, whose proportional divisions underlie the ordering of motions in the visible cosmos. His constructions thus combine the projects proposed as 'rational astronomy' and 'rational harmonics' in the *Republic*. Passages 2.4 and 2.5 are of relatively minor importance. They develop, out of beginnings made especially by Archytas, some ideas about the physical basis of sound and hearing, and in particular of the phenomenon of concordance, ideas that were at the root of studies in acoustics made subsequently by Aristotle and his followers (chapters 3–5).

The last excerpt in this chapter (2.6) is from the *Philebus*. It is taken from a passage that discusses the way in which the arts and sciences analyse and synthesise their subject matters. The study of music is taken as an example, alongside phonetics, the science that studies the way in which verbal sounds are to be classified and interrelated. The issues that Plato considers arise out of a tangle of metaphysical problems, which he couches in terms that are certainly of Pythagorean origin. But it is not at all clear that in this case he has Pythagorean harmonics in mind. Despite the efforts made by commentators to relate the ideas expressed here to one tradition or another of specialised musicological thought (see especially Gosling (1975), pp. 153–81), I would suggest that Plato is thinking here rather of the kind of understanding needed by a serious practical musician, not of the theoretical analyses of any school of harmonic science. That the points he makes were not taken by his Greek readers as specifically Pythagorean in their implications is suggested by apparent echoes of this passage in Aristoxenus: see particularly 7 *El. Harm.* 69.1–5.

Books on Plato are legion. For a good recent study of the major aspects of his work see Rowe (1984). There are useful chapters on the three dialogues quoted here, in Guthrie, *History*, vols. 4 and 5 (also includes extensive bibliographical references). Major commentaries accessible to readers who know no Greek include White (1979) and Annas (1981) on the *Republic*, Cornford (1937) on the *Timaeus*, Gosling (1975) on the *Philebus*. On Plato's cosmological theories see especially Vlastos (1975). There is as yet no book in English wholly devoted to Plato's ideas about music (one by Alan C. Bowen is in preparation), but valuable discussions will be found in the first three chapters of Lippman (1964).

2.1 *Republic* 530c–531c

d 'Motion,' I [Socrates] said, 'presents not just one but several forms, as it seems to me. A wise man, perhaps, will be able to name them all, but two are quite obvious even to us.'

'What kinds are they?'

'In addition to the one we have discussed,' I said, 'there is its counterpart.'

'What sort is that?'

'It appears,' I said, 'that just as the eyes are fixed on astronomy, so the ears are fixed on harmonic [*enharmonios*] motion, and that these two sciences are one another's sisters, as the Pythagoreans say and we agree, Glaucon.[1] Or what do we do?'

'As you say,' he said.

e 'Then since the task is a big one,' I said, 'we shall ask them what they say about these matters, and about anything else in addition to them. We for our part, while pursuing all these things, shall safeguard our own objective.'

'What is that?'

'That those whom we shall bring up shall not try to learn anything with no purpose [or 'completion' (*ateles*)], anything that is not always reaching out to that point at which everything ought to arrive, as we were saying just now
531a about astronomy. Or don't you realise that in harmonics too they do something quite different, like this: they measure heard concords and notes against one another, and so labour to no purpose, just like the astronomers.'[2]

'Yes, by the gods,' he said. 'Their behaviour is quite ridiculous, when they name some things "*pyknōmata*" and incline their ears as if hunting out a sound from next door, some of them asserting that they can still just hear a sound in between, and that that is the smallest interval, by which measurement is to be made, while others take issue with them, saying that the notes sounded
b are already the same, each group putting their ears ahead of their mind.'[3]

[1] On the 'sister sciences' see 1.19 Archytas frag. 1 with n. 44. Plato's conception of harmonic motion must be distinguished from that of Aristoxenus. For the latter, the voice 'moves' from point to point in the dimension of pitch, and when singing a note 'stands still' at a point. For Plato, following Archytas, a sounded pitch is itself a movement of a certain velocity: harmonic relations are ratios between speeds of movement, not distances between points. See particularly 7 Aristox. *El. Harm.* 8.13–12.34.

[2] Astronomy has been discussed at 528e ff. On Plato's view of its proper task see the introduction to this chapter. His parallel attitude to harmonics is made clear at 531c below.

[3] Glaucon has mistaken Socrates' intention: the theorists he describes here are distinguished from the Pythagoreans in the next paragraph. They are 'empiricists', comparable if not identical to the practitioners of 'harmonics based on hearing' mentioned at 3.7 Aristotle *Post. An.* 79a, and to the *harmonikoi* whom Aristoxenus treats as his own predecessors (e.g., 7 *El. Harm.* 2.30 with n. 9; see also his distinction between two equally repugnant approaches to harmonics, 32.18–31).

The objective that Plato ascribes to these theorists is that of identifying the interval that is the smallest detectable by ear, and by which other intervals can then be 'measured' (i.e., of which they can be treated as multiples). For this purpose they seem to have settled on something they conceived as a quarter-tone (an 'enharmonic diesis'): see 3.4, 3.8 Aristotle *Post. An.* 84b, *Metaph.* 1016b, and with an extra complication 3.9

'You are talking,' I said, 'about those worthy persons who bully the strings and interrogate them with torture, racking them on the *kollopes*. But I must not spin out the image too long by talking about the blows inflicted by the plectrum, about accusation and denial and the strings' false pretences, and so I shall abandon the image and say that I do not mean these people, but those whom we said just now we would question about *harmonia*.[4] They do the same as those concerned with astronomy: they seek the numbers in these heard concords, but do not rise to problems, to investigate which numbers are concordant and which are not, and why each are so.'[5]

'The task you mention is superhuman,' he said.

'But it is at any rate useful,' I said, 'in the quest for the fine and the good, whereas if pursued in any other way it is useless.'

2.2 *Republic* 616b–617d

Each group spent seven days in the meadow,[6] and on the eighth they had to

Metaph. 1053a. Cf. Aristoxenus' references to his predecessors' diagrams, 7 *El. Harm.* 2.15, 7.32, 28.1.

Pyknōmata are things that are dense or compressed. The related term *pyknon* (neuter form of the adjective *pyknos*) is used by Aristoxenus to designate the pair of small intervals at the bottom of an enharmonic or a chromatic tetrachord. Cf. also the usage at 8 Eucl. *Sect. Can.* 148.9–10. In the present passage the 'compressed' items are apparently the allegedly minimal intervals, or groups of them, and there may be a connection with the procedure called *katapyknōsis* which Aristoxenus attributes to the *harmonikoi*: see 7 *El. Harm.* 7.24 and n. 34.

It is not clear why Plato speaks of a 'sound in between' rather than of an interval. Perhaps the point is only that an interval cannot, as such, be heard. In that case, the investigator is claiming that he can produce a sound intermediate in pitch between the other two, and can distinguish it as such by ear. Alternatively the sound might be that of the 'beats' between adjacent pitches, whose presence is evidence that true unison has not been reached (this suggestion was made to me by Alexander Mourelatos). The idea is inherently plausible, but I know of no unambiguous reference to the phenomenon in the ancient sources.

[4] Socrates' ghoulish metaphor is based on the legal practice of interrogating slaves by torture. The *kollopes* are the devices for tightening the strings: hence the analogy of the rack. Socrates' reference to the 'false pretences' of the strings well exemplifies Plato's insistence on the untrustworthiness of sense-perception. It is clear that Socrates is not referring to the use of a monochord (the instrument presupposed in 8 Eucl. *Sect. Can.* propositions 19–20, described fully at 11 Ptol. *Harm.* Book 1 ch. 8): that has a single string, whereas Plato speaks of several, and its use did not involve adjustments of tension, but the division of the string by moveable bridges while the tension remained constant. In general, the monochord is associated with 'Pythagorean' treatment of intervals as ratios between magnitudes, whereas the *harmonikoi* mentioned by Aristoxenus (see n. 3 above) seem like him to have described them as quasi-linear distances between points of pitch. Probably the instruments involved in the 'experiments' described here were just the familiar *lyra* and *kithara*.

[5] The implication is that the Pythagoreans do look for a mathematical expression of concordant relations (by assigning ratios of numbers to them: see 1.1–1.8, and especially 1.12), but they proceed too empirically, merely looking for the ratios of numbers exemplified in perceived instances. They fail to explain what it is about certain *mathematical* relations that makes the numbers in those relations concordant. On these issues see the introduction to this chapter and the Appendix to chapter 1.

[6] This passage is from the myth in *Republic*, Book x, which recounts the experiences of Er the Armenian. Here the structure of the universe is described in a manner that draws on a conception of the harmony of the heavens (see 1.5 Aristotle *Metaph.* 985b with

stand up and go on; and on the fourth day after, they reached a place from
where they could look down from above on a straight beam of light, stretched
through the whole heaven and the earth like a pillar, very like the rainbow, but
brighter and purer. To this they came after going forward another day's
c journey, and from there, in the middle of the light, they saw stretched from
heaven the ends of heaven's chains. For this light is what binds heaven together,
holding its whole circumference in, like the undergirding of a trireme. And
from the ends is hung the spindle of Necessity, by which all the circumferences
are turned. The spindle's shaft and its hook are of adamant, but the whorl is
d a mixture of adamant and other substances. The nature of the whorl is like this.
In shape it is like those that we have here, but from what he [Er] said you must
think of it as if it were one large hollow whorl with the inside cut out all the
way through, in which another similar smaller one has been fitted, like the
boxes that are made to fit inside one another, and then a third and a fourth,
e and then four more. There are eight whorls in all, lying inside one another, with
their rims showing as circles on top, making the surface of one single whorl
around the shaft; and the shaft is driven right through the middle of the eighth.
The circle of the rim of the first and outermost whorl is the widest, that of the
sixth is second in width, that of the fourth is third, that of the eighth is fourth,
that of the seventh is fifth, that of the fifth is sixth, that of the third is seventh,
and that of the second is eighth.[7] The circle of the greatest is spangled, that of
617a the seventh is brightest, that of the eighth receives its colour from the seventh
shining upon it, those of the second and fifth are very similar to one another,
more reddish-yellow than those just mentioned, the third has the whitest
colour, the fourth is rather red, and the sixth is second in whiteness.[8] The
spindle as a whole is spun and revolves with the same motion throughout, but
within the revolving whole the seven internal circles turn round consistently in
the opposite direction to the whole. Of these the eighth travels the most swiftly,
b and the seventh, sixth and fifth are second in speed, each travelling at the same
speed as the others. Third in the speed of its back-circling motion, as it seemed
to them, is the fourth, while the third is fourth and the second fifth.[9] The

n. 15). Though the account is nearer to poetry than to science, its details are consistent with the observational astronomy of Plato's time, and with his closer analyses of planetary motions in the *Timaeus*. See Vlastos (1975), ch. 2.

[7] The widths of the rims determine the radial distances between the heavenly bodies; on their role in fixing the relative pitches of the sounds emitted see n. 9 below.

[8] These descriptions conceal a well constructed ordering of the circles of the heavenly bodies from a geocentric point of view. The order, moving from the smallest circle outwards, is Moon, Sun, then the five known planets (Venus, Mercury, Mars, Jupiter, Saturn), and finally the circle of the fixed stars. Most early Greek astronomers, like Plato, wrongly placed Venus before Mercury. Otherwise the order of radial distances is correct.

[9] Every circle revolves with the motion of the outermost. But the movement of the seven inner circles is modified by movements of their own, obliquely in the opposite direction (for the obliquity of the ecliptic and the zodiac see 2.3 *Timaeus* 36c–d), so that they travel (at various speeds, but slowly) 'backwards' in relation to the fixed stars, taking different periods of time to return to their original position against that background (see also the continuation of the *Timaeus* passage to 40d).

The 'backwards' angular velocities of the Sun, Venus and Mercury are equal. If Plato intended (as he did not explicitly state) that the circles' velocities determine the pitch of

spindle is spun upon the knees of Necessity. Up on top of each of the circles
rides a Siren, carried around with its revolution, each giving out a single sound,
a single pitch [*tonos*]: and from these sounds, eight in all, is made the concord
c of a single *harmonia*.[10] Round about at equal distances are seated three others,
each on a throne, the Fates, daughters of Necessity, clothed in white and with
garlands on their heads. They are Lachesis, Clotho and Atropos; and they sing
[*hymnein*] to the *harmonia* of the Sirens, Lachesis of what has been, Clotho of
what is, and Atropos of what will be.[11] With her right hand Clotho touches the
outer circumference of the spindle from time to time, and helps to turn it, while
Atropos does the same to the inner circumferences with her left: Lachesis
d touches inner and outer in turn, using first one hand, then the other.

2.3 *Timaeus* 34b–36d

c Now God did not devise the soul [of the universe] later [than its body], in the
way that we are setting off to speak of it later, for when he constructed them
he would not have allowed an older to be ruled by a younger; but since we
ourselves partake greatly in chance and randomness, we are speaking also in
this rather random way.[12] But he made the soul prior and older in both birth

their notes in the *harmonia* mentioned below, the relevant velocities could not be those of their apparent movements, either forwards or backwards, since three would then be the same (and in the latter case the fixed stars would have no note). But Mercury's circle is larger than that of Venus, and the Sun's is smaller still. Hence the actual speed of Mercury through space would have to be greater than that of Venus, and that of Venus greater than the Sun's. It is only on the assumption that these are the relevant velocities that greater speeds can be correlated with higher pitches in a graded series from the fixed stars down to the Moon. But this approach is in tension with Plato's mode of expression here and his attitude elsewhere: he seems to think of the 'proper' speeds of the planets as their backwards angular velocities (cf. 2.3 *Timaeus* 36d, *Laws* 822a–c). On the assumption that actual speeds determine pitch, the relative widths of the circles will regulate the pitch differences and make possible their assimilation to a scale. We would expect the relations to be those that appear in the construction of 2.3 *Timaeus* 35b–36b, but the mathematics needed to produce this result seems impenetrable.

[10] Fairly clearly the sense of *harmonia* here is 'octave scale'. Cf. particularly 1.12 Philolaus frag. 6 and the context in which part of it appears at 10 Nicomachus *Ench*. ch. 9; 7 Aristox. *El. Harm*. 36.32–4; 12 Arist. Quint. *De Mus*. 15.8–11. In assigning pitches to the Sirens' voices Plato does not explicitly link them with velocities, or with any quantitative feature of his system.

[11] Commentators have often remarked that a 'harmony' consisting of the eight notes of a scale, sounded together, would be better described as a cosmic cacophony. Plato makes a distinction. Though the scalar *harmonia* is indeed sounded, it is not itself the celestial music, but constitutes the permanent framework, the reservoir of elements and relations, on which that music is based. Melody itself is moving, dynamic; the melodies of the Fates are not eternally self-same, but are musical representations of events in time. The *harmonia*, by contrast, is eternal. It stands to the melodies rather as a preordained syntax, grammar and vocabulary might stand to the sentences of a language.

[12] This passage from Plato's account of the divine construction of the universe carried great authority in later Greek thought and was still a key text in the Renaissance. Its principal ancient devotees were the Neoplatonists and Neopythagoreans of the early Christian era: for examples in this volume see 9.3 Adrastus ap. Theon Smyrn. 64–5; 10 Nicomachus *Ench*. ch. 8; 12 Arist. Quint. *De Mus*. Book III chs. 1 and 24 (but Theon and Aristides were much more heavily influenced by the *Timaeus* than those passages by

and excellence, to be mistress and ruler of its subject, the body; and he put it
35a together from the following things and in the following way. From the Being
that is indivisible and always the same, and that which occurs divided among
bodies, he mingled together a third, intermediate form of Being; and he did this
also with the nature of the Same and that of the Different, putting together in
the same way something intermediate between the indivisible form of each and
that which is divided up among bodies. Taking the three things, he next
blended them all into a single form, fitting [*synarmottōn*] the nature of the
b Different, which is resistant to mixing, by force into the Same. When he had
mixed this with Being, and out of three had made one, he again divided the
whole into as many parts as was proper, each part being a mixture of the Same,
the Different, and Being.[13]

This is how he began to divide.[14] First he took away one part from the
whole, then another, double the size of the first, then a third, hemiolic with
respect to the second and triple the first, then a fourth, double the second, then
c a fifth, three times the third, then a sixth, eight times the first, then a seventh,
36a twenty-seven times the first.[15] Next he filled out the double and triple intervals,
once again cutting off parts from the material and placing them in the
intervening gaps, so that in each interval there were two means, the one
exceeding [one extreme] and exceeded [by the other extreme] by the same part
of the extremes themselves, the other exceeding [one extreme] and exceeded [by
the other] by an equal number.[16] From these links within the previous intervals
b there arose hemiolic, epitritic and epogdoic intervals;[17] and he filled up all the

themselves suggest). Plato attributes a soul to the universe primarily to account for the existence and orderliness of its movements: only soul can initiate motion (see especially Plato *Phaedrus* 245c–e, *Timaeus* 37b). Here he describes the structure that organises the self-moving but intangible soul-stuff which governs the universe's movements.

13 A full explanation of the metaphysics behind this paragraph cannot be attempted here. Very roughly, soul must be involved both in the world of unified and eternal Forms ('Being') and in that of plurality and change ('Becoming', here 'that which is divided up among bodies'). It must also partake of the natures of both uniformity (the 'Same') and multiplicity or variety (the 'Different'). These receive a special application below at 36c–d, where the uniform rotary motion of the fixed stars has the character of the Same, while the planets with their converse motions also share in that of the Different (see also n. 9 above).

14 The soul-stuff on which the divine Craftsman works is conceived as a long, undifferentiated strip. His task is to mark off sections of it according to mathematically fundamental, 'harmonious' proportions. (Thus he, at any rate, must have the answers to the questions raised at 2.1 *Republic* 531c.)

15 The word 'hemiolic', literally 'half-and-whole', means 'in the ratio 3:2'. So far the divisions are to be understood as combining two sequences, 1, 2, 4, 8 and 1, 3, 9, 27 (that is, 1, 1×2, 2^2, 2^3 and 1, 1×3, 3^2, 3^3). The members of each sequence form a geometric progression (see the next note).

16 The first kind of mean mentioned is 'harmonic' (as, for example, 8 stands to 6 and 12). The second is arithmetic (as, for example, 9 stands to 6 and 12). Taken with the geometric progressions of the original sequences, they exemplify the three kinds of proportion identified as relevant to music by Archytas in 1.20 frag. 2.

17 A hemiolic interval, in the ratio 3:2, is the musical fifth; an epitritic interval, 4:3, is the musical fourth; an epogdoic interval, 9:8, is the musical tone (the residue of a fifth when a fourth is subtracted).

The sequences are most conveniently represented if we multiply all the terms by 6.

epitritics with the epogdoic kind of interval, leaving a part of each of them, where the interval of the remaining part had as its boundaries, number to number, 256 to 243.[18] And in this way he had now used up all the mixture from which he cut these portions.

Next, then, he divided this whole compound along its length into two, and
c put them together at their centres like an X: then he bent each round to meet itself in a circle, fixing them to themselves and to each other at the point opposite to their first junction, and he made them revolve with a circular motion in the same direction on the same axis, making one of them the outer circle and the other the inner. To the outer movement he gave the name of the nature of the Same, and to the inner that of the Different.[19] And he made the

Then the insertion of harmonic and arithmetic means between terms in the series of doubles, 6, 12, 24, 48, gives

6, 8, 9, *12*, *16*, 18, 24, *32*, 36, *48*

and their insertion into the series of triples, 6, 18, 54, 162, gives

6, 9, 12, *18*, 27, 36, *54*, 81, 108, *162*.

Combining these sequences (many terms are of course duplicated) we get

6, 8, 9, *12*, 16, *18*, 24, 27, 32, 36, *48*, *54*, 81, 108, *162*.

The ratios between adjacent terms are easily calculated. In musical language the sequence of intervals is as follows: fourth, tone, fourth; fourth, tone, fourth; tone, tone plus *leimma*, tone, fourth; tone, fifth, fourth, fifth. The *leimma*, whose ratio is 256:243, is the residue of the fourth when two 9:8 tones have been subtracted (see 36b below). We thus have three octaves, each divided into two fourths separated by a tone (one of the fourths in the third octave is subdivided); beyond them lies a further octave plus a sixth, differently and oddly divided. Perhaps its first tone is to be construed as a disjunction.

[18] The expression 'number to number' draws our attention to the fact that this interval, the *leimma*, is not of multiple or epimoric ratio. It is therefore by some standards anomalous (see Appendix to chapter 1, and 1.21 Ptol. *Harm.* 30.9ff., with nn. 63–5).

Plato means that each interval of a fourth is now divided into two tones and a *leimma*, forming a tetrachord. Two such tetrachords with the *leimma* at the bottom of each, separated by a tone of disjunction, form a diatonic octave of the 'Dorian' species, in what was later called 'Pythagorean' intonation (cf. 1.12 Philolaus frag. 6). The Dorian species, with the small interval at the bottom of the tetrachord, corresponds roughly to our white-note scale from E to E. It is treated as fundamental and as the prime focus of analysis by all the theorists in this collection. (For the name 'Dorian', and those of the other species, see, for example, 12 Arist. Quint. *De Mus.* 15.10ff.) 'Pythagorean' intonation, in which the two tones of the diatonic tetrachord are each equal to the tone of disjunction (the interval by which a fifth exceeds a fourth) allows the whole diatonic series to be generated from a given note by upwards and downwards movements through fourths and fifths, the 'method of concordance' (see 7 Aristox. *El. Harm.* 55.3ff., 8 Eucl. *Sect. Can.* proposition 17: cf. the Appendix to chapter 1).

Assuming that Plato has a Dorian sequence in mind, each fourth should be divided into *leimma*, tone and tone, in that order from the bottom. This poses no problems for the first two octaves. In the third octave a difficulty arises, since we already have a tone in first place. Beyond the third octave a variety of divisions are consistent with what Plato says; none can be fitted to any scalar analysis that we meet elsewhere. Comparable problems will arise if we read the scale in the reverse direction. In any case, no Greek theorist seriously considers a musical scale extending beyond two octaves. Plato's motives are mathematical and metaphysical: he would have found it no surprise and no objection that his construction is only partially realised in the structures of human musical practice. See 9.3 Adrastus ap. Theon Smyrn. 64–5 and cf. 2.1 *Republic* 531c with n. 5.

[19] These two intersecting circles correspond respectively to that of the fixed stars and that on which the Sun, Moon and planets have their special movements (it is subdivided later to account for their differences). See 2.2 *Republic* 616d–617b with notes, and n. 13 above.

movement of the Same turn round sideways to the right, and that of the
d Different diagonally to the left.[20] He gave predominance to the revolution of
the Same and Like, for he allowed it to be single and undivided; but he divided
the inner revolution six times, making seven unequal circles, each based on the
interval of the double and the triple, so that there were three intervals of each
kind.[21] He commanded the circles to move in directions opposed to one
another,[22] three of them at the same speed, the other four travelling at speeds
different from those of each other and of the other three, but rationally related
to them.[23]

2.4 *Timaeus* 67a–c

When considering the third of our parts that is capable of sensation, that to do
b with hearing, we must say what are the causes through which its experiences
come about. Let us take it that sound in general is an impact of air,[24] coming
through the ears and impinging on the brain and the blood, and passed on until
it reaches the soul; and that hearing is a movement caused by it, beginning in
the head and ending in the region of the liver.[25] The kind that is swift is high-
pitched, and the kind that is slower is lower-pitched.[26] The kind that is
c homogeneous is even and smooth, while the opposite sort is rough.[27] A large
amount is loud, the opposite kind quiet.[28] Matters to do with concords of
sounds must be spoken of in the course of what I shall say later.[29]

[20] To an observer in the northern hemisphere looking south, the fixed stars, revolving westwards, move 'sideways' to the right. The 'backwards' movement of the other bodies (cf. n. 9 above) is to the left, but along a line that cuts the celestial equator obliquely (hence 'diagonally'). The line is the ecliptic: the planets follow paths along or near it, within the band of the zodiac.

[21] The revolution of the Same predominates, in that it is universal, but for all bodies except the fixed stars it is modified by their other movements (cf. n. 9 above). The 'intervals of the double and the triple' seem to characterise the relative sizes of the seven circles. The numbers intended are probably those of the original series, 1, 2, 3, 4, 8, 9, 27, which contain three 'double intervals' (2:1, 4:2, 8:4) and three 'triple intervals' (3:1, 9:3, 27:9).

[22] This cannot mean that the circles revolve in opposite directions to one another; they do not. The probable sense is that each circle is simultaneously subject to two opposed directions of revolution.

[23] Those whose speeds are equal are the Sun, Mercury and Venus (on this point, and for the identity and order of the others, see notes 8 and 9 above). That their speeds are 'rationally' related probably means not merely that they stand in ratios of whole numbers, but that these ratios are of a mathematically and musically intelligible kind, forming a harmoniously ordered series. It is fair to assume that the three kinds of proportion will again be at the heart of the matter.

[24] In his general theory of sound production Plato follows Archytas: see 1.19 Archytas frag. 1 and the references in n. 45 on that passage.

[25] On this part of the theory see n. 31 below.

[26] Again following Archytas as cited above, and see n. 47 on that passage.

[27] Detailed examinations of the various qualities of sound were later carried out by the Peripatetics. See especially 5 *De Audib.*, roughness is discussed at 803b. Cf. 3.17 Aristotle *De Gen. An.* 788a, 4.3 ps.-Ar. *Probs* XI.11.

[28] Archytas offered no way of distinguishing the determinants of loudness from those of high pitch. Plato's distinction hints at a form of the doctrine worked out by Aristotle: see 3.17 *De Gen. An.* 787a, 4.1 ps.-Ar. *Probs* XI.3; and cf. 4.23 *Probs* XIX.37.

[29] The reference is to the next passage quoted.

2.5 *Timaeus* 79e–80b

80a Further, the explanation for what happens in medical cupping-glasses, in
swallowing liquid, and in the case of things that are thrown (those that travel
through the air after being projected and those that travel along the ground) is
to be sought in the same way;[30] and so too in the case of sounds, those that are
swift and those that are slow, perceived as high and low in pitch, which
sometimes travel in disharmony [*anharmostoi*] because of the dissimilarity of
the motion set up by them in us, sometimes in concord [*symphōnoi*] because
of its similarity. For the slower sounds catch up with the movements of the
earlier and swifter ones as the latter are dying away and have already attained
similarity with the movements that the slower sounds impart to them as they
subsequently impinge on them; and when they catch them up they do not
disturb their motion by imparting a different one, but impart the beginning of
b a slower motion in conformity with that of the swifter sound, when the latter
is fading.[31] By attaching a similarity, they blend together a single experience out

[30] Plato's objective has been to explain various forms of motion, particularly that involved in breathing (79b–e), without invoking the existence of empty space into which something might move. He has done so by a theory of circular replacement: air leaving the body pushes the adjacent air around, so that eventually the circle is completed by other air entering the body where the first left. 'This goes on like the rotation of a wheel, since there can be no such thing as a vacuum' (79c). The theory has its origins in the writings of Empedocles.

[31] The simplest explanation of this account of concordance seems to be as follows. (i) Pitch of sound depends on the velocity of a movement of air from an initial impact: see 1.19 Archytas frag. 1. (ii) The hearing of a sound consists in another movement, initiated by the first, passing from the brain and blood in the head to the region of the liver (2.4 *Timaeus* 67a). (iii) When two sounds are produced simultaneously, the higher-pitched, being swifter, will reach the ear and initiate hearing first. (iv) The motion that constitutes our hearing of it will 'fade', become slower, during the time it persists in us. Then (v) the motion of hearing set up in us by the lower and slower sound may be similar, at the time it begins, to the now diminished velocity of the higher. When this is the case, the motion of the first is not 'disturbed', and the sounds are heard as concordant; otherwise they are heard as discordant.

The main difficulty is in the interpretation of 'similar'. If it means 'identical', we are left wondering why what is heard is not a unison. If it means something like 'proportional' (in a concordant ratio), then we have to explain why the first sound's motion must 'fade' before such a relation can be reached. In his notes on the passage, Cornford opts for the second, but is forced into an excessively elaborate explanation. I prefer the first. The heard pitch of a sound cannot be supposed to depend directly on the velocity of the inner movement, since that 'fades', and the pitch of a single note would be heard as a falling glissando. It must be a variable function of that velocity, directly dependent only on the speed of movement at the instant of the initiating impact on the ear. Then a second movement, identical in velocity at its first impact with the now faded movements of the first, will not be heard as a unison with it, but since it will not 'disturb' the existing movement of the first, it may reasonably be conceived as concordant with it. Sounds will be heard as forming a unison only if they are at the same velocity when initiating the movement of hearing. In a concord the pitches are perceived as different because of differences (in various ratios) between the velocities of the movements they generate in us at the time these movements begin, but as concordant because of the identity between the one initial velocity and that to which the other movement has now faded.

With this explanation of concordance contrast that implied by the report on Pythagoreans at 1.8 Porph. *Comm.* 107.15ff., but note the repeated terminology of 'similarity' and 'dissimilarity' (cf. also 'difference' at 4.24 ps.-Ar. *Probs* XIX.39).

of high-pitched and low-pitched movement.[32] Hence they provide pleasure to people of poor understanding, and delight to those of good understanding, because of the imitation of the divine *harmonia* that comes into being in mortal movements.[33]

2.6 *Philebus* 17a–e

Protarchus I think I more or less understand some of what you say, Socrates, but I still need to have some of it put more clearly.[34]

Socrates Well, Protarchus, the letters give a clear exemplification of what I am
b saying: since you have been taught them, try grasping it in them.[35]

Prot. In what way?

Soc. We can agree that vocal sound, that of people in general and that of each individual, is both a single thing that comes through the mouth, and again indefinite in number.[36]

Prot. Certainly.

Soc. And we are by no means yet experts by virtue of either of these things, because we grasp either the indefinite aspect of it or the unitary aspect. What makes each of us literate is knowing the number of the sounds, and their qualities.[37]

Prot. Perfectly true.

[32] On the blending of notes that form a concord see 8 Eucl. *Sect. Can.* 149.18–20 with n. 7.

[33] This is a characteristic Platonic contrast between the mere sensation of pleasure and the 'delight' that comes from intellectual understanding of its source in a correspondence with the divine.

[34] The context of this passage is philosophically complex. For helpful notes see Gosling (1975). Socrates has been describing, in terms that echo Pythagorean locutions, some aspects of the task of technical or theoretical understanding in any field. Every kind of subject matter is both a 'one' and a 'many', and comprises both a 'determinant' and 'indeterminacy' (cf. 1.9, 1.10 Philolaus frags. 1 and 2). That is, each field of knowledge deals with one kind, which is determinate, definable, but its instantiations, though all of this kind, differ from one another in indeterminately many ways. Science cannot pin down, enumerate and exhaustively classify all their possible variations. But between the one and the indeterminate many is a definite number of intermediate and intelligible forms. Understanding is not complete when it has grasped just the nature of the one kind, but must also enumerate the determinate forms into which it is articulated. Only then can it 'dismiss them into the indeterminate' (16c–e). It is not surprising that Protarchus is puzzled: the examples that follow are intended to clarify the doctrine.

[35] The 'letters', here as often, are the classifiable elements of articulate vocal sound, not those of writing.

[36] 'Of indefinitely many varieties', rather than merely 'having indefinitely many instances'. There is no determinate number of pronounceable sound-types, but a continuum containing indefinitely many subtly different possibilities.

[37] The sense is clarified at 18b ff. Though there is an indeterminate number of sound-types, they fall into a definite number of classes, corresponding to the letters of the alphabet (or of an ideal alphabet). Then, though types of pronunciation may vary indefinitely, some kinds of sound are members of the group corresponding to the letter A, others fall under the letter E, and so on. The expert knows how to classify sounds at this level, and how the classes fit together to form the one class 'vocal sound'. Below this level there are no real classes, only indeterminacy. (On the connection between recognising the several determinate sub-kinds and recognising the unity of the whole class, see 18c–d.)

Soc. And again, what makes a man *mousikos*[38] is the very same thing.

Prot. How is that?

c *Soc.* Sound, as considered in this art too, is somehow one.[39]

Prot. Of course.

Soc. But we posit two things, low pitch and high, and a third, sameness of pitch.

Prot. Yes.

Soc. But you would not yet be an expert in music by knowing only those things, though if you did not know them you would be virtually worthless in these matters.[40]

Prot. So I would.

d *Soc.* But suppose, my good friend, that you grasp the number and the qualities of the intervals related to high and low pitch of sound, and the boundaries of the intervals, and the number of *systēmata* that have arisen out of them.[41] These *systēmata* were identified by people in the past, and they handed down to us, their successors, the practice of calling them *harmoniai*;[42] and in the movements of the body they identified other,

[38] The word is multiply ambiguous. Depending on our overall interpretation of the passage, we might translate 'a musician', or 'an expert musicologist', or 'a sound judge of music', or just 'musically cultivated'.

[39] That is, there is a single aspect of sound, a single 'kind', which is the subject of musical expertise, just as there is another which is the field of the student of phonetics.

[40] Sound, considered as musical, is conceived in its character as exhibiting differences of pitch, and this gives an indication of the 'kind' mentioned above. But (a) the mere fact that two sounds differ in pitch is not enough to make their relation a musical one; and (b) there are other determinate classes of relation, within the general kind 'musical relation', which one must understand in order to be an expert.

[41] The four tasks identified here might be variously construed. Though the language of the passage has a Pythagorean ring, and though there are obvious Pythagorean features in Plato's approach to technical harmonics elsewhere, the present descriptions are too vague for us to be sure what he had in mind. On the whole, I think, the evidence is against a Pythagorean interpretation. Thus (a) we must grasp the number of intervals, presumably the number of distinct kinds of interval that can be used in music. How many there are will depend on how they are classified, for example by their ratios, or by their 'sizes' (tones, fourths and the like), or by the functional roles they have in a musical system (see 7 Aristox. *El. Harm.* 39.4–40.24). (b) We must grasp their qualities: Plato might have been expected to say 'ratios', but he does not. Possibly he has in mind a classification of intervals by the 'character' that each has and contributes to the music in which it occurs: see 12 Arist. Quint. *De Mus.* Book II chs. 13–14, especially 80.23ff. (c) We must identify the boundaries of the intervals. These are of course notes, but they may be identified either as points of pitch on a continuum (in the manner suggested by 7 Aristox. *El. Harm.* 8.13ff., 15.15ff., and certainly adopted by his 'harmonicist' predecessors), or as magnitudes standing in certain ratios (the Pythagoreans), or again by reference to the named notes of the system (which may not be conceived as fixed in their quantitative relations to one another: see, for example, 7 Aristox. *El. Harm.* 40.11ff.). (d) We must identify the *systēmata*, the legitimate combinations of notes and intervals that form the scalar frameworks for melodies. Plato's language here and in the next sentence indicates that their 'legitimation' is historical rather than mathematical.

[42] Different *harmoniai* are referred to under such names as 'Dorian', 'Phrygian', etc. by many writers: a notable example is *Republic* 398c–399c. It is likely that they were by then identified with *systēmata* differing in intervallic structure: see *GMW*, vol. 1, ch. 10, Appendix A, and cf. 12 Arist. Quint. *De Mus.* 18.5ff. On the association of the word *harmonia* with octave systems in particular see 10 Nicomachus *Ench.* ch. 9; with octave

similar inherent features which, they say, we must measure by numbers, and call rhythms and measures [*metra*],[43] while being aware that this is
e how we should investigate every one and many. For when you grasp them in this way, then it is that you have become an expert; and when you have grasped any other one by investigating it in this way, you have by so doing understood it. But the indefinite plurality inherent in any kind of thing makes you, in each case, indefinite in your understanding, not numbered among persons of repute, since you have never turned your attention to number in anything.[44]

systems in a special pattern of arrangements see 7 Aristox. *El. Harm.* 36.30–2, 12 Arist. Quint. *De Mus.* 15.9–20. Damon, whose work lies behind *Republic* 398c ff., is said at 12 Arist. Quint. *De Mus.* 80.29–30 to have 'handed down' certain *harmoniai*. It is possible that the kind of analysis Plato is thinking of here is Damonian (largely qualitative) rather than mathematical. (On Damon see *GMW*, vol. 1, pp. 168–9.)

[43] Compare *Republic* 399e–400c, which alludes to Damon's analysis of rhythms. In this part of the *Republic*, no quantitative conceptions are introduced into the account of *harmoniai* (398c ff.), but rhythms are discussed, as one would expect, in terms of long and short syllables, equality, and so on (400b–c). Similarly, here the idea of measurement 'by numbers' appears only in connection with rhythm and metre, not in the fields of intervals and *systēmata* (the references to number in the first sentence of this paragraph relate only to an enumeration of kinds, not to a numerical analysis of each kind). For an extended discussion of rhythmics see 12 Arist. Quint. *De Mus.* Book 1 chs. 13ff.

[44] The plays on the word 'number' here may suggest that the analyses intended above are after all mathematical: cf. 16d–e. But once again it is the enumeration of kinds and sub-kinds that is indicated, not an analysis of each in terms of number. There is no hint in Plato's parallel discussion of phonetics (17a–b, 18b–d) that the elements of articulate sound, or their interrelations, are to be described in quantitative terms.

CHAPTER 3

Aristotle

The philosophical and scientific researches of Aristotle (384–322 B.C.) are of astonishing scope. His work as a scientist touched on almost every aspect of the natural world, though his most important studies were in biology. As a philosopher he made minutely detailed analyses of our ways of thinking and speaking. He made important contributions to most of the sciences that existed in his time, but though he reflected on their methods and conceptual resources, he produced no original work, so far as we know, in either mathematics or harmonics. His significance in the history of Greek musicology lies elsewhere, and rests on four principal grounds.

First, there are his studies of the role of music in a civilised community. These appear mainly in the *Politics* (excerpts will be found in *GMW* vol. 1, ch. 11). Secondly, his works contain analyses of various terms used in the description of music, and some brief references to different forms of harmonic science and the concepts they employ, embedded as examples in investigations of much wider scope (see 3.1–3.10). More substantially, among his scientific and conceptual researches in biology and psychology are discussions of the physical nature, the production and the perception of sound (see 3.11–3.17). Here he analyses the processes involved in a sound's creation and transmission, and the physical basis of perceived differences between sounds, with special attention to differences of pitch. He explains how sounds and their properties are perceived, and raises knotty problems about the perception of concords. He discusses the mechanism of the voice in humans and other animals, again giving close scrutiny to the ways in which sounds of different pitches are produced. These studies build on foundations provided by Archytas (1.19) and Plato (2.4–2.5), bringing to them a sophisticated eye for unresolved difficulties and a true scientist's grasp on his biological and physical subject matter. They were further elaborated by his successors in the Lyceum (see particularly chapters 4–5).

Aristotle's fourth contribution is the most important, but it is not directly represented in the passages quoted here. It lies in his theoretical delineation of what a 'science' is, what constitutes scientific knowledge, and by what methods it can be attained by the human mind. The most systematic and influential of his writings on these topics is the *Posterior Analytics*, a work of great complexity to which a brief summary will not do any kind of justice. But it is essential to pick out a few central ideas.

In Aristotle's view the prime task of science is to explain. We have scientific understanding of some fact if we know not only that it is true, but why. Such understanding can be achieved only with respect to facts of a special sort; not just any 'matter of fact' can be scientifically explained. The ones that can are those that hold of kinds of things rather than merely of individuals (dogs in general, or the dog as such, not Fido), and hold of them universally, because they must. The scientist's aim is to reveal the grounds of this 'must', to show that it is because of what a thing of the relevant kind essentially is (what it must be, to be a thing of that kind) that it necessarily displays the feature in question.

The issues that the scientist raises, then, are ones like 'Why do cats have claws?'; 'Why do sounds diminish in volume as the distance from the source increases?'; 'Why do triangles have internal angles summing to two right angles?'. The answers will begin

from an account (a definition) of what it is to be a cat, a sound, a triangle. They will then demonstrate that anything so defined must, logically must, have the feature under investigation. These demonstrations (*apodeixeis*) are deductive, taking the form of syllogisms. If their premises are already soundly established, they are therefore proofs, though proofs of things already known at a pre-scientific level, and they count also as explanations so long as it is really because the subject is of the sort defined, and not because it falls under some other description, that it possesses the property attributed to it in the conclusion. Thus, to use an example of Aristotle's, one can prove deductively that all equilateral triangles have internal angles summing to two right angles. But it is not because they are equilateral triangles that they do so: it is because they are triangles. Hence though such an argument may be a proof, it is not an explanatory demonstration.

It is partly as a consequence of this last point that Aristotle insists on something I shall call the 'same domain' rule. One cannot 'demonstrate' or 'scientifically explain' features of something falling into one domain or under one genus by reference to the essences of things that fall under another (the thesis is briefly stated in 3.6). Thus, for instance, one cannot explain why twice the area of a square of side n is less than the area of a square of side 2n, just by demonstrating arithmetically that $2(n^2)$ is less than $(2n)^2$, even though these propositions are obviously linked. An explanation of a feature belonging to squares, as such, must be derived from a definition of what it is to be a square, not from propositions about numbers. Squares are geometrical figures, and numbers are not. They are items belonging to a distinct domain, things of a different sort or genus.

Aristotle recognises certain exceptions to this rule, in cases where one subject is 'subordinate' to another in a special sense (3.6, 3.7). In these cases the fact to be explained is stated as a fact about a subject conceived in a certain way, the way in which it is grasped through perception. It is explained by means of a demonstration that begins from a different analysis or definition of the subject, though it is the same subject, and this redefinition will typically also redefine the property to be explained. In Aristotle's view, 'empirical harmonics' falls under 'mathematical harmonics' in this sense. Thus it is a fact given to perception that when two notes are heard as standing an octave apart and are played simultaneously, their sounds seem to 'blend' in a characteristic way. Why is this? Aristotle holds that the 'Pythagorean' approach is broadly correct. It is because such blending is, from another point of view, the relatedness of two quantities that stand in a special sort of numerical ratio; and if the two notes are conceived as physical movements, rather than as auditory phenomena defined by their perceived qualities, they are indeed characterised by quantities standing in that sort of ratio (cf. 3.5, 3.11–3.14).

The harmonic theorist most indebted to Aristotle was Aristoxenus, who saw it as his mission to convert harmonics from a loosely connected jumble of empirical observations into a unified and genuinely explanatory Aristotelian science. The structure of explanations in this science was to be 'demonstrative' in precisely Aristotle's sense. Aristoxenus lays special stress on Aristotle's 'same domain' rule, and is emphatic in his refusal to follow Aristotle in treating harmonics as an exception to it. His reasons will emerge more fully in chapter 7, but they begin from a point that can usefully be sketched here.

We have seen that demonstrations start from definitions. Other forms of proposition are also involved at this initial level, and we can conveniently call them all 'principles', corresponding to the Greek term *archai*. Where do these principles come from? They are not directly given to perception, and yet perception, on Aristotle's view, is our only starting point for the acquisition of knowledge about the world. His answer is stated most concisely in *Posterior Analytics* II.19 and in *Metaphysics* I.1 (cf. also *De Anima*

II.5 and 12, III.4). Its details are problematic, but the gist is this. First, we perceive individual items, but we do not perceive them just as individuals. We perceive something as white, or as high-pitched, or as a hedgehog, and in these perceptions there is already implicit a kind of generality, a mode of classification. Such perceptions are stored in the memory. As similar cases are accumulated, so the similar individual internal representations come to coalesce into a representation of a thing of a certain sort, a kind, and are no longer just an aggregate of distinct items. At the highest level, through reflection on what it is that constitutes something as a thing of a given kind, we come to grasp what the essence of that kind is, and eventually to be able to define it in words. It is this sort of definition that will stand as a 'principle' in the project of demonstration.

What is crucial here is the relation of the principles to their origins in perception. They are abstractions from what we perceive. Now for Aristoxenus the subject matter of harmonics is musical sound as it presents itself to the hearing, and the task of science is to explain why certain kinds of sequences of heard sounds are melodically legitimate while others are not. In order to do this it must first show what it is for something to be 'melodically legitimate', what constitutes the essence or nature of 'the attuned' (*to hērmosmenon*). But this attunement is something that can exist only in sequences of sounds as grasped by the hearing, and the principles to be reached must therefore describe an essence belonging to a class of things heard, as such. It is quite wrong to look for explanations in descriptions of sounds that represent them from some other point of view, as physical movements of certain velocities, for example; for it is not in that character that they have the properties with which harmonics is concerned. A scientist who takes that route is in breach of the 'same domain' rule, and anything he says is irrelevant to harmonics. (On these issues see also notes 13 and 15 below.)

In adopting this approach Aristoxenus speaks as a purer Aristotelian than Aristotle himself. Where Aristoxenus insists on a species of phenomenalism (explaining the melodic phenomena by showing what is involved in being a phenomenon of that sort, and by coordinating them as exemplifications of a definable essence or nature), Aristotle seems content to look for a mathematical analysis of underlying physical causes, explaining the phenomena as resultants from events of a different order. No doubt both projects are legitimate in their separate ways, but it is clear that Aristoxenus' is much the more fruitful from a musical perspective. It was in general a feature of the 'Pythagorean' style of research, to which the 'mathematical harmonics' mentioned by Aristotle belongs, that until placed on a new footing much later by Ptolemy (chapter 11), it had rather little to say about specifically musical relations, much more about the mathematics of relative velocities or other such variables. When Aristotle wrote the *Posterior Analytics*, no such science as Aristoxenus envisages existed. On the one hand there was the observation and tabulation of empirical phenomena – descriptions of scale forms, the identification of concordant and discordant intervals, and so on. On the other there was the developing mathematics of ratios, and its application to quantitative variables envisaged as the causes of differently pitched sounds. Aristotle's remarks about harmonics seek merely to bring the two into an intelligible relation with one another. But it was his reflections on what a science should be, on its origins in human perception and on the abstract structure to which it should conform, that inspired Aristoxenus' invention of harmonics in an entirely new style.

The multitude of scholarly works on Aristotle cannot be reviewed here. There are useful general studies by, for example, Ross (1960), Lloyd (1968), and in the sixth volume of Guthrie's *History* (1981). On Aristotle's contributions to science see especially Solmsen (1960), and cf. Lloyd (1970); on the special issue of the 'same domain' rule see Lear (1982), Lennox (1984). There are relatively few accounts of Aristotle's influence on harmonic theory, but see chapter 4 of Lippman (1964), and

especially chapter 2 of Bélis (1986). For discussion of his theory of the physical basis of sound see Gottschalk (1968). Valuable recent bibliographies will be found in Guthrie (above), and in the four volumes of articles edited by Barnes, Schofield and Sorabji (1975–9). The latter scholars' bibliographies are revised and separately published in the Oxford Study Aids series (1980).

3.1 *Topics* 106a 9–32

The question whether a word is said in several ways,[1] referring to different kinds, or in one way only, must be investigated in the following manner. First, one must consider its opposite, and see if it is said in several ways, whether the discrepancy is in the kind referred to or in the name itself. For some opposites are plainly different even in their names: the opposite of the sharp in the sphere of sound is the heavy, but in solid bodies it is the blunt.[2] It is therefore plain that the opposite of the sharp is said in several ways. In that case, so too is the sharp, for each of these things [the heavy and the blunt] will have a different opposite: the sharp that is opposite to the blunt will not be the same as that which is opposite to the heavy, yet the sharp is opposite to each. Again, the opposite to the heavy in sound is the sharp, while in solid bodies it is the light. Hence the heavy is said in several ways, since its opposite is too...

[23] In some cases there is no discrepancy at all in the names, but the difference in kind is immediately apparent, as in the case of white and black. For sound is called white and black,[3] as also is colour. There is no discrepancy in the names, but the difference in kind is immediately apparent, for the colour and the sound are not called white in the same way. This is also clear if we consider sense-perception, for things that are the same in kind are objects of the same sense. We do not distinguish the white that applies to sound and that which applies to colour by the same sense, but one by sight, the other by hearing.

[1] That is, whether it is in a certain way ambiguous. Aristotle is offering suggestions as to how such ambiguities can be detected.

[2] *Oxys*, 'sharp', is standard Greek for 'high-pitched' as well as for 'pointed' or 'keen-edged'; its musical opposite, *barys*, 'low-pitched', means 'heavy' (not 'blunt') when applied to solid bodies. The metaphor of 'high' and 'low' in pitch is seldom found in Greek sources (but see 4.23 ps.-Ar. *Probs* XIX.37). The usual contrast is between 'sharp' and 'heavy', which are not even polar opposites. This suggests a way of conceiving (even of perceiving) pitch that is interestingly different from ours: cf. 3.15 *De Anima* 420a–b.

[3] In Greek as in English, most words describing quality of sound are metaphorical, having their primary application elsewhere. The physical basis of many such qualities, and their relation to things with qualities to which the relevant terms apply literally, are examined at length in 5 *De Audibilibus*. The words *leukos* and *melas*, 'white' and 'black', are often more aptly translated 'bright' (or 'clear') and 'dark', even as applied to colours. As applied to sounds their exact sense is uncertain and the usage is rare. The best clues we have are in 3.2 and 3.3 below: cf. 5 *De Audib.* 801b–802a on sounds that are 'bright' (*lampros*), where comparisons with colour are also made.

3.2 *Topics* 106b 4–9

Further, [the same thing follows from] a consideration of intermediates, if some things have an intermediate and others do not, or if both pairs have intermediates but not the same one.[4] For instance, the intermediate between white and black in colours is grey, but in sound there is none, or if there is, it is the dull, if we adopt what some people say, that a dull sound is intermediate.[5] Hence the white is homonymous, and so too is the black.[6]

3.3 *Topics* 107a 11–17

In the same way the white in body is a colour, but in sound it is the well-sounding.[7] Similarly with the sharp, for it is not said in the same way in all its applications. A sharp sound is a swift one,[8] according to what is said by the mathematical harmonicists,[9] while a sharp angle is one that is less than a right angle, and a sharp knife is one with a sharp edge.

3.4 *Posterior Analytics* 84b 37–9

As in other fields, a principle is something simple, but it is not the same thing in all areas: in weight it is the *mina*, in melody the diesis, and in other areas other things.[10]

3.5 *Posterior Analytics* 90a 14–23

In all these cases it is plain that the 'What is it?' and the 'Why is it?' are the same. What is an eclipse? – the deprivation of light from the moon by the interposition of the earth. Why is there an eclipse, or why is the moon eclipsed? – because the light fails when the earth is interposed. What is concord [*symphōnia*]? – a ratio of numbers between the high-pitched and the low-pitched. Why does the high-pitched form a concord with the low-pitched? –

[4] That is, the members of a pair of opposed terms are 'said in different ways' (ambiguous) if the condition described holds.

[5] *Somphos*, 'dull', lit. 'spongy', perhaps 'unresonant', is used of the voices of people with blocked noses, Hippocr. *De Morb.* 2.33, of an aulete producing a 'dead' sound, Dion. Halic. *De Comp. Verb.* 11. At 5 *De Audib.* 802a the author uses the term *phaios*, 'grey', of voices, in contrast to *lampros*, 'bright'. Aristotle seems to deny that this term can be applied to sounds.

[6] Things are homonymous if they have nothing in common except the word that refers to them.

[7] *To euēkŏon*, more literally 'well-heard', 'readily audible', hence 'clear'.

[8] See 1.19 Archytas frag. 1 with n. 47, and for the word 'sharp' see n. 2 above.

[9] Lit. 'the harmonicists (who pursue their science) on the basis of numbers'. For Aristotle's conception of mathematical harmonics see 3.5–3.7 below.

[10] 'Principle' translates *archē*: in the examples here it is equivalent to 'unit' or 'common measure' (cf. 3.8). For the diesis as the interval of which all others are to be computed as multiples see 3.8 and 3.9 with nn. 16 and 17.

because the high-pitched and the low-pitched stand in a ratio of numbers.[11] Does there exist a concord between the high-pitched and the low-pitched? – Is their ratio in numbers? Granted that it is, what then is the ratio?[12]

3.6 *Posterior Analytics* 75a 38–b17

It is not possible to give a demonstration by shifting from one class of things to another: for instance, one cannot demonstrate something in geometry by means of arithmetic[13]... [b12] Hence it is impossible to demonstrate by means of geometry that opposites are studied by a single science, or even that the product of two cubes is a cube: nor can any other science demonstrate something belonging to a different one, except where the subjects are so related that one is subordinated to the other, as optics is subordinated to geometry and harmonics to arithmetic.[14]

3.7 *Posterior Analytics* 78b 34–79a 6

There is another difference between the reason why and the fact that, in that each may be studied by a different science. This happens where the subjects are so related that one is subordinated to the other, as are optics to geometry, mechanics to solid geometry, harmonics to arithmetic and observed appearances to astronomy. Some of these sciences have almost the same names: thus there is mathematical astronomy and nautical astronomy, mathematical harmonics and harmonics based on hearing. In these cases it is the task of those

[11] Aristotle's point is that when we have an adequate grasp of what something is, we also know why it occurs on the occasions when it does. A proper understanding of what it is for something to be eclipsed incorporates an explanation, since eclipse is not just any deprivation of light, but one with a specific kind of cause. Similarly, when we know that concordance is a (special sort of) ratio between the numbers attaching to some variable (e.g., velocity) that characterises each of a pair of sounds, we know why two sounds are concordant when they are. See also the next two passages, and cf. 3.13. (For Aristotle's attribution of a 'velocity' theory of pitch to mathematical harmonicists see 3.3; for his own modification of the theory see 3.15 *De Anima* 420a–b and 3.17 *De Gen. An.* 786b–787a.)

[12] The last two questions give a way of breaking down the previous one, 'Does there exist...?'. Aristotle seems to mean that there is a concord if and only if (a) the relevant ratio is 'in numbers' (that is, its terms can be expressed as integers), and (b) the numbers form one of those ratios which are known to constitute concords. See the contrasts at 3.11 *De Sensu* 439b–440a between (a) numbers that are in a ratio and those that are not, and (b) those that are 'well-ratioed' and those that are not. But there Aristotle also uses the phrase 'not in numbers' to contrast with 'well-ratioed', and he suggests no clear answer to the question of what systematically distinguishes the ratios of the concords from other ratios between integers. Cf. 2.1 Plato *Rep.* 531c with n. 5.

[13] That a thing has property *p* is explained only if it is shown that it must be *p* in virtue of being, essentially, a thing of a certain (definable) kind. Different sciences are defined by their 'domains', that is, by the identity of the 'kind' that each studies. Then a science cannot 'demonstrate' (explain) *X*'s possession of a property if *X* is of a kind that belongs to the domain of a different science. (On the grounds and implications of this 'same domain' rule see the introduction to this chapter.)

[14] On these exceptions to the 'same domain' rule see the next passage.

who use perception to know the fact that, and that of the mathematical scientists to know the reason why: for the latter possess the demonstrations of the causes, and often do not know the fact that, just as people who study the universal often do not know some of the particular instances because they have not observed them.[15]

3.8 *Metaphysics* 1016b 18–24

To be one is to be the principle of being any number, since the first measure is a principle. For the first thing by which we acquire knowledge of each class is the first measure of that class. Thus the one is a principle of what can be known about each class. But the one is not the same thing in all classes: in one field it is the diesis,[16] in another the vowel or consonant: that of weight is different, and that of movement different again. But in every field the one is indivisible, either in quantity or in kind.

3.9 *Metaphysics* 1053a 12–17

In music the measure is the diesis, since it is the smallest, and in speech it is the

[15] In all these cases of exceptions to the rule, there is one science that describes a set of phenomena, another, of a mathematical sort, that explains what the first describes. In the case of harmonics, the former gives purely empirical descriptions of scale-systems etc., based on the judgement of the ear: the latter is conceived in a 'Pythagorean' mould, and will give explanations of why these sets of intervals, treated as ratios, fit together into a coherent system of proportions, of why some relations are concordant, and so on. For the contrast see especially 2.1 Plato *Rep.* 530c–531c, 7 Aristox. *El. Harm.* 32.18–33.1; compare 3.6, 3.9, and the quotations from Ptolemaïs and Didymus at 9.10–9.14. The sort of programme for mathematical harmonics that Aristotle has in mind is exemplified in 8 Eucl. *Sect. Can.*

Aristotle is not thinking of the application to a special subject matter of the propositions of an independent 'pure mathematics'. He means to identify the qualitative phenomena described with certain quantitative aspects of the subject. These quantities can then be thought of just in their character as quantities, without reference to the variables of which and the subject in which they are quantities, and their relations can be studied in isolation. The boundaries of the mathematical harmonicist's domain are determined by the scope of the phenomena falling into 'empirical' harmonics, and since the 'mathematician' conceives them in abstraction from their qualitative appearances, he may not always know, as Aristotle says, to what perceptible quality of sound a given mathematical description corresponds.

Though Aristoxenus follows Aristotle in his view of the nature and task of a science, he rejects the present thesis (see the introductions to this chapter and chapter 7). For him, the principal phenomena that a harmonic theorist must consider are not explicable through mathematics (7 *El. Harm.* 32.20ff., cf. 9.1ff., 12.4ff.). Many such phenomena are not essentially quantitative at all (e.g., 39.4–40.24, 48.15–50.14), and he will not allow harmonics to be an exception to the 'same domain' rule (cf. 44.15–18). Other writers go further, denying that even pitch itself is a quantitative characteristic: the best example is 6 Theophrastus ap. Porph. *Comm.* 61.22ff.

[16] Cf. 3.4 above. In both passages it is natural to understand 'diesis' in the sense 'quarter-tone', as usually in Aristoxenus, and to assume that Aristotle is thinking of the descriptive 'empirical' harmonics that represented intervals as distances on a line. The diesis is then the 'common measure' of which all other intervals are treated as multiples. Cf. 2.1 Plato *Rep.* 531a–b, 7 Aristox. *El. Harm.* 2.15, 7.32, 28.1. But something different is suggested in 3.9 below.

letter...But the measure is not always numerically one. Sometimes there are several, as for instance there are two dieses – not those given by the ear, but those found in ratios[17] – and several articulate sounds that we use for measuring [in phonetics].

3.10 *Metaphysics* 1093a 29–b4

Some people say that there are many such [numerical correspondences]: for instance the *mesai* are respectively 9 and 8, while the epic line has seventeen syllables, equal to the sum of those, and in its first part it moves with nine syllables and in the second with eight.[18] They point out also that the interval from alpha to omega in the alphabet is equal to that from the lowest note to the highest in *auloi*, whose number is equal to the whole system of the heavens.[19]

[17] Here Aristotle explicitly contrasts the dieses of 'empirical' harmonics (see previous note), which are allegedly equal quarter-tones, with those of ratio theory. In the latter, the tone (ratio 9:8) cannot be divided into equal halves or quarters (see 8 Eucl. *Sect. Can.* propositions 3, 16, 18), and the two small intervals at the bottom of, for example, an enharmonic tetrachord must be of different sizes. (Cf. also 1.12 Philolaus frag. 6 with n. 36.) But the sense in which these dieses constitute a 'measure' for other intervals within ratio theory must be different from that applicable to measurements of quasi-linear 'distance', and it is unclear how it could be applied. The tone, 9:8, can be resolved into unequal 'semitones' of, for example, 17:16 and 18:17, and each of these can also be unequally divided to form dieses (e.g., 33:32x34:33 and 35:34x36:35): see 12 Arist. Quint. *De Mus.* Book III ch. 1. The dieses might jointly be conceived as the 'measures' of the larger intervals, but there are four of them, not two, and others again will be needed to measure the residue of the fourth after two tones (the *leimma* of 256:243), or the various different intervals involved in other divisions of the tetrachord. (This remains true even if, as is possible, 'diesis' here means something more like 'semitone' than 'quarter-tone': see, for example, 9.2 Adrastus ap. Theon Smyrn. 55.) It seems likely that Aristotle was (a) familiar with the empiricists' use of the diesis as a measure, and (b) impressed by Pythagorean demonstrations that the dieses of, for example, an enharmonic tetrachord cannot be equal, and that he failed to notice that the ratio theory within which the latter proposition holds cannot accommodate a notion of 'measure' comparable to that of the empiricists.

[18] Aristotle is commenting on Pythagorean 'discoveries' of affinities between apparently disconnected items, in virtue of casual numerical correspondences (other examples abound in 12 Arist. Quint. *De Mus.* Book III: see especially chapter 6). The simplest way of representing the framework of concords within the octave in terms of whole numbers and their ratios is through the series 12, 9, 8, 6, where the extreme terms bound the octave (2:1), the pairs 12, 9 and 8, 6 bound fourths (4:3), the pairs 12, 8 and 9, 6 bound fifths (3:2), and 9:8 is a tone. The terms are usually taken to correspond to the notes *nētē diezeugmenōn*, *paramesē*, *mesē*, *hypatē mesōn*. Then the two middle notes (*mesai*) are 9 and 8, as Aristotle says. (The terminology of 'two *mesai*' may be Pythagorean, reflecting their claim that there can be no single mid-point in the octave. The thesis is an application of 8 *Sect. Can.* prop. 3.)

[19] The Greek alphabet has 24 letters. The Pythagorean system of the heavens contained ten bodies (see, for example, 1.5 Arist. *Metaph.* 986a 8–12), and the number 10 is formed from the *tetraktys* 1+2+3+4 (see 1.2 with n. 3). If these numbers are multiplied instead of being added, the result is 24. The reference to the notes of *auloi* is more problematic. It is sometimes taken as an allusion to the fact that there are 24 dieses (quarter-tones) in the two-octave system, but this involves a non-Pythagorean method of calculation, and fails to explain why *auloi* are specifically mentioned (cf. nn. 16–17 above). Aristoxenus listed five types of *aulos* classified by range (ap. Athenaeus *Deipn.*

3.11 *De Sensu* 439b 19–440a 4

We must now speak of the other colours, reviewing the number of ways in which it is possible for them to arise.[20] It is possible, first, that the white and the black are laid side by side in such a way that while each of them is invisible because of its smallness, the combination of the two becomes visible. This cannot appear either as white or as black, but since it must necessarily have some colour, and can have neither of those, it must be something mixed, a different kind of colour. In this way, then, it is possible to accept that there are more colours than just white and black, and that they are many in ratio: for they may lie side by side in the ratio of three to two or that of three to four or in other relations of numbers. (Some may be in no ratio whatever, but in some incommensurable relation of excess and deficiency.[21]) Thus, they may be in the same condition as concords [*symphōniai*]: the colours that depend on well-
440a ratioed numbers, like concords in their domain, are taken to be the pleasantest of colours (purple and red and a few others of that kind – few for the same reason that the concords are few), while those that are not in numbers are the other colours.[22]

3.12 *De Sensu* 447a 12–b 21

There is also this further problem about perception, whether it is possible to perceive two things simultaneously, in the same indivisible time, or not, if it is the case that the greater movement always drives out the smaller: that is why people do not perceive things presented to their eyes if they happen to be thinking hard, or frightened, or hearing a loud noise. Let this be assumed, and also that anything can be more readily perceived when it is simple than when it is mixed. Unmixed wine, for instance, is perceived more readily than mixed; so too are honey and colour; and *nētē* is perceived more readily alone than in

634e–f), and at 7 *El. Harm.* 20.32–4 he says that from the lowest note of the lowest to the highest of the highest is more than three octaves. A three-octave range in any one generic system of tuning will be covered in 22 notes, so that 24 notes will extend to a little over three octaves, which fits Aristoxenus' account. But the interpretation remains uncertain.

20 Aristotle is enquiring how the whole range of colours can be conceived to arise out of combinations of the two basic opposites, white and black (or light and dark).

21 This seems to mean what it says, that there may be relations that are not expressible as ratios of integers (like the relation between the side of a square and its diagonal). But see the next note.

22 Here the contrast is between a combination that is 'well-ratioed' (*eulogistos*) and one that is 'not in numbers'. The former cannot be those in just any ratio of integers, since such combinations are said to be few: *symphōniai* must mean 'concords' in its proper technical sense, and to be well-ratioed is to have one of the small number of ratios that correspond to the concords. But it is then odd to refer to the others as 'not in numbers', an expression that looks as if it means 'incommensurable', as in the parenthesis above. The puzzle reflects the pervasive difficulty of giving a clear mathematical characterisation of the 'concordant' ratios which distinguishes them satisfactorily from all others. Cf. n. 12 above, and see also 4.25 ps.-Ar. *Probs* XIX.41.

the octave, since the two elements obscure one another.[23] This effect is produced by things out of which a single whole comes into being.[24] If then the greater movement drives out the smaller, then if they occur simultaneously, even the greater must necessarily be less perceptible than if it occurred alone. For the smaller, in being mixed with it, has subtracted something, if indeed all simple things are more perceptible. If, then, the movements are equal while being different, there will be no perception of either, since each will obscure the other in the same way. It is then impossible to perceive a simple movement. Hence there will either be no perception, or a different perception generated from both together. And this is what does seem to be generated out of things that have been combined, in cases where a mixture is produced.[25] Out of some things, then, something is generated, while out of others it is not, the latter
447b being those falling under different senses. (For mixtures are formed from things whose extremes are opposites: it is impossible for a single whole to be produced from white and high-pitched, except incidentally, not in the way that concord is produced out of high-pitched and low-pitched.) Hence it is impossible to perceive the latter type simultaneously. For when the movements are equal they will obscure one another, since no single movement is generated out of them, while if they are unequal, a perception will be implanted by the greater.[26] Again, the soul must be more capable of perceiving two things simultaneously in a single perception if they are objects of a single sense, as are high and low pitch: for the movement of this single sense is more simultaneous than is that of two, such as sight and hearing.

It is impossible, however, to perceive two things simultaneously with a single sense if they are not mixed: for a mixture purports to be one thing; of one thing there is one perception, and one perception is simultaneous with itself. Thus, things that are mixed must necessarily be perceived simultaneously, since they are perceived with a perception that is one in its actuality: for the perception of what is numerically one is one in actuality, while that of what is one in kind is one potentially. Now if the actual perception is one, it will say that its objects are one, and hence they must have been mixed. Then when they are not mixed, the actual perceptions will be two. But the actual activity of a single faculty in

[23] Ps.-Ar. *Probs.* XIX.14 cites cases where we are liable to be deceived into thinking that two notes are in unison when in fact they are in octaves. Many of the questions in that book concern the phenomenon of 'correspondence' in the octave: see 4.17, 4.19–4.21, 4.24–4.26, 4.28 (*Probs.* XIX.13, 23, 24, 35, 39, 41, 42, 50); others in *Probs* XIX include 7, 13, 16–19, 34.

[24] That is, where two things impinging simultaneously on the sense are perceived as a single whole, not as distinct items. This corresponds to the commonest characterisation of concordance: see 8 Eucl. *Sect. Can.* 149.18–20 with note, and cf. 3.16 below.

[25] The perception of a true mixture is different from the perception of both of the two things that are mixed. The difference is often drawn on in attempts to distinguish concord from discord (see previous note).

[26] Similar considerations are at work (to a different purpose) in 6 Theophrastus ap. Porph. *Comm.* 63.15–28. Aristotle's point is that if the movements are unequal in strength, only the stronger will be perceived; if they are equal, neither will be perceived in its own right, but there may be the perception of a 'mixture'.

an indivisible time must be one, for the movement and exercise of a single faculty at a single moment must be one, and here there is just one faculty. Hence it is impossible to perceive two things simultaneously with a single sense.[27]

3.13 *De Sensu* 448a 9–13

Nor can things that are mixed be perceived simultaneously (for they are ratios of things in opposition, as is the octave, for instance, and the fifth), unless they are perceived as one. In that case one single ratio, that of the extremes, will be produced, but otherwise it will not: for there will be simultaneously the ratio of many to few or of odd to even, and the ratio of few to many or of even to odd.[28]

3.14 *De Sensu* 448a 20–b 2

Are people right or wrong about concord, when they say that the sounds do not arrive simultaneously, but only seem to, and that this fact escapes us when the time involved is imperceptible?[29] One could then say that this is why we seem to see and hear simultaneously, because the intervening times escape our notice. Or is this not true, and is it impossible for there to be any imperceptible time or one that escapes our notice, all times being capable of being perceived? For if, when someone perceives himself or anything else during a continuous stretch of time, it is impossible for him not to notice that he exists, and if within the continuous time there is a time so small as to be altogether imperceptible, it is plain that during that time he would fail to notice whether he exists, and whether he is seeing, or perceiving, and if he does still perceive, there could be no time in which he perceives and no object that he perceives, except in the sense that he perceives part of the object in part of the time...

[27] This argument raises obvious difficulties for a conception of discord as the simultaneous perception of two sounds that do not blend to form a mixture.

[28] This paragraph re-emphasises a theme of the previous passage. The obscure argument of the last sentence contrasts the case where two items (sounds) are perceived as a unified mixture, in which the terms are not separately grasped but only the character of the ratio, with that in which they are grasped individually. Aristotle apparently means that in the latter case there is no unified focus of attention. The higher and the lower are attended to separately. In attending to the higher we note its relation to the lower, and in attending to the lower we note its relation to the higher, but these two experiences are not fused into the perception of a single ratio. Since they are different modifications of one sense organ, they cannot occur at the same time.

[29] The question arises out of the difficulties of the preceding passages, but is also prompted by theories of pitch according to which a higher sound travels faster and so arrives sooner. See the references given at n. 9 to 1.4 Theon Smyrn. 59.4.

3.15 *De Anima* 419b 4–421a 6

Now let us first outline the facts about sound and hearing.[30] Sound exists in two ways, in actuality and in potentiality. For we say that some things, like sponge and wool, do not possess sound, and that others, like bronze and everything that is solid and smooth, do possess it, on the grounds that they are capable of making a sound: that is, they can implant an actual sound between the object and the hearing. Sound that occurs in actuality is always the sound of something, against something, and in something: for what makes it is an impact. Hence it is impossible for a sound to occur when there is just one thing, for the striker and the thing struck are different. Hence that which produces sound does so against something. Nor does an impact occur without movement. But as we were saying, sound is not an impact between any random objects, for when wool is struck it makes no sound, but bronze does, and so do all things that are smooth and hollow. Bronze does so because it is smooth, while hollow things make many impacts after the first, by reverberation, since that which is moved is unable to escape.

Hearing occurs in air, and in water too, though to a smaller degree: for neither air nor water is responsible for sound, but rather there must occur an impact of solid things against one another and against the air. This happens when the air stands fast when it has been struck, and is not dispersed. Hence if it is struck quickly and vigorously, it makes a sound; for it is essential that the movement of the striker should forestall the fragmentation of the air, as when something in rapid motion strikes a heap or an eddy of sand.[31]

Echo occurs when some air is unified by the vessel that contains it and prevents its fragmentation, and the air rebounds [from it] like a ball.[32] It seems likely that echo always occurs, but is not distinct, since what happens with sound is like what happens also with light. For light is always reflected (otherwise it would not be everywhere, but there would be darkness except where the sun shone directly), but it is not reflected in the way it is from water or bronze or any other smooth object – that is, so as to make shadow, by which we determined the presence of light.

It is rightly said that void is responsible for hearing, for people suppose the air to be void, and it is air that creates hearing, when it is moved as something

[30] With the general account of sound production offered here compare especially 1.19 Archytas frag. 1, 5 *De Audib.* 800a, 8 Eucl. *Sect. Can.* 148.3–8.

[31] Archytas (1.19 frag 1) and Plato (2.4, 2.5 *Timaeus* 67a–c, 79e–80b) seem to have thought of the impact as transmitting a volume of air from the source to the ear. The author of 5 *De Audib.* 800a speaks of the air at the source pushing against the air next to it, and so on, not itself being transmitted across the intervening space: cf. also 4.2 ps.-Ar. *Probs* XI.6 with n. 9. The present passage (this paragraph and the next three) could be intended either way. That that air 'stands fast' suggests the second view; its rebounding 'like a ball' in the next paragraph hints at the first. I do not think that the question can be settled with certainty. On the dispersal of sound cf. 5 *De Audib.*, e.g. 800b–801a.

[32] On echo see also 4.2, 4.11 ps.-Ar. *Probs.* XI.6, 23 (cf. 7, 8, 9, 25, 51, XIX.11).

420a continuous and one.[33] But because of its fragility it makes no sound unless what strikes it is smooth. In that case it becomes one at the same time, on account of the surface of the striker, for the surface of what is smooth is one.[34]

A thing is productive of sound, then, if it can move air as one unified thing continuously as far as the hearing.[35] Air is intrinsic by nature to hearing: because it [the organ of hearing] is in air, when the air outside is moved, that inside is moved too. For this reason the living creature does not hear everywhere [i.e., in all its parts], nor does the air penetrate everywhere: for it does not contain air everywhere, but it is the part that will be moved and will be filled with sound that does so. Air as such is soundless, because it is readily fragmented: but when it is prevented from fragmenting, its movement is sound. The air in the ears is walled up inside so as to be unmoved,[36] in order for it to perceive accurately all the different kinds of movement. This is why we also hear in water, since the water does not penetrate to the air that is naturally intrinsic to the hearing: it does not even enter the ear, because of the spirals. When that does happen, one does not hear, and nor does one when the membrane fails, as in the case of the film over the pupil of the eye. A sign of whether we hear or not is the continual echoing of the ear, like that of a horn: for the air in our ears is always moving with a movement belonging properly to it, whereas sound is external, not intrinsic to the ear. That is why people say that we hear by what is empty and echoing, because we hear by that which contains air that is enclosed.

Is it the thing struck or the striker that makes a sound? Or is it both, but in different ways? For sound is a movement of that which is capable of being moved in the manner in which things rebound from smooth surfaces when one strikes them. Not every thing struck, and not every striker makes sound, as we were saying – for instance if a needle strikes a needle – but what is struck must be even, so that the air rebounds and is set in motion as a single mass. The differences between things that make sounds are displayed in sound when it is actualised; for just as colours are not seen without light, so the sharp and the heavy[37] are not perceived without sound. These things are said on the basis of a transference from tangible things, for the sharp moves the perception to a

[33] Aristotle does not of course agree that air is 'void'. His point is that the source of sound must be at a distance from the ear, if the sound is to be heard, since we neither hear nor see by direct contact with the source. Cf. *De Anima* 432b. For an odd conception of the character of the 'void' across which we hear, see ps.-Ar. *Probs* XI.33.

[34] But compare 5 *De Audib.* 803b on the rough sounds produced by rough objects (also mentioned by Aristotle at 3.17 *De Gen. An.* 788a).

[35] Cf. 5 *De Audib.* 800a, 802a, 804a–b.

[36] This is qualified below. It moves perpetually within its chamber (it cannot escape) with its own intrinsic motion, and this accounts for the 'echoing' heard in the ear in the absence of external sound. The hearing of sound from elsewhere is an alteration in this movement due to an impact of air (or water) on the wall of the chamber (the ear-drum).

[37] That is, high and low pitch. The more literal translations are given for the sake of the explanation that follows. Cf. 3.1 above with n. 2. The immediate point is that the relevant difference between a source of low sounds and a source of high sounds cannot be perceived as such except when their sounds are actually being produced.

great extent in a short time, while the heavy moves it to a small extent in a long time. It is not the case that the sharp is swift and the heavy slow:[38] rather, the movement of the former acquires its quality because of its speed, that of the
420b latter because of its slowness. There seems to be an analogy with the sharp and the blunt in the field of touch; for the sharp pierces, as it were, while the blunt pushes, since the former produces movement in a short time and the latter in a long one, with the consequence that the one is swift, the other slow.

Let that be our outline of the facts about sound. Now voice is a kind of sound belonging to something alive [*empsychon*], for no inanimate thing has a voice, but is said to do so by way of a metaphor, as are the *aulos*, the *lyra*, and all other inanimate things that are capable of prolongation, melody and articulation. They are like voice, because voice has these features too. But many animals do not have voice, the bloodless ones, for instance, and fish, among those with blood. This is to be expected, since sound is a movement of air; but those that are said to have voice, like those in the Achelous,[39] make a sound with their gills or some similar part. Voice, however, is a sound belonging to something alive, and one made with not just any random part. But since all sound occurs when something strikes, and strikes something else, and does so in something, and this latter is the air, it is to be expected that only those that admit the air can have a voice. For nature makes use of what is breathed in for two tasks: just as it uses the tongue for both tasting and articulation, of which tasting is essential (and therefore belongs to more creatures), while articulate expression [*hermēneia*] is for the sake of living well, so it uses the breath both to preserve the inner heat, this being something essential (for a reason that will be explained elsewhere), and for the voice, so that living well may be achieved. The organ used for breathing in is the throat, and this part exists for the sake of the lung; for it is through the lung that land animals contain heat to a greater degree than do other creatures. The region around the heart needs respiration too, and most importantly. Hence it is essential that the air should be breathed in and enter. Thus voice is the striking of the air that is breathed in against what is called the windpipe, under the agency of the soul [*psychē*] in those parts. For not every sound made by an animal is voice, as we were saying (for one can also make a sound with the tongue, or as people do when they cough), but that which makes the impact must be both alive [*empsychon*] and accompanied by some act of imagination, for voice is a sound that has meaning. It is not an
421a impact made *by* the air that is breathed in, as a cough is, but *by means of* this air one strikes the air in the windpipe against the windpipe. An indication of this is that one cannot give voice when breathing in or when breathing out, but when holding one's breath, for the person who holds his breath makes a movement by means of it.[40] It is clear, then, why fish have no voice, for they

[38] As Archytas and Plato seem to have thought, along with many others: see 1.19 Archytas frag. 1 with n. 47, and cf. Aristotle's further discussion at 3.17 *De Gen. An.* 786b–787a.
[39] A sort of catfish.
[40] This curious doctrine perhaps suggests that Aristotle is thinking of 'giving voice' as the initial attack on an utterance, not its prolongation. This is consistent with his talk of an

have no throat; and they do not have this part because they do not admit the air, or breathe in. The reason why this is so is another story.

3.16 *De Anima* 426a 27–b 7

If a concord is a sound,[41] and if a sound and the hearing of it are in a way one,[42] while a concord is a ratio, then the hearing must necessarily be a ratio. For this reason either element in excess – either the high or the low – destroys the
426b hearing: similarly in flavours such excess destroys the taste, in colours what is exceedingly bright or shadowy destroys the sight, and in smelling the same applies to a powerful smell, whether sweet or bitter, since the perception is a ratio.[43] That is why, while things are pleasant when they are brought pure and unmixed into the ratio (things such as the high-pitched or the sweet or the salty: for they *are* pleasant in such circumstances), nevertheless what is mixed, concord, is more pleasant than the high or the low.[44] The perception is a ratio, and things in excess dissolve or destroy it.

3.17 *De Generatione Animalium* 786b 7–788b 2

On the subject of the voice, we must investigate the reasons why each of the following things are so: that some creatures are deep-voiced, others shrill-voiced, while others are well-pitched and properly proportioned to both the extremes, that some are large-voiced and others small-voiced, and that they differ from one another in smoothness and roughness, flexibility and inflexibility.[45] So far as high and low pitch are concerned, we should assume that the cause is the same as that involved in the change that they undergo when they pass from being young to being older. For all creatures when they are younger utter a higher sound, except for calves, which utter a lower one. The same thing happens in the case of males and females: in all other kinds of creature the female utters a higher sound than the male. (This is particularly

impact on the windpipe, and such an attack is indeed preceded by a 'catching' of the breath. Contrast 4.5 ps.-Ar. *Probs* XI.14. For further discussions of the role of windpipe see 3.17 *De Gen. An.* 788a, 5 *De Audib.* 800b–801a.

[41] This is often translated 'if sound (or 'voice') is a concord'. The issue, and the whole passage, are difficult: see my discussion of the problems in *Phronesis*, 1981. In the following notes I assume the interpretation proposed there.

[42] This is argued in the preceding passage: see especially 425b 25ff.

[43] The passage is best construed on the assumption that Aristotle is talking only of the special cases where what is perceived is a mixture and a ratio (cf. 3.12–3.14 above), not asserting that every perception is of that sort. Then an excess in either element of the mixture makes it impossible to perceive them in combination as a mixture (cf. 422a 24, 424a 28–32: see also 6 Theophrastus ap. Porph. *Comm.* 63. 15–28).

[44] When two sounds of different pitch are heard together, they may be heard as a mixture, or they may remain perceptually distinct. In both cases they are 'brought into the ratio', since they do in fact stand in some ratio to one another. In the latter case, each may be pleasant in itself, but the perception of a mixture is always pleasanter. In these descriptions the 'mixtures' are concords, while the 'pure and unmixed' elements are the constituents of a discord. See 3.12–3.14 above, and 4.24 ps.-Ar. *Probs* XIX.39 (cf. 38).

[45] Many of these questions and others like them are discussed in more detail in 5 *De Audib.*

apparent in human beings, since nature has given them this capacity [i.e., that of emitting voice] in the highest degree, because of all creatures they alone use speech, and voice is the material of speech.) But in cattle the opposite is the case: cows utter lower-pitched sounds than bulls. The reason why creatures have voice, and what voice and sound in general are, have been explained partly in our work *On Perception*, partly in the work *On the Soul*.[46]

Since low pitch is to do with a movement's being slow and high pitch with its being swift, there is a problem about whether it is the mover or the thing moved that is the cause of the slowness or swiftness. Some people say that a large thing is moved slowly and a small one swiftly,[47] and that this explains why some creatures have low-pitched voices and others high-pitched ones. Up to a point what they say is right, but not entirely. In general, it seems correct to claim that low pitch is to do with the thing moved having a certain bulk. But if that is so, it will not be easy to utter a sound that is both small and low-pitched, nor, similarly, one both large and high-pitched.[48] Further, it is believed
787a that a low-pitched voice belongs to a nobler nature, and that in melodies the low-pitched is better than those that are tense:[49] for being better consists in superiority, and low pitch is a kind of superiority. But low and high pitch in the voice are different from largeness and smallness of voice (for there are creatures that are both high-voiced and large-voiced, and similarly ones that are both small-voiced and low-voiced); and something similar is true of the pitch intermediate between these. Now in what way could one define largeness and smallness of voice, except by reference to the quantity or smallness of that which is moved?[50] Then if high and low pitch are to be in accordance with the definition mentioned, it will turn out that creatures with low-pitched voices will be the same as creatures with large voices, and creatures with high-pitched voices will be the same as creatures with small voices: but this is false. The reason is that the large and the small and the great and the little in quantity are sometimes spoken of absolutely, sometimes in relation to one another. Large voices occur when what is moved is a great quantity absolutely, small voices where it is little. Low-pitched and high-pitched voices occur where the distinction consists in relations between quantities. For if what is moved exceeds the strength of the mover, that which travels must necessarily travel slowly, while if it is exceeded, it moves swiftly. Something strong sometimes moves a great quantity, because of its strength, and so generates a slow movement: sometimes it generates a swift movement because of its superiority. By the same reasoning, weak movers that are moving things too large for their

[46] The main references to sound and hearing in these two works are collected in 3.11–3.16 above. Aristotle is probably thinking principally of 3.15. There is no detailed discussion of these topics in the *De Sensu* ('On Perception') as it has come down to us.

[47] See, for example, 4.4, 4.5, 4.8, 4.10, 4.12, 4.13 (ps.-Ar. *Probs* XI.13, 14, 19, 21, 32, 34); rather different explanations are suggested in, for example, 4.6, 4.7, 4.9 (*Probs* XI.15, 16, 20).

[48] This is also a difficulty for the theory of Archytas: see 1.19 frag. 1.

[49] Compare, for example, Aristotle's criticism of 'tense' (high-pitched) melodies at *Pol.* 1342a, and Plato's rejection of both the 'tense' and the 'slack' at *Rep.* 398e.

[50] Cf., for example 4.1 ps.-Ar. *Probs* XI.3.

power generate a slow movement, but those which because of their weakness move a small quantity generate a swift one.[51]

These, then, are the explanations of the following oppositions: of the fact that young creatures are not all high-voiced or low-voiced, nor are all older creatures or all males or all females, that both people who are ill and those that are healthy in body make utterances at a high pitch, and again that as people grow old their voices become higher, even though their time of life is opposite to that of the young.[52]

Most creatures when they are young, and most females, are high-voiced, because they move little air, on account of their lack of power.[53] A little air travels swiftly, and in sound what is swift is high-pitched. Bull-calves and female cattle, however (the former because of their youth, the latter because of the nature of femaleness), are not strong in the part with which they generate
787b movement, but the quantity they move is large, and so they are low-voiced. For what travels slowly is low-pitched, and a large quantity of air travels slowly. These creatures move a large quantity while the others move little, because the vessel through which the breath first travels has a large passage in the former, and is therefore compelled to move a large quantity of air, while in the others it is under better control.[54] As their age advances, this part which is the agent of movement grows stronger in both kinds of creature, so that they change into the opposite condition, and those of them that were high-voiced become lower-voiced, while those that were low-voiced become higher-voiced.[55] That is why bulls have higher voices than calves and female cattle. In all creatures the strength is in the sinews, which is why creatures in their prime are stronger: for the young have inferior joints and sinews. Again, in the young the sinews have not yet been stretched tight, while in the old the tension is already becoming slack.[56] Hence both are weak and deficient in the power needed for movement. Now bulls are especially sinewy, as is the heart: hence in them this part, with which they move the breath,[57] is taut, like a string made of sinew that has been

[51] A mass of air that is large absolutely, and so when moved generates a loud sound, may be moved by something so strong as to move it easily and quickly, giving a high note, or by something too weak to move it except slowly, when its pitch will be low. Similarly, even a small mass may be moved quickly or slowly. Hence all combinations of pitch and volume are after all possible.

[52] Similar issues are aired at 4.4–4.7, 4.10, 4.12, 4.13 (ps.-Ar. *Probs* XI.13–16, 21, 32, 34).

[53] See the passages cited at n. 47 above. Aristotle evidently means that though they have little power, it is enough to move quickly the small amount of air they emit (because their windpipe is narrow, as the sequel explains).

[54] The 'others' are the females and young of the other species. On the 'control' exercised by the windpipe see 5 *De Audib.* 800b–801a.

[55] The explanation is abbreviated. In most creatures, as they get older, the size of the mass of air to be moved increases more than does the strength with which they propel it, hence their voices become lower. Bulls, however, as Aristotle explains below, acquire unusually great strength in the relevant parts, so that their voices becomes higher.

[56] The old are included among those with higher voices also at 4.7, 4.13 ps.-Ar. *Probs* XI.16, 34.

[57] This doctrine is not repeated in our other Peripatetic sources. In 5 *De Audib.*, most notably, the breath's movement is correctly explained by reference to the bellows-like action of the lung (800a–b). For another theory see 4.5 ps.-Ar. *Probs* XI.14.

stretched tight. Bulls' hearts are shown to have a nature of this sort by the fact that bone grows in some of them, and bones follow [lit. 'seek'] the nature of sinew.

All creatures when castrated change to the female condition,[58] and because the strength of their sinews is slackened at its origin they emit a voice like that of females. This slackening comes about in a way comparable to what would happen if one were to stretch a string and make it tight by fixing a weight to it, as do women who weave at looms: they too stretch the warp downwards by fixing to it what are called *laiai* ['stones']. The testes are attached to the seed-bearing tubes in a similar way, and these are attached to the vessel which has its origin [*archē*] at the heart, next to the part that moves the voice. This is why, when the seed-bearing tubes alter, around the age at which they have become able to secrete the seed, this part too [i.e., that which moves the voice] alters with them. When it alters, so does the voice, more so in males, though
788a the same thing happens less obviously in females, and there occurs what some people call 'goat-bleating', while the voice is uneven. After this it settles into the low or high pitch of voice proper to the next phase of life. When the testes are removed, the tension of the tubes is slackened, just as is that of the string or the warp when the weight is taken off. When this is slackened, the source [*archē*] of the movement of the voice is loosened in the same manner. Thus it is for this reason that castrated creatures change into the female condition both in voice and in the rest of their form, because the origin [*archē*] from which the tension in their body arises is slackened. It is not, as some people suppose, because the testes themselves are a complex bundle of many principles [or 'origins', *archai*]: it is rather that small changes of condition become causes of great ones, not in their own right [lit. 'through themselves'], but when a simultaneous change in an origin [*archē*] goes with them. For though the origins are small in size they are great in power: that is what it is to be an origin [*archē*] – to be the cause of many things, while there is nothing else that is higher than it.[59]

The heat or cold of the region also contributes to the fact that some creatures are naturally so constituted as to have low-pitched voices, others so as to have high-pitched ones. For hot breath produces low pitch of voice, because of its thickness, while cold breath because of its thinness produces the opposite.[60] This is clear also in the case of *auloi*, for those who use hotter breath, and project it in the same way as people who are groaning [*aiazontes*], play at a lower pitch.[61] The cause of roughness of voice and of the voice's being smooth,

[58] This phenomenon is also noted in the passages mentioned in n. 56 above.

[59] The sense is that the testes do not, in themselves, have many different powers, but the one that they do exercise in their own right (as an *archē*, not because of the action of something else) affects the body in a large variety of ways.

[60] Cf. 4.1, 4.4, 4.5, 4.12 ps.-Ar. *Probs* XI.3, 13, 14, 32. But in some of those passages the internal heat is an agent by which the air is propelled. Here it serves only to 'thicken' the breath (as in part of 4.12).

[61] Repeated at 4.4 ps.-Ar. *Probs* XI.13. The phenomenon noted is probably that pitch is lowered if the player's mouth-cavity is made wider (as in yawning or groaning). That coldness goes with compression of breath and heat with its rarefaction is a doctrine as

and of all unevenness of that sort, is the fact that the part or organ through which the voice travels is either rough or smooth or in general even or uneven. This is clear when there is any moisture in the windpipe, or when roughness develops as a result of some illness, for in these cases the voice becomes uneven too.[62] Flexibility depends on whether the organ is soft or hard: a soft organ can be controlled and can take on all sorts of shape, while a hard one cannot. A soft organ can utter both a small sound and a large one, and hence also both a high-pitched sound and a low one, since it controls the breath easily, as it easily becomes large and small itself. But hardness cannot be controlled.[63]

788b Let that suffice, then, on the subject of the voice, concerning the things of which no account was given previously in the books *On Perception* and *On the Soul*.[64]

old as Anaximenes (6th century B.C.): see Plutarch *De Prim. Frig.* 947f. No doubt it is based on the observation that if you blow on your hand with a wide open mouth the breath feels hot, while a stream of breath propelled sharply through pursed lips feels cold. Plutarch reports Aristotle as denying Anaximenes' thesis that the compression or rarefaction causes the change in temperature. Rather, the heat felt in the one case is heat from inside ourselves, while the cold in the other is that of the air outside the lips pushed forward by the breath (cf. ps.-Ar. *Probs* XXXIV.7.)

[62] Cf. 4.3 ps.-Ar. *Probs* XI.11, 5 *De Audib.* 801a, 803b, 804a.

[63] Cf. 5 *De Audib.* 800a–b, 803a, where it may possibly be Aristotle's account that the author disputes.

[64] See n. 46 above.

CHAPTER 4

The Aristotelian *Problemata*

The *Problems* is a collection of questions on a host of different topics, each provided (sometimes rather tentatively) with an answer or answers. Questions belonging to the same general area are grouped together in 'books', two of which are relevant here. Book XI addresses problems to do with the production and reception of sound, particularly vocal sound, and Book XIX considers questions on the subject of music (there is inevitably some overlap). It seems likely that the work was compiled by students in the Lyceum during and after Aristotle's time, mainly in the late fourth century and the early third, both as a record of research and as an aid to further enquiry. The way in which the solutions to the puzzles are expressed seems exploratory rather than doctrinaire, and though the question-and-answer format is similar, the tone of the collection is quite different from that of the pedagogic 'catechisms' of later antiquity, exemplified in harmonics by the writings of Ptolemaïs (see 9.10–9.12) and Bacchius. Even within one book, the arguments and explanations presented are not all from the same hand. Sometimes we are offered alternative explanations for the same phenomenon. Often the theories put forward in answer to one question are incompatible with those suggested in the context of another.

Many of the problems discussed in Book XI are closely related to Aristotle's investigations of the way vocal sounds are produced, and how the pitch of a vocal utterance is physiologically determined. These include 4.1, 4.4–4.7, 4.12–4.13. Some touch on the causes of different qualifications of vocal sound (volume in 4.1, roughness in 4.3), and such issues are pursued in greater detail in 5 *De Audib*. Five of the passages quoted consider issues about the physical nature of sound in general, focussing especially on the way in which it travels and is modified by distance or by obstructions: these are 4.2, 4.8, 4.9, 4.11, 4.14.

The excerpts from Book XIX (other passages are quoted in *GMW* vol. I ch. 14) begin with a problem about the transmission of sound, closely related to that addressed in 4.14. After that the questions raised deal more specifically with issues proper to music. Several are concerned with the phenomenon of concordance, and offer explanations that depend on 'Pythagorean' ratio theory (4.19, 4.21, 4.24, 4.25, 4.28), referring occasionally to 'experiments' with instruments. Another group looks at issues to do with pitch, with explicit reference to the theory that higher pitch goes with greater velocity of movement (4.18, 4.22, 4.23). Three involve the vaguely conceived idea that higher pitch goes with smaller size (4.16, 4.17, 4.23). Two consider the phenomenon of sympathetic vibration (4.20, 4.26). One reflects on a teasing issue in the history of musical practice, the nature of the 'seven-note *harmoniai*' that were thought to have been current before the adoption of an eight-note octave system (4.27).

Few of the explanations offered are of great penetration or originality. They are valuable mainly as expressions of the state of acoustic science, at a fairly routine level, in the half-century or so after Aristotle's death, and for snippets of information that they convey in passing. Two of those in Book XIX (4.24, 4.26) are of special interest, both for the sophistication of their arguments, and for the way in which they hint at a theory of pitch certainly developed out of Pythagorean and Platonist beginnings, but distinct from the theories of relative velocity or 'force' that are usual in this tradition (and, with

modifications, in Aristotle). They provide a line of continuity between the presuppositions of the bizarre procedures outlined in 1.8, and the fully articulated thesis that higher pitch depends on greater frequency of impact, which is found in 8 Eucl. *Sect. Can.* 148–9.

In most respects the kind of acoustic science represented here is resolutely if informally quantitative. It conceives sound as constituted or caused by physical movements, and as varying in its most important features (especially pitch) in accordance with quantitative variations in some dimension of that movement. In this respect it is at one with the Pythagoreans, Plato and Aristotle, but there was one follower of Aristotle, Theophrastus, who attacked the whole basis of this approach to acoustics with all the ingenuity his distinguished intellect could muster. His arguments appear below in chapter 6.

Book XI

4.1 3 Why are those who are hot by nature always large-voiced? Is it because there must also be much cold air in them? For hot breath draws air towards itself, and the hotter it is the more it draws.[1] A large voice occurs when one moves much air, a high-pitched one when one moves it swiftly, and a low-pitched one when one moves it slowly.[2]

4.2 6 Why do voices seem higher-pitched from far away?[3] At any rate, those who imitate people shouting from a very long way off utter high-pitched sounds, similar to those produced when people make echoes: the sound of an echo seems higher, and it is far away, since it is refracted.[4] But then, since the high-pitched in sound is the swift and the low-pitched is the slow, voices from far away ought to appear lower-pitched. For all things that travel continuously travel more slowly the further they are from their origin, and ultimately they fall.[5] Is the fact, then, that those who imitate a far-away voice do so with a feeble and thin voice? A voice that is thin cannot be heavy [*bary*, 'low-pitched'], nor is it possible to utter a small and feeble sound heavily: it has to be high-pitched [*oxy*, 'sharp'].[6] Or is it not just that imitators imitate by these means, but also that the sounds themselves become higher?[7] The reason is that the sound is made by air in motion; and just as a sound is first made by that which moves the air, so the air in its turn must do the same, and some air must

[1] The idea is perhaps based on the observation that when a fire is burning, air is drawn towards it. See also 4.4, 4.5, 4.12.

[2] A simple statement of what may be called the 'standard' Peripatetic view, though other Problems give variants and alternatives. See also 3.17 Aristotle *De Gen. An.* 786b–787a.

[3] The question is raised again at 4.9; for related issues see also 4.8, 4.22.

[4] On echo sees 3.15 Aristotle *De Anima* 419b with n. 32. The notion that an echo is higher than the original sound is repeated at *Probs* XIX.11, but by implication contradicted at 4.11 and 4.26.

[5] Compare 4.22.

[6] On thinness of voice and its relation to high pitch see 4.9, and 5 *De Audib.* 803b; cf. 6 Theophrastus ap. Porph. *Comm.* 64.4–19.

[7] The first explanation has suggested that sounds are not really higher in pitch at a greater distance from the source. The fact is only that when we imitate distant sounds we make our voices thinner and so higher. The second explanation will assume that the alleged phenomenon is real, and will offer an account of it.

be the mover, some the moved. That is why sound is continuous, because air causing motion continually succeeds air causing motion, until the motion is exhausted, which in the case of bodies means that they fall: in the one case this happens when the air can no longer propel the missile,[8] in the other when it can no longer propel the air. For continuous vocal sound occurs when air is propelled by air, while a missile travels when a body is moved by air. Hence in the latter case it is the same body that travels throughout, until it falls, but in the former case it is different air at every stage.[9] Now what is smaller travels more swiftly at first, but over a shorter distance. That is why far-away sounds are higher-pitched and thinner.[10] For what is swifter is high-pitched – a point that has already been investigated. Similarly, the same reason explains why both children and the sick utter at a high pitch, while men and those who are healthy do so at a low pitch.[11] The reason why it is not apparent to people nearby that a sound is becoming lower or higher in pitch, and why, in general, the case is different from that of heavy bodies that are thrown, is that the thing thrown travels as one and the same thing throughout, while a sound is air propelled by air.[12] Hence the former falls in one place, while the voice falls on every side, just as if the object thrown were broken into an infinity of fragments as it travelled, and even went backwards.[13]

4.3 11 Why is the voice rougher in those who have been sleepless? Is it because the body is moister, due to lack of concoction, and especially in the upper region, which is also why the head is heavy? Since there is moisture in the area of the windpipe, the voice must inevitably be rougher. For roughness is caused by unevenness, while low pitch is caused by obstruction, since the movement is slower.[14]

4.4 13 Why do those who weep utter high-pitched sounds, while those who

[8] This reflects the idea that a projectile is kept in motion, after it has left the projecting agent, by being pushed by air that rushes into the gap behind it. Two accounts of the process are offered at Aristotle *Physics* 215a. (One of them corresponds to Plato's at *Timaeus* 79a–80c, part of which is quoted at 2.5.)

[9] Clearly the author's view is that the air moved when sound travels does not in this respect behave like a missile, by travelling to the hearer's ear. At the same time the conception is not strictly that of the transmission of vibrations through a stationary medium. The thesis is that the air adjacent to the source is indeed moved forwards, but only a little, and 'shunts' the air next to it, in its turn. See 5 *De Audib.* 800a, and cf. 3.15 Arist. *De Anima* 419b with n. 31. For other comparisons with the behaviour of missiles see 4.14, 4.15, 4.22.

[10] The author apparently means that as the movement travels it is dispersed (see the end of the passage), so that an initially broad and unified continuum of movement is fragmented into many 'thin' ones travelling in different directions, and each branching in turn. Then any one stream of movement is thinner the further it is from the source, and so, on the present theory, will move more rapidly. Cf. 4.8, 4.9, where a variant of this explanation is offered.

[11] See 4.5, 4.7, 4.10, 4.13.

[12] The difference from the case of missiles is clear, but it is obscure how this explains why the alleged phenomenon of altering pitch is not evident to people nearby.

[13] On the direction of the movement of sound see *Probs* XI.49, and cf. the observations about sounds of different pitch at 6 Theophrastus ap. Porph. *Comm.* 64.4–19.

[14] Cf. 3.17 Arist. *De Gen. An.* 788a, 5 *De Audib.* 801a, 803b, 804a.

laugh utter low-pitched ones?[15] Is it because some through their weakness move little breath, while others [move the breath] vigorously, which makes the breath travel swiftly?[16] What is swift is high-pitched; and what is propelled from a body that is under tension [*syntonos*] travels swiftly. Someone who laughs, by contrast, is relaxed [*dialelymenos*]. The weak utter high-pitched sounds, because they move only a little air, some of them only at the surface.[17] Again, those who laugh emit the breath in a hot state, while those who weep – just as grief is a chilling of the region of the chest – emit the breath in a cooler condition. Now what is hot moves much air, so that it travels slowly, while what is cold moves little. The same thing happens in the case of *auloi*; for those who play the *auloi* with hotter breath pipe a much lower pitch.[18]

4.5 14 Why do boys and the young of other animals utter higher-pitched sounds than mature adults, despite the fact that high pitch is vigour?[19] Is it because voice is movement of air, and the swifter is the higher-pitched? Now a little air is more easily and swiftly moved than a large quantity.[20] It is moved when it is either brought together or dispersed by something hot. Since inhalation is the drawing in of something cold, the air within us must be brought together in this process. But it is exhalation, when what is hot moves the air, that can become voice, since we utter voice when we exhale, not when we inhale.[21] Since the young are hotter than those who are older, and the channels they have inside them are narrower, they must have less air within themselves. Since what is moved is less, and there is more of what is hot and creates movement in them, for both reasons the movement of the air must be swifter;[22] and the swifter movement must sound at a higher pitch, for the reasons given previously.

4.6 15 Why do those who weep utter higher-pitched sounds, while those who laugh utter low-pitched ones?[23] Is it because those who weep make sounds while tightening [*synteinontes*] and contracting the mouth? The air inside them is moved swiftly by the tension [*syntonia*], and because it travels through a narrowed mouth it travels more quickly: thus the sound becomes high-pitched

[15] The question recurs at 4.6.

[16] The words rendered 'some', 'others', might be translated 'the former', 'the latter', but this hardly fits the context. What the sentence offers is a generalised account of two ways in which high-pitched sounds are produced: through weakness, because little air is moved (and a little air travels swiftly, see, for example, 4.5); and through strength or vigour, which propels the air faster. In the sentences that follow, the high pitches uttered by those who weep are explained in the first way, the low pitches of laughers both by their relaxation (lessening tension and vigour) and by the greater amount of air they move. See also 4.10.

[17] *Epipolēs*: cf. 12 Arist. Quint. *De Mus.* 7.5–6, 81.11–13.

[18] See 3.17 Arist. *De Gen. An.* 788a with nn. 60, 61.

[19] See 3.17 Arist. *De Gen. An.* 786b ff., with the references given at n. 52.

[20] Cf., for example, 3.17 Arist. *De Gen. An.* 787a, with 4.7, 4.8 below.

[21] Contrast 3.15. Arist. *De Anima* 420b with n. 40.

[22] For different explanations of the source of movement see 3.17 Arist. *De Gen. An.* 787b, 5 *De Audib.* 800a–b.

[23] See also 4.4 above.

for both reasons.[24] But those who laugh relax the tension and gape wide while doing so. Hence, since for this reason they expel the air over a wide front and slowly, it is not surprising that the voice they produce is low-pitched.

4.7 16 Why do people who cannot procreate, such as children, women, those who have grown old, and eunuchs, utter at a high pitch, while men utter at a low pitch?[25] Is it because just as a line and other thin things have only one dimension, while thick things have several, so also a thin voice must have only one dimension? It is easier to make and to move one thing than several: hence, since the people mentioned above have weak breath, it moves only a little air. Now the least air is that which has only one dimension, and this air will be thin for the reasons given. The voice arising from it will be of the same quality, and a thin sound is high-pitched [*oxeia*, 'sharp']. This, then, is why those who cannot procreate have high-pitched voices. But men are strong in the breath and so move much air, and since there is much of it, it must be moved slowly, and makes a low-pitched [*bareian*, 'heavy'] voice. For we saw that a thin and a swift movement makes a higher-pitched voice, and neither of these occurs in a man.[26]

4.8 19 Why is the lower-pitched voice more audible [than the higher] when heard from nearby, less audible from far away?[27] Is it because the lower voice moves more air, but to no great distance? Hence we hear it less from far away, since it is moved less far, but more from nearby, since more air falls on our organ of sensation.[28] The high-pitched voice is heard far away because it is thinner, and what is thin is extended to a distance.[29] Someone might add that the movement that produces it is also swifter, and that would be so if the breath moving the air were both compact [*pyknon*] and narrow. For a small amount of air is more easily moved (and it is a small amount that is moved by what is narrow), and what is compact makes more numerous impacts – the impacts that make the sound.[30] This can be seen in instruments, for the thinner strings are higher-pitched, if their other features are the same.

[24] That is, both because the tension creates vigorous propulsion, and because the passage is narrow. The latter point may reflect the idea that a smaller bulk travels faster (e.g., 4.5 above), or the observation that, for example, a river moves more rapidly when its channel becomes narrower: cf. 5 *De Audib.* 802b.

[25] The question is repeated at 4.13; cf. 3.17 Arist. *De Gen. An.* 786b ff., 5 *De Audib.* 803b.

[26] For the notion of 'thinness' see 4.2 above with nn. 6 and 10. Here this thinness is that of a narrow continuum of air in movement, as in the latter part of 4.2 and in 4.8, 4.9. The thesis that it reaches a single dimension (and moves faster because 'it is easier to make and to move one thing than several') seems to depend on a misplaced piece of mathematical sophistication. It is more clearly expressed in 4.9.

[27] See 6 Theophrastus ap. Porph. *Comm.* 63.20–22, 63.30–64.19; cf. 4.2 and 4.9.

[28] Cf., for example, 4.1 above.

[29] See n. 26 above.

[30] This hints at an alternative explanation of pitch, distinct both from the 'velocity' theory and from notions based on 'thinness' etc., correlating higher pitch with more frequent impacts made on the air by an agent. See especially 8 Eucl. *Sect. Can.* 148–9; cf. 4.24 ps.-Ar. *Probs* XIX.39, 5 *De Audib.* 803b–804a, 9.8 Heraclides ap. Porph. *Comm.* 30.1ff., and perhaps 1.8 Porph. *Comm.* 107.15ff. with n. 29. The word *pyknos* is used in the *Sectio* directly of the movements causing a sound, in the sense 'closely packed', 'frequent'.

4.9 20 Why does a voice seem higher-pitched to those standing further away, though high pitch is in swiftness of movement, and what travels further travels more slowly?[31] Is it because the voice's high pitch is not only in its moving swiftly, but also in its becoming a very thin sound, and to people standing further away the voice comes in a thinner and thinner condition, because of the smallness of the air that is moved?[32] For the movement dwindles: a dwindling number terminates in the one, and a dwindling body in one dimension.[33] In a body this is thinness, and so it is also in a voice.

4.10 21 Why do both those who have been exercising and those who are weak utter at a high pitch? Is it because the weak move little air, and a little air travels more swiftly than a greater amount, while those who have been exercising move the air vigorously, and air that is vigorously moved travels more swiftly?[34] In the voice, what is swift in its movement is high-pitched.

4.11 23 Why is it, given that voice is air that has been shaped and is in motion, that the shape is often broken up, while echo, which occurs when air like this collides with something solid, is not itself broken up, and we hear it clearly?[35] Is it because an echo is a rebounding, not a fragmentation? Hence the whole persists, and two parts, alike in shape, arise from it, since rebounding happens at a like angle. That is why the voice of the echo is like the original voice.[36]

4.12 32 Why do those who are worried utter at a low pitch, those who are afraid at a high pitch? Is it because in those who are afraid the region around the heart is chilled, as the heat sets off downwards, so that they move little air? But in those who are worried the heat travels upwards, as it does in those who are ashamed: for people also become worried as a consequence of shame. In those who are ashamed the heat travels upwards to the face: a sign of this is the fact that they become redder. Hence the heat concocts and thickens the air with which they utter. Air like this is propelled slowly, and in sound what is slow is low-pitched.[37]

[31] The question is an elaboration of the one at 4.2.

[32] Unlike the related 4.7, this suggests that velocity and thinness are independent conditions, either of which can produce high pitch on its own. The present author does not deny that the air moves more slowly as it gets further from its source. Cf. 4.2 with n. 6.

[33] Repeating the mathematical eccentricity of 4.7.

[34] This expresses more clearly the two alternative conditions of swift movement alluded to in 4.4 above (see n. 16).

[35] On echo see 3.15 Arist. *De Anima* 419b with n. 32. 'Shape' (*schēma*) probably does not refer to a cause of sound or of pitch (but see 5 *De Audib.* 800a, and cf. 4.7, 4.9, 4.16, with 6 Theophrastus ap. Porph. *Comm.* 64.4–19). The present author seems to be thinking of a sound's loss of clarity, and perhaps particularly of the distortion or obscuring of articulated sounds when speech or song is heard at a distance. See 5 *De Audib.* 801a–b, and 804a–b on 'cracked' sounds. Cf. *Probs* XI.25, 37, 51 (where the present question is repeated).

[36] This may deny the claim made in 4.2, that echo is higher in pitch than the original sound, but the author may be thinking only of its phonetic 'shape', not of its pitch.

[37] This incorporates two explanations. (a) In those who are afraid, the relevant bodily part loses heat, so that they move little air (the converse will hold of those who are worried). Heat is treated as an agent of movement: cf. 4.4. (b) Heat thickens the air, so that it cannot be moved so easily: cf. 3.17 Arist. *De Gen. An.* 788a.

4.13 34 Why do people who cannot procreate, such as children, women, those who have grown old, and eunuchs, utter at a high pitch, while men utter at a low pitch?[38] Is it because of weakness in the part that moves the air? For what is weak moves little air, and a little air travels swiftly, and what travels swiftly is high-pitched.[39] Or is it because the first channel through which the voice travels is small in those who cannot procreate, so that what propels the air from it is small, and the air, being little, travels swiftly through the wide opening of the upper windpipe, while in people in their manly prime this channel is expanded, like that to the testicles, so that the air propelled is greater too? Hence it passes through more slowly, and so becomes lower-pitched.[40]

4.14 52 Why is it that when one person utters and when many people utter together the sound is not equal, but neither does it travel further in proportion to the number of people uttering?[41] Is it because each person pushes forward the air next to him, but they do not propel the same air, except to a minor extent? It is like what would happen if many people were throwing, but each, or at least the majority, were armed with a different stone. In that case no missile will travel far, or not proportionately far [*kata logon*], and no more will it in the other case. For the voice that is so great is not that of one person, but of many. Hence from nearby the voice seems proportionately great (similarly, several missiles will hit the same place), but this will no longer be the case from far away.[42]

Book XIX

4.15 2 Why does the same person with the same voice transmit the sound further when singing or shouting together with others than he does when alone?[43] Is it because the result of several people's doing something as a group – compressing or pushing – is not the same number of times greater as is the number of people, but just as a line two feet long generates something not two times but four times as great, so when things are put together they are stronger in relation to their number than when they are separated?[44] Hence when people

[38] The question is repeated from 4.7.

[39] Cf., for example, 4.4, 4.10.

[40] This second explanation involves a version of the correlation between 'thinness' and velocity made at 4.2, 4.7 and elsewhere.

[41] The suggestion is that the sound from a group of voices is louder, but does not travel much further than that of a single voice. Contrast 4.15, which seems to assume that it does carry to a substantially greater distance.

[42] The explanation is admirably clear, but note that it does not raise the question whether the air moved behaves in all respects like a missile: cf. 4.2 above with nn. 8 and 9.

[43] The question is interestingly put: not 'Why does the sound of a group of voices travel further than the sound of one?', but 'Why does one person's voice travel further when it is associated with others than it does when alone?'. By contrast with 4.14, the assumption in the following sentences is that the distance is greatly increased, more than in proportion to the number of voices.

[44] That is, the square on a two-foot line is four times the square on a one-foot line; it is also twice the combined squares on two separate one-foot lines. The author probably has only a vague analogy in mind, not, for example, the precise hypothesis that the distance travelled is proportional to the square of the forces applied. (Aristotle's own

act as a group, the strength of their voice becomes united and pushes the air simultaneously, so that it advances many times further. For the voice of them all is many times greater than if each were taken by itself.

4.16 8 Why does the low-pitched [string] contain the note of the high-pitched?[45] Is it because the low-pitched [*bary*, 'heavy'] is greater? It is like an obtuse angle, while high pitch [*to oxy*, 'the sharp'] is like an acute [*oxeia*] one.[46]

4.17 13 Why is it that in the octave what is low-pitched is an answering sound of what is high-pitched, but what is high-pitched is not an answering sound of what is low? Is it because at the most the melody of both is in both, but if not, it is in what is low, since it is greater?[47]

4.18 21 Why is it that when people are singing, those who sing lower pitches, if they sing wrongly, do so more detectably than those who sing high? In rhythm, similarly, those who make mistakes in a slower rhythm do so more obviously. Is it because the time taken by something low-pitched is longer, and it is therefore more perceptible, since in the longer time it generates more sensation, while what is swift and high-pitched escapes detection because of its swiftness?[48]

4.19 23 Why is *nētē* double *hypatē*?[49] Is it, first of all, because when the

view, though heavily qualified, was probably that the distance, other things being equal, is directly proportional to the force. See his discussion of the relation between force and velocity, *Physics* 249b.)

45 This might refer to the higher harmonics heard simultaneously with a sound's fundamental pitch. But I have not found any unambiguous references to this phenomenon in Greek sources. Possibly the author is thinking principally of the sense in which a note 'contains' the sound of the note an octave higher: cf. 4.17, 4.19–4.21, 4.24–4.26, 4.28. More probably he is alluding to the fact that a string can produce not only its own pitch, but higher ones too when it is stopped at different points, as on a monochord – whereas it cannot produce lower ones.

46 The analogy is fairly unhelpful, though drawing instructively on ambiguities in the term *oxys*, 'sharp', 'high-pitched' (cf. 3.1, 3.3, Arist. *Topics* 106a9ff., 107a 11ff.), which may have influenced the theory that connected a sound's pitch with its 'shape': see n. 35 above. On the last interpretation offered in n. 45, the explanation might imply that a lower sound involves the movement of a larger body of air: cf., for example, 4.7–4.10.

47 'Answering' translates *antiphōnos*, used in many of the Problems for the 'correspondence' of the octave. The sense of the question is obscure. Perhaps the author is alluding to the sympathetic vibration of strings an octave apart. In 4.20 and 4.26 only the initiation of movement in the lower string by the higher is considered, not the converse. But the question may itself be inspired by the notion that the lower note 'contains' the higher and not conversely: see the preceding passage, and cf. *Probs* XIX.18. In the last sentence one might detect an allusion to the practice of accompanying a melody at a higher pitch, not a lower (cf. *Probs* XIX.12).

48 This is consistent with the theory that a lower-pitched note travels more slowly, but requires specifically that our hearing of it occupies more time. That thesis would be more plausible in connection with a theory linking pitch with frequency of impact: see 4.21, 4.24 with nn. 52 and 62–4.

49 The notes referred to are *nētē diezeugmenōn* and *hypatē mesōn*, which are an octave apart. (It is usually these notes that are intended when the terms *nētē* and *hypatē* are used without qualification.) No physical or mathematical answer to the question is explicitly given here (but see 4.21 and especially 4.24), only a string of examples. But the author seems to have some theory in mind, since the cases cited do not directly

string is plucked at half its length and as a whole, the notes form the concord of an octave? The case is similar in *syringes*, for the sound through the hole in the middle of the *syrinx* forms the concord of an octave with that through the whole *syrinx*. Again, in *auloi* the octave is formed by the double distance, and that is how those who bore *auloi* form it: the fifth, similarly, is formed by the hemiolic distance. Again, those who tune *syringes* mould the wax into the very end of the *hypatē* reed, but fill up the *nētē* reed as far as half-way: and similarly they form the fifth by the hemiolic distance and the fourth by the epitritic. Again, in triangular *psalteria* where the tension is equal, the concord of an octave is produced when the length of one string is double, that of the other half.[50]

4.20 24 Why is it that if one plucks *nētē* and then damps it, *hypatē* alone seems to sound in answer? Is it because the sound from this string is particularly close in nature to that note, because it is concordant with it? Hence through being increased by the presence of what is like it, it alone is apparent, while the others are undetectable because of their smallness.[51]

4.21 35 Why is the octave the finest concord? Is it because its ratios are between terms that are wholes, while those of the others are not between wholes? For *nētē* is double *hypatē*, so that if *nētē* is 2, *hypatē* is 1, if *hypatē* is 2, *nētē* is 4, and so on invariably. But *nētē* is the hemiolic of *mesē*: for the fifth, which is hemiolic, is not in whole numbers – if, for instance, the smaller term is 1, the greater is the same quantity and the half in addition. Hence wholes are not being compared [or 'combined', *synkrinetai*] with wholes, but parts are added. The case is similar with the fourth, for epitritic ratio is so much and one of the three in addition.[52] Or is it because the completest concord is that constituted out of both, and because the measure of melody ...[53]

4.22 35a ... in everything that travels the movement is most vigorous in mid-course, more gentle as it is beginning and ending. And when the movement is most vigorous, the sound of what travels is higher-pitched. This is why strings that are tightly stretched sound at a higher pitch, since the movement is swifter.

exemplify the relation mentioned. In the question the larger number attaches to the higher note, but in the instruments the greater distances attach to the lower. Probably he presupposes an account like that at 9.7 Aelianus ap. Porph. *Comm.* 33–5, cf., for example, 10 Nicomachus *Ench.* ch. 4.

[50] Hemiolic ratio is 3:2, epitritic 4:3. The 'hemiolic distance' is two thirds of the length, the 'epitritic' is three quarters. This passage is also quoted in *GMW* vol. 1; for remarks on the instruments see pp. 196–7, nn. 45–7.

[51] The explanation is impressionistic and vague. For a fuller account see 4.26.

[52] That is, if two quantities are in epitritic ratio (4:3), the greater is equal to the smaller plus one third of the smaller. The idea seems to be that the best kind of fit between two quantities is that in which the smaller fits into the greater an exact whole number of times, remaining undivided while accommodating itself to the greater. Though the explanation is purely mathematical, it can readily be linked with the physical hypotheses of 4.24 below. (As it stands, its main weakness is that all multiple ratios will apparently be equally concordant, which is inconsistent with the perceptual data.)

[53] Here the MSS text merges into that of the next passage without a break, making neither grammatical nor logical sense. Both the end of *Prob.* 35 and the beginning of *Prob.* 35a seem to be missing.

But if sound is a movement of air or of something else, it must be highest in pitch in the middle of its course. Hence if this does not happen, sound cannot be the movement of anything.[54]

4.23 37 Why is it, given that high pitch in sound goes with smallness and low pitch with large quantity (since what is low-pitched is slow because of the quantity, while what is high-pitched is swift because of the smallness), that it is harder work to sing high pitches than low ones, and few people can sing the upper notes [*ta anō*], and the *Orthioi* and *Oxeis nomoi* are difficult to sing because they are tightly stretched?[55] Yet it is a lesser task to move what is small than what is large, and hence this is true of air too.[56] Or is being naturally high-voiced not the same thing as singing high, but all naturally high-voiced things are so through weakness, because they cannot move much air but only a little, and a little air travels swiftly, while in singing, high pitch is a sign of power? For what travels vigorously travels swiftly, and hence high pitch is a sign of power. That is why healthy people have high voices.[57] It takes an effort to sing the upper notes [*ta anō*], but the low-pitched ones are below [*katō*].[58]

4.24 39 Why is correspondence pleasanter than unison?[59] Is it because correspondence is concordance at the octave? Correspondence arises when young children combine with men, whose pitches differ as do *nētē* and *hypatē*.[60] Now every concord is pleasanter than a simple sound (we have explained why already), and the octave is the pleasantest of the concords, while the sound of a unison is simple. People magadize[61] in the concord of the octave,

[54] The question and part of the answer are lost, and cannot confidently be reconstructed. No other passage suggests that the pitch of a sound becomes lower when it has passed the mid-point of its movement – rather the contrary (see 4.2, 4.9, cf. 4.8). But the argument might better be taken as a reductio ad absurdum: sound is not highest in mid-course, and hence it is not the movement of anything. The author could hardly be denying that sound is somehow transmitted, but he might be arguing that in this transmission no *thing* moves from place to place: see 4.2 with n. 9.

[55] On the *nomoi* see *GMW* vol. 1 ch. 15, Appendix A. The two named here were sung at a high pitch ('tightly stretched', *anatetamenoi*).

[56] This is the supposition behind the explanations of the high voices of weak people and animals, e.g., 4.4, 4.5, 4.7, etc.

[57] Weak creatures are 'naturally' high-voiced, in that they can utter only high pitches. Strong and healthy utterers are capable of singing high through the exercise of great force (the other way of accounting for greater velocity, e.g., 4.10), but they can also, presumably, restrain their vigour and sing low.

[58] The final remark has often been thought incomprehensible, and emendations suggested (e.g., 'the lower-pitched ones are easier'). Certainly it is difficult to find much point in the MSS text. The use of *anō*, however, whether or not *katō* is accepted too, is unusual enough to be interesting. Both terms occur also in 4.27, but metaphors of 'up' and 'down' in pitch are uncommon in classical Greek sources (see 3.1 Arist. *Topics* 106a with n. 2, and cf. 10 Nicomachus *Ench*. ch. 3 on the names *hypatē* and *neatē*).

[59] I read *antiphōnon*, 'correspondence' (that of the octave: see n. 47 above) for the MSS *symphōnon*, 'concordance' (only in this sentence; the rest of the text need not be disturbed). The author does indeed hold that concordance is pleasanter than unison, but that is not his topic here: see the third sentence (where the reference is to *Prob*. 38).

[60] See n. 49 above.

[61] 'Magadizing' was done by a two-part choir singing the same melody at different pitches, normally (perhaps always) an octave apart. See particularly *Probs* XIX. 17–18 (with nn. 30–8 at *GMW* vol. 1, pp. 194–5).

because just as in metres the feet exhibit the ratio of equal to equal or two to one or some other, so the notes in a concord have a ratio of movement to one another. In the case of the other concords the endings of one note or the other are incomplete, finishing at the half-way point.[62] For this reason they are not equal in power;[63] and since they are unequal, a difference is present to our perception, just as there is in choruses when some people sing louder than others at the end. Now it happens that *hypatē* has the same ending of the periodic movements in its notes: for the second blow on the air made by *nētē* is *hypatē*.[64] Since they finish at the same time, though their effect is not the same, the function they perform is one and common to them both, as in the case of people who play an accompaniment in subordination to the song. Though elsewhere these people do not play the same notes as the melody, still, if they finish on the same note, the pleasure they give with the ending is greater than the pain they give with the differences before the ending, because the common note, that arising from the octave, comes most pleasingly after differences.[65] Magadizing arises from opposed sounds; and that is why they magadize in the octave.[66]

4.25 41 Why is a doubled fifth or a doubled fourth not concordant, whereas a doubled octave is? Is it because the fifth is in hemiolic ratio and the fourth in epitritic? Where there are three numbers in sequence in hemiolic or epitritic ratio, the extremes will have no ratio to one another, for they are neither epimoric nor multiple.[67] But since the octave is in duple ratio, when it is

[62] The author conceives a sound as constituted by a sequence of impacts, those of a higher note being more frequent. The model he seems to have in mind is that of a string moving back and forth when it has been plucked: cf. 5 *De Audib.* 803b–804a, 9.7 Aelianus ap. Porph. *Comm.* 35. If two strings are tuned, for example, a fifth apart, the higher making three impacts to the lower's two, then when the higher string has completed a back-and-forth movement the movement of the lower is still incomplete. When the first movement of the lower is completed, the higher is part-way through its next oscillation. See also n. 64 below.

[63] That is, presumably, at the time when one string is making an impact on the air but the other has not yet completed a movement.

[64] Where two strings are tuned an octave apart, the higher makes two impacts to the lower's one. Then, although a movement of the higher is completed when that of the lower is not, every movement of the lower is completed at the same time as one of the higher. The theory relates frequency of impact to pitch, but need not imply that higher pitch is caused or constituted by the frequency. It may still be held to depend on velocity of transmission. (Thus the author of 5 *De Audib.* combines a velocity theory of pitch, 803a, with an account of concordance close to that of the present passage, 803b–804a: cf. 9.8 Heraclides ap. Porph. *Comm.* 30.1ff.) The only source that plainly makes pitch depend directly on frequency of impact is 8 Eucl. *Sect. Can.* 148–9. The present account of concordance has affinities with that discussed in 1.8 Porph. *Comm.* 107.15ff: see n. 29 on that passage, and note the comparable roles of the words 'dissimilarity' there and 'difference' here (cf. also 2.5 Plato *Timaeus* 79e–80b).

[65] On this form of accompaniment see *GMW* vol. 1, p. 201, n. 81.

[66] See n. 61 above.

[67] An example of a sequence of three terms in hemiolic ratio is 4:6:9, and one in epitritic ratio is 9:12:16. That all concords are either epimoric (n+1:n) or multiple (mn:n) in ratio is a proposition essential to 8 *Eucl. Sect. Can.* (stated at 149.20–4). At some time it became a fixed tenet of the 'Pythagorean' tradition (see 11 Ptol. *Harm.* Book 1 ch. 5; the divisions of the tetrachord reported for Archytas at 1.21 Ptol. *Harm.* 30.9ff give

doubled the extremes will be in quadruple ratio to each other.[68] Thus since concord exists between notes that are well-ratioed to one another, and since the notes bounding the interval [*dialeimma*] of the double octave have a ratio to one another while those bounding the double fourth or the double fifth do not, those bounding the double octave must be concordant, while the others are not, for the reasons given.[69]

4.26 42 Why is it that if one plucks *nētē* and then damps it, *hypatē* alone seems faintly to sound?[70] Is it because when *neatē* is ceasing and dwindling it becomes *hypatē*?[71] A sign of this is the fact that one can sing *neatē* from *hypatē*, for since it is an answering-song[72] to *neatē*, people grasp the similarity from it. And since an echo, too, is a sort of answering-song to a sound, and since when *neatē* is ceasing an echo identical with the note of *hypatē* is aroused,[73] it is to be expected that *neatē* seems to move *hypatē*, because of the similarity. For we know that *neatē* is not moving, since it has been damped: we see that *hypatē* itself is not damped, and hear its note, and hence we suppose that it is sounding. This happens to us in many cases where we are unable to attain precise understanding through either reasoning or perception.[74]

Again, it would not be surprising if when *neatē*, which is very tightly stretched, has been struck, the bridge were moved too. If it were moved, it would be reasonable to expect all the strings to be moved with it, and to make some sound. Now the note of *neatē* is alien to the other strings both when it is ceasing and when it is beginning, whereas it is the same as *hypatē* when it is ceasing.[75] When this note is added to its own proper movement, it is quite natural for it all to seem to belong to *hypatē*. It will be greater than the combined sound of the remaining strings, because those that have been as it were propelled by *neatē* sound weakly, while *neatē* sounds with all its own

epimoric ratios a still more pervasive role, as does Ptolemy himself). The argument of the present sentence is a special application of 8 Eucl. *Sect. Can.* proposition 4.

68 See 8 Eucl. *Sect. Can.* proposition 12.

69 There are obvious affinities between this passage and 8 *Sect. Can.* But the author of the latter never says that integral terms not in multiple or epimoric ratio have no ratio to each other: they are in 'epimeric' ratio, *Sect. Can.* 149.13, what Plato calls a relation of 'number to number' (2.3 *Timaeus* 36b). The contrast here between having 'no ratio' and 'being well-ratioed' echoes 3.11 Arist. *De Sensu* 439b–440a, and poses the same problems (see nn. 21 and 22 on this passage).

70 For the question, see 4.20.

71 The author may have in mind the 'impact' theory of 4.24. Then if as *nētē* dwindles alternate impacts become too faint to register on the hearing, what will be left is the impact-pattern of *hypatē*. See also the next paragraph.

72 This translates *antōidē* literally: cf. 4.17. *Neatē* is a variant spelling of *nētē*, found also, for example, at 1.12 Philolaus frag. 6.

73 This 'echo' is thus an octave lower than the original: contrast 4.2 with n. 4.

74 The author seems to mean that the *hypatē* string is in fact not sounding, but we think that it is, since no other source for the note can be observed, and in other cases of similar obscurity we also commonly jump to unwarranted conclusions.

75 This seems to imply a theory like that of 4.24: 'the second blow on the air made by *nētē* is *hypatē*'. In this paragraph, unlike the end of the preceding one, the explanation assumes that the *hypatē* string is indeed emitting a sound (alternative and incompatible explanations are common in the *Problems*).

power, it being the most vigorous of them. Thus it is to be expected that its own second impulse should be more powerful than that of the others, especially since only a slight movement has been generated in them.

4.27 47 When people in ancient times created the *harmoniai*, why did they retain *hypatē* but not *nētē*? Or was it not *nētē* that they took out, but the string now called *paramesē* and the interval of the tone?[76] They used *mesē* as the last string of the *pyknon* in the high-pitched part, which is also why they called it *mesē*. Or is it because *mesē* was the end of the upper [*anō*] tetrachord and the beginning of the lower [*katō*] one, and stood in the relation of a mean [*meson logon*] in pitch between the extremes?[77]

4.28 50 Why is it that if there are two similar vessels, equal in size, and if one is empty while the other is half-full, the sound gives the concord of an octave? Is it because the sound from the half-vessel is double that of the empty one?[78] After all, what difference is there between this case and that of *syringes*?[79] For it seems that a swifter movement is higher-pitched, and in larger things the air impinges more slowly: in things double the size it does so that number of times more slowly, and proportionately in other cases.[80] A leather bag of double size also gives the concord of an octave with one half its size.[81]

[76] For references to the ancient seven-note *harmoniai* and an indication of the puzzles surrounding them, see 1.12 Philolaus frag. 6 with n. 34, and particularly the passages of Nicomachus cited there. Here the reference is clearly to a system of two conjoined tetrachords (*mesōn* and *synēmmenōn*), together spanning a seventh. The question whether it was *nētē* (*diezeugmenōn*) or *paramesē* that was omitted seems largely terminological, but perhaps reflects the author's sense that to analyse the structure mechanically, as a version of the disjunct structure with the top note missing, is to misrepresent the manner in which the system is put together.

[77] The last sentence explains only why this note was called *mesē* ('middle note'), rather than answering the original question. Cf. *Probs* XIX.25, 32. It is genuinely midway between the extremes only in the conjunct system. On the terminology of 'upper' and 'lower' see 4.23 with n. 58.

[78] Cf. 1.4 Theon. Smyrn. 59.4 with n. 11.

[79] See 4.19: the reference here is evidently to the *polykalamos syrinx* or Panpipe.

[80] The theory seems to be that the sound is caused when a movement of the air inside the vessel or pipe causes movement in the air outside (either by striking it directly, or by transmitting motion through the material of the vessel itself, where the vessel is closed). Cf., for example, 1.19 Archytas frag. 1 (on *auloi*), 9.7 Aelianus ap. Porph. *Comm.* 34, 10 Nicomachus *Ench.* ch. 4.

[81] The bag is perhaps an inflated wine-skin, used like a drum. In that case the proposition is false, just as are those in the stories told at 10 Nicomachus *Ench.* ch. 6, 12 Arist. Quint. *De Mus.* Book III ch. 1.

CHAPTER 5

The Peripatetic *De Audibilibus*

The *De Audibilibus* is preserved only as a quotation in Porphyry's *Commentary on Ptolemy's Harmonics*. Porphyry quotes it at length but not in full, as he himself makes clear, and the missing passages have not survived elsewhere. It is plainly a work of the Peripatetic school, not later than the mid-third century B.C. (and probably rather earlier), but its authorship is uncertain. Porphyry ascribes it to Aristotle, and its general approach and the theories it propounds are certainly close to his, but few modern scholars have accepted the attribution. It has been variously ascribed to Theophrastus, Heraclides Ponticus and Strato. The most recent detailed discussion is that of Gottschalk (1968), who gives cogent reasons for rejecting Theophrastus and Heraclides as possible authors. His arguments for Strato and against Aristotle seem to me inconclusive. (Strato followed Theophrastus, Aristotle's immediate successor, as head of the Peripatetic school, a position he held from about 287 to 269 B.C. He was a physical theorist of some originality.)

I cannot pursue the issues in detail here, but two important ones may be briefly mentioned. First, Gottschalk draws attention to the treatise's view (especially in 800a) that sound is propagated not as a moving current of air, or as air particles travelling from a source to the ear, but by 'vibrations' ('pulsations' might give the sense better) through a medium that remains in its place. This analysis seems correct, or nearly so (since it is not clear that the medium is thought of as entirely stationary), and Gottschalk concedes that Aristotle's view was similar. But he holds that Aristotle could not have agreed, as the *De Audib.* asserts (especially 803b–804a), that the prime cause of any apparently continuous sound is a succession of pulses that are in fact detached from one another. I do not think that the Aristotelian texts are sufficiently clear-cut over this issue for us to decide the point (see n. 39 below).

Secondly, Gottschalk holds (on the basis of *De Audib.* 803b–804a) that this treatise has a theory of the determination of pitch close to that of 8 Eucl. *Sect. Can.* 148–9, in which a higher pitch is caused or constituted by the greater frequency with which 'vibrations' follow one another through the medium. In that case its view will be different from Aristotle's. But to sustain this interpretation it is necessary to treat the passage as offering a theory of pitch-causation, which it does not in fact appear to do, and to dismiss as mere carelessness two other passages (801a, 803a) whose plain implication is that pitch depends on the velocity of a sound's transmission, a view that Aristotle shares (see n. 40 below). I would not go to the stake for the thesis that Aristotle wrote the *De Audib.*, but I would judge that the case against Porphyry's attribution is still not proven. Certainly, however, if another author has to be found, Strato is much the strongest candidate (see especially Gottschalk, pp. 453–4).

The work, as we have it, opens (almost certainly the beginning is lost) with a discussion of the way in which sounds are produced and transmitted, and an account of the physiology of vocal utterance. Most of the remainder is concerned with the causes of various modifications of sound's perceptible qualities (for example, apparent nearness or remoteness, brightness, clarity, dullness, roughness and so on). In every case the author seeks to show that the cause is something that affects the manner in which the sound-pulses move, and he is guided by the principle that the object causing the

alteration has the same quality as that given by it to the sound, so that thin things create 'thin' sounds, rough ones rough sounds, etc. The principle may be naive, but it is pursued with some ingenuity, if not with scientific rigour. There is no attempt to explicate the relevant causal conditions mathematically, and only the accounts of pitch and concordance would readily have lent themselves to development in that direction.

The author's main interest is in the human voice. But in setting out his views on the mechanisms that cause qualitative change in vocal sounds, he frequently resorts to analogies with musical instruments, some of which are pursued at length. He has interesting reflections, in particular, on the properties of horns (probably the 'bells' attached to certain wind instruments), on the qualities of strings and of the reeds and other parts of *auloi*, and on the results of different playing techniques. While his understanding of the causes of the phenomena he mentions is limited, he is an acute observer, and makes useful contributions to our knowledge of the materials, structures and acoustic properties of Greek instruments.

5 The *De Audibilibus*

800a It is a fact that all voices and sounds occur when either bodies collide with bodies or the air collides with bodies, not by the air being shaped, as some people think, but by its being moved in just the same way – being contracted, expanded, caught up with and in collision – as the result of impacts made by the breath or by strings.[1] For when the breath that impinges on the air strikes the air next to it, the air is at once forcibly moved, pushing forward in the same way the air continuous with it, so that the sound is stretched out and remains the same throughout, to the limit of the distance to which the movement of the air goes on.[2] For when the movement of the air takes place, its force spreads out over a wider area, like the winds that blow from rivers and from the land.[3] Sounds that are smothered at their source are muffled and cloudy, while sounds that are clear extend to a greater distance, and fill up all the intervening space.

[1] For theories to do with the 'shaping' of the air see 4.7, 4.9 ps.-Ar. *Probs* XI.16, 20, and 6 Theophrastus ap. Porph. *Comm.* 64.4–19; cf. 4.11 *Probs* XI.23, 9.3 Adrastus ap. Theon Smyrn. 66.1–3. That sound is in some way due to movements of the air caused by impacts on it is the view of most ancient authors, first clearly articulated by Archytas: see 1.19 Archytas frag. 1 with the references at n. 45. 'In just the same way' probably means 'in just the same way throughout its course', not 'in just the same way as other objects that are moved'. The uniformity of the developing process is an important part of the theory, and the air's movement is in a significant respect not like that of a missile: see the next note.

[2] On this 'shunting' theory of the air's movement see 4.2 ps.-Ar. *Probs* XI.6 with n. 9. The author is trying to express the idea of the transmission, not of air itself, but of pulsations through a more or less stationary medium. The list in the previous sentence ('contracted, expanded...') refers to parts of a single process, not to alternative methods of generating movement. When something strikes the air adjacent to it, that air is contracted and expands again, and thus 'catches up' and collides with the next portion of air, which in turn repeats the process. How these 'portions' are to be distinguished from one another is unclear in view of the air's 'continuity' (see also 3.15 Arist. *De Anima* 419b–420a).

[3] Though the movement is continuous and 'the same throughout', it is not in a single line, but is continually dividing into branches (cf., for example, 4.2 ps.-Ar. *Probs* XI.6 with n. 10). It might be conceived as a spreading continuum, rather than a network of multiplying streams, but see 801a with n. 10. In either case the point is that sound travels all around, not in one direction only (see the same *Problem*, with n. 13).

We all breathe the same air, but the breath and the sounds that we emit vary in correspondence with the differences in the vessels involved, those through which the breath of each of us penetrates to the region outside.[4] These are the windpipe, the lung and the mouth. Very great variation in the voice is created by the impacts of the air and by the shapings of the mouth. So much is obvious; for this is the cause of all variations in sounds, and we see the same people imitating the voices of horses and frogs and nightingales and cranes and virtually all other animals, using the same breath and the same windpipe: they do it by propelling the air from their mouth in different ways. Many birds, too, when they have listened to the voices of others, imitate them by the means we have mentioned.

As to the lung, when it is small and dense and hard, it cannot admit much air into itself or propel much back again outwards, nor can it make the impact of the breath powerful or strong. For since it is hard and dense and constricted, it cannot sustain expansion to a great extent, nor can it squeeze out the breath
800b vigorously by contracting itself again from a large expansion, just as we cannot with bellows, when they are hard and incapable of being easily expanded or compressed. For it is this that makes the impact of the breath strong, when the lung contracts itself from a large expansion and squeezes the air out violently.[5]

This is plain; for none of the other parts of the body can make a powerful blow from a short distance. It is not possible for the leg or the hand to strike something violently, or to propel the thing struck a long way, unless the person lifts either limb to a good distance to make the blow. Otherwise the blow is hard because of the tension, but the thing struck cannot be driven far off, since catapults cannot throw things far, and neither can the sling or the bow, if it is hard and incapable of being bent, and if the string cannot be pulled back a long way. But if the lung is large and supple and springy, it can admit a lot of air, and expel it again, controlling it however it wishes because of its suppleness and because it contracts itself readily.[6]

When the windpipe is long and narrow, people emit their voice with difficulty, and with great force, because of the long distance that the breath travels. This is evident; for all creatures with long necks utter sounds with violence, as do geese, for instance, and cranes and cocks. This is even clearer in the case of *auloi*. For everyone has difficulty in filling the *bombykes*,[7] and uses great tension, because of the length of the distance involved. Again, because of the narrowness of the space, when the breath has been compressed inside and escapes to the outside, it is at once dispersed and scattered, just like

[4] The author is concerned to deny that differences in sounds are the results of qualitative differences in the air or breath used to generate them. He makes no use of the idea that in some circumstances the air inside us is thickened by heat (3.17 Arist. *De Gen. An.* 788a with n. 60). But he allows for some such modifications: see 801a.

[5] With this account of the propulsion of breath contrast 3.17 Arist. *De Gen. An.* 787b, and 4.5 ps.-Ar. *Probs* XI.14.

[6] These points are elaborated further at 802b–803a.

[7] *Bombykes* may be the pipes forming the body of the instrument, in which case the remark applies to *auloi* in general (cf. 6 Theophrastus ap. Porph. *Comm.* 63.1–11). But the reference may be to a special version, the *bombyx aulos*, on which see 8 Eucl. *Sect. Can.* proposition 19, n. 66.

streams carried through narrow ditches, so that the voice cannot be sustained or extend to a great distance. At the same time the breath of such creatures is inevitably hard to control and does not readily obey them.[8]

In the case of people in whom the hole in the windpipe is large, the breath can easily come through to the outside: but when it travels inwards it is dispersed because of the width of the space, and the voice becomes hollow and
801a unfocussed. Such people cannot create distinctions with their breath, because their windpipe is not firmly held together. People whose windpipe is uneven and does not have the same width throughout are inevitably involved in every sort of difficulty. For their breath must obey them unevenly, and must be contracted and then dispersed again at the next place. When the windpipe is short the breath must be expelled quickly and the impact on the air must be more powerful, and all such people must utter sounds at a higher pitch because of the speed with which the breath travels.[9]

However, it is not only the properties of the vessels, but also all sorts of incidental modifications that cause variation in voices. For when the lung and the windpipe are full of a large amount of moisture, the breath is pulled different ways and cannot come through continuously to the outside, because it meets obstructions, and becomes thick and moist and hard to move, as happens in the cases of catarrh and drunkenness.[10] If the breath is absolutely dry, the voice becomes harder and fragmented, for moisture, when it is slight, holds the air together and creates a kind of singleness of voice.[11]

Thus each of the differences in the vessels, and in the incidental modifications that affect them, generates voices of the kinds we have mentioned. Now voices appear to be in the places in which, on any occasion, they are produced, but we hear all of them when they impinge on our hearing. For the air propelled by the blow travels as a coherent whole for a certain distance, and then becomes progressively more fragmented, in small stages.[12] It is through this that we recognise all sounds, both those that are produced at a distance and those nearby. This is plain; for when someone takes a jar or an *aulos* or a *salpinx*, points it at another person, up against his ear, and talks through it, all the sounds seem to be very close to the ear because the air is not dispersed as it travels, and the voice is preserved in the same state by the instrument that surrounds it. Just as in painting, if one makes this part similar in colour to what is at a distance, and that part similar to what is nearby, the former appears to recede from the painting and the latter to stand out, though both of them are

[8] Cf. 3.17 Arist. *De Gen. An.* 787b.

[9] Despite the indications at 803b–804a, the author evidently holds a form of the 'velocity' theory of pitch: see also 803a.

[10] Cf. 4.3 ps.-Ar. *Probs* XI.11.

[11] That is, slight moisture unifies the breath with which we strike the external air, and without this unification a distinct sound cannot be made: cf. 3.15 Arist. *De Anima* 419b, and 802b below.

[12] The author probably means that the air moves 'as a whole' for only a very short distance indeed, perhaps corresponding only to the first stage of the 'shunting' movement. Its pulsation then splits into a progressively more ramified network: see 800a with nn. 2 and 3.

on the same surface, so it is with sounds and with the voice. For when this one strikes the ear after already becoming dispersed, while that one strikes it as a coherent whole, then though both of them arrive at the same place, the former seems to be at a long distance from the hearing and the latter to be nearby,
801b because the one is similar to what comes from a distance, the other to what is close at hand.[13]

Voices are clear in proportion to the precision of the sounds. For if these are not perfectly articulated it is impossible for the voices to be clear, just as it is for the seals on rings when they are not precisely engraved. That is why children cannot speak clearly, and neither can people who are drunk, or the old, or people with a natural speech-impediment, or in general those whose tongues and mouths are difficult to move. For just as bronze or horn, by resonating, make the sounds of an instrument less clear, so in the case of speech the impulses of breath that come out of the mouth create a great deal of unclarity when they are not evenly formed.[14] Not only do they display lack of clarity in themselves, but they also impede those sounds that are well articulated, since their movement in relation to the hearing is uneven. That is why when we hear one person we understand him better than when we hear many people saying the same thing simultaneously. The same is true of strings: when the *aulos* is played in consort with the *kithara* [we grasp the sounds] much less, because the sounds of the one are confused with those of the other.[15] This is particularly evident in the case of concords, for both the sounds are concealed, each by the other.[16]

Sounds lose clarity, then, for the reasons we have mentioned. They become bright in the same way as do colours, for in their case too the colours that are best able to move the faculty of sight are the brightest. In the same way we should take it that the brightest sounds are those that are best able to move the sense of hearing when they impinge on it. This class includes those that are clear, dense, pure, and capable of stretching out to a distance. For in the case of all other objects of sensation the strongest and densest and purest make the sensation clearer. This is evident; for all sounds eventually become silent, when the air has been dispersed. It is plain too in the case of *auloi*. For those

[13] This ingenious explanation obviously presupposes that our ears can detect the difference between 'dispersed' and 'coherent' movements. For a rather different comparison with colour see 6 Theophrastus ap. Porph. *Comm.* 63.30ff.

[14] Reading *asaphesterous* ('less clear') for the MSS *saphesterous* ('clearer') at line 11. On horn resonators see the detailed discussion at 802a–b. Here the point is that they add extra movements which obscure the original, so reducing its clarity (not, of course, its volume): see n. 23 below.

[15] Cf. ps.-Ar. *Probs* XIX.43, where the blend of sounds formed by a combination of *aulos* and voice is said to be more pleasant than that of combined voice and *kithara*, precisely because the former mingle more completely and are not heard as distinct entities. 'Blended' sounds are often said to be pleasanter than unmixed ones (e.g., 3.16 Arist. *De Anima* 426a–b, ps.-Ar. *Probs* XIX.38). This is perfectly compatible with the present author's statements about 'clarity': see also 802a.

[16] This reflects the usual conception of a concord as that in which two sounds blend so that neither is heard distinctly in its own right: cf., for example, 3.13, 3.14 Arist. *De Sensu* 477a–b, 448a, 8 Eucl. *Sect. Can.* 149.17–20.

mouthpieces that have their tongues angled obliquely give out a softer sound, but one that is not equally bright; for as the breath travels, it falls directly into a wide space, and no longer travels under tension or compression, but is scattered. But in tongues that are closely united, the sound is harder and brighter, if one presses on them more firmly with the lips, because the breath travels more violently.[17]

802a Bright voices occur, then, for the reasons we have mentioned. Hence it seems that so-called 'grey' voices are no worse than 'white' ones.[18] For rougher sounds that are a little confused, and which do not have too pronounced a brightness, are more suited to emotions and to the later times of life.[19] At the same time, they [bright sounds] are not so easy to control, because of the tension, for what is propelled violently is unmanageable:[20] nor is it easy to raise or lower the pitch as one wishes. In *auloi* and in other instruments, sounds are bright when the breath that emerges is dense and under tension, for the blows of the external air[21] must also be of the same character, and in particular the sounds must be conveyed to the ear while constituted in this way, as is the case with smells and light and heat. For all these things are less noticeable to the senses when they appear more diffuse, as also do flavoured juices mixed with water or with other juices. For that which offers a sensation of itself makes the powers of each other thing less clear.[22] If the resonances of horns attached to other instruments are dense and coherent when they strike the air, they make

[17] Both the text and the sense of this passage are disputed. I read *zeugōn* ('mouthpieces') for the MSS *deuterōn* in line 34, and *synkrotētikais* (perhaps *synkrotōterais*) for *synkroterais* in line 38. It has been suggested that the *auloi* whose tongues (*glōttai*) are angled obliquely are ones with a single or 'beating' reed (like that of the clarinet), formed by a cut in the side of the pipe, instead of the double reed of the commoner instruments. Alternatively, they might be what were called *plagiauloi* (e.g., 10 Nicomachus *Ench.* ch. 4, cf. Athenaeus *Deipn.* 175e), which may be the same as the instrument described at Apuleius *Met.* XI.9.6. This was played in a position like that of the modern flute, and probably had a double reed inserted at an angle to the main pipe. In the former case the contrast is with the double reed (*synkrotōtikos* will mean 'beating together'); in the latter the point of the comparison is unclear. More probably, I think, the key is in Theophrastus *Hist. Plant.* IV.2.4–5, quoted in *GMW* vol. 1 ch. 13, which in my view contrasts two kinds of double reed. One (used in older and simpler forms of *aulos*-playing) had the parts of its reed tightly closed together, generating greater projection. The other (used in the later, more elaborate style) retained a wider opening. The latter kind will correspond to the 'angled' reeds of the present passage, the two elements being angled apart from one another, the former to the 'closely united' ones, which make a 'harder' and 'brighter' sound, especially when compressed with the lips. (See also n. 20 below, and cf. 804a.) For a review of opinions and evidence see Düring (1934), pp. 169–70.

[18] On the application of these colour-words see 3.1–3.3 Arist. *Topics* 106a, 106b, 107a.

[19] For the idea that morally and emotionally different music is suited to different ages see, for example, Arist. *Pol.* 1342b, where the 'relaxed' *harmoniai* rejected by Plato (*Rep.* 398e) are accepted on these grounds: cf. 12 Arist. Quint. *De Mus.* 58.28–32. Here the point is that 'clarity' of sound is not always musically desirable: cf. n. 15 above.

[20] Thus the elaborate style of playing mentioned by Theophrastus (n. 17 above) is not possible with the kind of reed that makes 'bright' sounds.

[21] These are the blows made by each portion of air on the air next to it, and ultimately on the ear.

[22] Cf. 3.12, 3.16 Arist. *De Sensu* 447a–b, *De Anima* 426a–b, and 6 Theophrastus ap. Porph. *Comm.* 63.15ff.

the sounds dim.[23] Hence the horn should be even and smooth in its natural growth, and not have sprung up too quickly, for such horns must be softer and more porous, so that the resonances are dispersed and do not emerge from them as coherent wholes; nor do they ring out so well, because of the softness and diffuseness of the pores. Nor again should the natural growth be retarded, nor should its natural composition be dense, hard or rigid. For whenever the resonance meets an obstruction as it travels, it is checked there, and no longer comes through to the outside, so that from horns of this sort the resonances emerge muffled and uneven. Now the fact that the movement is in a straight line is plain from the example of masts, when people test them: for when they tap them at one end, the resonance travels continuously to the other, unless the piece of timber has a crack, in which case the sound advances up to this point, and is then dispersed and ceases. (It also bends round the branches, and is not able to go through them in a straight line.) This is quite obvious in the case of bronze objects too, when people file the folds or pleats of the drapings of statues, to close up the slits: it makes them give off a loud whistling noise. But if one ties a rope round them, the noise stops, for the vibration advances up to
802b that point, but when it strikes something soft, it comes to a halt there.

Baking the horns contributes greatly to pleasantness of sound, for those that have been baked have a resonance more like that of pottery, because of their hardness and the aridity produced by firing. But if one bakes them insufficiently, they give out a gentler resonance because of their softness, but cannot ring out so well. This is why people select horns of specific ages: those of old creatures are dry and hardened and porous, while those of young ones are thoroughly pliable and contain plenty of moisture. As we have said, the horn should be dry and of even density, smooth, and with a straight bore, for in this way the resonances too will best travel through it in a dense, smooth and even condition, and the blows on the external air will have the same character, since in strings, too, the best are those that are smoothest and most even in every part: their construction is even throughout, and the plaitings of the fibres are hard to discern.[24] In this way they make the most regular impacts on the air.

[23] The beginning of the sentence might mean 'unlike the other instruments, if the resonances of horns are dense...', treating the horn (*keras*) as an independent instrument, with different properties from others. (On either translation the structure of the Greek is a little awkward.) But there is nothing in the sequel to explain why the sound of a horn, so conceived, will be dim if its resonances are dense and coherent; one would expect the opposite. The distinction between the sounds (*phōnai*) and the resonances (*ēchoi*) suggests that clear and coherent reverberations of a resonator obscure the intrinsic sound (*phōnē*) of the instrument, making it *amauros*, 'dim'. This will apply to any instrument equipped with a resonating bell (as some *auloi* were, notably those called 'Phrygian' or *elymoi auloi*: see *GMW* vol. 1, p. 267, n. 31). See also 6 Theophrastus ap. Porph. *Comm.* 64.8ff, and compare the description of the ear resonating 'like a horn' at 3.15 Arist. *De Anima* 420a. If this is correct, the sequel treats the obscuring of the principal sound by the resonance as musically desirable, since the author recommends horns that will produce the coherent and 'dense' *ēchoi* mentioned here. This is consistent with the aesthetic view expressed at the beginning of the paragraph.

[24] A string on a Greek instrument was usually made of several strands of gut, twisted together.

The reed-tongues of *auloi* should be dense and smooth and even, so that the breath may pass through them in a smooth and even state, without being dispersed. This is why reeds that have been moistened and those that have been soaked in oil have a pleasanter sound, while the sound of dry ones is unpleasant.[25] For air that travels through something moist and smooth remains soft and even. This is evident; for the breath itself, when it contains some moisture, is much less impeded on the pair of reeds and is less dispersed. Dry breath is more obstructed, and makes the impact harder because of the force involved.[26]

Differences between sounds occur, then, for the reasons mentioned. Voices are hard if they impinge violently on the hearing: that is why they are especially painful. To this type belong those that are harder to move, and which are propelled with great force, for what yields swiftly cannot resist the blow, but is the first to recoil. This is evident; for the heaviest missiles travel with the most violent movement, as do streams that flow through narrow ditches. These are most violent around the narrow parts, where they cannot quickly retreat, but are pressed upon with great force. The same sort of thing happens with
803a voices and sounds. This is plain, for all violent sounds are hard, as are those of boxes and hinges when they are violently opened, and those of bronze and iron. The sound from anvils is hard too, especially when they beat iron that is already cooled and hard, and so is that from files, when they file and sharpen iron objects and saws, just as the most violent thunder-claps are hardest, and so are the rain-storms that people call 'furious' in their violence.[27]

Swiftness of the breath makes the voice high-pitched, and force makes it hard.[28] Hence not only do the same people sometimes utter higher sounds and sometimes lower, but also sometimes harder and sometimes softer. Yet some people suppose that it is because of the hardness of the windpipe that voices become hard: but they are wrong.[29] This factor no doubt makes some very small contribution,[30] but the main cause is the impact of the breath made violently by the lung. For just as some people's bodies are moist and soft, while those of others are hard and tense, so in the same way is the lung. That is why some people's breath comes out in a soft condition, while that of others comes out hard and forceful; for it is easy to see that the windpipe itself can provide

[25] On the preliminary softening of *aulos* reeds see Theophrastus *Hist. Plant.* IV.2.4.

[26] On the role of moisture in the breath see 801a above.

[27] With these reflections on hardness and violence compare 800a–b above. For a further discussion of the sound of the file see 803b.

[28] The author has thus made a further distinction among the conditions identified by Archytas (1.19 frag. 1) as the causes of high pitch (Archytas had treated violence and swiftness as facets of the same condition). Compare the attempts by Aristotle and others to adapt Archytas' account so as to distinguish the causes of high pitch and of loudness, e.g., 3.17 Arist. *De Gen. An.* 786b–787a, 4.1 ps.-Ar. *Probs* XI.3.

[29] Possibly the author has Aristotle in mind, though the evidence is not conclusive. The last sentences of *De Gen. An.* 788a (in 3.17) might hint at a proposal to make such variations in sound quality dependent on the texture of the windpipe. This, together with Aristotle's apparent neglect of the role of the lung, may have been enough to inspire the criticism.

[30] On this qualification see 803a–b with n. 35.

very little power. No windpipe is as hard as are *auloi*, but for all that, when the breath travels through the windpipe and through *auloi*, some people pipe softly and others harshly. This is clear also from the evidence of sensation itself, for if one strains the breath more violently, the voice at once becomes harder because of the force, even if it is quite a soft voice.[31] Much the same holds of the *salpinx*, which is why everyone, when they are merrymaking, relaxes the tension of the breath in the *salpinx*, to make the sound as soft as possible.[32]

This is evident also in the case of instruments. Thus tightly twisted strings make harder sounds, as we have said, and so do horns that have been baked.[33] And if one plucks the strings with the hands violently and not gently, they will necessarily make a more violent response in the same way again. But strings that are less tightly twisted and horns that are less baked make softer sounds, and so too do larger instruments. For the blows on the air are slower and gentler because of the size of the spaces involved, while those made by shorter instruments are hardest because of the tension of the strings. This is evident; for the sounds of a single instrument are hardest when one plucks the strings elsewhere than in the middle, since the parts of them that are next to the cross-bar and the string-holder are under greater tension.[34] Instruments made of
803b fennel stalks have gentler sounds, since the sounds, impinging on something soft, do not rebound with the same degree of force.[35]

Sounds become rough when there is not one single impact of all the air together, but it is fragmented into small and frequent impacts. For each portion of air falls separately upon the hearing, as though from a distinct impact, and creates a fragmented sensation, so that one sound fails while another impinges more vigorously, and the contact with the hearing is uneven, just as when something rough impinges on our skin.[36]

This is clearest in the case of the file. For because the impact on the air occurs in many small pieces, the sounds that come from them to strike the hearing are rough, and more so if the file is rubbed against something hard, as in the case of touch: for hard and rough things cause a more violent sensation. It is evident also in the case of things that flow; for of all liquids oil has the least perceptible sound, because of the cohesiveness of its parts.

Thin sounds are those that occur when the breath expelled is little in quantity. That is why the voices of children are thin, and those of women and

[31] 'Soft', *malakos*, here and throughout the paragraph, does not mean 'quiet': the contrast between soft and hard is thought of as quite distinct from that between quiet and loud.

[32] The *salpinx* (a kind of trumpet) was much more often used for martial purposes (for giving signals, etc.) than for strictly musical ones.

[33] See 802b above.

[34] The phenomenon is well observed. Possibly the author's remarks reflect the existence of techniques used by musicians to generate special effects. The illustrations in Paquette (1984), chs. 3 and 4, for example, show considerable variation in the positions at which the strings of the *kithara* and *lyra* were struck or plucked.

[35] This indicates the source of the 'very small contribution' to hardness of sound made by the constitution of the windpipe (803a above). Violence of movement is still the key, but it is liable to be reduced if the surface from which the movement rebounds is soft.

[36] Cf. 3.17 Arist. *De Gen. An.* 788a, 4.3 ps.-Ar. *Probs* XI.11, and 801a, 802b above.

eunuchs, and, in the same way, those of people enfeebled by illness or labour or lack of food; for because of their weakness they cannot emit much breath. It is clear, too, in the case of strings: for the sounds from thin strings are thin and narrow and hairlike, because the impact of the air also occurs on a narrow front.[37]

Thus whatever character may belong to the sources of the movement of the impacts of the air, the sounds that fall on the hearing will be of the same kind, diffuse or dense, for instance, or soft or hard or thin or thick. For as each portion of air moves the next in the same way, it makes the whole sound alike, as is the case also with high and low pitch. For the velocities of impact, each succeeding closely on another, ensure that the sounds remain similar in character to their sources.[38]

The impacts made on the air by strings are many and separate, but because of the smallness of the time between them the ear is unable to detect the gaps, and hence the sound seems to us single and continuous, as is also the case with colours. For separate bits of colour often seem to us to be joined to one another, when they are moving quickly.[39] The same thing happens with
804a concords: because the one set of sounds is included along with the other, and their cessations occur simultaneously, the intervening sounds escape our perception. For in all concords the impacts of the air belonging to the higher notes occur more frequently, because of the swiftness of the movement; but the last of the sounds strikes our hearing simultaneously with the sound from the slower movement. Thus since the ear cannot detect the intervening sounds, as we have said, we seem at the same time to hear both notes continuously.[40]

37 On 'thinness' of sound see particularly 4.2, 4.7–4.9 ps.-Ar. *Probs* XI.6, 16, 19, 20. On the conditions of femaleness, immaturity, weakness etc., see 3.17 Arist. *De Gen. An* 786b ff. with the references at n. 52.

38 Compare the opening paragraph of the treatise, 800a. The view summarised here, that the audible qualities of sound are 'the same' as the tangible qualities of the things that initiate and modify their movements, is the principal thesis that the author's discussions are designed to establish. The impacts mentioned in the last sentence are evidently those made by each portion of air on its neighbour during the propagation of a single sound, not successive impacts of a single sound-source on the air, as in the next paragraph.

39 The analogy with the perception of colours is elaborated at 9.8 Heraclides ap. Porph. *Comm.* 30.1.ff. For other discussions that depend on the thesis that a plucked string makes many discrete impacts on the air, though they are not perceived as such, see also 4.23, 4.24 ps.-Ar. *Probs* XIX.37, 39, 8 Eucl. *Sect. Can.* 148–9, 9.7 Aelianus ap. Porph. *Comm.* 35, cf. 1.8 Porph. *Comm.* 107.15ff. with n. 29. The claim that the gaps are undetectable implicitly contradicts the argument at 3.14 Arist. *De Sensu* 448a20ff., but the issue there is different: see also 3.11 *De Sensu* 439b, which allows the possibility of something's being perceptible as a continuum when it is in fact composed of separate but imperceptible fragments laid side by side.

40 The theory is similar to that implied most clearly at 4.24 ps.-Ar. *Probs* XIX.39 (see nn. 62 and 64 on that passage). The present author is trying to account for two phenomena at once: that we do not detect the gaps between impacts, and that in the special case of concord we notice only the 'blending' of coincident impacts, not the disruptive effect of those impacts that intervene between coincidences and fail to match. (The latter are the 'dissimilars' of 1.8 Porph. *Comm.* 107.15ff.: cf. 2.5 Plato *Timaeus* 79e–80b.) The author is not, however, offering a theory of the determination of pitch. There is nothing here to suggest that height of pitch is caused or constituted by greater frequency of impact (as in 8 Eucl. *Sect. Can.* 148–9). The *De Audib.* adopts the more common view

Thick voices, by contrast, are those that occur when the breath is expelled as a large quantity massed together. That is why those of men and of *teleioi auloi* are thicker, especially when one fills them with the breath.[41] This is clear; for if one compresses the pair of reeds, the sound becomes higher-pitched and thinner, as it does if one pulls down the *syringes*: and if one closes them, the bulk of the sound becomes fuller because of the quantity of breath, just as it does from thicker strings.[42] Voices are also thick in people whose voices are breaking, or whose throats are sore, or after vomiting, because of the roughness of the windpipe, and because the voice cannot escape, but striking against the windpipe is massed together there and gains bulk, and especially because of the moistness of the body.

Shrill voices are those that are thin and dense, such as those of grasshoppers, locusts and nightingales, and in general those thin ones that are not followed by any resonance from another source. For in general shrillness does not go with bulk of sound, or with relaxed and low-pitched tones,[43] nor with conjunctions of notes, but with high pitch and thinness and precision. That is why thin and tense instruments without horns make shriller sounds; for a resonance that comes off water, or in general one that follows from something else, does not preserve the precision of the notes.[44]

Cracked and broken voices are those that travel as coherent wholes up to a certain distance and are then fragmented. This is most obvious in the case of pottery: for any pot that is broken by a blow makes a cracked sound, since the movement is fragmented around the point of impact, so that the sounds that emerge are no longer cohesive. The same thing happens with broken horns, and
804b with badly plaited strings. For in all such cases the sound travels cohesively up to a certain point, and is then fragmented at the place where the material in

that pitch is fixed by velocity of transmission (see 801a, 803a), though his notion of what it is that is transmitted is different from that of some earlier sources (see 800a). But a string that moves in such a way as to generate greater velocity and hence higher pitch will also, as a secondary effect, generate more frequent impacts. (The contrary view, that the *De Audib.* does imply the theory of pitch-causation spelled out in the *Sectio Canonis*, is argued in Gottschalk (1968).)

41 *Teleioi* ('complete' or 'perfect') *auloi* appear in Aristoxenus' classification of *auloi* by their ranges of pitch (Athenaeus *Deipn.* 634f). They are the second-lowest of the types he mentions, and were used (according to Pollux *Onomastikon* IV.81) for solo performances without voices, and for accompanying paeans. On the notion that the pipe must be 'filled with breath' see, for example, 6 Theophrastus ap. Porph. *Comm.* 63.6–11.

42 On the results of compressing the reed mouthpiece see 801b. The nature of the process of 'pulling down' and 'closing' the *syringes* is problematic. The solution most commonly accepted is that of Howard (1893), according to which the *syrinx* of the *aulos* is a small hole not far below the mouthpiece, analogous to the 'speaker hole' on a clarinet, which could be opened to facilitate access to high harmonics. It was probably closed with a ring, which was pulled down to open the aperture. For other references see 7 Aristox. *El. Harm.* 21.1–5, Plutarch *Non Posse Suaviter* 1096a–b, ps.-Plut. *De Mus.* 1138a; and cf. Macran (1902) 243–4, Düring (1934) 172–3, and for a different interpretation Schlesinger (1939) pp. 54ff.

43 Cf. 4.2 ps.-Ar. *Probs* XI.6.

44 The paragraph is interesting in its connection of 'thinness' of sound with lack of resonance, a link that is not made in other discussions of the subject (see n. 37 above).

which it occurs is not continuous, so that there occurs not a unified impact, but a fragmented one, and the sound seems cracked.[45] These sounds are very similar to rough ones,[46] except that the latter are fragmented from each other into small portions, while most cracked sounds have cohesive beginnings, and later are divided into several parts.

Aspirated voices are those in which we expel the breath immediately along with the notes. Unaspirated voices, conversely, are those that occur without the expulsion of the breath.[47]

Voices are broken when people can no longer expel the air with a blow, but the region around the lung fails under the tension. For just as the legs and shoulders eventually fail under tension, so too does the region around the lung. The breath is lightly expelled because its impact has no force. At the same time, because of the roughening of their windpipe, the breath cannot travel outwards strongly and continuously, but is fragmented, so that their voices are broken.[48] Some people suppose that it is because of the stickiness of the lung that the breath cannot come through to the region outside, but they are wrong. For such people do make sounds; but they cannot make them carry, because the impact on the air is produced without tension: they speak only by forcing the breath from the pharnyx itself.

The affliction of stammerers resides neither in the veins nor in the windpipe, but in the movement of the tongue. For they have difficulty in changing its position when they want to utter a different sound. Hence they utter the same syllable for a long time, unable to utter the next, while their movement and their lung continue in the same impulse, because of the quantity and force of the breath. For just as the whole body of a person running violently is difficult to divert from its rushing course into movement in a new direction, so it is with each single part of the body.[49] That is why they often cannot utter a sound that succeeds another, but easily say what comes after that, when they have made a fresh start to the movement. This is obvious; for the same often happens to people who are angry, because the movement of their breath is so forceful.

[45] Cf. 802a above.

[46] Described at the beginning of 803b: cf. 802a.

[47] On aspiration see 12 Arist. Quint. *De Mus.* 41.16, 76.15, 79.10–11.

[48] Cf. 801a, 3.17 Arist. *De Gen. An.* 788a.

[49] The phenomenon, like others considered in the treatise, is well observed. The theory is clear in outline, but scarcely in detail. In particular it is not clear whether the author intends to make a connection between the incapacity of the tongue, which is first said to be responsible for stammering, and the incapacity to check the movement of breath from the lung. Perhaps the link is only that both are instances of the same general condition, an inability to divert any part of the body from its current mode of activity.

CHAPTER 6

Theophrastus

Theophrastus followed Aristotle as head of the Peripatetic school or Lyceum (much to the chagrin of Aristoxenus, so the gossip ran, who had hoped to succeed himself). His tenure lasted from 322 to about 287 B.C. He wrote voluminously, and though he shows little originality in strictly philosophical matters, where he seldom quarrelled with Aristotle's opinions, he nevertheless made two important contributions to learning. One was in botany, a science in which his achievements are comparable to those of Aristotle in zoology. His botanical handbook, the *Historia Plantarum*, survives more or less intact (an excerpt on the reeds used to make the mouthpieces of *auloi* is quoted in *GMW* vol. 1 ch. 13). The second was in the history of philosophy. His writings on the Presocratic cosmologists, now lost, were the main source for most later Greek accounts, on which the bulk of our own knowledge of them in turn depends.

Reports in various later sources give evidence of Theophrastus' lively interest in music. Porphyry's citation shows that his work *On Music* ran to at least two books, and his musical writings seem to have been extensive and diverse. (For references in other ancient writers to his views on music see n. 44 below.) Only a few fragments and brief paraphrases now survive, and of these much the longest and most interesting is the one translated below.

The passage begins and ends with tantalising hints of a positive theory about the psychological sources of the human impulse to melody-making, and about the basis of our capacity to give accurate melodic expression to the movements and passions of the soul. These hints, unfortunately, are not developed here. The bulk of the passage is a polemic against those who would explain our capacity for melodic accuracy by appeal to the thesis that pitch is a quantitative property of sound, and that it is because pitches are somehow numerable, or exist 'in numbers', that the soul can assess and produce them with precision. Music-making, on this view, is a sort of intuitive applied mathematics. Theophrastus does not confront the psychological aspects of this theory head-on. Instead, he attacks the general, underlying doctrine, common to Pythagoreans, Platonists and most Peripatetics alike, that pitch-differences are in essence quantitative. Against it he mounts subtle and complex arguments of two main sorts. In the first group (62.1–33) his arguments are abstract and dialectical, seeking to show that the mathematical theorists' position is incoherent. The second (63.1–64.24) introduces more empirical considerations, reviewing the data about the ways in which sounds of different pitches are physically produced and perceived. Theophrastus concludes that in every case where there are grounds for treating the sources of a given pitch as quantitatively large or numerically many, there are equally good grounds for treating them as small or few. He also argues that certain facts about our perception of sounds (in particular that we can hear concords, and more generally that we can hear notes of two different pitches simultaneously) are incompatible with the quantitative thesis. Finally, he seeks to show that various observations alleged to support the quantitative view are better explained on the hypothesis that differences of pitch are qualitative, and have nothing to do with quantity. Theophrastus sketches an alternative physical theory in which differences of pitch are due to differences in the 'shape' of the sounds' movements, not to differences of velocity, frequency of impact, or the like (compare 5 *De Audib.* 800a).

Towards the end of the passage (64.24–65.9), having disposed of theorists of those schools, he briefly turns his fire on a conception of pitch which he probably means to attribute to Aristoxenus, though something similar was espoused by the earlier 'empiricists' whom Aristoxenus calls *harmonikoi* (see also 2.1 Plato *Rep.* 531a–b). On this view pitches do not differ by being greater or smaller in respect of some variable, or by being composed of more or less numerous elements, but by being points differently placed on the continuum of pitch, separated by 'gaps' or intervals. But, Theophrastus argues, the contents of such gaps cannot constitute the difference between different notes, since it is precisely when those contents are absent that the musical distinctions become detectable. The differences must lie in distinctions between the notes themselves, not in what the empty space between them would hold if it were sounded.

These arguments against Aristoxenian views were revised and clarified, centuries later, in a trenchant passage of Ptolemy (11 *Harm.* Book 1 ch. 9). Some of the ideas set out in Theophrastus' attacks on the other group of theorists also find a later echo in 9.9 Panaetius ap. Porph. *Comm.* 65.21ff., though Panaetius' conclusions are a good deal more muted. In rejecting quantitative theories outright, Theophrastus seems to stand alone, at least in his own day (Porphyry, long afterwards, approved of his stance, as his remarks introducing the quotation show), but it is just possible that elements of his approach had their origins in the work of Damon (see n. 44 below). It is greatly to be regretted that so little of the musical writings of either of them has survived. Some discussion of the present passage will be found in Lippman (1964), ch. 4: see also Barker (1985).

6 Porphyry, *Commentary on Ptolemy's* Harmonics, 61.16–65.15 (Düring)

Perhaps there are a good many others who agree with me, though I cannot list them by name since I am not in possession of their writings. But Theophrastus will be adequate to stand for me in the place of them all, since he demonstrated the absurdity of the doctrine[1] by several arguments, powerful ones, as I at least am convinced, in the second book of his *On Music*. We should write down what he said, and reckon his remarks to be capable of correcting those who take the side of Ptolemy. They are as follows.

'For the movement productive of melody, when it occurs in the soul, is very accurate, when it [the soul] wishes to express it [the movement] with the voice. It [the soul] turns it [the voice], and turns it just as it wishes, to the extent that it is able to turn that which is non-rational.[2] Some people decided to refer its [the soul's] accuracy to numbers, saying that the accuracy of the intervals arises

[1] The doctrine is that pitch is a quantitative attribute of sound, representable by numbers. Porphyry is commenting on 11 Ptol. *Harm.* Book 1 ch. 3; cf. also 9.9 Panaetius ap. Porph. *Comm.* 65.21ff., which belongs to the same phase of Porphyry's discussion.

[2] The two opening sentences have often been misconstrued, as a result of wrong identifications of the referents of the pronouns. The nouns in parentheses give what I believe to be the only reading that is both satisfactory in sense and grammatically possible. 'That which is non-rational', *tēn alogon*, must be the voice. Just possibly the adjective should be rendered as 'wordless', rather than 'non-rational'. Some Arabic sources represent Theophrastus as saying that the soul has 'conformations' which it cannot express in words, and therefore expresses through melody and bodily movement. In that case the point will not be that the soul's capacity to 'turn' the voice melodically is limited, but rather that it extends even further than its power to represent its condition in words.

in accordance with ratios between numbers. For they said that the ratio of the octave is the same as that of the duple, the ratio of the fifth the same as that of the hemiolic, and the ratio of the fourth the same as that of the epitritic, and that for all the other intervals, in the same way, just as for the other numbers,
62 there is a ratio peculiar to each. Hence they said that music consists in quantity, since its distinctions exist on the basis of quantity.[3]

In saying this they seemed to some people to be more intelligent than the *harmonikoi*, who judge by perception, since they judged by the ratios of intelligible numbers.[4] They failed to realise that if the difference is a quantity, this same difference[5] arises in accordance with the difference in quantity, and must be a melody or part of a melody. Thus if one colour differs from another in quantity, as it must, it must be a melody or part of a melody,[6] if indeed melody and interval are a number, and if melody and the difference it contains exist because of the number. For if every interval were a quantity, and if melody arose from differences between notes, the melody would be as it is because it is a number. But if it were nothing but a number, everything numerable would participate in melody too, to the extent that it does in number.[7] If, however,[8] quantity belongs to notes in the same way as it does to colour, colour being something different [from the quantity itself], then a note is one thing and the quantity related to it another. But if a note is something other than a number,[9] then the lower and the higher note differ from one another either as notes or

[3] The ratios mentioned are the familiar ones, and the form of quantification that employs them is common to Pythagorean, Platonist and Aristotelian sources. Nothing said so far allows us to identify any single school as the focus of Theophrastus' comments. We can be sure only that he is thinking here neither of Aristoxenus nor of those predecessors whom Aristoxenus calls *harmonikoi* (cf. the reference in the next sentence). But their position is not immune from criticism either: see 64.24ff. below.

[4] These *harmonikoi* are probably those so called by Aristoxenus (see, for example, 7 *El. Harm.* 2.30, 5.7, 7.25, etc.), and correspond at least roughly to the empiricists discussed at 2.1 Plato *Rep.* 531a–b and the practitioners of the 'harmonics based on hearing' mentioned at 3.7 Arist. *Post. An.* 78b–79a. Theophrastus may well have Plato in mind, in view of the latter's mockery, in the passage cited, of those who 'prefer their ears to their intelligence'. The accuracy of the mind's independent activities, particularly in relation to number, is a characteristic Platonic theme, and this accuracy is contrasted with the vagueness and unreliability of the senses (e.g., *Phaedo* 74a ff., 78b ff., *Republic* 522a–531c, *Philebus* 55d ff.). Later the thesis became a commonplace of the 'Pythagorean' musicological tradition: see, for example, 9.11–9.13 Ptolemaïs and Didymus ap. Porph. *Comm.* 23.24ff., 25.9ff., 26.15ff., and compare 11 Ptol. *Harm.* Book 1 chs. 1–2.

[5] That is, difference of pitch.

[6] This clause is deleted by Düring (wrongly, I think), as a mistaken repetition from the previous sentence.

[7] This ends the first phase of Theophrastus' opening attack. If melody is constituted by pitch differences, and if differences of pitch are identical with differences of number, then any two items that differ in a numerable way must form 'a melody or part of a melody'. That is patently absurd. It seems clear that the shift in his line of criticism at this point does not mean that he is turning his fire on a second, distinguishable 'school'. Rather, he is offering his opponents (all those who adopt a ratio-based approach) first one, then another interpretation of their general thesis that pitch differences are differences of quantity.

[8] Reading *ei* ('if') with the MSS for Düring's emendation *ē*, and altering his next full stop to a comma.

[9] Reading *arithmos* ('a number') in line 13 with Wallis for the MSS *akoustos* ('audible'), but retaining *ē* in line 12 where he emends to *ho*.

in respect of their quantity.[10] If they differ in quantity, and if the higher is as it is by being moved in accordance with more numbers, the lower by being moved in respect of fewer,[11] what else could be the characteristic proper to sound? For every sound is such as to be grasped either as high or as low. For every sound is higher than this one and lower than that, so that the quantity of the one is smaller and that of the other greater, and hence it is a number. When this is taken away, what would be left to make it a sound? But if the sound is higher or lower than some other, the sound possesses quantity, while if it possesses something else, it will no longer be a sound.[12] But if it is as notes that high and low ones differ from one another, we shall no longer have any need of quantity: for their own intrinsic difference will be sufficient by itself for the generation of melodies, and knowledge of the differences will be possible.[13] For the differences will no longer exist in accordance with the quantities, but in accordance with the quality peculiar to the sounds, as is the case with colours. For no colour taken simply as such differs in quantity from another colour taken simply as such: the quantities can be equal, just as, if one mixed

[10] This sets up the two-pronged argument that follows. Theophrastus has shown that the simple identification of notes with quantities or numbers is absurd. But things that are not numbers (e.g., colours) may have numerical attributes. Since being a note is not the same as being a number, any numerical attribute of a note is distinguishable from those other features that make it a note. Then the difference between a high note and a low note may lie in either of these characteristics. Theophrastus proceeds to examine each possibility in turn.

[11] I have rendered these phrases as literally as possible. They are obscurely expressed, but plainly indicate Theophrastus' supposition that the theory he is considering must attach its numbers in some way to movements. Later (63.21–2) he seems to contrast the view that pitch is correlated with velocity of movement with one that treats the higher note as a greater 'plurality'. This suggests that phrases like the present one (which recur plentifully in the rest of the paragraph) presuppose a hypothesis akin to that of 8 Eucl. *Sect. Can.* 148–9, rather than the commoner 'velocity' theory of pitch. But in view of the apparent generality of his attack in this section, it would be odd if he had intended to limit its application, here at least, to just one rather unusual version of the quantitative style of theory. Probably his curious expressions, 'more numbers', etc., are meant to be interpretable within the framework of any theory relating pitch to quantitative features of movement. But see also 64.13–22 with n. 37.

[12] At the beginning of this sentence (lines 19–20) I read *ei d'* for the MSS *hē phōnē d'*, omitting *phōnē* as a repetition from the previous sentence, and I replace Düring's full stop after *estin* in line 20 with a comma. (Alexanderson's suggestion, replacing the first *ē* in 20 with *ei*, gives the same sense.) This completes the first half of Theophrastus' two-pronged argument (see n. 10 above). It considers the hypothesis that it is the quantitative aspect of a note that constitutes its pitch. His claim, summarily, is that nothing can be a note, or even a sound, without having a pitch. Hence, if the quantitative aspect were taken away, then on this hypothesis what was left would not be a sound at all, no matter what other attributes it had ('if it possesses something else'). Sound is not a pitchless material on which pitch, in the guise of some 'quantity', can be imposed – a sound must have some pitch just in virtue of being a sound. Yet, according to the dilemma proposed, the quantitative aspect is distinguishable from the features that make a thing a sound. Hence, the quantity cannot be the pitch, and the first of the two possible hypotheses fails. Theophrastus turns to the second.

[13] The additional point made by the last clause looks like a parenthesis. Theophrastus may be concerned to rebut any suggestion that mere qualities cannot be known, whereas quantitative attributes can (see n.4 above). Possibly he intends the stronger point that on the quantitative hypothesis the differences would not be knowable, perhaps because the alleged quantitative distinctions (between velocities, or between pluralities of impacts) could not as such be detectable or measurable by us.

with white an equal amount of black, the numbers of the white would not be said to be more than those of the black, nor those of the black than those of the white. Nor would they if bitter [were mixed] with sweet: for each of them is equal in the area over which they are spread, but the quantity is equally spread out in accordance with its own special quality.[14] Thus the high sound does not consist of more numerous [parts], or move in accordance with more numbers,[15] and neither does the low one: for it is as possible to speak in this way of the latter as of the former, since there is a characteristic magnitude that belongs to a low sound.[16]

63 This is clear from the force exerted when people sing.[17] For just as they need a certain power in order to give out a high sound, so do they also in order to utter a low one. In the one case they draw in the ribs and stretch out the windpipe, narrowing them by force: in the other they widen the windpipe, which is why they make the throat shorter, since the width contracts the length. In the case of *auloi* too, there is the similar fact that power is needed to blow forcibly into a narrower one or into a wider one, so as to fill it.[18] In *auloi*, in fact, the case is even clearer; for a high note requires less labour, since it arises through the holes that are higher up, while a low note demands greater force, if the breath is impelled through the whole [pipe], so that however much length is added, there is added the same amount of strength in the breath. In strings it is clear that there is equality in each of the two cases: for by whatever amount the tension of the thinner is tighter, by the same amount the one that seems slacker is thicker.[19] Hence by however much the sound from the thinner is

[14] This completes the second leg of the dilemmatic argument. If differences of pitch do not consist in differences of quantity, then the quantitative hypothesis is irrelevant. The distinctions will be irreducibly qualitative, like those between colours, where the quantitative aspects of a colour-patch have no effect on the quality constituting its colour.

[15] It is again unclear whether Theophrastus' phrases identify two distinct theories, or are different ways of encapsulating the same generalised position: see n. 11 above. If two different theories are intended, the first must be that of 8 Eucl. *Sect. Can.* 148–9, the second the familiar velocity hypothesis.

[16] This concludes Theophrastus' first and rather abstract line of attack. The last sentence serves also to introduce a second (63.1–64.24), which brings to bear detailed considerations in physical acoustics. The immediate point, which will be elaborated, is that while higher sounds may be characterised by greater quantities of some variables, lower sounds are 'greater' in other respects, and that considerations of 'magnitude' therefore cut both ways.

[17] Despite the new focus from here onwards on physical and physiological issues, Theophrastus is not now attacking a different school of thought (e.g., Peripatetic acoustic scientists instead of Pythagorean mathematicians). As the form of this sentence shows, he is offering further arguments against the same quite generalised position. A connection between the force or strength exerted and the height of pitch produced had commonly been postulated: see, for example, 1.19 Archytas frag. 1, 3.17 Arist. *De Gen. An.* 787a, 4.4, 4.6, 4.10 ps.-Ar. *Probs* XI.13, 15, 21. (But both Aristotle and the *Problems* writers partly anticipate Theophrastus by claiming that under different circumstances either strength or weakness can be a source of high pitch.)

[18] Cf. 5 *De Audib.* 804a, and for a more complex account 9.7 Aelianus ap. Porph. *Comm.* 33–4.

[19] There are ambiguities here. *Tasis*, 'tension', can mean 'pitch'; the words translated 'tighter' (*eutonōtera*) and 'the one that seems slacker' (*hē aneisthai dokousa*) could

stronger, by the same amount the other is heavier.[20] For it is from the larger that there comes the greater, encircling sound.[21] For how could some notes be concordant, if there were no equality?[22] For what is in excess is unblended, since what is beyond the measure becomes evident over and above the mixture. Hence with the stronger things in the blend there is mixed a greater amount of what is slacker, to produce equality of power.[23] Thus if there is a concord, there is also equality of the things from which it arises. For if the high note moved in accordance with more numbers,[24] how could consonance come about? Further, if, as they say, the higher note is also heard at a greater distance,[25] through its travelling further because of the sharpness of its movement, or through its arising as the result of plurality,[26] then this note could never be concordant with the low one, neither when it is heard alone, if indeed the concord consists in both of them, nor when it has ceased[27] (for because of this unnoticed cessation it must no longer be heard), nor, most importantly, when both of them are heard:[28] for then the higher is more vigorous, since it is capable of travelling further. Hence it obscures and overpowers the low note, so that it appropriates the perception for itself, even while the lower note is not reduced in power. But since concordance exists, displaying equality between the two notes, there is an equality in their powers, differing in the quality belonging to each. For what is higher is naturally more conspicuous, not stronger, and that is why it is apprehended at a greater distance than the lower, just as is white in relation to any other colour,[29] or as is anything else that is more strongly apprehended, not because one of the two is less what it naturally

as naturally mean 'higher-pitched' and 'the one that seems lower-pitched'. But Theophrastus' argument appears to require the translations I have given. The higher-pitched string is tenser but thinner, the lower-pitched slacker but thicker, so that each involves a greater amount of one variable and less of another. For a different way of attaching contrasted magnitudes to height of pitch see 10 Nicomachus *Ench.* ch. 4.

[20] 'Heavier' translates *baryteros*, the usual word for 'lower-pitched'. Theophrastus seems to be drawing on the ambiguities of the word: in being low-pitched a sound is heavy, and in that respect large. See 4.2 ps.-Ar. *Probs* XI.6, and cf., for example, 3.1, 3.15 Arist. *Topics* 106a, *De Anima* 420a–b.

[21] The text is in some doubt, and Theophrastus' expression is in any case crabbed, but the sense must be close to what I have given: cf. 64.4–20.

[22] This introduces a new point. The argument that follows presupposes both that a concord is a 'blending' of two notes (see, for example, 8 Eucl. *Sect. Can.* 149.17–20) and that there must be some kind of equality between them if the concord is to be heard as such (cf. 3.12–3.14, 3.16 Arist. *De Sensu* 447a–b, 448a–b, *De Anima* 426a–b).

[23] Theophrastus evidently takes his arguments to have shown that the lower ('slacker') note is greater in bulk, because its source is thicker, and it is 'heavier' and 'encircling'.

[24] See n. 11 above.

[25] See 4.8 ps.-Ar. *Probs* XI.19, cf. 4.2, 4.9 *Probs* XI.6, 20.

[26] On this disjunction see n. 11 above. I follow Düring in adding *tōi* in line 22, but the MSS may be right. In that case the sense is 'or because it becomes a plurality'.

[27] I follow Alexanderson (1969) in excising *ho baryteros* ('the lower') in line 24. The MSS reading gives the sense 'when the lower note ceases' (or 'is obliterated').

[28] Reading *outh' hote* in line 25 with Hoëg. The MSS reading (*oute*) gives the sense 'nor, most importantly, is it the case that both of them are heard'.

[29] Removing the accent on *tou* in line 32 with Alexanderson. On the greater conspicuousness of the high note cf. 4.18 ps.-Ar. *Probs* XIX.21; and for a related comparison with colour see 5 *De Audib.* 801a–b.

64 is,[30] or because it does not move in accordance with equal numbers, but because perception focusses more on the one than on the other, because of its unlikeness to its surroundings. Thus the low note penetrates [as far as the ear] too, but the hearing grasps the high note more quickly, because of its special quality, not because of the plurality it contains. And if indeed the higher note does move to a greater distance, it is not because it is moved in accordance with more numbers, but because of its shape, since the high sound travels more forwards and upwards, while the low note travels more equally all about.[31]

This is clear also from instruments. For those with a horn and those equipped with a bronze vessel[32] are more resonant round about, since the sound made is equal all around. Again, if one were to utter a high note while touching one's ribs, and then a low note, one would feel the movement about one's ribs more, with the hand, in the case of the low note.[33] And if one were to touch the tortoiseshell or the horn or arm of an instrument,[34] then when one struck the thin [string] and that which generates a low sound, once again one would feel the movement in the hollow body more when one struck the string that generates the lower sound. For the low note travels everywhere all around, while the high note travels forwards, or in the direction in which the utterer compels it to go.[35] Then if the low note moved all around to the same degree that the high note moves forwards, it would not move in accordance with fewer numbers, as is plain too from the facts about *auloi*. For the larger *aulos* is lower-pitched, the one, that is, in which there is more breath, through all of which the movement goes.[36] But neither can the high note be distinguished by its speed: for then it would occupy the hearing first, so that a concord would not arise.[37] If it does arise, both are of equal speed. Hence it

[30] At 63.33 I remove Düring's commas after *einai* and *mallon*, relocating them after *pephyke* and *estin*.

[31] Theophrastus' alternative explanation of the conspicuousness of high sounds is elaborated in the next paragraph. It is perhaps related to the notion that a high sound is 'thin', developed in 4.2, 4.7–4.9 ps.-Ar. *Probs* XI.6, 16, 19, 20: cf. 4.16 *Probs* XIX.8. For other uses of the idea of 'shape' see 4.11 *Probs* XI.23, 5 *De Audib.* 800a, 9.3 Adrastus ap. Theon Smyrn. 66.1–3.

[32] Theophrastus' expressions are not clear, but most probably refer to 'bells' of horn or bronze attached to the pipes of wind instruments. On this, and on the phenomenon of 'resonance', see 5 *De Audib.* 801b, 802a–b. For another interpretation of 'horn' see n. 34 below.

[33] With these claims compare, for example, the account of the physiological sources of low and high notes at 12 Arist. Quint. *De Mus.* 7.4–6, 81.7–13.

[34] The absence of an article before 'arm' (*ankōnos*, nominative *ankōn*) indicates that 'arm' and 'horn' here refer to the same thing, the member extending upwards from the soundbox (the 'tortoiseshell') of a stringed instrument. Each instrument had two such arms, joined by a crossbar to which the strings were attached. In the *lyra*, whose soundbox was often a tortoiseshell, the arms were sometimes actually made of horn, and were usually horn-shaped. The terms 'arm' and 'horn' are made equivalent in this sense by Pollux *Onomastikon* IV.62.

[35] This contrast seems not to be made elsewhere. On the direction of a sound's travel see particularly 4.2 ps.-Ar. *Probs* XI.6, 5 *De Audib.* 800a.

[36] See 63.6ff. above, with the passages cited at nn. 18 and 19.

[37] This summarises the argument of 63.20ff. The distinct attacks made in this sentence and the previous two on the theses: (a) that a lower note moves 'in accordance with fewer numbers', and (b) that a high note is 'distinguished by its speed', are the best evidence

is not some unequal numbers that give the explanation of the differences, but the sounds with their natural qualities, being naturally attuned together.[38]

Again, it is not the intervals, as some people say, that are the causes of the differences and hence their principles, since if these are left out the differences still remain.[39] For when something comes into being if certain things are left out, these are not the causes of its existence, not as productive causes, but [only] as things that do not prevent it.[40] For the unmelodic is not a cause of the melodic, since the melodic would not come into being if the unmelodic were not rejected, not would any other thing become characterised by knowledge if the opposite, the knower's ignorance, were not rejected. For ignorance is not the cause of one's knowing through its existence, but through not preventing it, when it is rejected, so that neither are the intervals the causes of melody as producing it, but as not preventing it. For if someone were to sing simultaneously the continuous series of intervening positions as well, the sound produced would certainly be unmelodic. Then while the unmelodic would arise if these were not rejected, it is not through their being left out that the melody arises, merely because they would prevent it if they were not left out.[41]

It is therefore a great help that melody revolves around these,[42] enabling us

for the view that Theophrastus has two different sorts of acoustic theory in mind (see n. 11 above).

[38] This sentence presents the conclusion of the whole passage so far; in the next paragraph the focus shifts. The repetition of 'natural' (*physei*) hints at the view that the pitches of sounds, conceived purely qualitatively, fit together in ways that derive from their own intrinsic characters (not from mathematics). To that extent Theophrastus seems to be in sympathy with Aristoxenus (and perhaps with Damon: see 2.6 Plato *Philebus* 17d with n. 42, and cf. n. 44 below).

[39] Here Theophrastus turns his attack on a different approach, that adopted by the *harmonikoi* mentioned by Aristoxenus, and with some qualifications by Aristoxenus himself (see particularly 7 *El. Harm.* 8.13ff., 15.13–32, and on the *harmonikoi* see n. 4 above). For them, pitches were points on a continuum. For a note to have a pitch was not for it to be of a certain magnitude, but for it to lie in a particular position in this linear dimension. It was therefore possible (if not perhaps wholly just) to represent them as holding that pitch-differences are actually constituted by the sizes of the gaps (*diastēmata*, 'intervals') between such points. This is obviously open to the objection (which is the nub of Theophrastus' argument here) that a sound's pitch cannot be merely its 'distance' from another pitch, but must be some intrinsic feature of the sound itself. See also 11 Ptol. *Harm.* Book 1 ch. 9.

[40] The distance on the continuum between two given notes is conceived as constituted by a range of intervening pitches. Theophrastus is arguing that these cannot be the 'cause' of the difference between the two given notes, since that exists whether the intervening pitches are sounded or not. He is also shifting towards the point, emphasised in the last sentence, that notes as employed in melody must be separated by gaps whose contents are 'left out'. It is by being absent that these contents 'do not prevent' properly melodic movement from pitch to pitch. But in not preventing it the intervening pitches cannot be said actively to cause it.

[41] For the direction of the argument see the previous note. The expressions 'the melodic' and 'the unmelodic' translate *emmeleia* and *ekmeleia*. In the usage of Aristoxenus and many other writers, a pitch is melodic if it fits, with all the other pitches in its context, into a properly structured scalar system, and unmelodic if it does not. Hence within the range of any system there are indefinitely many unmelodic pitches between any two melodic ones, and these must be 'left out' if the melodic structure is to emerge.

[42] Grammatically, 'these' can only be the melodic and the unmelodic. They are perceptible by us, and our perception of them guides us towards the identification of notes that are

to find the notes that are attuned to one another. But it is these notes that are the causes of the melody, while if the rejected intervals are made apparent, they are the causes of the unmelodic, whose principles they might be said to be, not those of melodic sound. Thus neither are the intervals causes of the melodic, but damage it, at least when they are made apparent, nor are the numbers causes,[43] by the notes' differing from one another in quantity. For the low notes are found to be equal to the high ones, in a different respect, in accordance with the way in which the labour involved is also equal in opposite respects. For people uttering low notes labour no less than those uttering high ones, but exert their effort in the opposite direction.

The nature of music is one. It is the movement of the soul that occurs in correspondence with its release from the evils due to the emotions; and if it were not this, neither would it be the nature of music.'[44]

properly attuned. The implication is that proper attunement does not consist in perceptible melodiousness: the latter is only a sign of the former. On the real basis of the melodic order see the last paragraph of the passage.

[43] The remainder of the paragraph recapitulates the arguments of 63.1–64.24.

[44] Or possibly 'if it did not exist, neither would the nature of melody exist'. This hint (all too brief) of Theophrastus' positive views indicates that it is the role and nature of certain psychic movements that constitute them, and derivatively their audible expressions, as music (cf. 61.22–26). Music's essence is not then to be described in terms of laws inherent in the audible phenomena themselves. (Cf. n. 42 above, and for some elaborations see Plutarch *Quaest. Conv.* 623a, Ael. Fest. Aphthonius *De Metris* 4.2, Philodemus *De Mus.* 3.35, 3.37.) The theory can be linked in a general way with Plato's ideas on music, emotion and character, particularly in *Republic* Book III and *Laws* Book II, though not of course with his penchant for mathematical analysis. Behind those passages of Plato lies Damon (see n. 42 to 2.6 Plato *Philebus* 17d). It is likely that Damon's analyses were primarily qualitative. He too claimed that music arises from a movement of the soul (Athenaeus *Deipn.* 628c), and one source assigns to him ideas about musical therapy that parallel those elsewhere associated with Theophrastus (Galen *De Hipp. et Plat.* 5.453 (DK37 A 8); for Theophrastus on therapy see Athenaeus *Deipn.* 624a–b, Apollonius *Hist. Mirabil.* 49.1, Aulus Gellius *Noctes Atticae* 4.13.1–2). Few, if any of the theories current in Theophrastus' time have been spared his attacks. He rejects the mathematical investigations of Plato and the Pythagoreans, much of the acoustic research of Aristotle and the Lyceum, the assumptions apparently underlying the representations of musical 'space' offered by the *harmonikoi* and Aristoxenus, and the insistence of the latter that musical principles are inherent in the musical phenomena themselves. Theophrastus may have written here as a solitary maverick, but it is at least possible that he is appealing implicitly to the authority of Damon, over the heads of his own contemporaries.

CHAPTER 7

Aristoxenus

Aristoxenus was the son of a musician. He was born in Tarentum, probably a little before the middle of the fourth century, while the great Tarentine philosopher, statesman, mathematician and musicologist Archytas was still alive. Aristoxenus will have heard much about him, though he can hardly have known him personally. We are told that he studied music and philosophy with his father, and later with distinguished Pythagoreans and others in mainland Greece. Some time after 330 he came to Athens and joined the school of Aristotle. To judge by the evident Aristotelian influence in his writings, this was an important turning-point in his career. By the time of Aristotle's death in 322 B.C. Aristoxenus was a figure of some distinction in the Lyceum and hoped to follow him as its head. The rudeness of his remarks about Aristotle when he learned that the school had been bequeathed to Theophrastus was startling enough to be remembered, and passed into the biographical tradition.

It seems unlikely that Aristoxenus stayed in the Lyceum under Theophrastus' presidency, but he may not have left Athens. We do not know where he went or how long he lived. The bulk of the *El. Harm.*, however, was certainly written later, and any pique he had felt against Aristotle must have passed. His only explicit remarks about his teacher in that work compare him favourably with Plato, and the treatise is thoroughly Aristotelian in conception. But the tale of his peevish insults is perfectly believable. He was notoriously humourless, acerbic and opinionated, outspoken and unscrupulous in speech as in writing. In his biographies of earlier philosophers he was not content with intellectual criticisms of those whose views he disliked, but spiced them with scandalous stories of dubious authenticity: Socrates and Plato especially were maliciously pilloried. In the *El. Harm.* we find nothing of quite that sort, but in criticising his predecessors he certainly pulls no punches. To judge by what he says, all previous writers on harmonics were incompetents or charlatans. He and he alone has understood how the subject should be pursued, and has grasped the truths it contains.

Aristoxenus was a tireless writer on a wide variety of philosophical and historical subjects (according to the *Suda* his works totalled 453 'books'). We hear, for instance, of at least three works on Pythagoras and the Pythagoreans, biographies of Socrates and Archytas, and writings on the soul, on principles of education and on political laws. In the field of music he wrote technical and historical studies of great scope and erudition, on which later musicologists drew extensively. For scholars of the Roman era he was simply 'the musician'. Of these works none has survived intact, though substantial quotations and paraphrases can be found in other people's writings. Among them were a very substantial treatise on the practice of music and its history, biographies of poets and composers, essays on instruments, on composition, on dancing, on choruses, and various collections of miscellaneous notes and discussions. Only two pieces have come down to us independently in the MSS traditions. One is a modest number of pages from his treatise on rhythmics. The other is the set of writings known as the *Elementa Harmonica*.

I shall nothing here about the *Rhythmics*: the surviving parts of the second book are translated in the Appendix to this chapter, and some comments will be found in the notes to 12 Arist. Quint. *De Mus.* Book 1 chs. 13ff. In harmonics his work was truly

revolutionary, as he would have been the first to agree. It sets aside the researches of the Pythagoreans as irrelevant and misdirected, and seeks in effect to establish a wholly new science that will study music on the basis of principles intrinsic to itself, not borrowed from physics or mathematics. So powerful was his novel conception of the subject, and so sophisticated and detailed were his studies, that his authority on matters of melodic analysis was accepted for centuries almost without criticism. Even later adherents to the 'Pythagorean' school tended to look to Aristoxenus (or to the simplified summaries of his views in the 'Aristoxenian handbooks' of the early Christian era) for accounts of the musical phenomena, though they sought to explain them differently and to develop them for different purposes. The one notable exception is Ptolemy, and even he accepts a good many Aristoxenian conceptions without question.

The *El. Harm.* as we have it is not a complete treatise. In the opinion of most scholars it is not even the remains of a single work, but incorporates parts of at least two, possibly more, and the question of the relationship between its parts has provoked much learned controversy. The most recent study (Bélis 1986) offers a detailed defence of the unitarian position, but though it is powerful, ingenious and erudite, I do not think it succeeds. The minutiae of the problem cannot be pursued here, but it may be helpful to set out a schematic summary of the work as it stands, and to add a few comments about the issues in dispute.

Book 1 begins with a brief description of the scope of harmonic science. Harmonics constitutes only a part of the multifaceted science of melody, and in particular it does not include the study of composition (1–2). After some reflections on the limitations of his predecessors' work (2–3), Aristoxenus devotes several pages to the presentation of his programme for harmonics, listing the subjects that the science should study, with comments about their importance and their previous neglect (3–7). The list includes some repetitions, probably because, as he indicates elsewhere, the topics were first to be sketched in outline and later considered in depth. The topics promised are the following. (i) The movement of the voice in 'space', that is, in the dimension of pitch. He adds that an understanding of this form of movement is essential if we are to grasp what it is to be a note, and that it presupposes understanding of five other subjects, tension, relaxation, height, depth and pitch. (ii) The greatest and smallest extensions (intervallic distances) that are melodically usable. (iii) Intervals and their differentiae. (iv) *Systēmata* (roughly, scales, concatenations of intervals) and their differentiae. (v) Musical melody: how it differs from other types of melody, and its division into genera. (vi) The nature of continuity and succession in *systēmata*. (vii) The genera, and the ranges within which the moveable notes alter in changes of genus. (viii) Intervals, and how they can properly be combined to form larger units. Here Aristoxenus emphasises that such combination is thoroughly orderly and subject to rules (ignored by his predecessors). The underlying principles must be found. (ix) From this emerges the study of *systēmata*, including the Complete or Perfect *Systēma*. They are to be enumerated and classified in detail. (x) Mixtures of genera, and the *systēmata* thereby formed. (xi) Notes. (xii) Regions of the voice, *tonoi*, and modulation, to be studied just as far as this bears on the nature of *systēmata*.

The list seems to be in three main parts: (i) – (ii) are preliminary studies of basic entities and concepts, (iii) – (vi) indicate topics of which an outline must be given before detailed work can begin, (vii) – (xii) are subjects to be pursued in depth, many of them having already been cursorily sketched.

Aristoxenus now sets off on his project, after reminding us of the place of harmonics in the wider study of music. In 8–10 he discusses the movement of the voice, and in 10–13 gives a careful study of the other five topics listed in (i). The subject proposed under (ii) is considered in 13–15. Next (still in 15) we find a preliminary definition of

interval, corresponding to (iii); in 16 a similar account of *systēma*, corresponding to (iv); and in 16–18 a discussion of the differentiae of both intervals and *systēmata*, as proposed in (iii) – (iv). Preliminary remarks on musical melody and on succession, (v) – (vi), are offered in 18–19; and in 19 we find a list of the different melodic genera, presented in a manner that echoes the proposal of (v).

So far Aristoxenus has followed his programme faithfully enough. At first sight 19–21 diverges from the sequence. It returns to one of the differentiae of intervals and gives an enumeration of the concords. This is followed by a definition of the tone in terms of concords, and a sketch of the ways in which the tone can be divided. In 22–7 we have a detailed study of the genera and the ranges of the moveable notes, plainly corresponding to (vii). In 27–9 (where the Book breaks off) there is an avowedly preliminary discussion of continuity and succession, emphasising again the orderliness underlying them, and finally a list of principles that govern the succession of intervals in *systēmata*. One might locate 27–9 under the heading of either (vi) or (viii). It seems fair to treat 19–21 as having been placed where it is (possibly after a gap in the surviving text) as a necessary preface to the quantitative investigation of the genera in 22–7. In that case, though the order of the discussion does not exactly reflect that of the programme set out, it comes very close. It is of course plain that it has not at this stage been completed.

Book II does not pick up where Book I left off. It starts with an introduction of its own, once again referring (though from a different angle) to the limited role of harmonics in the study of music (30–2). Introductory discussions continue in 32–4. The science is described as studying the way in which intervals can properly be placed in relation to one another in melody, a manner which is not random, but reveals the determinate orderliness of melody's nature. Then follow reflections on the methods appropriate to the science, on the roles in it of reason and of perception, and on the need for the rigorous demonstration (*apodeixis*) of subordinate propositions from principles. The procedures of Aristoxenus' predecessors are sharply criticised. These opening pages of Book II give the impression of being a new introduction to an independent work, and this impression is strengthened in the next passage, which sets out afresh the programme that a treatise on harmonics should follow (35–8).

The programme is not identical with that proposed in Book I. It lists as subjects for study (i) genera, (ii) intervals, (iii) notes, (iv) *systēmata*, (v) *tonoi*, (vi) modulation, (vii) melodic composition. In most cases Aristoxenus adds caustic remarks about the inadequacy of previous treatments. The list is a good deal more straightforward than that of the first book: it omits the two preliminary studies; it reorders the material somewhat; it avoids the repetitions of the earlier list; and very importantly, it adds melodic composition as a part of harmonics, whereas in Book I this had been explicitly excluded.

The next passage, 38–44, is again devoted to methodology. It involves a long polemic against some earlier approaches, and contains some of the most interesting of Aristoxenus' conceptual reflections.

The programme outlined in 35–8 begins to be tackled in earnest in 44, with a list of the genera. As in Book I, this is followed by a study of the concords and their relations to smaller intervals (44–6), after which the divisions of tetrachords in each genus are investigated in detail (46–52). All this seems to correspond to stage (i) of the programme.

In 53–4 Aristoxenus discusses the notion of melodic continuity, and states a set of principles that govern the way in which intervals may succeed one another. The passage is closely comparable to 27–9 in Book I. It is followed by a study of the ways in which discordant intervals can be constructed by movements through concords, and of a related method of confirming that the concord of a fourth comprises two and a half

tones. These sections can plausibly be assigned to stage (ii) of the programme, and with them Book II ends.

I shall not attempt a detailed summary of Book III. It opens with further reflections on the notions of continuity and succession, and a discussion of the ways tetrachords can be linked either in conjunction or in disjunction by a tone. After some additional preliminaries it proceeds to a set of theorems, or 'demonstrations', occupying the rest of the book, and based mainly on the principles stated in 54, at the end of Book II. Their task is to show which intervals can follow which in succession and which cannot, and how these rules are implied by the principles previously adopted. Towards the end of the book the rules are converted into statements about intervallic progressions that are possible (and those that are impossible) from a starting point in various specified notes. Finally, after the derivation of a few further, subordinate propositions, Aristoxenus announces the study of the different arrangements of small, incomposite intervals that go together to form greater intervallic magnitudes. Only some opening remarks about the arrangements of the fourth survive before the MSS break off.

The substance of Book III concerns the ways in which intervals can be combined. Such a study might be conceived as part of topic (ii) in the list in Book II, or alternatively as part of (iv), since the investigations of interval-combination and of the formation of *systēmata* cannot be sharply distinguished. The subject matter of (iii), notes, is missing, or at least is not given a full discussion of its own. Parts of Book III suffer from this omission, which may be due to the fragmentary state of the text. But there is a good deal to encourage the belief that Books II and III belong to the same treatise, and that only a few pages in the space between them have been lost. Evidently a great deal is missing at the end, and to make good the loss we must rely on the harmonic writings of later Aristoxenians, represented in this volume by 12 Arist. Quint. *De Mus.* Book I chs. 5–12. (Other summaries are found in the 'Aristoxenian handbooks' of Cleonides, Bacchius, Gaudentius and the unidentified author or authors known as Bellerman's Anonymous. The best of them is that of Cleonides.)

If we now compare Book I with Book II it seems obvious, in the first place, that there is a good deal of repetition and overlap, far more than would be expected in two segments of the same work. They have independent introductions, and distinct summaries of the programme that harmonic science should follow. There are also inconsistencies, of which that concerning melodic composition, mentioned above, is the most striking (but see n. 36 to Book II). Book II, furthermore, uses as a key concept the notion of melodic 'function' (*dynamis*), implying in particular that notes are to be understood as *dynameis*, not simply as pitches, and that the character of melodic relations in general is to be understood 'dynamically', by reference to the melodic role or character in which they are perceptually grasped, not just through representations of the sizes of intervals they involve. Book I says nothing of *dynamis*, and pursues its quantitative analyses without any hint that they will need reinterpretation. It explicitly defines a note as a pitch. The books also differ markedly in style, the second being much more leisurely, reflective and methodologically self-conscious. It seems to me that there are strong grounds for supposing that Books I and II are from different treatises, the first having belonged, perhaps, to an earlier and rather less sophisticated attempt at the project. Either book could have had a continuation of roughly the sort we find in Book III, but our Book III must in fact go with Book II, if only in view of its liberal use of the concept of *dynamis*.

These conclusions are strengthened a little by a couple of references in Porphyry. He quotes a good deal from Aristoxenus, and in citing passages from our *El. Harm.* he twice names the work he is drawing on. Quoting a paragraph from our Book I, he states that it comes in the first book of a treatise *On Principles*. Quoting a passage of our Book

II, he identifies its source as the first book of the *Harmonic Elements*. This sounds as if he thought of them as separate works. But this evidence must be used with caution. Aristoxenus himself sometimes uses the word *archē*, 'origin', 'beginning' or 'principle', in reference to a part of the work he is engaged in, or to a phase of its procedure. Thus, for example, at 16.11–12 (in Book I) he speaks of his initial conceptual sketches as *en archēi*, 'at the outset', 'among the preliminaries', and at 44.6–7 (in Book II) he discusses the importance of first establishing *archai*, 'principles', then demonstrating what follows from the *archai*, and of carefully distinguishing these phases. Further, he twice refers to the *stoicheia*, 'elements', as a second phase of an exposition of harmonics. At 28.34–29.1 (at the end of Book I), we are told that the ways in which intervals can be placed after one another will be demonstrated 'in the *stoicheia*'; and at 43.27–30 (towards the end of the introduction to Book II) he speaks of himself as 'on the point of setting out on the study concerning the *stoicheia*'. It is possible, in view of parallels elsewhere, that Aristoxenus gave the name 'elements' to that part of his project which derived theorems axiomatically from premises previously adopted, and described all the earlier material as 'on principles'. The reference to 'elements' in Book I certainly seems to look forward to a treatment of intervallic succession like that of the theorems in Book III. The reference in Book II, on the other hand, might suggest that 'the study of the elements' includes all detailed investigations of both sorts. In either case, Porphyry might have taken Aristoxenus' own words to imply that Book I dealt with principles while the bulk of Book II and its sequel were concerned with 'elements', without meaning to suggest that *On Principles* and *Harmonic Elements* were two different works. Nevertheless it seems clear to me, as I have said, that the two books must in fact come from treatises written at different times, and perhaps for slightly different purposes.

In the introduction to chapter 3 I said a little about Aristoxenus' debt to Aristotle's conception of science and its methods. Four points are of special importance. First, the data are not scientifically understood just by being observed and recorded. They must be explained. Secondly, such explanation takes the form of 'demonstration', the logical derivation of propositions from principles. Thirdly, the principles must themselves be established by a method that begins in perception, and proceeds by abstracting the essential features that constitute perceived cases as instances of the melodic categories under which our hearing grasps them. We are thus able to formulate rules which state, at the most general level, what features a sequence of sounds must present to perception in order to be melodic at all, and more specifically what characters it must display in order to be grasped as, for example, a tetrachord, a *pyknon*, a phrase in the enharmonic genus, and so on. The identification and coordination of the principles to which all melodic sequences must, as such, conform, amounts to a definition of the nature or essence of melody. Further specifications of the forms that things with this essence can take reveal its genera and species. Finally, by what I have called the 'same domain' rule, the terms in which the melodic phenomena are analysed and the principles by which they are explained must be ones that describe 'the melodic' and its features as such, and do not redescribe them as entities of other sorts. There is melody where there is a set of sounds that strike the ear in a certain way, and nowhere else. Its melodiousness lies wholly in its perceptible features, and if from another point of view the same set of sounds is a sequence of physical movements whose velocities stand in certain ratios, that is quite irrelevant. It is not by exemplifying these ratios that it is melodic: a sequence with the relevant perceptible properties would be a melody whether or not its sounds, or their causes, fell under that mathematical description. Aristoxenus has no interest in explaining why a certain combination of sounds strikes us as concordant, for example, in terms of descriptions drawn from physics and mathematics in the Pythagorean

manner. His task in this case is rather to explain the role of things heard as concords in systems that are perceived as melodically legitimate, and to identify the relations in which they stand to other elements, as these are grasped through a strictly musical mode of perception. Given that some sequences strike us as melodic while others do not, his project is not to explain this fact physiologically or psychologically. It is to state what must be true of the phenomena themselves as they appear to us, no matter what their causes or the biological conditions of our sensory apparatus, if they are to count for the musical ear as genuinely melodic. Having done so, he will show that these truths form a coherent pattern describing a single nature, that of melody. Agreed kinds of instance of the melodic or the unmelodic are then proved to be so precisely in virtue of their conformity to that nature or their inconsistency with it.

Two crucial points follow. One is that his science is rigorously phenomenalistic, concerned with the analysis and synthesis of what is heard as melodic, simply in its character as an object of perception. The physical basis of sounds and their production are issues quite foreign to harmonics – all that is to be studied is the form of the percepts themselves. Secondly, in order to undertake this investigation, it is necessary to develop a system of concepts and a corresponding terminology that belongs wholly to the domain of harmonics. The elements of melodic sequences and the various ways in which they can be organised and combined must be classified, named and related to one another. Little of this conceptual and terminological equipment had been needed by Pythagorean scientists. No doubt a good deal of the Aristoxenian repertoire was drawn from the language of practising musicians. Some of it he may have invented himself. Certainly he was responsible for the formal articulation of the character of such things as melodic genera, 'shades' of the genera, *tonoi*, *systēmata*, the *pyknon*, and so on, as well as for the theoretical deployment and analysis of many broader conceptions like 'movement of voice', 'melodic succession', and *dynamis* or 'melodic function'. Later harmonic theorists made remarkably few revisions to the system of concepts and classifications within which his analyses are framed.

Despite his great originality, Aristoxenus owed a debt – only grudgingly acknowledged – to a number of earlier writers (and presumably also to the practical musicians among whom he was brought up). These were not Pythagoreans, whose work he knew but considered irrelevant, and we have little information about them apart from the scathing references in the *El. Harm.* itself. Aristoxenus describes them generically as *harmonikoi.* Though he mentions several names, only Eratocles emerges as a theorist of any substance, and about him we know only what Aristoxenus tells us. He seems to be treated as a representative example of *harmonikoi* in general, just a little more acute than most.

What Aristoxenus tells us explicitly about them is mainly negative – their neglect of whole topics, their carelessness in observation, their recording of alleged facts without any attempt at demonstration or coordination under principles, and so on. Reading between the lines, it appears that these writers were 'empiricists', in the sense that they sought ways of describing what was presented to the ear, rather than arguing towards harmonic truths on mathematical grounds. To this extent they were primitive 'Aristoxenians'. They also shared with Aristoxenus a conception of pitch as a linear dimension on which notes appeared as points, rejecting the Pythagorean treatment of notes as quantities, and intervals as ratios between them.

Their main objective seems to have been the quantitative description of scalar structures as sequences of intervals measured by reference to a unit-interval, this unit being identified by perception, not defined by mathematics. (Compare 2.1 Plato *Republic* 531a–b, 3.4, 3.8 Aristotle *Post. An.* 84b, *Metaph.* 1016b.) Aristoxenus complains of their inaccuracies, but the difficulty of the project should not be

underestimated. Eratocles, if no one else, also tried to work out some of the relations between the structures he described. We hear in particular of his attempt to give an account of the alternative sequences formed by conjunction and disjunction (5.9ff.), and of his representation of the seven species of the octave through the cyclic reordering of intervals (6.19ff.). (These species are probably the 'octachords called *harmoniai*' of 36.30–1.) The *harmonikoi* attempted to illustrate their findings in diagrams, to which Aristoxenus refers several times, some of them constructed through a procedure called *katapyknōsis*, 'compression'. It seems likely that the diagrams showed the pitch of each note in a system as a point marked on a line, intervals being represented by the sizes of the intervening distances. Diagrams involving *katapyknōsis* were probably designed to facilitate comparisons between scalar structures, by displaying them all within the same theoretical stretch of pitch. This involved abstracting the structures from the pitch-relations in which they stood to one another in real musical compositions, and 'compressing' them all into the smallest possible overall range. There are clear suggestions that they also attempted to develop a system of notation, quantitative in form. Aristoxenus chides them for supposing that by notating a harmonic structure they had thereby understood it (39.4ff.).

Other aspects of their work are also mentioned, but these are the most significant. There are clear affinities between their aims and those of Aristoxenus, as he himself recognised. Most importantly, both sought to describe the melodic phenomena as they are perceived, not by reference to their unobserved character as physical movements or from principles proper to mathematics. Their faults, according to Aristoxenus, were those of inaccuracy and incompleteness, together with a crucial failure to grasp what sort of thing a science should be. At best they were just recording data. They made no serious attempt to explain, or to show how the melodic phenomena come together as interlocking expressions of a single, coordinated nature. The very idea of that 'nature', and of its expression in principles to which everything that is a melody conforms, is Aristoxenus' own, and his writings in harmonics are its articulation.

The only book in English wholly devoted to Aristoxenus is Macran (1902), containing a long introduction, text, translation and notes. The most recent edition of the text, with Latin introduction, Italian translation and notes, a selection of reports about Aristoxenus in Greek sources and an annotated index of Greek terms, is da Rios (1954). The classic work on him is Laloy (1904). The German edition of Westphal (1883, 1893) has an important commentary (this work is fairly described by Macran as 'exhaustive but diffuse and garrulous'). There are stimulating but often incautious discussions of many passages in Schlesinger (1939). A valuable recent contribution is Bélis (1986). All the books that offer surveys of Greek music and musical theory have something to say of him, of course, and other substantial studies appear in articles in scholarly journals; a selection of them will be found in the Bibliography.

7 The *Elementa Harmonica*

Book I

1 The science concerned with melody[1] has many parts and is divided into several species, of which the study called Harmonics must be considered one: in order it is first, and its character is like that of an element. For it is the study of first principles, which include[2] whatever is relevant to an understanding of *systēmata* and *tonoi*. The man who is proficient in this science should not consider anything beyond these as falling within his province; for that is the
2 end of this branch of study. Matters investigated at a higher level, where the science of composition makes use of *systēmata* and *tonoi*,[3] no longer belong to this science, but to the one which includes both this and the others through which all musical matters are investigated: and that is the science whose possession makes a man a musical expert.[4]

It turns out that those who have previously taken up the study of harmonics were concerned to be truly 'harmonicists' and no more, since they dealt only with the enharmonic, and never gave a thought to the other genera.[5] There is

[1] *Melos*, here translated 'melody', can mean: (i) song, broadly conceived to include melody, rhythm and words (a common usage, sometimes expanded by technical writers into the phrase *teleion melos*, 'complete melody', e.g., 12 Arist. Quint. *De Mus.* 28.8–10); (ii) melody conceived apart from the other elements (e.g., 35.24, 38.21 below), or (iii) the melodic series or scale on which a melody is based (e.g., 5.11–12, 39.20–3). If the sense here is (i), the other parts of the science will deal with, for example, words, rhythms, instruments (cf. 32.5–9). If it is (ii), it is less clear what the other parts are. Westphal suggested those dealing with the melodic uses of voice and instruments, and with composition, but these should belong to the 'higher level' studies of 1.24–2.6, not to 'parts' coordinate with harmonics. If it is (iii) the puzzle is harder still. The complex distinctions of later writers (e.g., 12 Arist. Quint. *De Mus.* 6.8–24) give little help. It seems likely that sense (i) is intended, and that the other parts are those of 32.5–9, but I draw this conclusion hesitantly, since Aristoxenus nowhere else uses *melos* in this way, usually making *mousikē* and its cognates do duty instead (e.g., 2.6, 32.7).

[2] Here Macran's text is better than that of da Rios; cf. the paraphrase at Porph. *Comm.* 5.27.

[3] *Systēma* is often translated 'scale', but in the *El. Harm.* includes structures smaller than that term suggests. It is any acceptable series of intervals, even as few as two (e.g., 15.34–16.1, 24.30; contrast 12 Arist. Quint. *De Mus.* 13.4–5). On *tonoi* see General Introduction, pp. 17–27. No systematic treatment by Aristoxenus survives: there are brief discussions at 7.10–23, 37.8–38.5, cf. 34.9–11. An allegedly Aristoxenian system is recorded in 12 Arist. Quint. *De Mus.* Book I chs. 10–11 (cf. Cleonides *Eisagoge* 203.4–204.15); for Ptolemy's discussions and independent constructions see 11 *Harm.* Book II chs. 7–11.

[4] 'Composition' here renders *poiētikē*; the more common word is *melopoiia* ('melodic composition'). In Book II, unlike the present passage, *melopoiia* is a part of harmonics (38.18–26); on the inconsistency see the introduction to this chapter and n. 36 to Book II. For *melopoiia* as a part of the science see 12 Arist. Quint. *De Mus.* 7.12, 28.10ff. On the scope and limits of harmonics see 31.16–32.8 below, and cf. the Aristoxenian discussions in ps.-Plut. *De Mus.* 1142f–1144e. On the musical 'expert' cf. 31.31–32.8.

[5] The coupling of 'harmonicists' and 'enharmonic' gives a pun: the usual noun for 'the enharmonic genus' is *harmonia*, though the adjective is always *enharmonios*. The accusation is repeated in 35.7, cf. 4.4–5, ps.-Plut. *De Mus.* 1143e–f. It can apply only to the 'empirical' school of *harmonikoi*. Pythagoreans and Platonists, whose work Aristoxenus treats as irrelevant (cf. 32.19–23) had certainly analysed the diatonic (see

evidence of this: their diagrams are of the enharmonic *systēmata* only, and no one has ever yet seen any for those in the diatonic or chromatic.[6] And yet their diagrams displayed melodic order in its entirety, though they used them to discuss only enharmonic octachord *systēmata*. Concerning the other magnitudes and arrangements in this genus itself and the remaining ones, no one has even tried to acquire understanding:[7] instead, they have separated off just one magnitude, the octave, of just one of the three genera that melody as a whole comprises, and devoted all their study to it. The fact that they were quite unsystematic in their work, even about the things they did happen to touch on, became clear in our earlier remarks, when we were investigating the opinions of the harmonicists:[8] but this will become even easier to see now, as we explain in detail how many parts there are in the science and what the function of each of them is. We shall find that they did not even touch on some of them, and dealt inadequately with others; and this point will thus become clear to us simultaneously with our getting an overall view of the outline of what the science is.[9]

First of all, then, the prospective student of melody must analyse the movement of the voice, its movement, that is, with respect to place [*topos*], for there is not just one variety of this movement.[10] The voice moves in the kind of movement I have mentioned both when we speak and when we sing (since high and low are obviously present in both of these, and movement with respect to place is that through which high and low come about), but the two movements are not of the same form. Up to now no one has ever carefully defined what the distinguishing feature of each of them is: and yet if this is not defined, it is not at all easy to say what a note [*phthongos*] is. Anyone who does not want to be forced into the position of Lasus and certain of the followers

especially 1.12 Philolaus frag. 6, 1.21 Ptol. *Harm.* 30.9ff. on Archytas, 2.3 Plato *Tim.* 35b–36b).

6 These diagrams are again mentioned at 7.32, 28.1. They seem to have represented the continuum of pitch as a line, divided by quarter-tones, on to which sequences of intervals could be mapped. They are associated with the procedure called *katapyknōsis*: see 7.22–33, 28.1, 38.3–5, 53.3–6; cf. also 12 Arist. Quint. *De Mus.* 12.12ff.

7 A 'magnitude', *megethos*, is the size of an interval (computed by Aristoxenus in tones and their fractions, where a tone is the difference between a fourth and a fifth: see, for example, 21.20–23). An 'arrangement', *schēma*, is an ordering of intervallic magnitudes. One structure differs in *schēma* from another of equal magnitude if their constituent intervals are the same in size and number, but are differently ordered (see 74.9–16). For the substance of Aristoxenus' complaint see 6.19–31.

8 The work referred to (whether an independent essay or a part of the present treatise) has not survived. See also 5.8, 6.17, 6.31. Aristoxenus often chides his predecessors for incompetence in their methods: see 3.32, 5.6–27, 6.11–33, 7.3–5, 7.22–33, 32.29–31, 36.18–37.1, 37.8–38.5, 39.4–43.9.

9 From here to 8.12 Aristoxenus reviews the parts of harmonics, in a programme which the sequel follows, so far as it survives, though not quite exactly. On the list of parts and its differences from the programme of Book II (34.34–38.26) see the introduction to this chapter.

10 Treatment of change of pitch as a form of local movement is characteristically Aristoxenian, contrasting with conceptions derived from the Pythagoreans. Aristoxenus' fuller account is at 8.13ff.

of Epigonus,[11] who thought that a note has breadth, must say something rather more precise about it: and once this has been defined, many of the subsequent issues will become clearer.[12] But to understand them it is necessary, in addition to what has been mentioned, to discuss relaxation, tension, depth, height and pitch, so as to say how they differ from one another. No one has said anything about these: people have grasped some of them not at all, others only confusedly.[13]

4 Next, we must speak of extension between high and low, and say whether it can or cannot be indefinitely increased and diminished, or can in one sense and not in another.[14] When that has been determined, we must speak of the interval in general, and then it must be divided in as many ways as it can be.[15] The next subject is the *systēma*: having considered it in general we must say into how many divisions its nature requires it to be distinguished.[16] Then we must give a general account of melody, and a sketch of its nature, the nature, that is, of the melody proper to music; for melody has several different natures, but only one out of all of them is that of the kind that is attuned [*hērmosmenon*] and capable of being sung.[17] But in drawing together a

[11] Lasus of Hermione was a distinguished late sixth-century musician, said to have written the first book on music (see the entry under his name in the *Suda*, cf. Martianus Capella *De Nupt.* IX.936), credited with experiments in acoustics (see 1.4 Theon Smyrn. 59.4) and innovations in musical practice, especially in Athens (ps.-Plut. *De Mus.* 1141b–c, Herodotus *Hist.* VII.6, Athenaeus *Deipn.* 455c, 624e–f). Epigonus, another sixth-century musician, was born in Ambracia but worked mainly in Sicyon. Innovations attributed to him include (perhaps mistakenly) the invention of the forty-stringed *epigoneion* (Athenaeus *Deipn.* 183c–d, 637f, Pollux *Onomastikon* IV.59).

[12] The thesis that a note has 'breadth' cannot have been designed, as Macran thought, to challenge the doctrine that notes are dimensionless points (for later criticisms of this Aristoxenian view see 6 Theophrastus ap. Porph. *Comm.* 64.24ff., 11 Ptol. *Harm.* Book I ch. 9). No such doctrine is implied in any theorist before Eratocles (probably late fifth century), and sophisticated discussions of the reality of dimensionless points are unlikely before Zeno (mid-fifth century). Three possibilities suggest themselves. (i) The thesis was an attempt to explicate Pythagorean attributions of quantitative properties to notes. (ii) It reflects commonplace Greek descriptions of notes as 'sharp', *oxys*, or 'heavy', *barys*, in a way later developed by Peripatetics who linked a sound's pitch with its 'shape' (see 4.16 ps. Ar. *Probs* XIX.8). (iii) It seeks to explain the way in which a limited number of notes 'fill up' a stretch of musical space, so that no more can melodically be fitted between them (compare Aristoxenus' idea that the moveable notes have ranges of variation which abut, but do not overlap, so that though each is dimensionless, it is the sole occupant of a determinate region of pitch, which no other note can enter: see 23.26ff.). This last interpretation gives point to Aristoxenus' remarks here, especially in view of the importance he assigns to the analysis of scalar succession and continuity (cf. n. 19 below), but the issue cannot be decided with certainty.

[13] For the discussion see 10.11–13.30. Even Aristoxenus' account 'confused' some later writers: see 12 Arist. Quint. *De Mus.* 6.28ff.

[14] See 13.30–15.12.

[15] In general, 15.24–34; its 'divisions', 16.16–34.

[16] In general, 15.34–16.1; its 'divisions', 17.1–18.4.

[17] See 18.5–19.16. The term *hērmosmenos*, contrasted with *anharmostos* ('ill attuned') is common in Aristoxenus and other musical theorists, as is the noun *to hērmosmenon*, 'the attuned', 'attunement'. A series of notes or intervals is *hērmosmenos* if it conforms to the proper rules of harmonic order and so reflects the nature of *to hērmosmenon*; a series that breaks the rules may be said to 'destroy *to hērmosmenon*' (e.g., 54.19–21). 'Capable of being sung' translates *melōidoumenon*, another common term, literally

conception of this variety of melody and dividing it off from the others, it will be necessary to touch a little on the nature of the other varieties too. Once musical melody has been distinguished as far as it can when we have not yet studied each detail of it, but only sketched it in broad outline, this overall whole must be divided and analysed into as many genera as it seems to have.[18] After this we must speak of continuity and succession, and say what they are in *systēmata* and how they come about in them.[19]

We must then set out those distinguishing features of the genera which lie in the moveable notes, and the ranges [*topoi*] within which these move.[20] No one previously has ever had even the least understanding about any of these, but we ourselves must start from the beginning in our investigation of all the things mentioned: nothing of any value has been handed down to us about them. After that we must speak of intervals, the incomposite first and then the composite. And when we come to composite intervals, which in a way are the same as *systēmata*, we must find something to say about the synthesis of incomposite intervals.[21] Most of the harmonicists did not even realise that this is a subject to be considered, as became clear in our earlier work. The school of Eratocles[22] got as far as saying simply that the melodic sequence splits into two as it moves in either direction from the interval of a fourth, but without distinguishing whether this division originates from all fourths, or saying what the reason for it is,[23] and without enquiring in what way the other intervals are put together with one another, and whether there is some definite principle governing the synthesis of every interval with every other, and governing the ways in which *systēmata* arise from them and in which they do not, or whether this is quite indeterminate.[24] No account of these things, with or without

'that which is sung', but implying 'that which is properly or legitimately melodic'. Compare 28.6ff., where Aristoxenus claims that it is 'impossible' to sing certain sequences: he means not 'physically impossible', but 'impossible in the context of properly formed melody'.

18 See 19.16–29.

19 The terms *synecheia* ('continuity') and *to hexēs* ('succession') are largely interchangeable (but see n. 129 below). Roughly, two notes are successive in a given *systēma* if, in that context, no note lies between them. Similar definitions cover continuity or succession between *systēmata*. But Aristoxenus' detailed discussions are complex. The one promised here is at 27.15–29.1; see also especially 52.33–53.32, 58.14–61.4.

20 See 21.31–27.14.

21 The discussion of incomposite intervals is perhaps that of 21.20–31; see also 45.34–46.18, 60.10–61.4. On their synthesis see 29.1–34, cf. 52.8–33, 53.33–55.2, and the theorems of Book III.

22 On the 'earlier work' see n. 8 above. Eratocles probably belongs to the late fifth century. Only the *El. Harm.* (here and in the next two pages) give details of his work (Porph. *Comm.* 3.5 is the merest echo of Aristoxenus). It treats him and his school as having occasionally, if haphazardly, seen a little further than most *harmonikoi*.

23 This probably refers to the alternative progressions from one tetrachord to another, in conjunction or in disjunction by a tone: see especially 58.14–59.33. Aristoxenus' complaint is that Eratocles failed to say whether the alternatives are both available after just any four-note sequence spanning a fourth, or only in special cases. He himself holds that they are open only at the junctions of tetrachords bounded by fixed notes. Several arguments in Book III seem expressly designed to prove this: see 65.31ff., 66.9ff., 71.23ff.

24 Aristoxenus is firmly committed to the view that these matters are orderly, subject to

demonstration,[25] has ever yet been given by anybody. Though the putting together of melody has in fact a remarkable orderliness, music has been accused by some people of extreme disorder, because of those who have handled incompetently the science we are discussing: yet none of the objects of perception displays so great or so fine an order. That this is so will become clear to us when we have actually embarked upon the science: but we must now mention its remaining parts.

6 When we have demonstrated the way in which the incomposite intervals are put together with one another, we must discuss the *systēmata* which are constructed out of them, the complete *systēma* as well as the others;[26] and we must demonstrate from a consideration of them how many they are and what they are like, displaying both the differences between them in respect of magnitude and the differences displayed by each of the magnitudes in respect of arrangement, combination and position, so that none of the magnitudes, arrangements, combinations or positions involved in melodies may be left without demonstration.[27] No one else, up to now, has ever got to grips with this part of the science. Eratocles did attempt an enumeration in just one section of it, without demonstration; but we saw previously, when we were examining that matter in its own right, that what he said amounted to nothing, but was all false, and his perception of the phenomena was erroneous. No one, as we said before, has dealt with any of these matters at all, except that Eratocles, taking one *systēma* in one genus, attempted to enumerate the arrangements of the octave, displaying them by moving the intervals around cyclically, but without demonstration.[28] He did not understand that unless the arrangements of the fifth and the fourth have also been demonstrated, along with the form of combination [*synthesis*] according to which they can be melodically put together, his account will generate many times the seven

fixed principles. Here he refers especially to that enunciated at 29.6–14, 53.33–54.21, which is the basis of many of the theorems about the 'synthesis' of intervals in Book III.

25 Demonstration (*apodeixis*) involves the logical derivation of propositions from appropriate principles. The principles themselves are not demonstrable (44.3–15), but whatever depends on them must be demonstrated in order to be explained. These points are dependent on Aristotle's arguments, especially in the *Post. An.*: see the introduction to chapter 3.

26 The expression 'complete (or 'perfect', *teleion*) *systēma*' is used by some authors of the octave (e.g., 12 Arist. Quint. *De Mus.* 14.16), but here probably refers to the two-octave structure of the GPS (see General Introduction, p. 13 and cf. 11 Ptol. *Harm.* Book II ch. 4).

27 Aristoxenus promises to proceed, after his treatment of proper sequences of incomposite intervals, to demonstrate the patterns taken by sequences on a larger scale, in *systēmata* such as the fourth, fifth, octave and double octave. The surviving theorems of Book III prepare the ground; Aristoxenus is just setting out to address the larger subject when the text breaks off. (That Book III corresponds in this respect to the proposal made here does not of course prove that it belongs, in its present form, to the same treatise as Book I. See the introduction to this chapter.)

28 These cyclic rearrangements of the octave form the system of *harmoniai* mentioned again at 36.30–2: see the General Introduction, p. 15 and, for example, 12 Arist. Quint. *De Mus.* 15.9–19.

arrangements.[29] We explained previously that this is so:[30] let us therefore put these matters aside and specify the remaining parts of the science.

7 When the *systēmata* have been enumerated in each of the genera, with reference to each of the types of distinction we have mentioned, the same thing must be done for mixtures of the genera.[31] Most of the harmonicists did not even realise that this is something which should be studied: they had not even come to understand what mixture is.[32] The proper sequel to this is a discussion of notes (*phthongoi*), since intervals are not enough by themselves to give an understanding of notes.[33]

Each of the *systēmata* is placed and sung in some particular region (*topos*) of the voice; and although this makes no difference to the *systēma* considered in itself, it makes not just a casual difference, but just about the greatest difference there is to the melody produced in it. The practitioner of the present science must therefore discuss regions of the voice both in general and in detail, so far as this is appropriate – that is, to the extent that the nature of the *systēmata* themselves indicates. And we must speak of the mutual affinities of *systēmata* and regions of the voice, and of *tonoi*, not by concentrating on *katapyknōsis*, as the harmonicists do, but by studying the melody that occurs between one *systēma* and another, and in which *tonoi* they lie when the melody can move from one to another. Certain of the harmonicists have briefly touched on this part of the science, quite accidentally, while not actually addressing themselves to this matter, but seeking to compress [*katapyknōsai*] the diagram; but to put it generally, not one of our predecessors has left us a clear account
8 of it. Yet to speak generally, it is this part of the study of modulation that is relevant to the theory of melody.[34]

[29] To generate only seven arrangements from the seven intervals in the octave it must be assumed that the intervals occur only in certain sequences within the fourth and the fifth out of which the octave is composed. Aristoxenus undertakes to show, in Book III, that various arrangements of intervals within the fourth and the fifth, and various sequences crossing the boundaries of tetrachords, are illegitimate. See 62.34ff., 63.5ff., 63.21ff., 65.30ff., 66.9ff., 71.23ff. Without such demonstrations, the number of permissible arrangements of the octave cannot be restricted to seven.

[30] See 2.29–30.

[31] The reference is probably to *systēmata* in which each tetrachord contains notes belonging properly to different genera. See 27.9–11, 52.13–32, cf. 12 Arist. Quint. *De Mus.* 29.5–7. It can hardly be to *systēmata* containing whole tetrachords of different genera, since these would inevitably break the fundamental rule stated at 29.6–14, 53.33–54.21. But contrast 11 Ptol. *Harm.* 38.33ff., 75.9ff., 80.6ff.

[32] Macran found this sentence puzzling, but Aristoxenus may merely be reminding us of the harmonicists' neglect of genera other than the enharmonic. They cannot have understood 'mixture' if they had not even distinguished the genera adequately.

[33] The names of the notes are not listed in what remains of Aristoxenus: see, for example, 12 Arist. Quint. *De Mus.* 7.15ff. Aristoxenus defines 'note' at 15.15–16 as 'the incidence of the voice on one pitch', which sheds no light on the nature of the problems indicated here. The issue is treated in an altogether more sophisticated way in Book II: see especially 36.2–14, 47.8–50.14, and cf. 39.4–40.24.

[34] For references to studies of *tonoi* and of modulation, missing from our texts of the *El. Harm.*, see n. 3 above. Two issues seem to be raised here. First, do *systēmata* of different forms or arrangements (like those of the so called *harmoniai* of 36.30–2) belong properly to different relative pitches, and if so which? The matter is to be studied in so far as it is implicit in 'the nature of *systēmata*', that is, so far as the relations between such

Those, then, are the parts of the science called harmonics, and that is all there are. Studies of a higher level than these should be thought of as belonging to some more inclusive science, as we said at the outset.[35] Concerning them we must say on the proper occasion what they are and how many there are, and what each is like: but now we must try to follow through the one which is first.

First of all, then, we must discuss the different kinds of movement with respect to place [*topos*], and try to understand what they are.[36] While every vocal sound can move in the manner mentioned, there are two forms of this movement, the continuous and the intervallic. In the continuous form the voice seems to perception to traverse a space [*topos*] in such a way as never to stand still even at the extremities themselves, at least so far as its representation in perception is concerned,[37] moving continuously to the point of silence; whereas in the other, which we call intervallic, it seems to move in the opposite way. During its course it brings itself to rest at one pitch and then at another: it does this continuously (I mean continuously in respect of time), passing over the spaces bounded by the pitches, but coming to rest on the pitches themselves and
9 sounding them alone, and is described as singing, and as moving in intervallic motion. Each of these descriptions is to be understood with respect to the representation of perception. Whether it is actually possible or impossible for the voice to move and then to come to rest upon a single pitch, is a question belonging to a different enquiry, and for the purposes of the present science an account of the motion involved in each of these is unnecessary.[38] For whatever

arrangements are determined by harmonic laws and structures themselves, not, for example, by the available ranges of voices or instruments (cf. II Ptol. *Harm.* Book II ch. 7). Secondly, we are to discover in which *tonoi* these differently arranged *systēmata* lie when modulation between them is possible. This suggests that *tonoi* are not themselves conceived as incorporating different arrangements. They are probably identical interval-sequences at different relative pitches (roughly analogous to similar scales in different keys), and movement to a different *tonos* thus projects a different arrangement of intervals onto a given range of pitch. See also n. 33 to Book II.

The procedure of *katapyknōsis* is again criticised at 28.1–2, 38.3, 53.4–5. The word is related to *pyknos*, and means 'compression'. The objective was probably to represent a complete collection of *systēmata* (e.g., the seven *harmoniai*) in as 'compressed' a diagram as possible (cf. n. 6 above), placing each in the same theoretical range of pitch. This will make comparison between interval structures easier, but will do nothing, as Aristoxenus says, to display the pitch-relations in which they stand when modulation between them is permissible.

35 See 1.22–2.6.

36 As promised at 3.5. The careful discussion that follows was the model for many later accounts: cf., for example, 10 Nicomachus *Ench.* ch. 2, 12 Arist. Quint. *De Mus.* 5.24ff., neither of which is as clear and consistent as the present passage; the Aristoxenian basis of both is obvious. Aristides' Platonist affiliations are suppressed at this stage of his treatise, and Nicomachus' Pythagoreanism only serves to confuse the issues. Properly 'Pythagorean' analyses need to take a different approach: cf. Ptolemy's treatment at II *Harm.* 10.5ff.

37 Aristoxenus often emphasises that his object is to describe the phenomena as perceived, not the physical events or processes that might give rise to them. See especially the continuation of this passage at 9.3ff., and, for example, 32.19–28.

38 See the previous note; the 'different enquiry' deals with the physics of sound. Aristoxenus concedes that from the physicist's point of view, what he calls 'rest' may be a form of movement, but he insists that this fact, if it is one, is irrelevant to harmonics. The opposite view, of course, is taken by Pythagorean and Platonist writers.

the truth about that, it at any rate makes no difference to the way we distinguish melodic movement of the voice from movements of other kinds. The straightforward fact is that when the voice moves in such a way as to appear to the hearing to stand still nowhere, we call this movement continuous: but when it appears to stand still in one position and then seems to pass over some space, and having done this appears again to stand still at another pitch, and goes on appearing to perform this process of alternation continuously, we call this kind of movement intervallic. We say that continuous movement is the movement of speech, for when we are conversing the voice moves with respect to place in such a way that it seems never to stand still. In the other form, which we call intervallic, its nature is to move in the opposite way; for it does seem to stand still, and everyone says that the person who appears to be doing this is no longer speaking, but singing. For this reason we avoid bringing the voice to a standstill when we are talking, except that some affliction may sometimes force us into this kind of movement;[39] but in singing we do the opposite, avoiding the continuous and pursuing vocal immobility as far as we possibly can. For the more we make each of our utterances single and stationary and the same, the more precise the melody seems to perception. It is clear then from what we have said that there are two kinds of movement of the voice with respect to place, and that of these the continuous is that of speech, the intervallic that of melody.[40]

It is evident that in singing the voice must make its tensions and relaxations imperceptibly, and when uttering the pitches themselves must make them apparent; for its progress across the space of the interval which it traverses, whether relaxing or increasing tension, must not be detected, whereas it must give out the notes that bound the intervals clearly and without movement. Since this is evidently so, we must now speak of tension and relaxation, height and depth, and in addition to these, pitch.[41]

Tension, then, is the continuous movement of the voice from a lower region to a higher, and relaxation is that from a higher region to a lower.[42] Height is

The text of the last clause is uncertain. I take 'the motion involved in each of these' to refer to the kinds of physical movement alleged to be involved in (a) the apparent movement of the voice in pitch and (b) its apparent rest, not, as Macran argues, to those involved in intervallic and continuous vocal movement respectively.

39 'Affliction', *pathos*, might refer to either physical or emotional disturbance, or to both.

40 This sentence and the next paragraph give valuable information about current styles of singing. Aristoxenus implies that precision and stable 'rest' at a pitch is a virtue in performance: indeterminate pitching and portamento are not musical. Cf. 6 Theophrastus ap. Porph. *Comm.* 64.32ff., 11 Ptol. *Harm.* 10.5ff. An intermediate kind of vocal movement, in addition to those of speaking and singing, is identified at 12 Arist. Quint. *De Mus.* 5.25–6.7.

41 This announces a study of the topics proposed at 3.27ff.

42 The conception of high pitch as involving increased tension (*epitasis*), low pitch as involving relaxation (*anesis*), was well entrenched in the Greek language. Aristoxenus makes the term *epitasis* refer to a progressive process, not to a state ('increasing tension', not 'being tense'), and treats *anesis* similarly. Despite the argument of 11.3ff., he applies the terms primarily to the vocal sound itself, not to the physical means of its production: it is the vocal sound, not the vocal organ, that is tensed or relaxed, even when it conducts these processes in silence. For a confused echo of this passage see 12 Arist. Quint. *De Mus.* 6.27ff.

the product of tension, depth that of relaxation. It might seem odd to those who consider these matters superficially that we are positing four items here and not two: most people, at any rate, say that tension is the same as height
11 and relaxation the same as depth. Hence it is perhaps worth one's while to be shown that their views about these things are confused. We should seek to understand this by looking at what exactly the occurrence is that we bring about when, in the process of tuning each of the strings, we relax or tense them. It is plain, at any rate to those not wholly inexperienced with instruments, that when we tense a string we are drawing it towards height, and when relaxing it we are drawing it towards depth: but while we are altering the string and drawing it towards height, it is not possible for the height – that height, at any rate, which the tension will produce – to exist somehow already.[43] The height will exist after the tension has drawn the string to the proper pitch, and the string stands still and no longer moves. This will be the case when the tension has stopped and no longer exists, since the string cannot be moved and stand still at the same time; and we have made it clear that tension exists when the string is moving, and height when it has come to rest and moves no longer. We shall say just the same about relaxation and depth, except that these relate to movement towards the opposite region. It is obvious from what has been said that relaxation differs from depth as does the producer from what is produced, and that tension and height differ in the same way. Thus our discussion has shown that tension is different from height and relaxation from depth: and we must now try to understand that the fifth item, which we call pitch,[44] is
12 different from each of those mentioned. What we mean by pitch is something like the voice's stability, or standing still. Let us not be disturbed by the opinions of those who reduce notes to movements, and who say quite generally that sound is movement,[45] as though we should be obliged to say that it sometimes happens that movement does not move, but is stationary and at rest. It makes no difference to us if pitch is called evenness or sameness of movement, or if some yet more learned name than these is invented. We shall say, none the less, that the voice stands still when perception exhibits it to us not setting off towards the high or the low; and all we are doing is attaching this name to that sort of qualification of the voice. The voice appears to do this when it sings; for it moves while it makes an interval, and stands still on a note. If when it moves in what we call motion it is acquiring a difference in speed in the motion of which these people speak, and if when being at rest in what we call rest its speed remains constant and retains one and the same pace,[46] that

[43] Note that 'height' and 'depth', as most commonly, translate *oxytēs* (lit. 'sharpness') and *barytēs* (lit. 'heaviness') respectively. The metaphors of 'up' and 'down', and of movement between high and low, are not built into ordinary Greek in the way that those of tension and relaxation are.

[44] *Tasis*, a word from the same stable as *epitasis*, *teinein*, *tonos*, etc., all originally to do with stretching and tightening. A *tasis* is a stable state of tension, and may be attributed straightforwardly to a string. As a property of a note it is its pitch.

[45] Another reference to physical theories current in Pythagorean, Platonist and Aristotelian circles: see nn. 37 and 38 above.

[46] This reflects the 'velocity' theory of Archytas (1.19 frag. 1) and many subsequent acoustic theorists.

need make no difference to us. For it is clear enough what we call movement and rest of the voice, and what they call movement. That is enough about this for the moment: it has been explicated more fully and clearly elsewhere.[47]

13 That pitch is neither tension nor relaxation is altogether obvious, since pitch, as we say, is rest of the voice, while we have already found that the others are kinds of movement. We must now try to understand that pitch is something different from the remaining items, depth and height.[48] It is evident from what we have said that the voice comes to rest both when it reaches depth and when it reaches height: from our next remarks it will become clear that though pitch has been specified as a kind of rest, that does not make it the same as either of them. It must be understood that for the voice to be at rest is for it to remain at one pitch; and this will happen to it whether it is at rest at depth or at height. If pitch is present in both cases, since it has been shown that the voice must rest both upon the low and upon the high, and if height never coexists with depth nor depth with height, pitch is evidently something different from each of them, as being something common to both. What we have said is then enough to show that these five, pitch, height and depth, tension and relaxation, are different from one another.

Now that these points have been grasped, we must next consider extension 14 between high and low, to enquire whether it is unlimited or limited in each direction.[49] It is not hard to see that if considered solely in relation to voice, it is not unlimited, since for each voice, whether it is that of an instrument or that of a human being, there is a determinate maximum and a determinate minimum range which it can traverse melodically. For the voice cannot augment the extension between high and low without limit of largeness, nor contract it without limit of smallness, but in both directions comes somewhere to a standstill. Each of these must be determined in relation to two things, that which utters sound and that which discriminates it, these being the voice and the hearing. What the one of these is incapable of producing and the other of discriminating must be reckoned as falling outside the extension which is usable and capable of occurring vocally. In their progress towards the small, voice and perception seem to reach the limits of their competence at about the same time. The voice cannot distinctly produce an interval even smaller than the smallest diesis, nor can the hearing detect one, in such a way as to grasp what part it is either of a diesis or of any of the other intervals which are known.[50] In their progress towards the great, the hearing would perhaps

[47] It is not discussed further in any surviving passage of Aristoxenus.

[48] See n. 43 above.

[49] This is the enquiry proposed at 3.34–4.3. The issue is not whether there is a highest and a lowest note, but whether there is a maximum and a minimum interval. That previous theorists had, for their own special purposes, been interested in the notion of a minimum (detectable) interval is made clear by 2.1 Plato *Rep.* 531a–b. The latter part of 10 Nicomachus *Ench.* ch. 2 is a muddled reinterpretation of this passage.

[50] This is not to deny that such intervals could be both sounded and detected, but they could not be identified as constituting any particular size of interval, and hence are melodically unusable. (Though Aristoxenus insists that the smallest usable interval is the enharmonic diesis, a quarter-tone, he is perfectly prepared to employ smaller intervals, down to one twelfth of a tone, in his theoretical calculations.) Aristoxenus'

appear to extend further than the voice, but not by much.[51] At any rate, whether we are to take the limit of extension in each direction as being the same when we consider the voice as when we consider the hearing, or to take it as being the same in their progress towards the small and different in their progress towards the great, the extension will still have a greatest and a least size, either common to that which utters and that which discriminates, or peculiar to each. It is clear, then, that considered in relation to the voice and the hearing, extension between high and low will not progress indefinitely in either direction. Whether it will turn out, on the other hand, that progress towards the greater can continue indefinitely if the constitution of melody is considered in itself alone, is a matter for a different enquiry, irrelevant to our present concerns; and we must therefore try to investigate it on a later occasion.[52]

Now that this is understood we must say what a note [*phthongos*] is.[53] To put it briefly, a note is the incidence of the voice on one pitch: for it is when the voice appears to rest[54] at one pitch that there seems to be a note capable of being put into a position in a harmonically attuned melody [*melos hērmosmenon*]. That, then, is the sort of thing a note is.

An interval [*diastēma*] is that which is bounded by two notes which do not have the same pitch, since an interval appears, roughly speaking, to be a difference between pitches, and a space capable of receiving notes higher than the lower of the pitches which bound it, and lower than the higher of them.[55] Difference between pitches lies in their having been subjected to greater or lesser tension.[56]

That is what one might say to outline a definition of the interval: and a
16 *systēma* is to be understood as something put together from more than one interval.[57] The student should try to accept each of these in the right spirit, without quibbling over whether the account offered of each is exact or only rather approximate. Rather, he should try to grasp it sympathetically, on the assumption that enough has been said for the purpose of instruction when the account has the capacity to put him on the road to understanding the thing that is spoken of. For it is difficult, perhaps, in all cases where we are dealing with things that stand at the beginning, to articulate an account that contains an

claim here is in part self-confirming. Though we could tell if the voice could not produce intervals as small as some heard in melodic use on instruments, how could we establish that the ear was *less* discriminating than the voice?

[51] A proposition that could be established by listening to instruments, or more fully on the basis indicated at 20.32ff.

[52] This wholly theoretical issue is not discussed in the sequel. There is one relevant remark about the notion of a least interval at 46.16–18: cf. 9.3 Adrastus ap. Theon Smyrn. 63–5. In the present sentence I read ⟨*ei*⟩ *symbēsetai* with Bellerman and Macran.

[53] As promised at 7.7–10; this brief discussion breaks the sequence of the original list. On the issues raised by the definition of 'note' see n. 33 above.

[54] Reading *hotan hē phōnē phanēi hestanai* with Meibom.

[55] This is the preliminary definition promised at 4.3–4: cf. 12 Arist. Quint. *De Mus.* 10.16–19. On the conception of an interval as an empty space bounded by 'points', see 11 Ptol. *Harm.* Book 1 ch. 9.

[56] See 10.24ff.

[57] The generalised definition promised at 4.6. See also n. 3 above.

exhaustive and accurate interpretation, and not least in the case of these three, the note, the interval and the *systēma*.

Having defined these things in this way, we must first try to divide the interval, and next the *systēma*, into as many useful divisions as their nature permits.[58] The first division of intervals is in respect of their differences in magnitude;[59] the second in respect of the differences between concordant and discordant intervals;[60] the third in respect of the differences between the composite and the incomposite;[61] the fourth in respect of genus;[62] and the fifth in respect of the differences between the rational and the irrational.[63] The remaining differences should be ignored for now, since they are of no use for the purposes of the present study.[64]

17 One *systēma* can differ from another in respect of all the same kinds of difference except one. It is obvious that *systēmata* can differ in magnitude, and in the concordance or discordance of the notes which bound the magnitude. But it is impossible for the third of the differences between intervals which we mentioned to exist between one *systēma* and another, since clearly it cannot be the case that some *systēmata* are composite and some incomposite, at any rate not in the same way in which some intervals are composite and some incomposite. The fourth, that in respect of genus, must exist also in *systēmata*, since some of them are diatonic, some chromatic and some enharmonic. The fifth must exist in them too, since some of them are bounded by an irrational interval and others by a rational one. Beside these we must add three other divisions. One divides them in respect of conjunction, disjunction, and the combination of these two, since any *systēma* which equals or exceeds a certain magnitude is either conjunct or disjunct or a mixture of both – this last can be

[58] This corresponds to the rest of what is proposed in 4.3–8.

[59] See n. 7 above.

[60] For definitions or characterisations of concordance see particularly **2.5** Plato *Tim.* 80a–b, **4.25** ps.-Ar. *Probs* XIX.41, **6** Theophrastus ap. Porph. *Comm.* 65.15ff., **8** Eucl. *Sect. Can.* 149.17–20, **9.2** Adrastus ap. Theon Smyrn. 51, **11** Ptol. *Harm.* 10.25–8, **12** Arist. Quint. *De Mus.* 9.26–10.6, Cleonides *Eisagoge* 187.19–188.2; cf. **3.12**, **3.13**, **3.16** Arist. *De Sensu* 447a–b, 448a, *De Anima* 426a–b, **4.21**, **4.24**, **4.26** ps.-Ar. *Probs* XIX.35, 39, 42, **5** *De Audib.* 803b–804a. Aristoxenus makes no attempt at a definition: concord and discord are basic 'givens' of musical experience. The concords are enumerated at 20.1ff., 44.28ff.; for another important thesis about them see 55.3–10.

[61] The notions of the composite and the incomposite are elaborated at 60.10ff. For references to related discussions see n. 21 above.

[62] Some magnitudes belong only to one genus (e.g., the quarter-tone to enharmonic). Others belong to a particular genus when they are incomposite (e.g., the incomposite ditone is proper only to enharmonic).

[63] The surviving parts of *El. Harm.* make no use of this distinction. The terminology is familiar in ratio theory (cf., for example, **12** Arist. Quint. *De Mus.* 11.4ff., which is a set of 'Pythagorean' remarks set in an otherwise 'Aristoxenian' context). Aristoxenus' meaning is clarified in *El. Rhythm.* II.21: see the Appendix to this chapter, with Macran (1902), pp. 237–40. Briefly, an interval is rational if (a) it can be sung, and (b) it is recognisable as either a concord or a tone, or as commensurable with them in such a way that their common measure is an interval actually used in melody. Cf. da Rios (1954), n. 5 to p. 24 of the translation. A feeble account of the matter is given at Cleonides *Eisagoge* 189.2–8.

[64] That is, for the purposes of the science of harmonics. For what two of the 'irrelevant' distinctions might be, see **12** Arist. Quint. *De Mus.* 11.14–17, 21–23.

shown to occur in certain cases.[65] Then there is that which divides them into gapped and continuous, since every *systēma* is either continuous or gapped.[66]
18 There is also the division into simple, double and multiple, since any given *systēma* is either simple or double or multiple. What each of these is will be shown in the subsequent discussion.[67]

With these definitions and preliminary distinctions behind us, we must now consider melody, and try to outline what its nature is.[68] We have already said that the movement of the voice in it must be intervallic, so that musical melody has been distinguished in this way from the melody of speech, at any rate. (For there is indeed said to be a kind of melody which belongs to speech, that constituted by the tone-patterns that occur in words, since tension and relaxation belong naturally to speech.[69]) But harmonically attuned melody [*to hērmosmenon melos*] must not only consist of intervals and notes: it demands also a way of putting them together which is of a special kind, and not haphazard, since it is plain that the property of being constituted out of intervals and notes is of wider scope, belonging also to that which is harmonically ill-attuned [*anharmoston*]. Since this is so, it must be granted that the most important factor, and the one carrying the greatest weight in the pursuit of the correct constitution of melody, is that which deals with the process of combination and its special peculiarities. It is clear, then, that musical melody differs from the melody that occurs in speech by its use of the intervallic movement of the voice, and from harmonically ill-attuned and faulty
19 melody by a difference in the way the incomposite intervals are put together. What this way is will be shown in the sequel. For the present let me say at least this much, in a general way: that although what is harmonically attuned puts its intervals together in many different ways, nevertheless there is a feature which we shall assert to be one and the same in everything that is harmonically attuned, whose power is such that its removal removes with it the attunement [*to hērmosmenon*]. This will become clear as the investigation proceeds.[70] Let us then accept this as distinguishing musical melody from the other kinds: but it must be understood that this distinction has been drawn only in outline, since the details have not yet been explained.

65 On conjunction and disjunction see n. 23 above. A *systēma* contains a mixture of both if (like the GPS) it is composed of tetrachords of which some are conjoined, some disjoined.

66 'Continuous' translates *synechēs*, 'gapped' *hyperbaton*. A *systēma* is gapped or transilient if it omits notes which the general principles of continuity and succession would assign to it. See 12 Arist. Quint. *De Mus.* 13.7–9, 14.26–7; cf. 29.14–17, 79.24–5, 81.15–17, and for possible examples 18.18–20, 18.23–5; cf. also ps.-Plut. *De Mus.* 1134f ff. and 1137b–d.

67 Not in the surviving parts of the *El. Harm.* But see 40.20–1, which indicates that the distinction is that between the non-modulating and the modulating. Cf. 12 Arist. Quint. *De Mus.* 13.9–11, Cleonides *Eisagoge* 201.14–18.

68 This corresponds to the promise to give a general account of melody, 4.8ff.

69 Pitch-relations between syllables were an important part of the pronunciation of spoken Greek, but the surviving scores suggest that the patterns of this 'speech-melody' had little effect on the way in which words were set to music.

70 For references to this crucial principle see n. 24 above.

Following on from these remarks, we should distinguish melody considered as a whole into the number of genera into which it appears to be divided.[71] It appears to be divided into three, since any given melody which is attuned [*hērmosmenon*] on a single basis[72] is either diatonic or chromatic or enharmonic. Of these the diatonic, since human nature comes upon it first, must be reckoned the first and oldest, the chromatic second, and the enharmonic third and most sophisticated, since perception becomes accustomed to it at last, with difficulty, and through much hard work.[73]

After distinguishing the genera into this number of types, we must next try to investigate one of the two classes distinguished by the second on our list of the divisions of intervals.[74] These classes were discord and concord; and concord is what must now be considered. One concordant interval can apparently differ from another in several respects, of which one is magnitude; and we must therefore explain what the distinctions of magnitude appear to be. The smallest concordant interval seems to be determined by the intrinsic nature of melody, since there are many melodic intervals smaller than the fourth, but they are all discordant. Thus the smallest concord is determined in accordance with the intrinsic nature of voice, but the greatest does not seem to be so determined. So far as the nature of melody is concerned, the concordant appears to increase without limit, as does the discordant: for when any concordant interval is added to the octave, whether it is greater than the octave or smaller or equal to it, the resulting whole is concordant.[75] Considered in this way, then, there seems to be no greatest concordant interval: but in respect of our own practice, by which I mean the practice engaged in both through the human voice and through instruments, there does appear to be a concord which is greatest. It is the double octave and a fifth, since our compass does not extend as far as three octaves.[76] We must define this compass by reference to the sound and limits of some one instrument.[77] No doubt the highest note of the 'maiden' *aulos* makes with the lowest note of the 'extra-complete' *aulos*
21 an interval greater than the three octaves mentioned, and that interval would also be exceeded by the one formed by the highest note of a performer piping

[71] As promised at 4.20–1.

[72] Accepting Macran's conjecture *tōn eis tauto hērmosmenōn.*

[73] The notion that the diatonic is the most natural genus, the enharmonic the most recondite, is a commonplace of Greek authors: see, for example, 12 Arist. Quint. *De Mus.* 16.10–18. 'Most sophisticated' translates *anōtaton*, lit. 'highest', the reading of most MSS; one MS, followed by Westphal, has *neōtaton*, 'most recent'. The sense of *anōtaton* might be 'highest in worth' (cf. 23.4ff. and ps.-Plut. *De Mus.* 1145a–c), so that Da Rios' translation 'più elevato degli altri' is possible, but fits the context less well.

[74] The list is at 16.17ff.

[75] Cf. 45.20ff. This principle is followed by other Aristoxenian sources, and by Ptolemy. As he notes (11 *Harm.* 13.1–23) it was one to which many Pythagoreans could not subscribe, since their assumptions entailed that the interval of an octave plus a fourth was not concordant.

[76] Many other writers, according to Porph. *Comm.* 96.11–12, also enumerated eight concords. Ptolemy, because he restricts his attention to the scope of the double octave, is only concerned with six (11 *Harm.* 11.1–5).

[77] Aristoxenus does not explain why, but is perhaps applying a principle similar to that stated at 68.13–69.28.

with the *syrinx* drawn down and the lowest note of one playing the *aulos* in the usual way:[78] the voice of a small child would also be related in the same manner to that of a man. It is from cases like these that we get to know the greater concords: for it is from different ages and different dimensions that we have discovered that the triple octave, the quadruple octave, and even greater intervals than these are concordant. Our discussion has shown, then, that as we move towards the smaller, it is the intrinsic nature of melody that makes the fourth the smallest of the concords, whereas when we move towards the greater, the greatest concord is determined in one way or another by our own capacities: and it is easy to see that there will turn out to be concordant intervals of eight magnitudes.[79]

Now that we have explained these points, we must try to define the interval of a tone. The tone is the difference in magnitude between the first two concords.[80] It is to be divided in three ways, since the half, the third and the quarter of it should be considered melodic. All intervals smaller than these are to be treated as unmelodic.[81] Let the smallest of them be called the least enharmonic diesis, the next the least chromatic diesis, and the greatest the semitone.

Given these definitions, we must try to understand the source from which the differences between the genera arise, and the manner in which they do so.[82] We must concentrate on the smallest of the concords, that which is, at least usually, occupied by four notes:[83] it is from this feature that it takes the name which it was given in antiquity.[84] Of the several arrangements that there are we must concentrate on one, that in which there is an equal number of elements which move and of elements which remain constant with changes of genera; and this is the case in an interval such as that from *mesē* to *hypatē*. For there the two bounding notes remain fixed when changes of genus occur, whereas the two between them move. So let us accept this suggestion. But there are several

[78] On the classification of *auloi* used here and on the *syrinx* of the *aulos* see 5 *De Audib.* 804a with nn. 41 and 42.

[79] Reading *oktō megethē* with Macran. The eight are: fourth, fifth, octave, octave plus fourth, octave plus fifth, double octave, double octave plus fourth, double octave plus fifth.

[80] That is, the fourth and the fifth: cf. 45.34–46.1. This definition of the tone, in various forms, is adopted by most Greek writers on harmonics, Pythagoreans included (it is implied as early as 1.12 Philolaus frag. 6, and is explicit, in its Pythagorean version, at, for example, 8 Eucl. *Sect. Can.* proposition 13).

[81] Cf. 46.3ff. Smaller intervals can legitimately be used in theoretical calculations (e.g., 25.11–20), but not as melodic steps. Nevertheless, the three fractions of a tone mentioned here, together with their combinations, do not include all the small melodic intervals that Aristoxenus treats as acceptable and familiar. In particular, they do not include the interval of $\frac{3}{8}$ tone found in the hemiolic chromatic (51.1ff.). More generally, see 26.13ff., 48.9ff., 60.13ff.

[82] As promised at 4.25ff.

[83] At 61.5ff. Aristoxenus offers an argument to show that differences of genus can involve only differences in the intervals inside the tetrachord.

[84] That is, *dia tessarōn*, 'through four (strings)'; cf. *dia pente*, 'through five' for the fifth, *dia pasōn*, 'through all', for the octave. Some early writers use a different terminology: see 1.12 Philolaus frag. 6.

groups of notes in which this arrangement of the fourth is contained, each of which is specified by its own set of names. One of them, however, that which consists of *mesē*, *lichanos*, *parhypatē* and *hypatē*, is probably the most familiar to students of music, and it is a convenient focus for the study of the way in which differences between the genera come about.[85]

It is clear that it is the tensions and relaxations of the notes whose nature it is to move that are responsible for the differences between the genera;[86] but we must say what the range [*topos*] of the movement of each of these notes is.[87] The total range in which *lichanos* moves is a tone, since it does not stand at less than the interval of a tone from *mesē*, nor at a greater interval than a ditone. The smaller of these is accepted by those who have already understood the diatonic genus, while those who have not yet grasped it can be led through a survey of examples to accept it too.[88] The greater is accepted by some but not by others, and we shall go on to explain later the reason why this is so. The fact that there is a form of melodic composition which demands a ditone *lichanos*,[89] and that far from being the most contemptible it is perhaps the finest, is not at all evident to most of those who are concerned with music nowadays, though it might become so if they were led towards it through a survey of examples. But what we have said is plain enough to those who are familiar with the first and second groups of ancient styles.[90] It is to be expected that those who are used only to the style of composition at present in vogue rule out the ditone *lichanos*, since most people nowadays use higher ones. The reason is their endless pursuit of sweetness: that this is their objective is shown by the fact that they spend most time and effort on the chromatic, whereas when they do occasionally come to the enharmonic they force it close to the

85 The text of this paragraph poses problems, but the general sense is not in doubt; cf. 46.19ff. All later Greek theorists agree on the central point, that the differences between genera are found in the variations in size of the three small intervals that make up the distance between fixed notes a fourth apart, that is, in the different positions of the two moveable notes between such fixed points. (On the tetrachords of Archytas, which have only one moveable note each, see the Appendix to chapter 1.) In any *systēma* of a single genus, all such tetrachords will be identical in interval structure, so that any one of them may be taken as representative. Aristoxenus is followed by most writers in using for this purpose the tetrachord *mesōn*.

86 For an argument to this effect see 61.5ff.

87 That is, we must find the limits of the movement of each moveable note in either direction. This is the stated purpose of the long passage from here to 27.14 (as more briefly at 46.24–47.8). It is not that of giving a complete analysis of each generic division, even though much of the material necessary for such an analysis is presented here. Aristoxenus addresses this latter task, for the six most familiar divisions, at 50.19–52.32.

88 This sort of proposition is not one that can be demonstrated from higher principles. It is a generalisation from the facts of perception, as grasped by the musically educated ear. For the structure of the tense diatonic, which Aristoxenus has in mind, see 51.29–31.

89 That is, a *lichanos* separated from *mesē* by an incomposite ditone. This is Aristoxenus' preferred form of the enharmonic: see, for example, 50.23–5.

90 The reference must be to styles of music current before the middle of the fifth century, but its exact intention is uncertain. Possibly the two styles correspond, as Marquard thought, to (a) the form of the enharmonic described by Aristoxenus in the present work, whose tetrachords are divided $\frac{1}{4}$, $\frac{1}{4}$, 2, and (b) the form treated by Aristoxenus (ps.-Plut. *De Mus.* 1134f–1135a) as its precursor, the *spondeion* of Olympus, in which the fourth is divided into only two intervals, $\frac{1}{2}$, 2.

chromatic, and the melody is correspondingly pulled out of shape.[91] This will have to be enough about these matters: let the range of *lichanos* be agreed to be a tone, and that of *parhypatē* the smallest diesis, since it never comes closer to *hypatē* than a diesis and is never more than half a tone away from it. For the ranges do not overlap, but their point of conjunction is their limit, since when *parhypatē* and *lichanos* reach the same pitch, as the one is tensed and the other relaxed, their ranges have their limit, the range below it being that of
24 *parhypatē*, the range above it that of *lichanos*.[92]

Let this be accepted as an account of the overall ranges of *lichanos* and *parhypatē*. We must now speak of their ranges in the particular genera and shades.[93] The proper way of analysing the fourth, and whether it can be measured by reference to any of the smaller intervals or is incommensurable with all of them, is dealt with in our passage on determining intervals by means of concords.[94] But since it appears to be two and a half tones, let it be assumed that this is its magnitude.[95] Let us call '*pyknon*' that which is composed of two intervals which, when put together, cover an interval smaller than that which makes up the remainder of the fourth.[96] Given these definitions, let us take the

[91] On the 'sweetness' of the chromatic compare the interpolated passage at 12 Arist. Quint. *De Mus.* 92.24–6, and, for example, Anon. Bell. 26; for the corrupting influence of sweetness see, for example, Plato *Laws* 802c–d. Aristoxenus does not claim that modern musicians have abandoned the enharmonic, only that they play it in a decadent version, close to the chromatic: cf. ps.-Plut. *De Mus.* 1145d. It seems likely that they replaced the true ditone preferred by Aristoxenus with the perceptibly sweeter major third, represented by Pythagoreans by the ratio 5:4 (the ditone is 81:64): see 1.21 Ptol. *Harm.* 30.9ff. with the Appendix to chapter 1, and compare the tables of divisions of the enharmonic at 11 Ptol. *Harm.* Book II ch. 14. Such a version, according to Aristoxenus, is still genuinely enharmonic, though aesthetically inferior: see 26.9–28, 28.13–15, 48.33–49.18. Aristoxenus does not trace his contemporaries' lack of enthusiasm for the 'best' enharmonic to the difficulty of singing quarter-tones. That is a separate issue. Later writers (e.g., 12 Arist. Quint. *De Mus.* 16.14–18) often account in that way for the disuse of the genus as a whole. Aristoxenus comments on the matter elsewhere (ps.-Plut. *De Mus.* 1145a–c).

[92] The lowest *lichanos* is a ditone below *mesē* (in enharmonic), and the highest *parhypatē* is a semitone above *hypatē* (in tonic chromatic and in diatonic: see 51.8–11, 23–31). Assuming that the fourth between *hypatē* and *mesē* spans $2\frac{1}{2}$ tones (see 24.4–10), the locus of the highest *parhypatē* therefore coincides with that of the lowest *lichanos*. Aristoxenus seems to treat it as a matter of principle that the ranges of *lichanos* and *parhypatē* cannot overlap (nor can those of other notes). He does not enlarge on the grounds of this claim, but cf. n. 12 above.

[93] A shade (*chroa* standardly means 'shade' in the sense 'shade of colour') is a particular version or nuance of a genus. The special aesthetic character of any one genus remains perceptible while the two moveable notes in the tetrachord shift within ranges which Aristoxenus proceeds to specify: cf. 48.15–49.21. The placing of the two moveable notes at any definite positions within the ranges proper to a genus constitutes a shade of that genus.

[94] See 55.3ff.

[95] Cf. 46.1–2. Aristoxenus offers a way of confirming this proposition at 56.13ff.

[96] The term *pyknon* means 'compressed'. Properly speaking it is an adjective, whose noun (*systēma*, as indicated at 24.30 and 29.1–2) is generally omitted in this usage. It applies to the pair of intervals at the bottom of a tetrachord between fixed notes, in cases where the two of them together constitute an interval smaller than the one interval making up the remainder of the fourth. In effect, they must add up to less than $1\frac{1}{4}$ tones: cf., for example, 25.6–8, 51.14–22, and on the aesthetic significance and status of the concept see 48.15–31.

smallest *pyknon*, placed next to the lower of the fixed notes. This will be the one composed of two enharmonic or two of the smallest chromatic dieses. The two *lichanoi* thus specified will be the lowest in each of the two genera, one in the enharmonic, the other in the chromatic:[97] for we have explained that, taken overall, the enharmonic *lichanoi* are the lowest, the chromatic next, and the diatonic the highest. After these, consider a third *pyknon* placed next to the same note,[98] and then a fourth one, which is a tone:[99] fifthly, from the same note take the *systēma* composed of a semitone and an interval one and a half times as great,[100] and sixthly, that composed of a semitone and a tone.[101]

We have already spoken of the *lichanoi* bounding the first two *pykna* listed. The one that bounds the third is chromatic, and the *chrōma* in which it is is called hemiolic.[102] That bounding the fourth *pyknon* is chromatic, and the *chrōma* in which it is is called tonic.[103] The *lichanos* bounding the fifth *systēma* mentioned, which was specified as greater than a *pyknon*, since the two intervals are equal to the one,[104] is the lowest diatonic: that which bounds the sixth *systēma* mentioned is the highest diatonic.[105] Thus the lowest chromatic *lichanos* is higher than the lowest enharmonic by a sixth part of a tone, since the chromatic diesis is greater by a twelfth part of a tone than the enharmonic diesis. A third part of anything must exceed a quarter of the same thing by a twelfth part, and the two chromatic dieses must evidently exceed the two enharmonic ones by twice that amount. This is a sixth part, an interval smaller than the least of the melodic intervals. Such intervals are unmelodic, since we call 'unmelodic' any interval that is not placed in a *systēma* in its own right.[106] The lowest diatonic *lichanos* is higher than the lowest chromatic by a semitone and a twelfth part of a tone. We said that it is a semitone from the *lichanos* of the hemiolic chromatic, and from there to the enharmonic *lichanos* is a diesis:[107] from the enharmonic to the lowest chromatic *lichanos* is a sixth part

[97] The lowest chromatic *lichanos* is that of the soft (*malakon*) chromatic, whose *pyknon* is made up of two intervals of $\frac{1}{3}$ tone each (see 50.28–51.1). An enharmonic diesis is $\frac{1}{4}$ tone (cf. 21.24–31). Aristoxenus sometimes uses the expression 'chromatic diesis', without qualifying it as 'smallest', to refer to the interval of $\frac{1}{3}$ tone.

[98] This is the *pyknon* of the hemiolic chromatic, in which each of the two small intervals is $\frac{3}{8}$ tone: see 51.1–4.

[99] The *pyknon* of the tonic chromatic, in which each of the two small intervals is a semitone: see 51.8–11.

[100] Called by the general name *systēma* because it exceeds the magnitude properly called *pyknon*. The sequence of intervals $\frac{1}{2}$, $\frac{3}{4}$ belongs at the bottom of a tetrachord in the soft diatonic: see 51.24–8.

[101] These are the two lowest intervals of a tetrachord in tense diatonic: see 51.29–31.

[102] *Chrōma* is a variant for *chroa* ('shade'), used in the case of chromatic systems only. 'Hemiolic' means 'half and whole': one number is hemiolic in relation to another if it is one and a half times as great. (The word is used very commonly in Pythagorean writers to refer to the ratio 3:2.) This shade is called 'hemiolic' because its *pyknon* is one and a half times as great as that of the enharmonic, and each of the intervals making up its *pyknon* is one and half times an enharmonic diesis.

[103] Because the two intervals of its *pyknon* add up to a tone.

[104] See n. 96 above.

[105] The 'lowest' and 'highest' are more technically called 'soft' (*malakon*) and 'tense' (*syntonon*) diatonic respectively; see the passages cited at nn. 100 and 101 above.

[106] That is, it cannot be used as a melodic step, though it may be referred to, as here, in comparisons between the sizes of legitimate melodic intervals: cf. 21.24–7.

[107] The word diesis, unqualified, refers to the enharmonic diesis, $\frac{1}{4}$ tone.

of a tone, and from the lowest chromatic to the hemiolic *lichanos* is a twelfth
26 part of a tone. Now a quarter is composed of three twelfth parts, so that it is clear that from the lowest diatonic to the lowest chromatic *lichanos* is the interval stated.[108] The highest diatonic *lichanos* is higher than the lowest diatonic by a diesis.

This establishes the ranges of each of the *lichanoi*, for every *lichanos* lower than the chromatic is enharmonic, every *lichanos* lower than the diatonic[109] down to the lowest chromatic is chromatic, and every *lichanos* lower than the highest diatonic down to the lowest diatonic is diatonic. For it must be understood that the *lichanoi* are unlimited in number.[110] Wherever you arrest the voice in the range that accommodates the *lichanos* will be a *lichanos*, and no place in the *lichanos* range is empty or incapable of receiving a *lichanos*. Hence the present controversy is of no little importance. Other people argue only about the interval in question, for instance whether the *lichanos* stands at a ditone or is higher, as if there were only one enharmonic *lichanos*. But we not only say that there is more than one *lichanos* in each genus, but also add that they are unlimited in number.

Let this, then, be our account of the *lichanoi*. For *parhypatē* there are two ranges, one common to the diatonic and the chromatic, the other peculiar to the enharmonic; for two genera share the same *parhypatai*. Every *parhypatē*
27 lower than the lowest chromatic is enharmonic, and all the rest down to this limit are chromatic and diatonic.[111] Of the intervals, that between *hypatē* and *parhypatē* is melodic if it is either equal to or smaller than that between *parhypatē* and *lichanos*, while that between *parhypatē* and *lichanos* is either equal to that between *lichanos* and *mesē*, or unequal to it in either direction.[112] This is because the *parhypatai* are common to both of the genera mentioned, since there can be a melodically proper tetrachord with the lowest chromatic *parhypatē* and the highest diatonic *lichanos*.[113] What we have said makes clear the size of the range of *parhypatē*, both when it is divided into its varieties and when these are taken together.

It is not easy at the outset to present an accurate account of continuity and

[108] That is, a semitone plus a twelfth part of a tone, totalling $\frac{7}{12}$.

[109] Da Rios adds *barytatēs* ('lowest') to qualify the words 'chromatic' and 'diatonic', but the additions are unnecessary.

[110] On the issues raised here see particularly 47.8–49.21, 68.13–69.29, and cf. nn. 91 and 93 above.

[111] On the thesis that chromatic and diatonic have the same range for their *parhypatai* see also 52.1ff. The claim seems puzzling when applied to the lower chromatic *parhypatai*, but Aristoxenus explains immediately that there can be properly melodic tetrachords combining these with high diatonic *lichanoi*. The list of shades is not exhaustive. There are, in principle, indefinitely many, and in particular, *systēmata* with chromatic or diatonic *lichanoi* can 'borrow' one another's *parhypatai*.

[112] That is, the lowest interval of the tetrachord must be no greater than the second interval; the second may be smaller than, equal to or greater than the highest. The rules are repeated, with further examples, at 52.8–32. Compare 11 Ptol. *Harm.* 32.23–5, 33.22ff.

[113] This gives an instance of the kind not exemplified in the regular divisions so far mentioned, that in which the second interval is greater than the highest. (From the bottom, the intervals are $\frac{1}{3}$, $\frac{7}{6}$, 1: cf. 52.25–32.)

succession, but we must try to give a rough indication.[114] The nature of continuity in melody seems to be similar to that which in speech relates to the putting together of letters. For in speaking it is natural for the voice, in each syllable, to place some one of the letters first, others second, third and fourth, and so on for the other numbers. It does not place just any letter after any other: rather, there is a kind of natural growth in the process of putting together. In singing, similarly, when the voice places intervals and notes in succession, it appears to maintain a natural principle of combination, and not to sing every interval after every other, either when the intervals are equal or when they are unequal.[115] Continuity is not to be sought in the way in which
28 the harmonicists try to set it out in their compressions [*katapyknōseis*] of the diagrams, displaying as successive with one another those notes which happen to be separated from one another by the smallest interval.[116] So far is the voice from being able to sing twenty-eight dieses consecutively that do as it will it cannot even add a third diesis, but in its progress upwards sings at least the remainder of the fourth – anything less is impossible.[117] And that is either eight times the smallest diesis, or less than that by some extremely small and unmelodic amount.[118] In its progress downwards from two dieses it cannot sing

[114] This discussion of *synecheia* and *to hexēs* was promised at 4.22–5. The task is to find principles governing the ways in which intervals may be placed in succession to form a continuous sequence of scalar steps. Book III deals with many points of detail; more general issues are discussed here, at 52.33–53.32, and at 58.14–61.4.

[115] The analogy between the orderings of elements in speech and in melody is often repeated: see, for example, 9.2 Adrastus ap. Theon Smyrn. 49 with n. 12. On equality and inequality see nn. 120 and 125 below.

[116] On *katapyknōsis* see n. 34, and on the diagrams n. 6 above. Aristoxenus' point is that the extended series of quarter-tones (dieses) on to which they mapped their *systēmata* does not represent a sequence of intervals that are melodically successive. No actual *systēma* or melody could include such a series. (The issue is briefly revisited at 53.2ff.) It is unlikely that the *harmonikoi* supposed that it could. If they called their dieses 'successive', it will have been only in the sense that no melodic points of pitch can lie inside any diesis, since it is allegedly the smallest identifiable interval (cf. 2.1 Plato *Rep.* 531a–b). But this too would be a mistake, from Aristoxenus' point of view, since the enharmonic diesis is not even a common measure of all his melodic intervals, and the diagrams could not represent such steps as one third of a tone: cf., for example, 2.7ff.

[117] The reference to 28 dieses poses a problem, in view of Aristoxenus' claim that his predecessors dealt only with enharmonic octachords (2.15–18, 36.30–2). At 2.22–5 he says explicitly that they concerned themselves only with octave *systēmata*. Since an octave contains 24 dieses, not 28, some editors have emended the text here to fit. But the MSS reading may after all be right, since there is a strong possibility that the analysis of 'ancient scales' recorded at 12 Arist. Quint. *De Mus.* 18.5ff. is derived from Aristoxenus' own account of his predecessors' practices, and one of the scales, the Dorian, exceeds the octave by a tone. A diagram of 28 dieses would then have been needed. But the issue is surprisingly complex: see Barker (1982). Of course, Aristoxenus does not mean that it is physically impossible to sing less than a ditone after two dieses: it is melodically impossible. Cf. 46.8–18, and for the derivation of the rule see 67.10–25.

[118] This may be a dismissive reference to the Pythagorean thesis that the fourth is not exactly $2\frac{1}{2}$ tones (e.g., 8 Eucl. *Sect. Can.* proposition 15). Alternatively, it may allude to the fact, which Aristoxenus freely admits, that some divisions of the tetrachord whose highest interval is slightly less than a ditone are nevertheless legitimately enharmonic (see n. 91 above). In such divisions the small intervals at the bottom could still be called 'dieses', or 'enharmonic dieses', even though each would be very slightly greater than a quarter of a tone.

less than a tone.[119] We should not then press the question whether continuity arises sometimes from equal and sometimes from unequal intervals,[120] but we must look for our evidence in the nature of melody, and try to concentrate on understanding which interval the voice naturally and melodically places after which. If after *parhypatē* and *lichanos* it is impossible to sing any note closer than *mesē*, then *mesē* will succeed *lichanos*, whether the interval which it bounds is twice or many times the size of that between *parhypatē* and *lichanos*. What we have said, then, shows clearly the way in which continuity and succession are to be sought: how they occur, and which interval is placed after which, and which is not, will be shown in our *Elements*.[121]

Let us assume[122] that given a *systēma*, whether *pyknon* or non-*pyknon*, no interval less than the remainder of the first concord can be placed next above it, and no interval less than a tone next below it.[123] Let us also assume that each of the notes which are melodically successive in each genus will either form with the fourth note in order from it the concord of a fourth, or will form with the fifth note in order from it the concord of a fifth, or both, and that any note of which none of these things is true is unmelodic relative to those with which it forms no concord.[124] Let us further assume that given that there are four

[119] For this rule see 67.10–25.

[120] That is, the laws of succession cannot be captured in any rule of the form 'so many equal intervals must be followed by so many unequal ones' or the converse. Melodic succession has its orderly regularities (notably those derived in Book III), and a few of them can be expressed in terms of equalities and inequalities (see n. 125 below), but the principles in which they are grounded are not concerned with the rationing out of equal and unequal intervals.

[121] The title apparently refers to a treatment comparable to that of Book III and its lost continuation. The word 'elements' seems to have been adopted from the fourth century onwards as the title of mathematical or quasi-mathematical works in which theorems were derived from axioms (the best known example is Euclid's treatise in geometry: cf. Proclus *In Eucl.* 66–8). Aristoxenus' third book is plainly modelled on this pattern. But the matter is not wholly straightforward: cf. 43.28, and see the introduction to this chapter.

[122] This introduces an assortment of principles, to be used in deriving theorems about permissible and impermissible sequences of intervals: cf. 53.33ff. They are not themselves derived deductively from anything else, but stand as principles abstracted from the experience and judgement of the well trained expert. They are generalisations about the conditions found to be fulfilled in all cases that are perceived as melodically acceptable. See particularly 43.25–44.20, and cf. 33.1ff. But some of them are in fact derivable from others, especially from the second, which is fundamental to the theorems of Book III.

[123] By *systēma* Aristoxenus here means the pair of small intervals at the bottom of a tetrachord between fixed notes. 'Non-*pyknon*' represents *apyknon*, 'uncompressed'. For a different use of the term see 12 Arist. Quint. *De Mus.* 9.14ff.

[124] 'Fourth note in order' means 'fourth note inclusive'. This second principle is of crucial importance in Book III; see also 53.33–54.21. It is itself derivable from the thesis, nowhere quite explicit in Aristoxenus, that any extended *systēma* can be analysed as a series of similar tetrachords in conjunction or in disjunction by a tone. That thesis, however, could not be derived strictly from the present principle (since a series of pentachords in which no note stands at a fourth from any other would be consistent with this principle), but since Aristoxenus plainly assumes throughout that any extensive *systēma* can indeed be analysed into similar tetrachords (cf. 58.15ff.), it is fair to conclude that the present principle is grounded in reflection on the tetrachordal structure framed by the traditional system of fixed notes.

intervals in the fifth, of which two are usually equal (those constituting the *pyknon*) and two unequal (the remainder of the first concord, and the amount by which the fifth exceeds the fourth), the unequal ones are placed next to the equal ones in the opposite order above and below.[125] Let us assume that notes standing at the same concordant interval from successive notes are in succession with one another.[126] Let us assume that in each genus an interval is melodically incomposite if the voice, in singing a melody, cannot divide it into intervals.[127] Let us also assume that no concord can be divided into magnitudes all of which are incomposite.[128] Let consecution be movement through successive notes, each of which, apart from the extremes, has on each side a single incomposite interval: and let direct consecution be that which maintains the same direction.[129]

[125] Fifths enter Aristoxenus' analyses primarily as the distance between a note in a tetrachord, and the note in the equivalent position in the tetrachord successive with it, where the tetrachords are disjoined by a tone. Given that the two tetrachords are divided in the same way, as they must be in order to obey the second principle above, every such fifth within the pair of tetrachords will contain the same intervallic magnitudes, though their arrangements will differ (see n. 7 above). The two arrangements that Aristoxenus has in mind here are, in the enharmonic genus, $\frac{1}{4}$, $\frac{1}{4}$, 2, 1 (ascending), and $\frac{1}{4}$, $\frac{1}{4}$, 1, 2 (descending): the two unequal intervals come in the opposite order as one descends from the equal ones from that in which they come as one ascends from the equal ones. Aristoxenus here indicates that something can properly be said about succession in terms of equal and unequal intervals, despite his strictures at 28.17ff. See also 72.29–74.8.

[126] This principle can be derived from the assumption that every extended *systēma* is composed of similar tetrachords in conjunction or disjunction (see n. 124 above), but not from the second principle taken by itself.

[127] This amounts to a definition of 'melodically incomposite interval'. The conception is relative to the accepted series of named notes, not to the sizes of intervals. Thus, for example, the interval between *mesē* and *lichanos*, no matter what its size in a particular genus, is always melodically incomposite. See 60.10ff.

[128] The notion 'incomposite magnitude' is different from that of a 'melodically incomposite interval'. An incomposite magnitude is one that cannot be divided exactly into smaller magnitudes each of which can serve as a melodic interval. Some of the smaller melodically incomposite intervals found in tetrachords (e.g., enharmonic dieses) are incomposite magnitudes, but the larger ones (i.e., those that stand highest in the tetrachord) never are, at least in the divisions that Aristoxenus quantifies. They can always be broken down into sub-magnitudes that can serve as melodic intervals in some legitimate system.

[129] The text of the last few lines is confused, but the general sense is clear. Consecution (*agōgē*) is not further discussed in the *El. Harm.* It is plainly related to 'continuity' and 'succession' (*synecheia* and *to hexēs*, see n. 19 above). Macran argued that the new term *agōgē* is needed to designate 'direct' scalar succession, since to describe notes as *hexēs* may mean only that they belong to the same series, not necessarily that they are adjacent. But in most cases Aristoxenus does use *hexēs* to mean 'directly successive'. Only in 58.14–60.17 are there ambiguities, of which Aristoxenus does not seem fully aware, and which he makes no attempt to solve by use of the word *agōgē*. More probably the point is that *synecheia* and *to hexēs* are terms used in the study of *systēmata*, while *agōgē* belongs to the analysis of melodies themselves: see 12 Arist. Quint. *De Mus.* 16.18–17.2. Only there could there be a role for the present distinction between 'consecution' and 'indirect consecution': the latter, as defined here, seems irrelevant to the study of *systēmata*.

Book II

30 It may be as well to explain in advance the kind of study this is, so that knowing beforehand the road, as it were, which we are to travel, and understanding which part of it we are on, we may walk the more easily, and not adopt unwittingly a misconception of the project.[1] This, as Aristotle used to recount, was what happened to most of those who heard Plato's lecture on the good.[2] They all came, he said, supposing that they were going to acquire one of the things which people commonly consider good, such as wealth, health, strength – in general, some astounding happiness or other. But when the discourse turned out to be about the mathematical sciences, about numbers and geometry and astronomy, and its conclusion to be that the good is one,[3] it seemed to
31 them, I imagine, altogether contrary to their expectations, so that some of them belittled it while others found fault. And why? Because they did not know in advance what the subject was, but like eristics rushed open-mouthed towards what was nothing but the name.[4] But I take it that if an overall account of the subject had been presented in advance, the prospective listener could either have abandoned it, or if he found it attractive, he could have continued to hold the conception of it which he had acquired. Aristotle himself, so he said, used to give introductions for just these reasons, explaining to those who intended to listen to him what the investigation was, and what it was about.[5] And to us too, as we said at the beginning, foreknowledge seems preferable.

People sometimes make mistakes in either of two directions. One group imagines that this science is of huge significance, some of them even supposing that listening to a discourse on harmonics will make them not just experts in music, but morally better men. These people have misunderstood what we said in our public lectures: 'What we are trying to do is to show for each kind of melodic composition and for music in general that such and such a type damages the character while such and such another improves it': and while misunderstanding that, they have altogether failed to hear our qualification 'in so far as music is capable of yielding such benefits'.[6] Others imagine it to be insignificant, of no importance, and yet profess themselves not to be ignorant

[1] On the relationship between Books I and II see the introduction to this chapter. This opening sentence already shows the more expansive style and the greater willingness to linger over general methodological issues that characterise the second book.

[2] The reference to a 'lecture' on the good has provoked endless controversy. For a brief discussion see Guthrie *History* vol. 5 (1978), pp. 424–6.

[3] The sense might alternatively be 'and that limit is the good, a unity', conceivably an allusion to *Philebus* 16c. See Guthrie, p. 424, n. 2.

[4] 'Eristic' is the abusive title given by Plato and others to those whose arguments were based on verbal tricks and ambiguities, and who aimed at victory in debate rather than truth. See, for example, Plato *Rep.* 454a–b, *Sophist* 225c, 231e, Aristotle *Soph. El.* 171b35.

[5] The tenses in this sentence seem to indicate quite clearly that by the time of writing, Aristotle was dead.

[6] It is not clear how far Aristoxenus accepted theories concerning the effect of music on character, but the present passage shows that he did not reject them altogether. See the Aristoxenian discussion in ps.-Plut. *De Mus.* 1142f–1144e.

of what it is.[7] Neither of these positions is correct. The science is not to be 32 despised by anyone of intelligence: that will become clear as our discussion progresses: nor is it so important as to be sufficient on its own for everything, as some people think. Many other things are needed, as we are constantly saying, to make a man a musical expert. Harmonics is only a part of the musician's accomplishment, as are the sciences of rhythm, metre and instruments.[8]

We must therefore give an account of harmonics and its parts.[9] It is to be understood, speaking generally, as the science which deals with all melody, and enquires how the voice naturally places intervals as it is tensed and relaxed. For we assert that the voice has a natural way of moving, and does not place intervals haphazardly. We try to give these matters demonstrations which conform to the appearances, not in the manner of our predecessors, some of whom used arguments quite extraneous to the subject, dismissing perception as inaccurate and inventing theoretical explanations, and saying that it is in ratios of numbers and relative speeds that the high and the low come about. Their accounts are altogether extraneous, and totally in conflict with the appearances.[10] Others delivered oracular utterances on individual topics, without giving explanations or demonstrations, and without even properly enumerating the perceptual data.[11] We, on the other hand, try to adopt initial principles

[7] Macran emends *apeiroi*, 'ignorant', to *empeiroi*, 'cognisant', and translates 'and yet prefer to remain totally unacquainted even with its nature and aims'. But the people Macran's version describes would hardly have attended Aristoxenus' lectures, and if we take *boulomenoi einai* in the sense 'professing to be', the text can stand. (For this usage see, for example, Plato *Rep.* 595c8, Aristotle *Eth. Nic.* 1119b34, 1125b33.)

[8] Cf. 1.22–2.7 with nn. 1 and 4 on that passage.

[9] The list does not begin until 35.25. It is prefaced by an important excursus on method, and on the theoretical grounds of the science.

[10] This refers to the procedures of Pythagoreans and Platonists, and it assumes their allegiance to the 'velocity' theory of pitch first enunciated in 1.19 Archytas frag. 1. Their explanations are 'theoretical' in the sense that they base their analyses of scalar structures, and their accounts of the reasons why some arrangements of notes and not others are harmonically proper, on propositions from the mathematics of ratio and proportion, not on laws inductively abstracted from the musical phenomena. (See particularly 2.1, 2.3 Plato *Rep.* 531c, *Tim.* 35–6, and cf. the Appendix to chapter 1. The best example of the genre Aristoxenus has in mind, though its theory of pitch is different, is 8 Eucl. *Sect. Can.*) Their claims conflict with the 'appearances' in two ways. First, melodic sequences do not present themselves to perception as strings of items that differ in quantity, and in perceiving them as forming a melody we do not perceive their relations as constituted by numerical ratios; yet that is how these theorists describe them (cf. 6 Theophrastus ap. Porph. *Comm.* 61.22ff.). Secondly, their principles give rise to various specific theorems that are flatly at odds with what, on Aristoxenus' view, perception reveals. For examples see 8 Eucl. *Sect. Can.* propositions 14, 15, 16, 18, and 11 Ptol. *Harm.* Book 1 ch. 6; cf. also 9.10–9.14 Ptolemaïs and Didymus ap. Porph. *Comm.* 22.22ff. Aristoxenus holds that these people's procedures are beside the point: cf. 9.2ff., 12.4ff., ps.-Plut. *De Mus.* 1144e. On the Aristotelian basis of his view, and the way it differs from that of 3.6, 3.7 Arist. *Post. An.* 75a–b, 78b–79a, see the introductions to chapter 3 and to this chapter. The word 'extraneous' in the present sentence indicates Aristoxenus' reliance on the 'same domain' rule discussed there.

[11] Aristoxenus now turns his fire on a different school of theorists, those *harmonikoi* whose empirical researches he recognises as precursors of his own. For the sharp distinction between such 'schools' see 2.1 Plato *Rep.* 531a–c, cf. 3.7, 3.9 Arist. *Post. An.*

33 which are all evident to anyone experienced in music, and to demonstrate what follows from them.[12]

Taken as a whole, our science is concerned with all musical melody, both vocal and instrumental. Its pursuit depends ultimately on two things, hearing and reason. Through hearing we assess the magnitudes of intervals, and through reason we apprehend their functions [*dynameis*].[13] We must therefore become practised in assessing particulars accurately. While it is usual in dealing with geometrical diagrams to say 'Let this be a straight line', we must not be satisfied with similar remarks in relation to intervals. The geometer makes no use of the faculty of perception: he does not train his eyesight to assess the straight or the circular or anything else of that kind either well or badly: it is rather the carpenter, the wood-turner, and some of the other crafts that concern themselves with this. But for the student of music accuracy of perception stands just about first in order of importance, since if he perceives badly it is impossible for him to give a good account of the things which he does

78b–79a, *Metaph.* 1053a, 6 Theophrastus ap. Porph. *Comm.* 62.1–3, and see also 9.10–9.14 Ptolemaïs and Didymus ap. Porph. *Comm.* 22.22ff., whose classification of theorists is more complex. This remark about the neglect of demonstrations (*apodeixeis*: see the introduction to chapter 3) is echoed in what they say about people they call *organikoi*, but it is a frequent complaint of Aristoxenus against all his 'harmonicist' predecessors. So too is the accusation that they failed to perceive the phenomena accurately. They adopted a very limited form of empiricism which does nothing to explain its findings; and they were bad empiricists, incapable even of getting the facts right. See, for example, his remarks about Eratocles, 6.12ff.

[12] This neatly summarises Aristoxenus' methodological outlook. The initial principles must be accepted without demonstration (this is often insisted on in Aristotle's *Post. An.*, e.g., 72b5ff.). They are abstracted from perceptual experience ('evident' in this sentence translates *phainomenas*, whose full sense here is probably 'perceptually evident'), not however that of just anybody, but of people whose perceptual discrimination has been trained to accuracy. (Cf. 33.9–26. On the process of abstraction or 'induction' see Arist. *Post. An.* 99b15ff., cf. 88a2–5.) Their truth is also to be checked against perceptual experience, not against metaphysical postulates or scientific hypotheses about the causes of experience. What can be demonstrated from them, however, must be so demonstrated, and thus displayed in its proper relation to the unified nature of melodic attunement, if it is to count as scientifically understood: cf. Arist. *Post. An.* 71b16ff., 76a33–4. On these issues see also 43.27–44.20 below.

[13] The roles to be assigned to perception and to reason are much discussed in later authors, particularly in their comparisons between Aristoxenian and Pythagorean method. (See especially Ptolemaïs and Didymus, cited in n. 11 above, and 11 Ptol. *Harm.* Book I chs. 1–2.) This sentence, and the comparison with geometry that follows, is probably the source of what Didymus says at 9.14 Porph. *Comm.* 28.6–19, but the scope of reason here goes beyond the demonstrative derivation from first principles that he and Ptolemaïs discuss, and which Aristoxenus uses throughout Book III. Intellectual understanding of the *dynameis* of notes and intervals is indeed articulated through such derivations, but must be preceded by rational reflection on perception, to generate principles and definitions. See 38.27–44.20, 47.8–50.14. These passages are also central to the study of Aristoxenus' conception of *dynamis*, melodic 'function' (see the introduction to this chapter, and especially 39.4–40.24 for the thesis that an understanding of *dynameis* is the major goal of the science). This pivotal conception is not even mentioned in Book I. Macran emends 'their functions' to 'the functions of the notes', on the grounds that Aristoxenus never ascribes function to intervals. But Aristoxenus' usage is more flexible than that: see, for example, 39.26ff., 69.6–18. When he is dealing with intervals without reference to function, he usually makes the point by speaking of their 'magnitudes': see, for example, 34.1ff., 36.5, 69.6.

not perceive at all.[14] This will become clear in the course of our investigation itself.

Further, we must not forget that musical understanding involves the simultaneous grasp of one thing that remains constant and another that changes, and that this holds, to speak broadly, throughout music as a whole and all its parts. For instance, we perceive differences of genera when the bounding notes remain fixed while the intermediate ones change:[15] or again,
34 while the magnitude remains constant we call one interval that between *hypatē* and *mesē*, and another that between *paramesē* and *netē*, since the functions [*dynameis*] of the notes can change while the magnitude remains the same.[16] Again, there can occur several arrangements of the same magnitude, of the fourth, for example, or the fifth, and the rest: and in the same way, when a given interval is placed in one position modulation occurs, while when the interval is placed in another, it does not.[17] Further, we see many similar things happening in matters to do with rhythms. For instance, while the ratio remains constant – the ratio by reference to which the genera are distinguished – the magnitudes of the feet are altered by the character of the tempo [*agōgē*], and while the magnitudes remain constant the feet become dissimilar: and the same magnitude can function either as a foot or as a conjunction of feet. It is clear that distinctions of division and of arrangement also depend on a fixed unit of magnitude. In general, while rhythmic composition employs many varied kinds of movement, the feet by reference to which we indicate rhythms have simple movements that are always the same.[18] Since the nature of music is like this, it is necessary also in matters concerning harmonic attunement [*to hērmosmenon*] to train our reason and our perception to assess properly that

[14] The intention of the last phrase is unclear. Aristoxenus might be referring to the very small, melodically unusable intervals by which, for example, the chromatic diesis exceeds the enharmonic, or to the functional attributes of notes and intervals which are 'apprehended by reason'. More probably the phrase is dismissive: the man cannot give a good account of things which he does not even perceive correctly.

[15] The 'bounding notes' are the fixed notes bounding a tetrachord: cf., for example, 21.32ff.

[16] *Hypatē* here is *hypatē mesōn*, *nētē* is *nētē diezeugmenōn*. The magnitude of the interval is in each case a fourth. They are functionally different, in that they play different roles in the melodic structure. If a note is *mesē*, for instance, that implies that it stands to other notes in the system in a certain set of relations, different from those in which *nētē* stands to its melodic surroundings.

[17] On modulation see 7.10ff. with n. 34. If 'interval' here means 'magnitude', the point is straightforward. For example, if in a melody in tense diatonic we move upwards by a semitone from a note that the context has established as *hypatē*, there need have been no modulation. If we do so from *parhypatē*, the note reached does not belong to the original *systēma*, and a modulation of some sort must be involved. But 'interval' may here be contrasted with 'magnitude' and defined functionally, by its place in a *systēma*. In that case, the point is that if we move, for example, through an interval whose sequel identifies it as that between *mesē* and *paramesē*, then if its lower boundary is the *mesē* of the *systēma* to which the previous part of the melody belonged, there has been no modulation. If in relation to that *systēma* this boundary is a note other than *mesē*, modulation has taken place.

[18] Of Aristoxenus' works on rhythm, only a few pages survive (see the Appendix to this chapter). For a treatment based on this see 12 Arist. Quint. *De Mus.* Book I chs. 13ff.; and for the special issues raised here see Arist. Quint. 32.13 with n. 165.

which remains constant and that which changes.[19] What is called harmonics, then, straightforwardly considered, is a science of the kind we have described; and it can be divided into seven parts.[20]

One of them, and it is the first, is that which distinguishes the genera, and makes clear what remains constant and what changes when these distinctions occur.[21] No one has previously dealt with this matter in any adequate way; for they paid no attention to two of the genera, and studied nothing but the enharmonic.[22] Those who worked on instruments, admittedly, did perceptually discriminate each of the genera, but none of them ever even addressed himself to the question when it is that a form of the chromatic begins to emerge from the enharmonic. For their perceptual discrimination did not grasp each of the genera in every one of its shades, because they were neither experienced in every style of melodic composition, nor trained in defining such differences accurately.[23] They did not even grasp the fact that there are particular ranges [*topoi*] belonging to the notes which move in changes between the genera.[24] These, then, are the reasons why the genera have not previously been defined, and it is obvious that they must be defined if we are to understand the differences which occur among melodies.

That, then, is the first of the parts of harmonics. The second is the discussion of intervals, in which we must not overlook, so far as we can avoid it, any of the distinctions that exist among them.[25] Broadly speaking, one may say that the majority of them have never been studied: but we must realise that whenever we are in the presence of one of the distinctions which have been overlooked and not scientifically investigated, we shall fail to grasp the differences in actual melodies which depend on it.

Since intervals by themselves are insufficient to give us an understanding of notes – every magnitude of interval, broadly speaking, being common to several functions [*dynameis*] – the third part of the science as a whole must be that which deals with notes, and explains how many there are, by what means they are recognised, whether they are pitches, as most people suppose, or functions, and also just what a function is. Of such things as these, not one is thoroughly and faultlessly understood by those who work on subjects of this sort.[26]

[19] Compare the importance assigned to memory, 38.26–39.3. This paragraph may owe something to Aristotle's reflections on change, e.g., *Physics* 189b30ff.

[20] The list that follows is clear and straightforward, by contrast with the rather ramshackle presentation of a programme in Book I, 3.5–8.2. See the introduction to this chapter.

[21] See 46.19–52.32.

[22] Compare 2.7ff.

[23] It is not clear whether these people are instrumental performers or theorists who based their propositions on facts about instruments: for the latter see 39.8–11, 41.24–43.24, and cf. 9.12, 9.13 Ptolemaïs and Didymus ap. Porph. *Comm.* 25.14–16, 26.6–14. On the perceptual basis of discrimination between genera see 48.9–49.21, and cf. 23.4–23.

[24] On these ranges see especially 22.24–27.14, and cf. 46.24–47.8.

[25] Nothing in Book II corresponds to the elaborate set of distinctions at 16.16ff. Probably some material has been lost just before the discussion of concords at 44.27ff.

[26] Since intervals of the same size are often bounded by functionally different pairs of notes, the character of a note is not exhausted by a description of the magnitudes of

The fourth part is the study of *systēmata* – how many there are, what they are like, and how they are put together out of intervals and notes. This part has not been investigated by earlier writers in either of the two possible ways. For neither has there been any research into the question whether *systēmata* are put together in just any way from intervals, no combination being contrary to nature, nor has anyone completely enumerated all the distinctions between *systēmata*.[27] Of the melodic and unmelodic[28] our predecessors have given no account whatever, while as to the distinctions between *systēmata*, some people did not even attempt to enumerate them, devoting their research only to the seven octachords which they called *harmoniai*;[29] whereas others made the attempt, but by no means achieved a complete enumeration, these being people
37 like the followers of Pythagoras of Zacynthos and Agenor of Mytilene.[30] And yet the order which relates to the melodic and unmelodic is similar to that concerned with the combination of letters in speech: from a given set of letters a syllable is not generated in just any way, but in some ways and not in others.[31]

The fifth part concerns the *tonoi* in which the *systēmata* are placed when they occur in melody.[32] Nobody has said anything about them, neither how they are to be discovered, nor what one must look to in order to establish their number. The exposition of the *tonoi* by the harmonicists is just like the way the days of the month are counted, where, for example, what the Corinthians call the tenth the Athenians call the fifth, and others again the eighth. In just the same way, some of the harmonicists say that the Hypodorian is the lowest of the *tonoi*, the Mixolydian a semitone higher, the Dorian a semitone above that, the Phrygian a tone above the Dorian, and similarly the Lydian another tone above the Phrygian. Others add below the ones mentioned the Hypophrygian

which it is a boundary. See particularly 47.8–50.14, cf. 68.13–69.28. These passages, together with the hint given here, make it clear that for Aristoxenus in Books II and III a note is a 'function', and not merely a point of pitch. Contrast 15.13–23: in Book I the notion of melodic function does not appear. In what survives of this work Aristoxenus does not fulfil his undertaking to explain 'just what a function is'. Nor does he say how many note-functions there are, or list their names; see the lists in, for example, 10 Nicomachus *Ench.* chs. 11 and 12, and in 12 Arist. Quint. *De Mus.* Book I ch. 6.

27 That is, no one has investigated the principles on which they are constructed, and from which theorems about the legitimacy or illegitimacy of particular types may be demonstratively derived; nor have they even listed all the types to which such theorems would refer (cf., for example, 32.29–31). The main principles are enunciated in 29.1ff., 53.33ff.; details are derived in Book III. On the main kinds of distinction between *systēmata* see 17.1ff.

28 That is, of the principles mentioned in the previous note.

29 *Hepta octachordōn*, 'seven octachords', is the emendation accepted by most editors for the MSS *heptachordōn*, 'heptachords': see 2.15–18, and for the principle underlying these theorists' constructions see 6.19–31. On these same *harmoniai* see 12 Arist. Quint. *De Mus.* 15.8ff.

30 This Pythagoras is not the celebrated sage of that name, but a musician probably active in the mid fifth century. He invented a complex instrument called the *tripous* (Athenaeus *Deipn.* 637b–f), to whose construction and performance an understanding of different patterns of attunement would have been essential. Agenor belongs to the early fourth century, and was a respected teacher and performer, but nothing specific is known about him.

31 For this analogy see also 27.18ff.

32 Aristoxenus' treatment of the subject is lost: see nn. 3 and 34 to Book I.

aulos; while others again, with an eye to the boring of the finger-holes of *auloi*, separate the three lowest *tonoi*, the Hypophrygian, the Hypodorian and the Dorian, by three dieses from one another, and the Phrygian from the Dorian by a tone, placing the Lydian at a distance of another three dieses from the Phrygian, and the Mixolydian at the same distance from the Lydian. But what
38 the grounds are on the basis of which they have persuaded themselves to separate the *tonoi* in this way, they have not told us.[33] It will become clear in the course of our investigation that *katapyknōsis* is unmelodic and altogether useless.[34]

Since some melodies are simple and others modulating, modulation must be discussed. First we must say what modulation is and how it comes about – where by 'modulation' I mean what happens when the order belonging to the melody undergoes a certain kind of qualification – and then how many modulations there are in all, and at how many intervals they occur. No one has said anything about any of these matters, with or without demonstration.[35]

[33] The names of the *tonoi* mentioned in these two lists are familiar from later accounts of structures analysed in Aristoxenus' time or before him. They appear in lists of *harmoniai* (e.g., 12 Arist. Quint. *De Mus.* 15.11ff.) and of *tonoi* (e.g., 11 Ptol. *Harm.* Book II ch. 10, 12 Arist. Quint. *De Mus.* Book I ch. 10). Those writers regularly mention at least seven *tonoi*, of which Hypolydian is missing here. Their ordering from low to high reflects that of other lists, except that Hypodorian is usually below Hypophrygian, and Mixolydian is oddly placed in the first series (cf. the controversies hinted at in ps.-Plut. *De Mus.* 1136d). This order is usually associated with a conception of each as an identical interval-series, whose origin is placed at a different relative pitch in each *tonos*: that is, the 'origins' of the various *tonoi* (whether their *proslambanomenoi* or their *mesai*) stand at set intervals above and below one another. This also has the effect that each projects onto a given range of pitch a different series of intervals (cf. n. 34 to Book I). No other writer gives the spacings mentioned here. In the first series those between Dorian, Phrygian and Lydian are the usual ones (e.g., 11 Ptol. *Harm.* Book II ch. 10), but the others are eccentric. The second group is based (or so I suggest) on three pairings, Mixolydian with Dorian, Lydian with Hypodorian, Phrygian with Hypophrygian, in which the members of each pair are separated by a fourth – an interval at which modulation is easy and acceptable (see, for example, 12 Arist. Quint. *De Mus.* 22.15ff.). The reference to *auloi* (cf. 41.24–43.24) suggests that the pairings correspond to three kinds of pipe, where each was designed to be playable in both of two *tonoi* a fourth apart: this arrangement would minimise the number of extra finger-holes required to admit a modulation, and would effectively allow for a shift, in the lower *tonos*, between the tetrachords *diezeugmenōn* and *synēmmenōn* (see 11 Ptol. *Harm.* Book II ch. 6). The relations between the pitches of the Dorian-Mixolydian pipe and the Phrygian-Hypophrygian one are probably determined by the conventionally accepted interval between Dorian and Phrygian; the Lydian-Hypodorian pipe is pitched exactly midway between the other two. These suggestions are wholly conjectural, and do not solve all the problems. But the heavy irony of the last sentence relieves us of the task of finding any fully systematic theory behind these spacings. Aristoxenus' plain insinuation is that there is none. (Armed with her complex and original theories about *auloi* and their scales, Schlesinger was often prepared to explain away claims made by Aristoxenus about them, arguing that he was largely ignorant of the instrument. But even her ingenuity failed her here, and she could only conclude that Aristoxenus was somehow misrepresenting the facts: Schlesinger (1939), p. 193.)

[34] The proper disposition of *tonoi*, according to Aristoxenus, is determined by the rules governing the possibility of modulation: see the next paragraph. That *katapyknōsis* does nothing to relate them in this way is already argued at 7.22–33.

[35] The sort of modulation (*metabolē*) intended here is probably what was later called 'modulation of *systēma*', whose study is inseparable from that of the *tonoi* (see 11 Ptol.

The last part of the science is that concerned with melodic composition itself. Since many forms of melody, of all sorts, come into existence in notes which are themselves the same and unchanging, it is clear that this variety depends on the use to which the notes are put: and this is what we call melodic composition. Thus the science concerned with harmonic attunement, after working through the parts which have been mentioned, will find its completion here.[36]

It is clear that understanding melodies is a matter of following with both hearing and reason things as they come to be, in respect of all their distinctions: for it is in a process of coming to be that melody consists, as do all the other parts of music. Comprehension of music comes from two things, perception
39 and memory: for we have to perceive what is coming to be and remember what has come to be. There is no other way of following the contents of music.[37]

As to the objective that people assign to the science called harmonics, some say that it lies in the notation of melodies, claiming that this is the limit of the comprehension of each melody,[38] while others locate it in the study of *auloi*, and in the ability to say in what manner and from what origin each of the sounds emitted by the *aulos* arises.[39] But to say these things is a sign of complete misunderstanding. So far from being the limit of harmonic science,

Harm. Book II ch. 7, 12 Arist. Quint. *De Mus.* Book I ch. 11). No full discussion survives in the *El. Harm.*: cf. 7.10–8.2.

[36] The thesis that the study of composition (*melopoiia*) is part of harmonics contrasts with the standpoint of 1.34–2.6 (cf. ps.-Plut. *De Mus.* 1142f–1143a, 1143c): see the introduction to this chapter. But the difference may be one of perspective, rather than a flat contradiction. In 12 Arist. Quint. *De Mus.* composition is placed outside harmonics and then inside it within a single page of text (6.21, 7.12), and in the context there need be no real inconsistency. For a glimpse of what a treatment of composition within harmonics might involve see 12 Arist. Quint. 28.10–30.24.

[37] With this paragraph compare 33.1–34.30, ps.-Plut. *De Mus.* 1143f–1144c. The emphasis on 'coming to be' may conceal a thrust against Pythagorean theory, whose mathematical representations deal only with relations between notes located at specific pitches: they say nothing of the melodic implications of different sequences of movement through such pitches. The laws of harmonics are, centrally, laws of sequence, and are not fully captured in the abstract presentation of frozen structures. Macran comments on the uncomfortable brevity of this paragraph, but it is unnecessary to postulate a gap in the text before the word 'comprehension'.

[38] What this implies, as the sequel shows, is that we have 'understood' a melody when we have found a way of writing down what is heard in a way that represents the sizes of its intervals. Almost everything we know about notation in this period comes from this passage. It was evidently a tool of theorists rather than composers. It may well have been associated with the harmonicist's diagrams (see 2.11–18, 7.31–2, 28.1–6, cf. 12 Arist. Quint. *De Mus.* 12.5ff.), and perhaps one of their main aims was to develop a usable notation.

[39] This refers to attempts to infer proper harmonic relations from the physical structure of instruments (cf. n. 33 above). Aristoxenus' criticisms (41.24ff.) seem apt enough, but for the contrary view see Schlesinger (1939), pp. 57ff., 193–5. These theorists are perhaps the *organikoi* of 9.12, 9.13 Ptolemaïs and Didymus ap. Porph. *Comm.* 25.14–16, 26.6–14. Their procedures were probably related to the researches in physical acoustics undertaken by Pythagoreans and Peripatetics: for relevant references to *auloi* see, for example, 1.19 Archytas frag. 1, 4.19 ps.-Ar. *Probs* XIX.23, 5 *De Audib.* 800b, 802a, 804a, 6 Theophrastus ap. Porph. *Comm.* 63–4, 10 Nicomachus *Ench.* chs. 4 and 10, 9.7 Aelianus ap. Porph. *Comm.* 33–4, 9.10 Ptolemaïs ap. Porph. *Comm.* 23.1.

notation is not even a part of it, unless writing down metres is also a part of the science of metre. But if what applies there – that the man who can write down the iambic metre is not necessarily the one who best understands what the iambic really is – applies also to melodies (for the man who has written down the Phrygian *melos* is not necessarily the one who understands best what the Phrygian *melos* really is), then it is clear that notation cannot be the limit of the science in question.[40] That what we have said is true, and that the practitioner of notation needs nothing more than a perceptual grasp of the magnitudes of intervals, will be clear to those who consider the matter.[41] A person who sets out signs to indicate intervals does not use a special sign for each of the distinctions which exist among intervals – for instance, for the several divisions of the fourth produced by the differences between the genera, or for the several arrangements produced by alteration in the order of the combination of the incomposite intervals.[42] We shall say the same thing about the functions [*dynameis*] which the natures of the tetrachords create, for the interval from *nētē hyperbolaia* to *nētē* and that from *mesē* to *hypatē* are written with the same sign, and the signs do not distinguish the differences in their functions; so that their scope extends only to the magnitudes, and no further.[43] But we said at the beginning that the mere discrimination of magnitudes by the senses is no part of a complete understanding of the subject,[44] and what we are about to say will make the fact even easier to see. For through the magnitudes as such, no knowledge is forthcoming of the functions of either the tetrachords or the notes, or of the distinctions between the genera, or, to put it briefly, of the distinctions between the composite and the incomposite, of the simple and the modulating, of the styles of melodic composition,[45] or, in a word, of anything else at all.[46] If the so-called harmonicists adopted this supposition out

[40] Aristoxenus returns to these points at ps.-Plut. *De Mus.* 1143c–d. To write down the iambic metre is to represent the relative quantities of its syllables (on the science of metre see 12 Arist. Quint. *De Mus.* 40.28ff.): to write down the Phrygian *melos* is to notate the pattern of notes and intervals, the *systēma*, within which the Phrygian melodies can be composed. On the ambiguity of *melos* see n. 1 to Book 1.

[41] The point is that the capacity to notate requires only the identification of intervallic magnitudes, and that this tells us at most only one of several things relevant to understanding the role and implications of a given interval in its place in a melody: see the immediate sequel.

[42] This implies that the same sign or signs are used for the same size of interval, no matter to what genus its subdivisions belong, and no matter what order they come in.

[43] The text of this sentence is problematic, but the general sense is the same whichever editorial emendation is adopted (I translate that of Laloy, followed by da Rios). The intervals in question are both fourths. They are therefore identically notated even though their positions in the system, and their implications for the nature and the musical dynamics of their melodic surroundings, are entirely different.

[44] The remark might be a pardonable exaggeration. More probably it has the precise sense that perception of magnitudes constitutes raw data from which we may proceed to understanding: cf. 33.1ff.

[45] On the different 'styles' see 12 Arist. Quint. *De Mus.* 28.10–30.24.

[46] On the general point being made here compare 33.1ff., 47.8–50.14, 68.13–69.29. Can anything be inferred from this passage about the notation at which Aristoxenus' remarks are directed? One possibility is that the signs merely expressed the sizes of intervals, perhaps by numbers indicating so many dieses. This would fit Aristoxenus'

of ignorance, there would be nothing perverse about their procedure, but their ignorance must have been powerful and profound. But if they propounded the doctrine while fully aware that notation is not the limit of the present science, aiming to please the general public and to give them some end-product visible
41 to the eye, then they are to be condemned, instead, for gross perversity in their method. The reasons are, first, that they suppose that the layman should be set up as judge of the sciences – since for the same person to be learning and judging the same thing would be an absurdity – and secondly that in putting a perceptible product in the position of the limit of understanding, as their conception has it, they are reversing the proper order, since the limit of everything visible is understanding: for that is the ruling principle and judge of everything. If anyone supposes that hands, voice, mouth and breath are much more than inanimate instruments, he is quite mistaken; and if the understanding is buried somewhere in the soul, and is not immediately tangible or apparent to most people, as are the products of the hands and other things like that, this is no reason for imagining that what we have said is wrong. We shall have missed the truth if we make that which judges neither the limit nor the ruling principle, and make into the limit and ruling principle that which is judged.

No less absurd than this is the conception relating to *auloi*. The greatest and most preposterous of errors is to make the nature of harmonic attunement depend on an instrument. It is not because of any of the properties of instruments that harmonic attunement has the character and arrangement which it does. It is not because the *aulos* has finger-holes, bores,[47] and other
42 such things, nor because it admits operations of the hands, and of other parts naturally adapted to raising and lowering its pitch, that the fourth, the fifth and the octave are concords, or that each of the other intervals has its own appropriate magnitude. For even though all these factors are present, auletes

polemic, and corresponds to the method of marking metres: compare the manner in which the set of 'ancient scales' is introduced at 12 Arist. Quint. *De Mus.* 18.5ff. But no other source encourages the belief that a purely intervallic notation existed. A second possibility is that it designated each note by a symbol indicating its place in a series of pitches set out in a diagram (cf. n. 38 above). One might object that this procedure would after all reveal, for example, the difference between intervals mentioned at 40.7–8, but this would not be so on a diagram that represented all the different arrangements of the octave in the same theoretical range of pitch (see n. 34 to Book I, and cf. 47.8–25 with n. 70). Such a system might be the source of the notational diagram at 12 Arist. Quint. *De Mus.* 12.15ff. The third possibility, assumed in Macran's notes, is that it is the 'Alypian' system, or an early version of it. But it cannot possibly be said of this system that it discriminates nothing but magnitudes. (A discussion of Alypian notation cannot be undertaken here. It is preserved in the *Eisagoge* of Alypius, written in about the third century A.D. See also 12 Arist. Quint. 23.6ff., with the diagrams. For modern accounts see, for example, Monro (1894), pp. 67–77, Macran (1902), pp. 45–61, Gombosi (1939), chs. 3 and 5, Winnington-Ingram (1956), Henderson (1957), pp. 358ff., Barbour (1960), Pöhlmann (1970), pp. 141ff., Chailley (1979), pp. 120–139, with the tables on pp. 184–190.)

47 'Bores' translates *koilias*. The word generally refers to the main pipe of an instrument, and editors have found the fact that it is plural here troublesome. Probably the sense is roughly 'the *aulos* (taken generically) has a plurality of different sizes of bore...'

for the most part fail to attain the proper order of attunement, and for all these efforts produce the proper results only rarely, despite employing such techniques as separating and bringing together,[48] increasing and decreasing tension with the breath,[49] and all the other causal expedients. It is clear, then, that it is no more correct to say that excellence is inherent in *auloi* than to say that what is bad is so. But this ought not to have been so, if there were any value in basing harmonic attunement on an instrument, since one would only have to submit a melody to the *aulos* for it to be at once immutable, infallible and correct. But in fact neither *auloi* nor any other instruments will ever provide the foundation for the nature of attunement. Each of the instruments participates, so far as it can, in the marvellous organisation which belongs to the nature of attunement as a whole, under the direction of perception, to which are referred both these and all other matters to do with music. If anyone imagines that because he sees that the finger-holes are the same every day, and that the strings have the same tension, he will therefore find the attunement permanently fixed in them and maintaining the same organisation, he is
43 thoroughly simple-minded. Just as there is no attunement in the strings unless someone brings it to them by manual adjustments, no more is there in the finger-holes, unless someone brings it to them by manual adjustments. It plainly needs no arguing, since it is obvious, that no instrument tunes itself, but that perception is the authority in this matter. It is strange that when people have seen these facts they have not abandoned this sort of conception, noticing that *auloi* change, and never remain the same; for any sound produced by the *aulos* alters in accordance with the agencies through which it is produced. It is clear, then, that there is no reason for basing the study of melody on the *aulos*, since this instrument cannot establish the true order of attunement: and even if someone supposed that the basis should be located in some instrument or other, it should not have been located in *auloi*, since they are especially susceptible to variation, introduced through the craft of *aulos*-making, through manual techniques, and through their own peculiar nature.[50]

These, then, are the things that should be dealt with in an introduction to the science known as harmonics: but now that we are on the point of setting out on the study that deals with the elements,[51] we must notice in advance the

[48] Plutarch, *Non Posse Suaviter* 1096a, indicates that the effect of bringing the two pipes of a pair of *auloi* together was to lower the pitch, that of drawing them apart was to raise it. The method probably depended on the way these movements altered the positioning of the two double reeds between the player's lips: cf. Schlesinger (1939), p. 62.

[49] See, for example, 5 *De Audib.* 804a, cf. 3.17 Aristotle *De Gen. An.* 788a, 4.4 ps.-Ar. *Probs* XI.13, Theophrastus *Hist. Plant.* IV.11.4–5.

[50] *Auloi* are in general less reliable even than stringed instruments, since though both have to be attuned in advance by physical adjustments made on the basis of perception, once a *lyra* or a *kithara* is tuned, normal methods of playing do not introduce further variations in the pitch produced by each string. With the *aulos* it is different: the same fingering on the same instrument will not always produce the same sound. Cf. Plato *Philebus* 55–6 with Barker (1987).

[51] This may be a reference to axiomatic derivations, like those of Book III: cf. 28.33–29.1. Alternatively, it may merely mark the advance from introductory discussions to more detailed ones. See the introduction to this chapter.

following points.[52] This study cannot be successfully completed unless three conditions which I shall mention are already fulfilled. They are firstly, that the 44 appearances are accurately perceived; secondly, that those of them which are prior are correctly discriminated from those which are derivative; and thirdly, that what follows and is implied is viewed together in the proper manner.[53] Since every science which consists of more than one proposition should adopt first principles from which the things dependent on these principles can be demonstrated, we must adopt such principles, bearing the following two points in mind. We must ensure, first, that each of the fundamental propositions is true and evident, and secondly that each is such as to be accepted by perception as belonging among the primary parts of harmonic science: for whatever demands demonstration is not fundamental.[54] And in general we must be very careful, as we set out, not to slip into extraneous territory by beginning from a conception of sound as a movement of the air,[55] and equally not to turn back too soon and leave out many things which belong to the subject.[56]

Melodies fall into three genera, the diatonic, the chromatic and the enharmonic.[57] The differences between them will be specified later: but let us assume that every melody is either diatonic or chromatic or enharmonic, or a mixture of these, or common to them.[58]

The second distinction to be made is that some intervals are concordant and others discordant.[59] The two distinctions between intervals which seem to be best known are their differences in respect of magnitude, and the difference between the concordant and the discordant: but the latter distinction is

[52] With the reflections of this paragraph compare 32.10–33.26. Aristoxenus' attachment to the principles of Aristotle's *Posterior Analytics* is again clear.

[53] On the need for accurate perception see, for example, 33.1ff. On the distinction between what is prior and what is derivative and to be demonstrated, see n. 12 above. The third condition seems to be new: Macran's translation, 'thirdly, our conclusions and inferences must follow legitimately from the premises' misrepresents the Greek (though no doubt Aristoxenus would accept the point). The suggestion seems to be that when legitimate inferences have been made, their results must be 'viewed together', that is, understood in their proper relations to one another and to the initiating principles, so as to generate a grasp on the 'nature of attunement' as a single, coordinated whole. For somewhat similar ideas in a different theoretical context see Plato *Philebus* 14c–19a, and compare *Rep.* 531c–d.

[54] See n. 12 above, with its references to the *Post. An.* 'By perception' is almost a parenthesis. Perception cannot decide whether or not something is in this sense primary. The phrase serves to remind us that the content of the principles is abstracted from perception and must not conflict with it.

[55] Cf. 32.20–8. The reference to 'extraneous territory' points to the claims of Aristotle's 'same domain' rule: see the introduction to chapter 3.

[56] 'Turn back too soon': a metaphor from the race-course.

[57] Macran marks a lacuna before this sentence, which lacks a connective. The transition is certainly abrupt, but what is lost may be no more than a phrase or two. The programme of Book II places the study of the genera first (35.1). Details appear later (46.19ff.), and Aristoxenus may have given only a bare outline here. Compare the procedure of Book I (outline at 19.17ff., details at 21.32ff.).

[58] On mixtures of genera see 7.3, cf. Cleonides *Eisagoge* 189.15–18. A melody 'common to the genera' is one using only notes that several genera share, and in the extreme case only fixed notes.

[59] A more substantial passage has probably been lost before this sentence. The 'first distinction' will have been that in respect of magnitude (see the next sentence and cf. 16.16ff.).

45 included in the former, since any concordant interval differs in magnitude from any discordant one. There being many distinctions between concords, let us first consider the best known of them, that in respect of magnitude.[60]

Let us assume that there are eight magnitudes of concords. The smallest is the fourth: that it is the smallest is determined by the nature of melody itself. This is shown by the fact that we sing many intervals smaller than the fourth, but all of them are discordant. The second is the fifth: every magnitude which there may be between these two will be discordant. The third is the sum of the two concords mentioned, the octave, all magnitudes between the fifth and the octave being discordant. What we have said so far has been handed down by our predecessors: but we must give an analysis of the remainder for ourselves. We must explain first that the addition of any concordant interval to the octave makes the magnitude resulting from them concordant. This is a quality intrinsic and peculiar to the concord of the octave, for whether the concord added is smaller than it or equal or greater, the result of the combination is concordant.[61] This is not the case with the first concords,[62] for if either is added to its equal it does not make the whole concordant, nor does either if it is added to the interval composed of itself and an octave, but the result of these combinations will always be discordant.

46 The tone is that by which the fifth is greater than the fourth:[63] the fourth is two and a half tones.[64] Of the parts of the tone the following are melodic: the half, which is called the semitone, the third part, which is called the least chromatic diesis, and the quarter, which is called the least enharmonic diesis. No interval smaller than that is melodic.[65] But two things must not be overlooked: first, that many people have mistakenly supposed us to be saying that a tone can be divided into three equal parts in a melody. They made this mistake because they did not realise that it is one thing to employ the third part of a tone, and another to divide a tone into three parts and sing all three.[66] Secondly, we accept that from a purely abstract point of view there is no least interval.[67]

The differences between the genera are found in a tetrachord such as that

[60] With the subsequent discussion compare 19.30–21.19. Aristoxenus again makes no attempt to analyse or even to describe the phenomenon of concord (see n. 60 to Book I).

[61] Compare the use made of this principle at 11 Ptol. *Harm.* 13.3ff.

[62] That is, the fourth and the fifth, which are not only first in the list above, but (a) first in magnitude, being smallest, and (b) first in that they are primary, all other concords being combinations of them. The expression 'the first concords' is common in this sense.

[63] Compare 21.22, 62.1–11.

[64] Cf. 24.4–10. Aristoxenus tries to show how this (controversial) proposition may be confirmed at 56.13ff.

[65] Compare 21.23–31.

[66] The translation of this last phrase, which I have borrowed from Macran, expands the Greek slightly, but makes its sense clear. Quarter-tones and thirds of a tone occur in melody, but no more than two such small intervals are ever successive (cf. 28.6–17, 62.34ff.). Hence, no tone is divided into three or more equal parts by notes proper to a single *systēma*.

[67] Cf. the complementary proposition at 15.7–12.

between *mesē* and *hypatē*, where the extremes remain fixed, and either one or both of the intermediate notes move.[68] Since a moving note must move within some range [*topos*], we must find the determinate range of each of the notes mentioned. It appears that the highest *lichanos* is that which lies at a tone from *mesē*, and creates the diatonic genus; and the lowest is that at a ditone from *mesē*, which belongs to the enharmonic. Hence it is clear that the range of *lichanos* is a tone. That the interval between *parhypatē* and *hypatē* cannot be
47 less than an enharmonic diesis is obvious, since the enharmonic diesis is the smallest of all melodic intervals: that it too will increase to double the size remains to be shown. When *lichanos* in its descent and *parhypatē* in its ascent reach the same pitch, the range of each evidently arrives at its limit, so that it is plain that the range of *parhypatē* is not greater than the smallest diesis.[69]

Some people find it baffling that the note remains *lichanos* when any one of the intervals between *mesē* and *lichanos* is altered. Why, they ask, is there one interval between *mesē* and *paramesē*, and similarly between *mesē* and *hypatē* and between all the other notes which do not move, and yet we must insist that the intervals between *mesē* and *lichanos* are many? It is better, they say, to change the names of the notes, and no longer to call the other ones *lichanoi*, supposing that the one at a ditone from *mesē*, or one of the others, no matter which, is given that name. Notes which bound different magnitudes, they say, must be different notes; and the converse must also be true, that notes bounding equal magnitudes must be included under the same names.[70] Against these views we have advanced the following arguments. First, to recommend that each different note has a magnitude of interval peculiar to it is revolutionary: for we see that *nētē* and *mesē* are different from *paranētē* and *lichanos* in respect of function [*dynamis*], as are *paranētē* and *lichanos* from
48 *tritē* and *parhypatē*, and these again from *paramesē* and *hypatē*, which is why the members of each pair are given their own special names: but the interval involved in every case is the same, a fifth.[71] Hence it is clear that it is impossible for differences in the magnitudes of intervals always to follow upon differences in notes.[72] That the converse relation does not hold either can be seen from the

[68] See 21.31ff.

[69] For a more elaborate study of the ranges of *lichanos* and *parhypatē* see 22.24–27.14.

[70] The proposal initially seems to be that N and M are different notes, deserving different names, if and only if they stand at different distances from a given fixed note, e.g., *mesē*. This is contrary to Greek practice, since, for example, *lichanos* may stand anywhere between a tone and a ditone below *mesē*. Its identity as *lichanos* depends on its melodic function (*dynamis*), as Aristoxenus explains below. But the proposal turns out to be still more radical: it makes a note's identity depend only on the sizes of the intervals surrounding it, without reference to the names of the other notes that are these intervals' boundaries. Aristoxenus has no difficulty in making the suggestion seem nonsensical, but it may have had a real basis in the harmonicists' diagrams and their attempts at notation. See 39.30–40.24 with n. 46.

[71] The intervals in question are those from *nētē diezeugmenōn* to *mesē*, from *paranētē diezeugmenōn* to *lichanos mesōn*, from *tritē diezeugmenōn* to *parhypatē mesōn*, and from *paramesē* to *hypatē mesōn*.

[72] By implication this answers the question, raised at 36.9–12, whether notes are pitches or *dynameis*.

remarks I am about to make.[73] First, if we seek individual names corresponding to every increase and decrease in the intervals of the *pyknon*, we shall evidently need an unlimited number of names, since the range of the *lichanos* is divisible into an unlimited number of sections.[74] Secondly, if we try to concentrate on the equal and the unequal, we shall be abandoning the distinction between like and unlike, so that we shall not apply the word '*pyknon*' to anything except one single magnitude, nor, obviously, the words 'enharmonic' and 'chromatic', which are also in fact determined within certain ranges. But it is clear that none of these suggestions corresponds to the representations of sense perception: for perception assigns the names 'chromatic' and 'enharmonic' with an eye to the common feature of a single form, and not to the magnitude of a single interval.[75] I mean that it assigns the form of the *pyknon* to every case in which the two intervals together cover a range [*topos*] smaller than the one,[76] since in all *pykna*, though they are of different sizes, the sound of something compressed [*pyknos*] is evident to perception;[77] and it assigns the form of the chromatic to every case where the chromatic character is perceptually evident. Each of the genera moves with what perception apprehends as its own
49 characteristic movement while using not just one division of the tetrachord, but many. Thus it is clear that the genus can remain constant while the magnitudes change, since up to a certain point the genus does not change when the

[73] He has shown that if N and M are different notes, they do not necessarily stand next to intervals of different magnitudes. He will now argue that if N and M are associated with different magnitudes of interval, it does not follow that they are different notes.

[74] Cf. 26.13ff. For the application of the word *pyknon* see also 24.11–14, 50.15ff.

[75] The points made here, and elaborated in the following sentences, are of the first importance. To call something *pyknon* or *apyknon*, or enharmonic or chromatic, is to describe it in aesthetic-perceptual terms, not quantitative ones. What makes something, for example, a *pyknon* is the fact that it presents a certain character to the ear, not that it is of a certain magnitude. Analysis will reveal that this character does indeed belong only to groups of intervals within a determinate range of size: they must span less than $1\frac{1}{4}$ tones. But this is not a conceptual truth or one based on mathematical principles; it is discovered empirically by the study of interval-groups that are heard as possessing the relevant perceptual character. Secondly, analysis shows that the character of being *pyknon* is not determined to a specific magnitude, but to a range. Similarly, note-names such as 'chromatic *lichanos*' belong to notes placed within a definite range, but not at a fixed point relative to other notes (this applies even more forcefully to more general names like *lichanos*). The characters in respect of which different instances are 'like' one another are not paired with specific magnitudes. Hence, concentration on quantitative equalities obscures the facts about 'likeness' and 'unlikeness'. Cf. 39.4ff., 68.13ff., and on the application of these ideas to concords and discords see 55.3ff.

[76] The two lowest intervals of the tetrachord constitute a *pyknon* when they jointly cover a range smaller than the one remaining interval. See the passages cited in n. 74 above.

[77] Since anything equal to or greater than $1\frac{1}{4}$ tones is not a *pyknon*, the distinction corresponds roughly to our perception of a difference in kind between intervals smaller than a minor third and those equal to or greater than it (within the boundaries of a fourth). Although the Greeks reserved the term 'concord' (*symphōnia*) for fourths, fifths, octaves, etc., it does not follow that they lacked a sense of the distinction between what are generally called 'concords' and 'discords' in modern usage. For the idea that there is a continuum of qualities, rather than a set of sharp divisions, between octaves, other concords, and smaller 'melodic' intervals, see 11 Ptol. *Harm.* 15.10ff.

magnitudes do, but remains the same:[78] and while the genus remains constant it is reasonable to suppose that the functions [*dynameis*] of the notes do too.[79] After all, with which of the people who argue about the shades of the genera should one agree? Not everyone looks to the same division when tuning the chromatic or the enharmonic, so why should the note a ditone from *mesē* be called *lichanos* rather than one a small amount higher? To perception it seems to be the enharmonic in both of these divisions, yet plainly the magnitudes of the intervals are not the same in each of them. But the form of the tetrachord is the same, so that we must say that the boundaries of the intervals are the same too.[80] To speak quite generally, so long as the names of the bounding notes remain constant, the higher of them being called *mesē* and the lower *hypatē*, the names of those inside the boundary also remain constant, the higher being called *lichanos* and the lower *parhypatē*; for perception always treats the notes between *mesē* and *hypatē* as *lichanos* and *parhypatē*. But to suppose that equal intervals ought to be bounded by notes of the same name, or unequal ones by notes with different names, is to quarrel with the evidence of the senses. For the interval between *hypatē* and *parhypatē*, as it occurs in melody, is sometimes equal and sometimes unequal to that between *parhypatē* and *lichanos*.[81] It is obvious that two successive intervals cannot each be bounded by notes with the same names, unless the note in the middle is to have two names.[82] The absurdity is also obvious where the intervals are unequal, for it is not possible for one of the names to remain fixed while the other changes, since they get their names from their relation to each other:[83] for just as the fourth note from *mesē* is called *hypatē* by virtue of its relation to *mesē*, so the

[78] The analysis of 'ranges' at 22.24–27.14 was designed to reveal the points at which the genus does change; the results are summarised at 46.24–47.8.

[79] While a note retains the same perceived character and melodic function, it remains the same note. See 47.26ff., cf. 36.6–12.

[80] Cf. 23.1–23, 26.10–27, ps.-Plut. *De Mus.* 1143e–f. Every *lichanos* lower than the chromatic range is enharmonic (26.9–11). Since the lowest chromatic *lichanos* is two thirds of a tone above *hypatē* (24.15–21), and no *lichanos* is less than a semitone from *hypatē* (22.27–30), we may infer that the range of movement of enharmonic *lichanos* is one sixth of a tone.

[81] Equal in most of the divisions analysed by Aristoxenus, unequal in the soft diatonic (51.24–8).

[82] On the system Aristoxenus is criticising, a note with equal intervals above and below would have both the name proper to a note below such an interval, and that proper to a note above it.

[83] Aristoxenus' precise meaning is obscure. If we focus on the thesis that one note-name cannot remain fixed while another changes, the point might be this. Take four consecutive notes, M, N, O, P. If the size of the interval between O and P changes, both O and P must acquire new names. But then so must N, since even though its relation to M and O has not changed, what was O now has a different name, and notes get their names from their relations to other (named) notes. Alternatively, we might focus on the posited inequalities of the intervals between M, N and O. If the note-name 'N' is taken to have the meaning, for example, 'note separated from its neighbour by a tone', then this note must have another name too, designating its relation to the other note. But it is hard to resist the conclusion that Aristoxenus must somehow be misrepresenting his victims' intentions.

one which follows *mesē* is called *lichanos* by virtue of its relation to *mesē*.[84] Let that, then, be our reply to this difficulty.

Let us use the term '*pyknon*' for every case where, in a tetrachord whose extremes form the concord of a fourth, the two intervals put together occupy a smaller range [*topos*] than the one.[85] Of the divisions of the tetrachord, the ones which are divided into familiar magnitudes of interval are themselves familiar and most noteworthy.[86] One of these divisions is enharmonic, in which the *pyknon* is a semitone and the remainder a ditone.[87] Three divisions are chromatic, those of the soft, the hemiolic and the tonic chromatic.[88] The division of the soft chromatic is that in which the *pyknon* consists of two of the smallest chromatic dieses, and the remainder is measured by two units of measurement, by the semitone three times, and by the chromatic diesis once.[89]
51 It is the smallest of the chromatic *pykna*, and this *lichanos* is the lowest in this genus. The division of the hemiolic chromatic is that in which the *pyknon* is one and a half times that of the enharmonic, and each of its dieses is one and a half times the corresponding enharmonic diesis.[90] It is easy to see that the

[84] Aristoxenus here relies on the ways in which note-names were actually assigned in normal Greek practice, and his remarks might cut little ice with people proposing a new system. It is unclear whether he means to imply that *mesē* is the primary note from their relations to which others get their designations, but the fact that *mesē* was conceived as in some sense foundational is well documented, if little understood. See, for example, Aristotle *Metaph.* 1018b29, ps.-Ar. *Probs* XIX.20, cf. 11 Ptol. *Harm.* 64.16ff., Cleonides *Eisagoge* 201.14–202.5. Cleonides clearly states that it is from their relations to *mesē* that the other notes' *dynameis* are derived.

[85] See n. 76 above.

[86] Aristoxenus proceeds to analyse six divisions of the tetrachord, according to genera and shades. He repeats many of the results given in Book I (22.24–27.14), but the two passages have different purposes. In the first book his aim is to mark the extent and boundaries of the ranges inhabited by the moveable notes, overall and in each genus. Here he seeks to identify the points in these ranges at which the moveable notes lie in the genera and shades most commonly used in practice (cf. 11 Ptol. *Harm.* Book I ch. 16, Book II ch. 16). He is not claiming that these divisions are the only legitimate ones, or even the best. Within each range there is an indefinite number of permissible positions for each note (see, for example, 26.11ff., 47.8ff., and for some examples 52.12ff.). Only his enharmonic and his tense diatonic have a special status, in that their moveable notes have respectively the lowest and the highest possible positions in the tetrachord. It seems clear, then, that Aristoxenus is at least trying to analyse the phenomena of actual musical practice. For the analyses to be successful it would not, of course, be necessary for them to represent exactly what one might hear in any performance. The test would be whether an expert musician would agree, on listening attentively to melodies played with just the intonation Aristoxenus describes, that they were indeed based on the attunements at which he would aim, when playing in 'familiar' musical styles.

[87] This is Aristoxenus' preferred enharmonic, the *pyknon* being divided into two equal dieses. From the bottom of the tetrachord it runs $\frac{1}{4}$, $\frac{1}{4}$, 2.

[88] 'Soft', *malakon*, implies a lowering of pitch; in this shade the moveable notes lie as low as they can in a chromatic tetrachord. 'Hemiolic' means 'half and whole': one interval is hemiolic with respect to another if it is one and a half times as great. (The word is standardly used by Pythagoreans and Platonists to refer to the ratio 3:2.) The usage here is explained at the beginning of 51. 'Tonic' refers to the interval of a tone, the compass of the *pyknon* in this shade.

[89] The (smallest) chromatic diesis is one third of a tone (see, for example, 21.29–30). This division is therefore $\frac{1}{3}$, $\frac{1}{3}$, $\frac{11}{6}$. Aristoxenus computes the upper interval as $\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{3}$ in order to represent it in terms of 'familiar' magnitudes.

[90] An enharmonic diesis is a quarter-tone. This division is therefore $\frac{3}{8}$, $\frac{3}{8}$, $\frac{7}{4}$.

hemiolic *pyknon* is greater than that of the soft chromatic, for the former falls short of being a tone by an enharmonic diesis, the latter by a chromatic diesis. The division of the tonic chromatic is that in which the *pyknon* consists of two semitones and the remainder is three semitones. Up to this division, both the notes move, but after this *parhypatē* stays still, since it has travelled through its whole range [*topos*],[91] while *lichanos* moves through an enharmonic diesis, and the interval between *lichanos* and *hypatē* becomes equal to that between *lichanos* and *mesē*, so that in this division the *pyknon* no longer occurs.[92] The *pyknon* disappears in the division of the tetrachord simultaneously with the first occurrence of the diatonic genus. There are two divisions of the diatonic, those of the soft diatonic and the tense.[93] The division of the soft diatonic is that in which the interval between *hypatē* and *parhypatē* is a semitone, that between *parhypatē* and *lichanos* is three enharmonic dieses, and that between *lichanos* and *mesē* is five dieses.[94] That of the tense diatonic is that in which the interval between *hypatē* and *parhypatē* is a semitone, and each of the others is
52 a tone.[95] There are thus as many *lichanoi* as there are divisions of the tetrachord, and two fewer *parhypatai*, since we use that which stands at a semitone both for the diatonic divisions and for that of the tonic chromatic. Of the four *parhypatai*, the enharmonic one is peculiar to the enharmonic genus, while the other three are common to the diatonic and chromatic.[96]

Of the intervals in the tetrachord, that between *hypatē* and *parhypatē* is, in melody, either equal to or smaller than that between *parhypatē* and *lichanos*, but never greater.[97] That it can be equal is evident from the division of the enharmonic and those of the chromatic, and that it can be smaller is evident from those of the diatonic; but one could also grasp this from the chromatic divisions, if one took as *parhypatē* that of the soft chromatic, and as *lichanos* that of the tonic chromatic: for such divisions as these are also perceived as melodic.[98] An unmelodic result would come from taking them in the opposite

[91] See 23.25ff.

[92] In tonic chromatic, *lichanos* lies a tone above *hypatē*. When *lichanos* is moved up by a quarter-tone, the magnitude spanned by the two lower intervals of the tetrachord together becomes equal to the remaining interval, that between *lichanos* and *mesē*. Hence the pair of intervals at the bottom no longer constitutes a *pyknon* (see 24.11–14, 48.26–31, 50.15–19).

[93] 'Soft' translates *malakon*: see n. 88 above. 'Tense' translates *syntonon*, as literally as possible: it indicates a high or 'tense' pitch for the moveable notes. Translators often render it as 'sharp', which is more natural English, but blurs the associations of the Greek.

[94] That is, $\frac{1}{2}$, $\frac{3}{4}$, $\frac{5}{4}$.

[95] This may fairly be called the 'standard' form of diatonic. It corresponds fairly closely to the form most commonly discussed by Platonists and Pythagoreans (e.g., at 1.12 Philolaus frag. 6, 2.3 Plato *Tim.* 35b–36b), though since they do not allow that the fourth is $2\frac{1}{2}$ tones, or indeed that the tone can be halved (e.g., 8 Eucl. *Sect. Can.* propositions 15 and 16), they do not represent the lowest interval as exactly a semitone. In general, when an author mentions the diatonic without qualification, it is usually to this form that he is referring, in either its Aristoxenian or its Pythagorean guise. For a useful compilation of different quantifications of the generic tetrachords see 11 Ptol. *Harm.* Book II ch. 14.

[96] See 26.29–27.1.

[97] This important and underived rule is also stated at 27.2ff.

[98] The division would be $\frac{1}{3}$, $\frac{2}{3}$, $\frac{3}{2}$.

order, if one took as *parhypatē* that which stands at a semitone, and as *lichanos* that of the hemiolic chromatic, or as *parhypatē* that of the hemiolic, and as *lichanos* that of the soft chromatic; for such divisions are perceived as harmonically ill-attuned [*anharmostoi*].[99]

The interval between *parhypatē* and *lichanos* may be equal, in melody, to that between *lichanos* and *mesē*, or unequal in either direction.[100] It is equal in the tenser diatonic, and smaller in all the others: but it is greater when one uses as *lichanos* that of the tensest diatonic, and as *parhypatē* one of those which are lower than that which stands at a semitone.[101]

After this we must give an account of succession [*to hexēs*], first explaining
53 in outline the method according to which we should agree that the task of defining it is best pursued. To speak generally, succession is to be sought by reference to the nature of melody, and not in the way in which continuity used to be explicated by those who looked to *katapyknōsis* for their evidence. They appear to treat the proper course [*agōgē*] of melody with contempt, as is clear from the quantity of dieses they place in succession, since the voice cannot put together as many as three. Thus it is evident that succession is not always to be sought in the smallest intervals, nor in equal or unequal ones, but that it is the nature [of melody] that we must follow.[102] It is not easy at this stage to give an accurate account of succession, before we have expounded the ways in which intervals are combined;[103] but the fact that there is such a thing as succession could be made clear even to someone totally ignorant of the subject through an argument of the following kind. The claim that there is no interval which we divide *ad infinitum* in melody is one that commands assent: there is some greatest number of parts into which melody divides each of the intervals. Whether we say just that this commands assent or that it is necessarily true, it is plain that the notes bounding the parts which make up this number are successive with one another. Among such notes are, apparently, those which we have used from earliest times, such as *nētē* and *paranētē*, and those which immediately succeed them.[104]

99 These improper divisions would be $\frac{1}{2}$, $\frac{1}{4}$, $\frac{7}{4}$, and $\frac{3}{8}$, $\frac{7}{24}$, $\frac{11}{6}$.

100 Stated also at 27.5–7.

101 Aristoxenus gives a specific example at 27.9–11, where the *parhypatē* is that of the lowest chromatic: the division would be $\frac{1}{3}$, $\frac{7}{6}$, 1. On the rules stated in this passage see 11 Ptol. *Harm.* 32.23ff., cf. 33.22ff.

102 On *to hexēs* see 27.15ff. On the sense of the remark about *katapyknōsis* see 28.1ff., and nn. 34 and 116 to Book I. On the voice's inability to put together more than two dieses, see 28.6ff. with n. 117, cf. 46.8ff. On the last sentence see 28.18ff. with n. 120.

103 This is the task undertaken in Book III.

104 The proposition is concerned with intervals defined by the *dynameis* of their bounding notes, not with 'magnitudes'. Aristoxenus appeals to the fact of experience that in any melodic system there is some determinate number of notes that stand between any given pair of notes. This implies that there are some notes that count as standing next to one another, between which, in that system, no note can be inserted. The size of the intervals between adjacent notes, and even the number of notes within a given interval, will vary from one system to another. (For a more elaborate expression of this point see 60.10ff.) What determines the maximum number of subdivisions in a given interval is the number of melodic functions that can be expressed, in any one system, by notes between its boundaries. Aristoxenus seems confident that the system of fixed notes marking the boundaries of conjoined and disjoined fourths, each fourth being divided into three

We must next give an account of the first and most indispensable of the
54 conditions that bear upon the melodic combination of intervals. Let it be accepted that in every genus, as the melodic sequence progresses through successive notes both up and down from any given note, it must make with the fourth successive note the concord of a fourth or with the fifth successive note the concord of a fifth. Any note which fulfils neither of these conditions must be considered unmelodic relative to all the notes with which it fails to form concords in the numerical relations mentioned.[105] But one must not overlook the fact that the principle we have stated is not enough by itself to ensure that the construction of *systēmata* out of intervals is melodic.[106] It is quite possible that even though the notes form concords in the numerical relations specified, the *systēmata* are unmelodically constructed; but if this condition is not fulfilled there is no help to be had from the others. Let this, then, be posited as first in the order of principles: if it is not fulfilled, the harmonic attunement [*to hērmosmenon*] is destroyed.[107]

Similar to it, in a way, is the principle which governs the positioning of tetrachords in relation to one another. Tetrachords that belong to the same *systēma* must have one or other of two properties. Either they must be concordant with one another, so that each note of the one forms some concord with the corresponding note of the other, or they must both be concordant with the same tetrachord, each of them being continuous with the one with which they are concordant, but not in the same direction.[108] This prlnciple, also, is not enough by itself to ensure that tetrachords belong to the same *systēma*, since
55 other conditions are needed in addition, of which we shall speak later:[109] but without this one the rest are useless.

intervals by two moveable notes, exhausts the number of distinct melodic functions that there can be.

105 This repeats the fundamental law stated at 29.6ff., used as the most important axiom in Book III.

106 One further principle is stated below; for others see 29.1ff. All of these, however, are very closely related to the previous one. The only plainly independent principles on which Aristoxenus relies are those governing the order of greater and smaller intervals in a tetrachord, stated at 52.8ff.

107 Yet on occasion in Book III (e.g., 66.22–5) he speaks as if conformity to this one rule were sufficient to make a sequence melodically permissible.

108 That is, if they are not concordant with one another, they must both be concordant with one that lies between them, and is continuous with the lower end of one and the upper end of the other: cf. 59.16–60.9. (Tetrachords are said to be concordant with one another when each note of one is concordant with the note in the equivalent position in the other.) In the GPS every tetrachord is concordant with every other; in the LPS the first tetrachord is not concordant with the third, but each of them is concordant with the second.

109 It is not clear what these conditions are. Aristoxenus might have in mind the rule that successive tetrachords must be either conjunct or disjoined by a tone (58–9), but this, as he makes clear himself, is derivable from the principle stated at 54.2ff. Macran suggests the rule that conjunctions and disjunctions must alternate in any extended *systēma*, but this will not do as it stands, since it does not apply to the LPS. (There is no suggestion in Aristoxenus, as in 11 Ptol. *Harm.* Book II ch. 6, that the LPS is not properly a single system, but combines parts of two through a form of modulation. It is natural to read, for example, 63.20ff. as implying that where progressions to disjunction and to conjunction are legitimate alternatives, the two systems generated are of equal status.)

Of the magnitudes of intervals, those of the concords appear to have either no range of variation [*topos*] at all, being determined to a single magnitude, or else a range which is quite indiscernible, whereas those of the discords possess this quality to a much smaller degree. Hence perception relies much more confidently on the magnitudes of concords than on those of discords.[110] The most accurate way of constructing a discordant interval will therefore be by means of concords.[111] Thus, if we have the task of constructing from a given note a discord such as a ditone downwards (or any of those which can be constructed by means of concords) we should construct from the given note a fourth upwards, from there a fifth downwards, then another fourth upwards, and then another fifth downwards. In this way the ditone downwards from the given note will have been constructed. And if the task is to construct the discord in the opposite direction, the concords should be constructed the opposite way round.[112]

Further, if a discord is subtracted from a concordant interval by means of concords, the remainder will also have been found by means of concords. For instance, let the ditone be subtracted by means of concords from the fourth. It is clear that the notes bounding the remainder by which the fourth exceeds the ditone have also been constructed, by means of concords, in their relation to
56 one another. The notes bounding the fourth are themselves concordant. From the higher of them we find a note concordant at a fourth above, from that note another a fifth below, then again one a fourth above, and then from that note another a fifth below. The last of these concordant intervals falls on the higher of the notes bounding the remainder in question: and it is thus clear that if a discord is subtracted from a concord by means of concords, the remainder will also have been found by means of concords.

The question whether we were correct in the assumption, which we made in

110 Aristoxenus assimilates concordant and discordant intervals to the class of 'functionally' apprehended phenomena. They are not constituted by their magnitudes as such, but by the character under which they are experienced: cf. 39.30–40.24, and especially 48.15ff. But just as the special sound of the *pyknon* is perceived only in intervals within a certain range, so is that of each specific concord and discord. Aristoxenus suggests here, correctly, that there is a small but nevertheless real range of variation in magnitude within which the sound of a discordant interval will continue to be heard as 'functionally' the same discord. The range assignable to any given concordant interval, by contrast, is either so small as to be indiscernible, or else is actually of zero extension, the characteristic sound of, for example, the fifth being restricted (in fact, but not a priori) to one determinate magnitude.

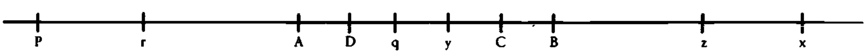
111 This method of tuning a complete scale by the construction of fourths and fifths is still familiar. Aristoxenus uses it for theoretical purposes only, but it is certainly derived from the ways in which practical musicians tuned their instruments. The 'method of concords' appears as freely in Pythagorean writings as in Aristoxenian. (See, for example, 8 Eucl. *Sect. Can.* proposition 17; it links also with the mathematical operations of 2.3 Plato *Tim.* 35b–36b. On its relevance to the divisions of Archytas see the Appendix to chapter 1.) The procedure cannot be used, in Aristoxenus' system, to construct intervals other than the semitone and its multiples. He therefore seems to have no method that is reliable even by his own standards for showing that the other intervals in his tetrachordal divisions are of exactly the sizes he asserts. Yet he insists elsewhere that the incapacity of the method of concordance to construct these intervals is no good reason for denying them a place in melody (ps.-Plut. *De Mus.* 1145b–c).

112 See the diagrams given in n. 114 below.

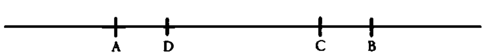
our introduction,[113] that the fourth consists of two and a half tones, can be investigated most accurately in the following way. Take a fourth, and starting from each of its extremes in turn, mark off a ditone by means of concords. It is clear that the remainders are equal, since equals have been taken from equals. Next, take a fourth upwards from the lower bounding note of the upper ditone, and take another fourth downwards from the upper bounding note of the lower ditone. It is clear that next to each of the notes which bound the resulting *systēma* there will be two consecutive remainders which must be equal, for the reasons stated before. When this construction has been set up, we must bring to the judgement of perception the outermost of the notes that have been located. If they appear to perception as discordant, it will be evident that the
57 fourth is not two and a half tones: but if they sound the concord of a fifth, it will be evident that the fourth is two and a half tones. For the lowest of the notes constructed was tuned to make the concord of a fourth with the upper bounding note of the lower ditone, and the highest of the notes constructed has turned out to form the concord of a fifth with the lowest; so that since the difference is a tone, and since it is divided into equal parts, each of which is a semitone and is also the excess of a fourth over a ditone, it is clear that the fourth consists of five semitones. It is easy to see that the extremes of the *systēma* constructed will not form any concord but the fifth. In the first place it must be understood that they do not form the concord of a fourth, since there is an excess added in both directions to the fourth which was originally taken. Next we must state that it cannot accommodate the concord of an octave. The magnitude formed by the sum of the remainders is less than a ditone, since the fourth exceeds the ditone by less than a tone: for everyone agrees that the fourth is greater than two tones and smaller than three. Thus the total added to the fourth is less than a fifth, which makes it clear that the combination of them cannot be an octave. But if the extremes of the notes constructed form a
58 concord greater than the fourth but smaller than the octave, the concord which they form must be the fifth: for this is the only concordant magnitude between the fourth and the octave.[114]

[113] See 24.4–10, 46.1–2.

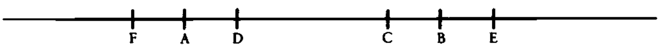
[114] The argument is most easily followed with the help of diagrams. First, we take a fourth and construct ditones inwards from its extremities, by the method given at 55.13ff.



AB is the original fourth. We construct a ditone downwards from B by taking a fourth upwards to x, a fifth down to y, a fourth up to z, and a fifth down to D. We construct a ditone upwards from A by taking a fourth down to p, a fifth up to q, a fourth down to r, and a fifth up to C. Then D is a ditone below B, and C is a ditone above A.



The remainders, that is AD and CB, are evidently equal. We next take the fourth upwards from D, to E, and the fourth downwards from C, to F.



Since CF is a fourth and CA is a ditone, FA is the remainder of the fourth when a ditone

Book III

1 *Successive tetrachords are either conjunct or disjunct*[1]

Let it be called 'conjunction' when two tetrachords, successive in the melodic series and with the same arrangement [*schēma*], have in the middle a common note; and let it be called 'disjunction' when between two tetrachords, successive in the melodic series and with the same arrangement, there lies a tone. That one or the other of these must apply to successive tetrachords is clear from the assumptions we laid down. When the notes four places apart in a continuous series form the concord of a fourth they will produce conjunction,
59 and when those five places apart form a fifth they will produce disjunction. But the notes must fulfil one or other of these conditions, so that successive tetrachords must have one or the other of the properties mentioned.[2]

is subtracted. So is AD, since DB is a ditone and AB a fourth. So, similarly, are CB and BE. Hence FA, AD, CB and BE are all equal.

We now test interval FE by ear. If it is a fifth, then the fourth must be $2\frac{1}{2}$ tones. This is because FC was constructed as a fourth, FE is a fifth, and the difference between a fourth and a fifth is a tone (see 45.34–46.1). Their difference, CE, is divided into equal parts CB and BE, which are therefore exact semitones, and each of these is the difference between a fourth and a ditone. Thus the fourth is equal to a ditone plus one of these semitones.

The rest of the argument is designed to show that if FE is a concord it must be that of the fifth. The claim depends straightforwardly on the assumption that the fourth is somewhere between two and three tones, and that the fifth is the only concord between the fourth and the octave. No Greek theorist supposed otherwise.

Aristoxenus' argument is not a mathematical proof, and is not intended to be one. It requires us not only to accept its reasoning, but to take an instrument and actually go through the procedure in practice, to construct the various steps by ear, and to judge the concordance of the final interval, FE, by the same means. Its task is not to prove anything, but to show us how we can satisfy ourselves, on the basis of our own perception, that what he has said is true. (It is ultimately on that criterion that all his basic, undemonstrated principles must rest. Their grounds lie in perceptual experience, and cannot be presented as arguments in a written treatise.) His construction could not convince the Pythagoreans, who believed they could refute it mathematically (e.g., 8 Eucl. *Sect. Can.* proposition 15). Aristoxenus in turn would no doubt have considered their arguments irrelevant (cf. 32.18ff.). Despite its ingenuity, however, his procedure is plainly open to criticism even in its own terms. The obvious comments are those of Ptolemy, in the latter part of his discussion of the conflicting Aristoxenian and Pythagorean positions on this issue (11 Ptol. *Harm.* 21.21–24.29).

[1] The third book is quite unlike the others. It consists of a set of theorems deriving propositions from principles already adopted. The theorems are what Aristoxenus, following Aristotle, calls *apodeixeis*, 'demonstrations', and are thought of as explaining why the propositions are true, as well as proving them (see the introductions to chapter 3 and to this chapter). Each proposition is first stated, then demonstrated (I follow Macran in italicising the initial statements, which I have also numbered, in order to bring out this pattern; it parallels that of Euclid in 8 *Sect. Can.* and in the *Elements of Geometry*). The first four theorems are introductory, establishing claims about rather general concepts and structures, to be drawn on in the sequel. Three of them take time off from the business of formal demonstration to clarify problematic features of the propositions under discussion, apparently in response to questions raised by Aristoxenus' students.

[2] On succession (*to hexēs*) in general see 27.15ff., 52.33ff., and on succession between notes see 60.10ff. Here Aristoxenus is concerned with successive tetrachords, that is,

Some of my hearers have already raised problems about succession: (i) what succession in general is; (ii) whether it comes about in only one way or in several; and (iii) whether both conjunct and disjunct tetrachords are equally successive. The answers I have usually given to these questions are as follows. In general, those *systēmata* are continuous[3] whose boundaries are either successive or coincident. There are two ways in which *systēmata* are successive. In one, the higher boundary of the lower *systēma* is the same as the lower boundary of the higher *systēma*. In the other, the lower boundary of the higher *systēma* is successive with the higher boundary of the lower *systēma*.[4] In the former case the *systēmata* of the successive tetrachords share in common a certain locus [*topos*], and are necessarily alike in arrangement.[5] In the latter they are separated from one another, and the forms [*eidē*] of the tetrachords can be alike:[6] this is so when a tone is placed in the middle, but not otherwise. Thus two tetrachords which are alike are successive if either there is a tone between them or their boundaries coincide. Hence successive tetrachords which are alike must be either conjunct or disjunct.[7]

tetrachords between which no note intervenes (cf. 59.13ff., but an ambiguity enters his treatment at 59.33). His main focus in this book is on enharmonic tetrachords between fixed notes, of the form $\frac{1}{4}$, $\frac{1}{4}$, 2, though he also discusses others, particularly that of the tense diatonic, $\frac{1}{2}$, 1, 1. The present proposition is said initially to apply to tetrachords 'with the same arrangement', and though this qualification is not mentioned again it must remain in force. If we take the enharmonic series $\frac{1}{4}$, $\frac{1}{4}$, 2; $\frac{1}{4}$, $\frac{1}{4}$, 2; $\frac{1}{4}$, $\frac{1}{4}$, 2, comprising three conjoined tetrachords (as in the LPS), and begin from the lower boundary of the first ditone, we find a tetrachord of the form 2, $\frac{1}{4}$, $\frac{1}{4}$ (not of course between fixed notes) followed at the distance of a ditone by one of the form $\frac{1}{4}$, $\frac{1}{4}$, 2. These should count as 'successive', since no note lies between them, but they neither share a note nor are separated by a tone. But for any two successive tetrachords with the same arrangement (whether between fixed notes or not) Aristoxenus' proposition does hold, and is derivable, as he says, from the fundamental law stated at 29.6ff., 53.33ff. In the notes that follow I shall call this law L.

3 'Continuous' represents *synechē*; I do not think that the sense is relevantly different from *hexēs* ('successive').

4 At first glance this account looks circular, defining 'successive' by reference to itself. But what Aristoxenus is doing is defining 'successive *systēmata*' (among which successive tetrachords are included) by reference to successive or coincident boundaries, i.e., notes.

5 This is true only if Aristoxenus refuses the title 'tetrachord' to certain sequences that superficially deserve it. The diatonic octave from *hypatē mesōn* to *nētē diezeugmenōn* consists of two tetrachords between fixed notes, disjoined by a tone. Its structure is $\frac{1}{2}$, 1, 1; 1; $\frac{1}{2}$, 1, 1. But the sequence from the upper boundary of the second tone (*mesē*) to the upper boundary of the fourth tone (*paranētē*) might also be construed as a tetrachord, a fourth comprising four notes. It shares a boundary with the tetrachord below it, but is different in arrangement. Aristoxenus cannot mean to reserve the term 'tetrachord' for those between fixed notes: in any one sequence, all such tetrachords have the same arrangement or 'form', and he speaks also (in the next sentence) of ones that do not. But he will not allow that the tone separating tetrachords between fixed notes can properly be included in any tetrachord: see 61.5–34, and cf. n. 9 below.

6 *Eidos* ('form') is interchangeable with *schēma* ('arrangement'): see 74.9–13.

7 Aristoxenus reiterates the first proposition of the book. He does not say that tetrachords disjoined by a tone must be alike, but that only such tetrachords, and conjunct ones, can be. (If he had intended the stronger claim, it would be open to counter-examples similar to those mentioned in n. 5 above.) In order to accommodate the notion of successive but dissimilar tetrachords, Macran thought it necessary to suppose that the sense of 'successive' has altered in this paragraph. The supposition is unnecessary. Successive but dissimilar tetrachords, separated by intervals other than the tone, are exemplified,

We assert also that between successive tetrachords there must be either no
60 tetrachord at all, or not one that is dissimilar.[8] Thus between tetrachords of the same form a dissimilar tetrachord cannot be placed, while between tetrachords which are dissimilar but successive there can be placed no tetrachord at all.[9] From these remarks it is clear that tetrachords of the same form can be placed in succession with one another in the two ways which have been mentioned.[10]

2 *An incomposite interval is one bounded by successive notes*

If the bounding notes are successive, no note has been left out; if none has been left out, none will intervene; if none intervenes, none will divide the interval; and what does not admit of division will not be composite. For everything composite is composed of parts, into which it may also be divided.[11] But because of the ambiguous nature of the magnitudes of intervals, people become confused about this proposition, and ask in surprise how the ditone can ever be incomposite, when it is capable of being divided into tones, and how the tone can ever be incomposite, when it is capable of being divided into two semitones: and they say the same thing about the semitone. Their lack of understanding arises from their failure to see that some of the intervallic magnitudes are shared in common by a composite and an incomposite interval: it is for this reason that the incomposite is defined not by reference to the magnitude of an interval, but by reference to its bounding notes. When the ditone is bounded by *mesē* and *lichanos* it is incomposite, but when it is
61 bounded by *mesē* and *parhypatē* it is composite. This is why we say that

for example, in the enharmonic sequence discussed in n. 2 above. But a change in the sense of 'successive' is coming: see the next note.

8 Here the sense of 'successive' has certainly changed, quite without warning. As Macran says, it now means 'in the same line of succession', that is, 'in the same *systēma*', rather than 'directly successive'. The shift of sense is all the odder, in that the last sentence of this paragraph, and the following one, return to the previous use of 'successive', again without warning. It is tempting to suppose that the present sentence and the next are misplaced, or that the text is in some other way defective.

9 Aristoxenus means that there can be no tetrachord lying immediately above a second and immediately below a third, if the second and third are dissimilar. Such tetrachords must be separated by some interval or group of intervals that is not a tetrachord. There can, of course, be a tetrachord together with some other intervals between two dissimilar tetrachords. The rule is plainly intended to be derivable from L. So it is, with one qualification: it breaks down in diatonic sequences if we permit the tone that disjoins tetrachords between fixed notes to count as part of a tetrachord. Thus in the sequence 1; $\frac{1}{2}$, 1, 1; $\frac{1}{2}$, 1, 1; 1; $\frac{1}{2}$, 1, 1; $\frac{1}{2}$, 1, 1, where the first and sixth tones are disjunctive, we could construct an anomaly by dividing it differently, as, for example, 1; $\frac{1}{2}$, 1, 1; $\frac{1}{2}$, 1, 1; 1, $\frac{1}{2}$, 1; 1; $\frac{1}{2}$, 1, 1. Here the first and third tetrachords are dissimilar, but are separated by a tetrachord. To eliminate this possibility, we must again refuse to allow the disjunctive tone a place in any tetrachord: cf. n. 5 above.

10 'In succession' now goes back to meaning 'in direct succession', and the two 'ways' are those of conjunction and disjunction by a tone.

11 This paragraph defines 'incomposite interval'. It could be used, conversely, to define 'successive notes'. As the sequel shows, the distinction between 'interval' and 'magnitude' is important here, since not every interval between successive notes is an incomposite magnitude, and 'successive notes' cannot be defined by reference to incomposite magnitudes. Cf., for example, 27.33ff., 39.4ff., 47.8ff., 49.22ff.

incompositeness does not depend on the magnitudes of intervals, but on the bounding notes.[12]

3 *In variations of genus it is only the parts of the fourth that change, while the interval peculiar to disjunction is unaltered*[13]

Anything harmonically attuned is divided so as to form conjunction or disjunction, at any rate when it is composed of more than one tetrachord. But the only incomposite parts from which conjunction is put together are those of the fourth, so that at least in this case it is necessarily only the parts of the fourth that will change. Disjunction contains in addition to these the tone peculiar to itself. If, therefore, it can be shown that this characteristic feature of disjunction does not change in variations of genus, we are evidently left with the conclusion that the change involves only the parts of the fourth themselves. Now the lower of the notes bounding the tone is the upper bounding note of the lower of the tetrachords that lie in disjunction, and we have said that this is unchanged in variations of genus. The higher of the notes bounding the tone is the lower bounding note of the higher of the tetrachords that lie in disjunction, and we have said that this, similarly, is unchanged in variations of genus.[14] Thus, since it is clear that the notes bounding the tone are unchanged in variations of genus, we are evidently left with the conclusion that in such variations it is only the parts of the fourth which change.

62
4 *In each genus there are, at the most, as many incomposite [magnitudes] as there are in the fifth*

Every genus is employed melodically in either conjunction or disjunction, as we have said. Conjunction has been shown to consist only of the parts of the fourth, while disjunction adds the one interval which is peculiar to it, that is, the tone. When the tone is added to the parts of the fourth, the total is the fifth. It is thus plain that since none of the genera, taken in a single shade, can consist of more incomposite [parts] than those which the fifth contains, there will be in each genus, at the most, as many incomposite [magnitudes] as there are in the fifth.

Some people have often found difficulty with the question why, in this

[12] A given interval is to be specified by naming its bounding notes, a given magnitude by identifying its size. The interval between *mesē* and *lichanos* is always incomposite, but the magnitude in which it is instantiated (which may be as great as a ditone) is not.

[13] Macran deletes the second part of this proposition, but on inadequate grounds.

[14] This has not been stated in so many words. Aristoxenus might be relying on the assumption that the tetrachords disjoined lie between fixed notes, but an argument could be made out on the basis of 58.14. If disjunction is conceived as a way of proceeding from one tetrachord to another where the second is of a form that could have been reached, alternatively, by conjunction, then this second tetrachord must be of the same form as the first (59.19–22); and the only incomposite interval that can separate similar tetrachords has the magnitude of a tone (59.23–7). Hence the interval of disjunction is always a tone, and is unaffected by changes of genus.

proposition, the words 'at the most' are added, and why it is not simply shown that each of the genera consists of as many incomposite [magnitudes] as there are in the fifth. My reply to them is that sometimes each genus will be constituted out of fewer incomposite [magnitudes], but never from more. For this reason just this point is demonstrated first, that it is impossible for any of the genera to be constituted out of more incomposite [magnitudes] than there are in the fifth; and it is shown in the sequel that each of them can sometimes be constituted from fewer.[15]

63 5 *No* pyknon, *neither a whole* pyknon *nor a part of one, is melodically adjacent to another* pyknon

The result will be that a given note will neither form with the fourth note in order from it the concord of a fourth, nor form with the fifth note in order from it the concord of a fifth: and notes so placed have been shown to be unmelodic.[16]

6 *Of the notes bounding the ditone, the lower is the highest note of a* pyknon, *the higher the lowest note of a* pyknon

Since in conjunction the *pykna* form the concord of a fourth with one another, the ditone must necessarily lie between them: and similarly since the ditones form the concord of a fourth with one another, the *pyknon* must necessarily lie

[15] Macran takes this proposition to be about incomposite intervals, not magnitudes. The Greek contains no noun that would settle the question. He supposes that the *systēmata* that contain fewer incomposite intervals are 'defective or transilient scales' such as those of 12 Arist. Quint. *De Mus.* 18.5ff., ps.-Plut. *De Mus.* 1134f–1135b. But (a) it would be odd to call such *systēmata* 'genera'. (b) They contain fewer intervals than there are, in principle, in the magnitude they span, but this is not always a fifth. (c) The reference here to 'the sequel' is certainly to 72.29–73.16, which explicitly refers back to this passage. The use of the word *megethos* ('magnitude') there, and the character of the argument, guarantee that the principal reference is to magnitudes, not to intervals. But in fact neither noun, on its own, quite captures the sense. The question is: 'How many different magnitudes characterize the incomposite intervals of a *systēma* in any given genus?' The answer is: 'At any rate no more than can, in principle, characterise those within a fifth'. The fifth includes the three incomposite intervals of a representative tetrachord, plus the tone of disjunction. If these are all of different magnitudes, the answer to the question will be 'four'. But often they are not all different. In the enharmonic, for example (where the fifth contains two quarter-tones, a ditone and a tone), the answer is 'three', and in tense diatonic (a semitone and three tones) it is 'two'.

[16] By '*pyknon*', in this book, Aristoxenus usually means the enharmonic *pyknon* of two quarter-tones, but in most cases his arguments could be adapted to apply equally to chromatic *pykna*. The proof here is formally incomplete. A sequence of four quarter-tones (two enharmonic *pykna*) plainly breaches the law in question, which is L. A sequence of only three quarter-tones would be consistent with L if it continued with a step of $\frac{11}{4}$ tones and then repeated itself. In such a sequence every note stands at a fifth from the fifth note in order from it. What is breached is the (inexplicit) assumption that every extended *systēma* is analysable as a string of successive tetrachords: in the sequence offered no note stands at a fourth from any of the others. This proposition is used in the derivation of several later ones: see 63.34ff., 65.30ff., 66.9ff., 67.10ff., 70.15ff., 71.23ff., 72.13ff.

between them. This being so, the *pyknon* and the ditone must occur alternately, so that it is evident that the lower of the notes bounding the ditone will be the highest note of the *pyknon* lying below it, and the higher will be the lowest note of the *pyknon* lying above it.[17]

7 *Each of the notes bounding the tone is the lowest note of a* pyknon

In disjunction, the tone is placed between tetrachords of such a kind that their bounding notes are the lowest notes of *pykna*; and it is by these that the tone is bounded. The lower of those that bound the tone is the higher of the notes bounding the lower tetrachord, and the higher of those which bound the tone is the lower of the notes bounding the higher tetrachord. Thus it is evident that each of the notes bounding the tone will be the lowest note of a *pyknon.*[18]

8 *Two ditones will not be placed in succession*

64 Suppose that they are. Then below the upper ditone will follow a *pyknon,* since we saw that the lower bounding note of the ditone is the highest note of a *pyknon.* Above the lower ditone will also follow a *pyknon,* since we saw that the upper bounding note of the ditone is the lowest note of a *pyknon.* This being so, two *pykna* will be placed in succession, and since this is unmelodic, it will also be unmelodic for two ditones to be placed in succession.[19]

[17] Though this proposition is apparently true only of tetrachords in conjunction, Aristoxenus states it quite generally, and uses it in derivations concerned with the disjunct series. See the next proposition and the next note.

[18] The disjunctive tone appears (in enharmonic) between tetrachords of the form $\frac{1}{4}$, $\frac{1}{4}$, 2. That its upper bounding note is always the lowest note of a *pyknon* is clear. In the series in which the tone occurs, however, its lowest bounding note is not a part of a *pyknon* at all. The interpretation of Aristoxenus' standpoint raises complex issues, and similar problems arise over some other propositions, particularly the next one. They are discussed in Barker (1985). Briefly, the solution is, I think, that Aristoxenus is concerned with the ordering of intervals in a *systēma* principally as a means to a further end, that of articulating the *dynameis* of notes. Here, a note identified as 'the upper boundary of a tetrachord in a genus that contains a *pyknon*' has as one of its *dynameis* the capacity to be the lowest note of a *pyknon.* The point of this interpretation will appear in cases where this proposition and its predecessor are used in other derivations. See particularly the next proposition, and 65.31ff., 66.9ff., 71.23ff.

[19] In the postulated series of two (incomposite) ditones, the successive *pykna* mentioned, on either side of the shared note, will plainly not actually appear. How then can their alleged existence be used to rule out the sequence of ditones? Further, the proposition that the argument relies on (proposition 6) superficially implies only that the ditone can have a *pyknon* above it as well as below, not that it must, since in disjunction the interval is a tone. The argument must turn on the idea that 'having a *pyknon* above it' is a property or *dynamis* of a note (the note at the top of an enharmonic tetrachord), not of a magnitude (the ditone) as such; and the rule 'two *pykna* cannot occur in succession' is grounded in a principle of the form 'No note with the *dynamis* of admitting a *pyknon* immediately above it can also have the *dynamis* of admitting a *pyknon* immediately below it (and conversely)'. Since the upper bounding note of any incomposite interval with the magnitude of a ditone has the *dynamis* of admitting a *pyknon* above it, and the lower bounding note of such an interval has the *dynamis* of

9 *In the enharmonic and chromatic, two tones will not be placed in succession*

Suppose that they are, first in the ascending series.[20] Now if the upper bounding note of the added tone is melodic, it must form either the concord of a fourth with the fourth note in succession from it, or that of a fifth with the fifth note. If neither of these conditions holds of it, it must be unmelodic. And that neither condition will hold is plain. If the *lichanos*, which is fourth in order from the added note,[21] is enharmonic, it will be four tones from that note: if it is chromatic, either of the soft chromatic or of the hemiolic, it will stand from it at an interval greater than the fifth; while if it is that of the tonic chromatic, it will form the concord of a fifth with the added note. But this it should not do: it should form the concord of a fourth with the fourth note or that of a fifth with the fifth. Neither of these happens: hence it is clear that the upper bounding note of the added tone is unmelodic. If the second tone is added below the first, it will make the genus diatonic.[22] Hence it is plain that in the enharmonic and the chromatic, two tones will not be placed in succession.

10 *In the diatonic, three tones will be placed in succession, but not more*

The note bounding the fourth tone will form neither the concord of a fourth with the fourth note from it, nor that of a fifth with the fifth note.[23]

11 *In this same genus, two semitones will not be placed in succession*

Let the added semitone be placed, first, below the existing semitone. Then the note bounding the added semitone will form neither the concord of a fourth

admitting a *pyknon* below it, these two notes cannot coincide. If they did, the resulting note would have two inconsistent *dynameis* (for fuller discussion see Barker 1985). It remains curious that Aristoxenus chose to prove the proposition in this abstruse way, since it can obviously be demonstrated much more simply: the postulated sequence of two incomposite ditones is bound to be in breach of L.

[20] Aristoxenus means: 'Suppose first that an interval of a tone is added above the tone already present in an enharmonic or a chromatic series (that is, above the tone of disjunction).'

[21] The added note is a tone above *paramesē*. A tone below *paramesē* is *mesē*; next comes *lichanos*, a ditone below *mesē* in enharmonic, $\frac{11}{6}$ in soft chromatic, $\frac{7}{4}$ in hemiolic chromatic, $\frac{3}{2}$ in tonic chromatic (see 50.25ff.).

[22] To place a tone below the disjunctive tone is to locate *lichanos* a tone below *mesē*. It has this position only on the tense diatonic (51.29ff.). More to the point, a tetrachord whose *lichanos* lies a tone below *mesē* cannot contain a *pyknon*, and the *pyknon* is absent only in diatonic divisions (51.19–22).

[23] Three tones occur in succession when the tone of disjunction is placed above a tense diatonic tetrachord. A fourth successive tone would indeed be in breach of L, whereas three are not, but Aristoxenus offers no positive argument to show that three are acceptable. Mere conformity to L is not enough to establish melodic legitimacy (54.11ff.).

with the fourth note from it, nor that of a fifth with the fifth note. Thus this positioning of the semitone will be unmelodic. But if it is placed above the existing semitone, the genus will be chromatic; so that it is evident that in the diatonic genus two semitones will not be placed in succession.[24]

We have now shown which kinds of equal incomposite magnitudes can be placed in succession, and how many, and which equal ones, by contrast, are such that they cannot be placed in succession at all. We must now speak of unequal intervals.

12 *A* pyknon *is placed next to a ditone, both above it and below*

It has been shown[25] that in the conjunct series these intervals are placed alternately. Hence it is evident that each of them will be placed both above and below the other.

13 *A tone is placed next to a ditone only above it*

Suppose it to be placed below it. The consequence will be that the highest note
66 of a *pyknon* and the lowest note of a *pyknon* fall on the same pitch, for we saw that the lower bounding note of the ditone is the highest note of a *pyknon*, and that the higher bounding note of the tone is the lowest of a *pyknon*. If these notes fall on the same pitch, two *pykna* are necessarily placed in succession. Since this is unmelodic, a tone placed immediately below a ditone must be unmelodic too.[26]

14 *A tone is placed next to a* pyknon *only below it*

Suppose it to be placed on the opposite side. Then the same impossibility will result, for the highest and the lowest note of a *pyknon* will fall on the same pitch, so that two *pykna* are placed in succession. Since this is unmelodic, the placing of a tone immediately above the *pyknon* must be unmelodic too.[27]

[24] Aristoxenus considers the placing of a semitone first below, then above the semitone in the sequence $\frac{1}{2}$, 1, 1. The first such placing breaches L; the second generates a *pyknon* in the tetrachord, which cannot then be diatonic (51.19–22). It will in fact be a tetrachord in tonic chromatic ($\frac{1}{2}$, $\frac{1}{2}$, $\frac{3}{2}$).

[25] The reference is to 63.5ff.

[26] The second sentence refers again to 63.5ff. The reasoning is parallel to that of 63.34ff., and raises similar problems: see n. 19 above. But whereas the previous proposition could have been proved directly from L, this one cannot, and neither can the others mentioned at the end of nn. 16 and 18. The nature of the reasoning in these cases, and the fact that they are concerned with the location of the tone, indicate that Aristoxenus is seeking to repair the omission of which he accuses Eratocles at 5.9ff. (cf. 6.21ff.). A *systēma* can 'split in two', i.e., proceed to disjunction by a tone as an alternative to conjunction, not after just any interval of a fourth, but only at the ends of tetrachords bounded by fixed notes. Clearly the other possibilities cannot be ruled out on the basis of L alone. The present proposition shows that disjunction immediately below the ditone is unacceptable.

[27] The reasoning parallels that of the previous proposition. This one rules out disjunction immediately above a *pyknon*.

15 *In diatonic, a semitone on both sides of a tone is not melodic*

The consequence will be that neither will the fourth notes in order form the concord of a fourth, nor will the fifth in order form that of a fifth.

16 *A semitone on both sides of a group of two tones or of three tones is melodic*

Either the fourth notes in order will form the concord of a fourth, or the fifth in order will form that of a fifth.[28]

17 *From the ditone there are two progressions upwards and one downwards*

It has been shown that both the *pyknon* and the tone are placed above it, but there will be no more progressions than these upwards from the interval
67 mentioned, for the only incomposite interval which remains is the ditone, and two ditones are not placed in succession. Plainly, then, there will be only two progressions upwards from the ditone. Downwards there is one; for it has been shown that one ditone will not be placed next to another, and that a tone will not be placed immediately below a ditone, so that only the *pyknon* is left. It is clear, therefore, that there are two progressions upwards from a ditone, one to the tone, the other to the *pyknon*, and one downwards, to the *pyknon*.[29]

18 *From a* pyknon, *conversely, there are two progressions downwards and one upwards*

It has been shown that below a *pyknon* there is placed a ditone or a tone. There will be no third progression, since of the incomposite intervals there remains only the *pyknon*, and two *pykna* are not placed successively. Hence it is evident that there will be only two progressions downwards from a *pyknon*; and there will be only one upwards, to the ditone. For one *pyknon* is not placed next to another, nor is a tone placed immediately above a *pyknon*, so that only the ditone is left. It is clear, therefore, that from a *pyknon* there are two progressions downwards, to the tone and to the ditone, and one upwards, to the ditone.[30]

[28] This again concerns the diatonic. The proof repeats the anomaly mentioned in n. 23 above, treating L as a sufficient and not just a necessary condition of melodic propriety.

After this sentence the MSS add: 'From the semitone there are two progressions upwards and two downwards'. Most editors excise the sentence, mainly because there is in fact, in diatonic, only one progression in each direction from the semitone, that to a tone. Probably the interpolator had in mind the progressions implied in proposition 16, the sequences of two and three tones. But the sentence can hardly stand, since the progressions mentioned in all the succeeding propositions are to the next interval only.

[29] This proposition concerns the enharmonic genus. Two ditones cannot be successive, 63.34ff. A tone cannot lie just below a ditone, 65.31ff.

[30] Two *pykna* cannot be successive, 62.34ff. A tone cannot lie just above a *pyknon*, 66.9ff.

19 *From a tone there is one progression in each direction, to a ditone downwards and to a* pyknon *upwards*

It has been shown that neither a tone nor a *pyknon* is placed immediately below it, so that only the ditone is left; and it has been shown that neither a tone nor a ditone is placed immediately above it, so that only the *pyknon* is left. It is clear, therefore, that from a tone there is one progression in each direction,
68 downwards to the ditone and upwards to the *pyknon*.[31]

The case will be similar in the chromatic genus, except that the interval between *mesē* and *lichanos* which occurs in each of the shades is adopted instead of the ditone, and so is each of the magnitudes of the *pyknon*.[32] It will also be similar in the diatonic; for from the tone common to the genera there will be one progression in each direction, downwards to the interval between *mesē* and *lichanos*, whatever it may happen to be in each shade of the diatonic, and upwards to that between *paramesē* and *tritē*.[33]

This proposition, too, has puzzled some people. They are surprised that the conclusion is not the opposite of this: for it seems to them that there is an indefinite number of progressions in each direction from the tone, since there is apparently an indefinite number of magnitudes belonging to the interval between *mesē* and *lichanos*, and to the *pyknon* too.[34] To this we have answered, first, that the same feature could be detected in the earlier propositions, just as much as in this one. It is obvious that one of the alternative progressions from the *pyknon* will have an indefinite range of magnitudes, as will one of those from the ditone; for an interval such as that between *mesē* and *lichanos* has an indefinite range of magnitudes, and an interval such as the *pyknon* has the same property as the interval just mentioned. Nevertheless, from the *pyknon* there are two progressions downwards and from the ditone there are two upwards; and similarly from the tone there is one progression in

[31] A tone (in enharmonic) cannot lie just below or above a tone, 64.11ff. A *pyknon* cannot lie just below a tone, 66.9ff. A ditone cannot lie just above a tone, 65.31ff.

[32] All Aristoxenus' demonstrations concerning ditones and enharmonic *pykna* can indeed be modified without difficulty to apply to chromatic intervals in each shade of the genus.

[33] This sentence has interesting features. The interval it considers (the disjunctive tone) is not specified by its magnitude, as in the preceding propositions, but by its bounding notes. This is unavoidable, since diatonic uses the undivided magnitude of a tone in two other positions as well, and of them Aristoxenus' statement would be false. But in identifying this interval by the *dynameis* of its bounding notes he is not being forced into a methodological anomaly: the boot is on the other foot. That is, he is interested in the magnitudes of intervals primarily for the light they shed on the *dynameis* of notes, and on what a note's identity implies about the intervals surrounding it. Some notes in some *systēmata* are such that the magnitudes of the intervals surrounding them are different from those surrounding any other notes in a representative fifth of that genus (tetrachord plus tone). Hence, these notes can be adequately specified, for convenience, by reference to their relation to those intervals (e.g., 'the note below the ditone'). In diatonic the expression 'the notes bounding the tone' does not adequately specify two particular notes, as it does in the other genera. Hence, Aristoxenus identifies the notes in question by their 'dynamic' names, not as a makeshift device, but as something quite in line with his main purposes. See also nn. 18, 19, 26 above, and Aristoxenus' reflections in the next paragraph.

[34] See, for example, 26.9ff., 48.9ff.

69 each direction. This is because we must specify the progressions in relation to a single shade of a single genus. Everything in music must be specified and assigned to its position in the sciences in accordance with the respect in which it is determinate, and in so far as it is indeterminate it must be passed over.[35] Now in respect of the magnitudes of intervals and the pitches of notes, the facts about melody seem to be in some ways indeterminate, but in respect of functions [*dynameis*], forms and positions they appear to be determinate and ordered.[36] To take an obvious example, the progressions downwards from the *pyknon* are determinate both in function and in form, and are only two in number, for the one which proceeds by a tone leads the form of the *systēma* into disjunction, while that which proceeds by the other interval, whatever its magnitude may be, leads it into conjunction. From these considerations it is clear also that from the tone there is one progression in each direction, and that the two progressions taken together are causes of a single form of *systēma*, that of disjunction. But it is clear from what we have said, and from the nature of the facts themselves, that if anyone tries to investigate the progressions from intervals not by reference to a single shade of a single genus, but by reference to all the shades of all the genera at once, he will topple into indeterminacy.

20 *In the chromatic and the enharmonic, every note participates in a* pyknon

Every note in these genera bounds either a part of a *pyknon* or an interval such
70 as that between *mesē* and *lichanos*. No argument is needed concerning those which bound parts of the *pyknon*, since it is obvious that they participate in a *pyknon*. Concerning those which bound the tone it has been shown above that each is the lowest note of a *pyknon*.[37] Of those which bound the remaining interval, the lower has been shown to be the highest note of a *pyknon*, and the higher the lowest.[38] Thus, since these are the only incomposite intervals, and each of them is bounded by notes both of which participate in a *pyknon*, it is clear that every note in the enharmonic and the chromatic participates in a *pyknon*.

[35] 'In so far as', reading *hēi* for *ei*, with Macran. For the general sentiment cf., for example, Plato *Philebus* 16c ff. (quoted in part at 2.6).

[36] For the sense in which the facts concerning magnitudes are indeterminate see particularly 48.33–49.18. As Macran remarks, 'functions, forms and positions' seems not to have a closely technical sense here. The relevant notions of function and form are exemplified in the next sentence. 'Position' will include the location of a note in the series of named *dynameis*, the location of the disjunctive tone in relation to *mesē*, and so on. Aristoxenus' main point is that the aesthetically important qualitative characteristics of the melodic series are determinate, while many quantitative ones are not: cf. 39.4–41.24.

[37] This was shown at 63.21ff.

[38] The 'remaining interval' is the ditone, or its equivalent in each shade of the chromatic. The reference is to 63.6ff.

21 *The notes situated in the* pyknon *have three positions*

This is easy to see, since neither a *pyknon* nor part of a *pyknon* is placed next to a *pyknon*. For it is obvious, in view of this principle, that the notes mentioned will not have a greater number of positions.[39]

22 *It is to be shown that from the lowest note alone there are two progressions in each direction, and that from the others there is only one in each direction*[40]

It was shown above that from the *pyknon* there are two progressions downwards, one to the tone and the other to the ditone.[41] But the fact that there are two progressions from the *pyknon* is the same as the fact that there are two progressions downwards from the lowest of the notes which lie in the *pyknon*, since that note is the boundary of the *pyknon*. It was also shown that from the ditone there are two progressions upwards, one to the tone, the other to the *pyknon*.[42] But the fact that there are two progressions from the ditone is the same as the fact that there are two progressions upwards from the higher of the notes which bound the ditone, for that note is the boundary of the
71 ditone; and it is also the lowest note of a *pyknon*, for that, too, has already been shown.[43] Hence it is clear that from the note mentioned there are two progressions in each direction.

23 *It is to be shown that from the highest note there is one progression in each direction*

It was shown previously that there is one progression from a *pyknon* upwards,[44] and saying that there is one progression upwards from a *pyknon* is no different from saying that there is one from the note which bounds it, for the reasons given in connection with the previous proposition. It has also been shown that from a ditone there is one progression downwards,[45] and there is no difference between saying that there is one progression downwards from the ditone, and saying that there is one from the note which bounds it, for the reason previously stated. And it is clear that the note which is the lower

[39] For the principle see 62.34ff. The three positions are those of the lowest, middle and highest notes of a *pyknon*. Later authors describe them as *barypyknoi*, *mesopyknoi* and *oxypyknoi* respectively; notes not belonging to a *pyknon* are called *apyknoi*. See, for example, 12 Arist. Quint. *De Mus.* 9.13ff., Cleonides *Eisagoge* 186.1ff.

[40] 'Lowest' here, and 'highest' and 'middle' in the next two propositions, mean 'lowest etc. in the *pyknon*'. In taking the trouble to transform his theorems from statements about interval-sequences to statements about the properties of notes, Aristoxenus again reveals that his main preoccupation is with the latter.

[41] Shown at 67.10ff.

[42] Shown at 66.27ff.

[43] At 63.6ff. Marquard, followed by Macran, added at the beginning of 71 a supplement which improves the passage's precision (rather pedantically) and parallels 71.16–20. But it is hardly necessary.

[44] At 67.10ff.

[45] At 66.27ff.

boundary of the ditone is the same as that which is the upper boundary of the *pyknon*, and is the highest note of a *pyknon*.[46] From this, then, it is evident that from the note mentioned there is one progression in each direction.

24 *It is to be shown that from the middle note, too, there will be one progression in each direction*

Since some one of the three incomposite intervals must be placed next to the note in question, and since, lying where it does, it has a diesis on each side, it is clear that neither a ditone nor a tone will be placed next to it, either above or below. For if a ditone is so placed, either the lowest or the highest note of a *pyknon* will fall on the same pitch as the note in question, which is the middle note of a *pyknon*, so that three dieses will occur in succession, no matter on which side the ditone is placed. If a tone is placed there, the result will be the same, since the lowest note of a *pyknon* will fall on the same pitch as the middle note of a *pyknon*, so that three dieses will be placed in succession. Since they are unmelodic, it is clear that there will be one progression from the note in question in each direction.[47]

It is now evident that from the lowest of the notes that lie in the *pyknon* there will be two progressions in each direction, and that from each of the others there will be one progression in each direction.

25 *It is to be shown that two notes which differ in respect of their positions in a* pyknon *cannot melodically be placed on the same pitch*

First, let the highest and the lowest be placed on the same pitch. The consequence of this will be that two *pykna* are placed in succession. Since this is unmelodic, so too is the incidence on the same pitch of notes which differ in the *pyknon* in this respect. And it is plain that notes which differ in the remaining possible way will not melodically share the same pitch. Three dieses are bound to be placed in succession, whether it is the highest or the lowest which shares the same pitch with the middle note.[48]

[46] See 63.6ff.

[47] The central premisses of the argument are the propositions proved at 62.34ff. and 63.6ff. The form of the reasoning and the difficulties it raises run parallel to those of 63.34ff.

[48] The reasoning is again similar: the new premise is that proved at 62.34ff. But Aristoxenus seems to have made an uncharacteristic slip, since in fact it is possible for two notes differing in their position in the *pyknon* to fall on the same pitch. This is true of *paramesē* and *paranētē synēmmenōn* in the tonic chromatic, each of which is a tone above *mesē*. The former is the lowest note of a *pyknon*, the latter the highest. What Aristoxenus should be asserting, I suggest, is that two different positions in the *pyknon* cannot both belong to the same functionally designated note. No transformation of a *systēma* will allow the same note to stand sometimes in one place in a *pyknon*, sometimes in another.

26 *It is to be shown that the diatonic genus consists of either two, three or four incomposite [magnitudes]*[49]

It has already been shown that each of the genera consists of as many incomposite [magnitudes], at the most, as there are in the fifth, and these are
73 four in number. Now if three of the four are equal and the fourth unequal – and this is what happens in the highest diatonic – the diatonic genus will have been constructed out of only two magnitudes.[50] If *parhypatē* is lowered to make two of them equal and two unequal, the diatonic genus will have been constructed out of three magnitudes, one smaller than a semitone, one a tone, and one greater than a tone.[51] If all the magnitudes in the fifth are unequal, the genus in question will have been constructed out of four magnitudes.[52] Thus it is apparent that the diatonic consists of either two, three or four incomposite [magnitudes].

27 *It is to be shown that the chromatic and the enharmonic consist of either three or four [incomposite magnitudes]*

Since the incomposite [magnitudes] in the fifth are four in number, if the parts of the *pyknon* are equal the genera in question will have been constructed out of three magnitudes – the part of the *pyknon*, whatever size it may be, the tone, and an interval such as that between *mesē* and *lichanos*. If, on the other hand, the parts of the *pyknon* are unequal, the genera in question will have been constructed out of four magnitudes, the smallest being an interval such as that between *hypatē* and *parhypatē*, the second one such as that between *parhypatē* and *lichanos*, the third a tone, and the fourth an interval such as that between *lichanos* and *mesē*.

The question has been raised why these genera cannot be constructed out of
74 two incomposite [magnitudes], like the diatonic. There is no doubt what the adequate and obvious explanation is for the fact that this cannot happen. Three equal incomposite intervals are not placed in succession in the enharmonic or the chromatic, whereas in the diatonic they are.[53] This is why the diatonic alone is sometimes constructed out of two incomposite [magnitudes].

After this we must say what difference of form [*eidos*] is, and what sort of difference it is. It is unimportant whether we use the term 'form' or the term

[49] This proposition is closely related to the argument at 62.1ff. (see n. 15). My additions of the word 'magnitudes' are subject to the same qualification as is made there.

[50] The fifth in the highest diatonic takes the form $\frac{1}{2}$, 1, 1, 1.

[51] This will be one of the diatonic shades that 'borrows' a chromatic *parhypatē*. The tetrachord in the example given at 27.9 has the form $\frac{1}{3}$, $\frac{7}{6}$, 1; to reach the fifth we add the disjunctive tone: cf. 52.30.

[52] The structure of a fifth in the lowest diatonic is $\frac{1}{2}$, $\frac{3}{4}$, $\frac{5}{4}$, 1 (see 51.24ff.).

[53] In the enharmonic there cannot be three consecutive dieses (62.34ff.), nor even as many as two consecutive ditones (63.33ff.) or tones (64.11ff.). The last of these proofs applies equally to the chromatic; the first applies also to the parts of chromatic *pykna*. Aristoxenus articulates no exact equivalent of the second to apply to the chromatic, but one could easily be constructed: cf. 68.2ff., 69.29ff.

'arrangement' [*schēma*], since we use both of these words to refer to the same thing. This difference occurs when in the same magnitude, composed of incomposite parts that are the same in magnitude and in number, the order of these parts is altered.

28 *Given this definition, it is to be shown that there are three forms* [eidē] *of the fourth*

First there is that in which the *pyknon* is at the bottom, secondly that in which a diesis lies on each side of the ditone, and thirdly that in which the *pyknon* lies above the ditone. It is easy to see that the parts of the fourth cannot be placed in relation to one another in more ways than these.[54]

[54] Aristoxenus is setting off to review the 'arrangements' of the fourth and the fifth, and the ways in which they can be put together to form an octave, so filling the gap alleged to have been left by Eratocles (6.19ff.). Probably he went on to consider more extensive *systēmata*, including the GPS and the LPS, thus completing the fourth part of the programme outlined in Book II (36.15ff.). In view of the existence of summaries, in authors such as Cleonides and Arist. Quint., of Aristoxenian doctrine on the remaining topics in his programme (*tonoi*, modulation, melodic composition, 37.8–38.26), it is likely that a treatise of the scope envisaged was indeed finished. It may not, of course, have reached these later scholars in quite the form in which it left Aristoxenus.

Appendix: Aristoxenus' *Elementa Rhythmica*

Of Aristoxenus' writings on rhythmics only fragments survive. Some appear as quotations or paraphrases in later sources, particularly in the work of the eleventh-century Byzantine scholar Michael Psellus. Part of the second book of the *El. Rhythm.* is preserved independently, and this is translated below. The sciences of rhythmics and metrics are outside the main scope of this volume, but the fragment is important both for the light it sheds, indirectly, on Aristoxenus' harmonics, and because it is a major source for the section on rhythmics in chs. 13–19 of 12 Arist. Quint. *De Mus.* Book 1. In writing the notes for that section I found it necessary to refer repeatedly to passages in the second book of *El. Rhythm.*, and it seemed better to present it here in full, rather than loading the notes with translated excerpts. Since it is on the whole clearly written, and because the subject is of only secondary interest here, I have provided no commentary other than the remarks to be found in the notes to Aristides Quintilianus. The text is that printed in Westphal (1893).

Elementa Rhythmica, Book II

1 In the preceding passage we explained that there are many kinds [*physeis*] of rhythm, what each of them is like, for what reasons they came to acquire the same name, and what it is that underlies [is the 'matter' for] each of them. Now we must speak specifically of the rhythm that is located in music.

2 We also explained in the preceding passage that it is to do with durations [*chronoi*, lit. 'times'] and the perception of them, but this must be said again now, for it is in a way the first principle of knowledge concerned with rhythms.

3 One must understand that there are these two natures [*physeis*], that of rhythm and that of the *rhythmizomenon* [lit. 'that which is made rhythmic'], these being related to one another in the same way as are shape and what is shaped.

4 For just as a body takes on many kinds of shapes, if all or some of its parts are disposed in different ways, so each of the *rhythmizomena* takes on many forms, in accordance not with its own nature, but with that of the rhythm. For the same utterance [*lexis*], when disposed into durations that differ from one another, takes on differences of a sort that are equal to the differences in the nature of the rhythm themselves. The same can be said about melody too, and about anything else whose nature it is to be made rhythmic [*rhythmizesthai*] by the sort of rhythm that is constituted out of durations.

5 We must lead the perception onwards [*epagein tēn aisthēsin*] from this point in the comparison we have mentioned, in an attempt to get a coherent grasp also of each of the things referred to (I mean rhythm and the *rhythmizomenon*). For of the bodies whose nature it is to be shaped, none is identical with any of the shapes; the shape is rather a disposition of the parts of the body, arising from the fact that each of them is placed [*schein*] in a particular way, which is why it is called 'shape' [*schēma*]. Rhythm, similarly, is not identical with any of the *rhythmizomena*: it is rather one of the things that dispose the *rhythmizomenon* in a particular way, and make it like this or like that in respect of durations.

6 The things that we have mentioned also resemble one another in that they do not arise by themselves. For it is obvious that shape cannot arise in the absence of that which will receive it; and rhythm, similarly, cannot arise without that which will be made rhythmic [*to rhythmisthēsomenon*], and which cuts up time [*chronos*], since time does not cut itself up, as we said earlier, but needs something else that will divide it. The *rhythmizomenon* must therefore be capable of being broken up into recognisable parts, by which it will divide time.

7 From what we have said, and from the perceptual evidence [*to phainomenon*] itself, there follows the statement that rhythm arises when the division of the durations takes on an organisation of some determinate sort: for not every organisation of durations is rhythmical [*enrhythmos*].

8 Thus, even without argument, it is convincing to say that not every organisation of durations is rhythmic. But we must lead the mind onwards [*epagein tēn dianoian*] by means of comparisons and try to gain understanding from them, until there grows up in addition the conviction that arises out of the subject matter itself. The facts concerning the combining [*synthesis*] of letters and that of intervals are familiar to us, that is, that in talking we do not combine the letters in every way, nor do we so combine the intervals in singing: rather, there are only a few ways in which the things mentioned are combined with one another, and many in which the voice cannot combine them in an utterance, and which perception does not accept favourably, but rejects as intolerable. It is for this reason that what is well attuned [*to hērmosmenon*] is arranged into far fewer forms, and what is ill attuned [*to anharmoston*] into far more. It will become apparent that the same is true of things to do with durations: for many of their proportions [*symmetriai*] and organisations [*taxeis*] are experienced as alien to perception, few as conformable to it and capable of being organised into the nature of rhythm. The *rhythmizomenon* is in a way common to both unrhythmicness [*arhythmia*] and rhythm: for the *rhythmizomenon* is by nature capable of receiving both kinds of structure [*systēma*], the rhythmic and the unrhythmic. We can put it well by saying that we should consider the *rhythmizomenon* to be such that it can be reorganised into all sorts of magnitudes of durations, and all sorts of combinations of them.

9 Time [*chronos*, 'duration'] is divided by the *rhythmizomena* by means of the parts of each of them. There are three *rhythmizomena*: diction, melody and bodily movement. Thus diction will divide time by means of its own parts, such as letters and syllables and words and everything of that sort; melody by its own parts, notes and intervals and *systēmata*; and movement by points and figures [*sēmeia* and *schēmata*] and anything else like this that is a part of movement.

10 Among durations, let us give the name 'primary' [*prōtos*] to that which is capable of being divided by none of the *rhythmizomena*, 'two-unit' [*disēmos*] to that which is measured twice by the primary, 'three-unit' [*trisēmos*] to that measured three times, 'four-unit' [*tetrasēmos*] to that measured four times. The names will be formed similarly in the cases of the remaining magnitudes.

11 We must try to gain an understanding of the character [*dynamis*] of the primary duration in the following way. It is characteristic of things that appear vividly to perception that they do not allow the speeds of their movements to increase indefinitely, but as the durations are brought close together they come somewhere to a standstill – the durations in which the parts of the things moved are placed. I am speaking of things moved in the way that the voice is moved in speaking and singing, and the body in making a gesture [*sēma sēmainon*] and dancing, and in being moved in the other movements of that sort. Since this appears to be the way these things are, it is clearly necessary that there exist some durations that are smallest, in which the singer will locate each of the notes: and the same argument clearly applies also to syllables and gestures [*sēmeia*].

12 We shall then give the name 'primary duration' to the duration in which there can in no way be placed either two notes or two syllables or two gestures. The way in which perception grasps this duration will become clear when we consider the arrangements [*schēmata*] of feet.

13 We also speak of an 'incomposite' duration, with reference to the usage of rhythmic composition [*rhythmopoiïa*]. The fact that rhythmic composition and rhythm are not the same is not easy to make clear at this stage, but let it be accepted on the basis of the following comparison. Just as we have seen that in the nature of melody melodic composition is not the same as *systēma* or *tonos* or genus or modulation, so we should assume that the case is the same with rhythms and rhythmic compositions, since we find that melodic composition is a use of *melos*, and in the science of rhythmics we say that rhythmic composition is, similarly, a kind of use. We shall understand this more clearly as our investigation proceeds.

14 We shall speak then of an 'incomposite' and a 'composite' duration with an eye to the usage of rhythmic composition, in the following way. If a duration of some magnitude is embraced by a single syllable or a single note or gesture, we shall call that duration 'incomposite'. But if the same magnitude is embraced by several notes or syllables or gestures, this duration will be called 'composite'. One may take an example [*paradeigma*] of what we are saying from the science that deals with attunement [*to hērmosmenon*]: for there the same magnitude is made composite by the enharmonic, incomposite by the chromatic, and sometimes the same genus makes the same magnitude both composite and incomposite, though not in the same place in the *systēma*. For the example differs from the matter under consideration [*to problēma*] in that it is by rhythmic composition that the duration is made incomposite and composite, whereas the interval is made so by the genera themselves, or by the organisation [*taxis*] of the *systēma*. Let the general definition of incomposite and composite duration be specified, then, in this way.

15 Now that the matter under consideration [*to problēma*] has been divided up in this way, let us describe as incomposite without qualification [*haplōs*] a duration that is divided by none of the *rhythmizomena*, as composite in the same sense a duration that is divided by all the *rhythmizomena*, and as composite in one way but incomposite in another a duration that is divided by some *rhythmizomenon* but not divided by another. Thus a duration that is incomposite without qualification is such as to be embraced neither by several syllables, nor by several notes, nor by several gestures. One that is composite without qualification is that which is embraced by more than one of all of these. One that is mixed is that which has the feature of being occupied by one note but by several syllables, or conversely by one syllable but by several notes.

16 That by which we indicate the rhythm, and make it known to perception, is a foot, either one foot or more than one.

17 Some feet are constituted from two durations, the up and the down [*anō* and *katō*], some from three, two up and one down or one up and two down, and some from four, two up and two down.

18 The fact that a foot cannot be constituted from just one duration is obvious, since a single unit [*sēmeion*] does not make a division of time: for without a division of time a foot is not thought to arise. It is the magnitudes of the feet that are responsible for the fact that a foot may accommodate more than two units. For the smaller feet, possessing a magnitude that is easily grasped by perception, are easily understood [*eusynoptoi*] through just the two units; but the opposite is true of the large ones, for since they have a magnitude that is hard for perception to grasp, they require more units, so that the magnitude of the whole foot, being divided into more parts, may become easier to understand. It will be explained later why there do not occur more than four units which the foot uses in accordance with its own character.

19 In what we are now saying one must not make the mistake of supposing that a foot is never divided into a number of parts greater than four. For some feet are divided into a number double or many times the quantity stated. But the foot is not divided of itself into more than the quantity stated: rather, it is divided into divisions of these kinds by rhythmic composition. We must understand that the units [*sēmeia*] that preserve the character of the foot are distinct from the divisions brought into being by the rhythmic composition. And to these remarks we must add that the units belonging to each foot remain equal in number and size, while the divisions brought into being by rhythmic composition take on a great deal of variation. This will be clear also in our later discussion.

20 Each of the feet is defined either by some ratio [*logos*], or by an irrationality [*alogia*] of a kind that will be intermediate between two ratios familiar to perception. What we are saying can be made clear in the following way, if two feet are taken, one of which has its up-beat equal to its down-beat, each of these being of two units, while the other has a down-beat of two units and an up-beat half that size, and if a third foot is taken beside these, having a *basis* [i.e., 'down-beat'] equal, once again, to both the others, but an *arsis* [i.e., 'up-beat'] with a magnitude intermediate between those of the other *arseis*. For such a foot will have an up-beat that is irrational [*alogos*] with respect to the down-beat. The irrationality will be between two ratios that are familiar to perception, the equal and the duple. This foot is called the irrational *choreios*.

21 Here too one must avoid making a mistake, through ignorance of the way the rational [*rhēton*] and the irrational [*alogon*] are understood in matters concerning rhythms. Thus just as among intervallic elements, what is rational in respect of melody was understood as being that which, in the first place, is sung, and is, secondly, recognisable in magnitude, either in the way in which the concords and the tone are recognisable, or in the manner of those which are commensurate with these, while what is rational only in respect of the ratios of numbers was understood as being that of which it is also true that it cannot be sung, so also we must suppose the rational and the irrational in rhythms to be like this. For one thing is understood as being rational in respect of the nature of rhythm, another only in respect of the ratios of numbers.

A magnitude of time that is understood as rational in rhythm must therefore be, first, one of those that enter into the rhythmic composition, and secondly a rational part of the foot in which it is placed. That which is understood as rational in respect of the ratios of numbers must be thought of as being like the twelfth part of a tone in matters to do with intervals, or any other such thing that is used in comparisons between intervals.

It is clear from what has been said that the *arsis* which was taken as intermediate will not be commensurate with the *basis*: for there is no common measure of them that is rhythmic [*enrhythmos*].

22 Let there be set out the following seven distinctions between feet.

First, that in which they differ from one another in magnitude.

Second, that in which they differ in genus.

Third, that in which some feet are rational, others irrational.

Fourth, that in which some are incomposite, some composite.

Fifth, that in which they differ from one another in division.

Sixth, that in which they differ from one another in arrangement [*schēma*].

Seventh, that in which they differ by antithesis.

23 Thus one foot differs from another in magnitude when the magnitudes of the feet, those that the feet embrace, are unequal.

24 They differ in genus when the ratios of the feet differ from one another, as when one has the ratio of the equal, another that of the duple, another some other ratio of rhythmic durations.

25 The irrational differ from the rational in that the up-beat duration is not rational in relation to the down-beat duration.

26 The incomposite differ from the composite in not being divided into feet, whereas the composite are divided.

27 They differ from one another in division, when the same magnitude is divided into parts that are unequal either in both their number and their magnitudes, or in one or the other.

28 They differ from one another in arrangement, when the same parts of the same magnitude are not arranged in the same way.

29 Those that differ from one another by antithesis are the ones which have the up-beat duration and the down-beat duration in the opposite order. This distinction exists in those which are equal, but have an unequal ordering of up-beat and down-beat durations.

30 There are three genera of feet that admit continuous rhythmic composition: the dactylic, the iambic and the paionic. The dactylic is that in equal ratio, the iambic that in duple, and the paionic that in hemiolic.

31 The smallest of the feet are those in the three-unit magnitude: for the two-unit magnitude would have a foot whose articulation [*sēmasia*] was completely crowded together [*pyknos*]. These feet in a three-unit magnitude are iambic in genus, for in the three there will be only the ratio of the duple [2:1 or 1:2].

32 Second come those in the four-unit magnitude. These are dactylic in genus, for in the four there are found two ratios, that of the equal [2:2] and that of the triple [3:1 or 1:3]. Of these that of the triple is not rhythmic, while that of the equal falls into the dactylic genus.

33 Third in magnitude are those in a five-unit magnitude. For in the five there are found two ratios, that of the quadruple [4:1 or 1:4] and that of the hemiolic [3:2 or 2:3]. Of these, that of the quadruple is not rhythmic, while that of the hemiolic will make the paionic genus.

34 Fourth are those in a six-unit magnitude. This magnitude is common to two genera, the iambic and the dactylic, for in the six there are found three ratios, the equal and the duple and the quintuple. The last mentioned is not rhythmic, and of the others the ratio of the equal will fall into the dactylic genus, that of the duple into the iambic.

35 The seven-unit magnitude does not have a division constituting a foot. For there are three ratios found in the seven, but none of them is rhythmic. Of these, one is the epitritic [4:3 or 3:4], the second is that of five to two, the third that of the sextuple.

36 Hence the fifth must be those in an eight-unit magnitude. These will be dactylic in genus, since...

[Here the MSS break off.]

CHAPTER 8

The Euclidean *Sectio Canonis*

The little treatise called the *Sectio Canonis* consists of a short introduction and twenty propositions presented and argued in the manner of theorems. It is attributed to Euclid in the manuscripts, and by Porphyry, who quotes it at length (*Comm.* 98.14–103.25 has the first sixteen propositions, and there are brief extracts elsewhere). The text Porphyry used was not quite identical with ours, and includes – probably wrongly – one proposition that our MSS lack (see n. 19 below). Parts of the treatise are also quoted by Boethius, again not quite in the form that our MSS transmit. If Euclid is indeed the author, the work will have been written around 300 B.C., but the attribution has been debated. Some scholars have doubted that it is all the work of a single writer, or even of a single period, speculations encouraged by the possibility that Porphyry and Boethius did not know the *Sectio* as a whole, in the form in which it has reached us (see especially Barbera 1984b). There are no good reasons, however, for denying Euclid's authorship of the main part of the treatise, at least as much as Porphyry quotes. Grounds have been suggested for thinking that either propositions 17–18 or propositions 19–20 come from a different pen, but they are not strong (see n. 57 below). The introduction is perhaps more questionable. Certainly it is an intelligent piece of writing, and some such preface is needed, including some of its special detail, if the theorems are to be understood. But it may seem too abbreviated, and some of its arguments, as they stand, too weak, to be the work of the careful author of the theorems. Porphyry and his sources paraphrase parts of it without suggesting that it is Euclid's. Possibly it is a later summary or paraphrase of the original introduction. My present inclination is to treat it, tentatively, as genuine, and (despite Barbera's arguments) to accept the whole essay as the work of a single hand.

The programme of the *Sectio* is simple. The introduction sets out a theory of the physical causation of sounds and their pitches, designed to justify the treatment of pitches as relative quantities, and the intervals between them as numerical ratios. It goes on to give a threefold classification of ratios, and an argument for assimilating concordant intervals to ratios of two of these classes only. There follow nine theorems that are purely mathematical, proving various propositions about ratios as such, and about the 'intervals' (here quantitative differences in any dimension, not necessarily musical intervals) between terms in such ratios. At proposition 10, musical conceptions begin to be introduced. Drawing on the mathematics of the first nine theorems, on the arguments of the introduction, and on a number of assumed facts of musical experience, propositions 10–13 demonstrate the ratios of the principal musical concords, and that of the tone. Propositions 14–16 are proofs of subsidiary theses that follow from a 'Pythagorean' treatment of intervals as ratios, but which are quite at odds with Aristoxenian methods of analysis. Proposition 17 shows how to locate certain notes in the enharmonic genus by means of concords, and proposition 18 has a further anti-Aristoxenian argument about the intervals thereby formed. Finally, propositions 19–20 show how to divide the string of a monochord in ratios that give a system in the diatonic genus: this is the 'division of the *kanōn*' from which the work gets its name.

The author is plainly concerned to give systematic, formal proofs of propositions that are basic to the Pythagorean and Platonist tradition. It should be noted that his derivations are not purely 'rational' or mathematical. They depend crucially on

accepted facts of empirical observation, and on the physical and conceptual considerations laid out in the introduction (cf. the remarks on 'Pythagorean' writers and 'kanonists' in 9.10–9.14 Ptolemaïs and Didymus ap. Porph. *Comm.* 22.22–28.26). The arguments will not stand, therefore, as substitutes for musical experience. They represent an attempt to translate the facts of this experience into the language of mathematics, where their implications and interrelations can be rigorously studied.

Two authors whose work the writer certainly knew are Archytas and Euclid himself. Proposition 3 is a version of an important theorem that Archytas had proved, and several propositions treat theorems proved in Euclid's *Elements* as familiar. But he departs from Archytas in his analyses of both the enharmonic and the diatonic genus (proposition 17, proposition 20). His divisions correspond to those of Philolaus (1.12 frag. 6) and Plato (2.3 *Tim.* 35b–36b), and to those of later Platonising Pythagoreans such as those drawn on by Theon of Smyrna (Adrastus in 9.2–9.3, Thrasyllus in 9.4–9.5), and Nicomachus in 10 *Enchiridion.* (See the introductions to chs. 9 and 10.) Again, though his opening sentences are evidently reminiscences of 1.19 Archytas frag. 1, he differs from Archytas, and from the prevalent tradition in general, in the theory of pitch that the introduction proceeds to outline. There are hints of the same idea elsewhere, but no other source states it plainly and explicitly (see n. 2 below), and its development into a fully articulated hypothesis may be Euclid's own achievement.

Two weaknesses in the treatise have to be emphasised. One is the unsatisfactory nature of the considerations linking concords with multiple and epimoric ratios, at the end of the introduction. No Greek author found a very plausible way of establishing this relation, but its existence was a persistent Pythagorean doctrine (see especially 11 Ptol. *Harm.* Book 1 chs. 5–6). It led to serious puzzlement about the status of the octave plus fourth (ratio 8:3), which the present author avoids by not mentioning the interval at all (one would expect to meet it in proposition 12). The second weakness is in the use that proposition 11 makes of the introduction's thesis about concords. Its argument involves an elementary logical fallacy, surprising in a work of such formal rigour, and the proposition that hangs on it is crucial to the sequel. There is, in fact, no way in which proposition 11 can be established legitimately from premisses on which the writer of the *Sectio* could rely.

Despite these flaws, the treatise is a neat attempt at the construction of a fully argued mathematical harmonics, though one of rather limited scope. Its relation to previous Pythagorean researches is perhaps comparable to that between Euclid's *Elements* and its precursors in geometry. What is most original about it is not what it asserts, nor even how each theorem is argued, but rather how the whole system of theorems is coordinated into a single scheme.

Not a great deal has been written on the *Sectio*, though most modern studies of Greek harmonics mention it at least in passing. See particularly Lippman (1964), pp. 153–6, Mathiesen (1975), Barker (1981b), Barbera (1984b). On the important Archytan theorem of proposition 3 see Burkert (1972), pp. 444–6, Knorr (1975), ch. 7. Mathiesen and Barbera include useful references to other discussions.

8 The *Sectio Canonis*

148 If there were stillness and no movement, there would be silence: and if there were silence and if nothing moved, nothing would be heard. Then if anything is going to be heard, impact and movement must first occur.[1] Thus since all sounds occur when some impact occurs, and since it is impossible for an impact

[1] These sentences are clearly echoes of 1.19 Archytas frag. 1. Archytas' influence is detectable at several other points in the treatise, notably in proposition 3.

to occur unless movement has occurred beforehand – and since of movements some are closer packed, others more widely spaced, those which are closer packed producing higher notes and those which are more widely spaced lower ones – it follows that some notes must be higher, since they are composed of
149 closer packed and more numerous movements, and others lower, since they are composed of movements more widely spaced and less numerous.[2] Hence notes that are higher than what is required are slackened by the subtraction of movement and so reach what is required, while those which are too low are tightened by the addition of movement, and so reach what is required.[3] We must therefore assert that notes are composed of parts, since they attain what is required through addition and subtraction. Now all things that are composed of parts are spoken of in a ratio of number with respect to one another, so that notes, too, must be spoken of in a ratio of number to one another.[4] Some numbers are spoken of in multiple ratio with respect to one another, some in epimoric ratio, and some in epimeric ratio, so that notes must also be spoken of in these kinds of ratio to one another:[5] and of these, the multiple and the epimoric are spoken of in relation to one another under a single name.[6]

[2] Various other sources draw attention to the fact that apparently continuous sounds are generated by sequences of discrete impacts made by a body (e.g., a plucked string) on the air, and that the impacts are more 'closely packed' or frequent in the case of a higher note. ('Closely packed' translates *pyknos*, elsewhere 'compressed', the adjective used in Aristoxenus, in the present treatise at proposition 18, and in many later authors, to refer to the pair of small intervals at the bottom of an enharmonic or chromatic tetrachord.) See particularly 4.24 ps.-Ar. *Probs* XIX.39, 5 *De Audib.* 803b–804a, 9.8 Heraclides ap. Porph. *Comm.* 30.1ff., cf. 1.8 Porph. *Comm.* 107.15ff. But none of these authors states or implies that it is the greater frequency of impact that causes or constitutes the higher pitch. Greater frequency may simply be taken as a concomitant of greater velocity, and it is explicit in 5 *De Audib.* 803a that higher pitch depends on greater speed of transmission, as in Archytas and Plato. Only the present author clearly articulates the view that frequency of impact is actually responsible for a note's perceived pitch.

[3] The verbs for slackening and tightening, *anienai* and *epiteinein*, also commonly have the sense 'lowering in pitch', 'raising in pitch', and so can be directly applied to notes as well as to strings (cf. n. 42 to Book I of 7 Aristox. *El. Harm.*). The present thesis is that the 'slackening' of a string 'slackens' the note (lowers its pitch) by reducing the rapidity of the string's back-and-forth movements, and so reducing the frequency of its impacts on the air.

[4] One of the attractions of the author's theory of pitch is that the variable by which pitch is determined alters discretely, not continuously. In a given time, a sounding string must have struck the air some whole number of times, and the relation between the impact-frequencies of two sounding strings will always be expressible as the ratio between two whole numbers. No surds can be involved. This is indicated here by the expression 'a ratio of number'. It becomes crucial in proposition 3 and in the harmonic conclusions drawn from it in propositions 16 and 18. The locutions 'are spoken of' etc., here and in the rest of the introduction (rather than, for example, 'are in a ratio of number' etc.) are quite natural Greek, but are probably chosen partly in order to fit with the linguistic nature of the considerations put forward in the last sentence of this paragraph. See n. 6 below.

[5] Multiple ratios are of the form $mn:n$. Epimoric ratios have the form $n+1:n$. Epimeric ratios have the form $n+m:n$ where m is greater than 1, and is neither equal to n nor a multiple of it. More complex classifications were later devised: see for instance Theon Smyrn. 76.8–80.14 (cf. 9.2 Adrastus ap. Theon Smyrn. 56).

[6] This sentence has been construed in several ways. On one view, 'these' are ratios, and there is a single name that refers jointly to multiples and epimorics. The author does not

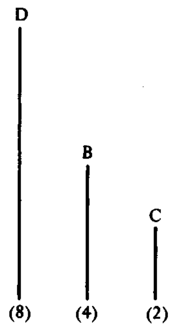
Among notes we also recognise some as concordant, others as discordant, the concordant making a single blend out of the two, while the discordant do not.[7] In view of this, it is to be expected that the concordant notes, since they make a single blend of sound out of the two, are among those numbers which are spoken of under a single name in relation to one another, being either multiple or epimoric.[8]

identify the name. Jan in his introduction (in *MSG*) suggested *kreittōn*, 'more powerful' or 'better', on the authority of Porph. *Comm.* 98.3–6, but Porphyry uses the word as a description rather than a title. A second view holds that 'these' are notes, and that the single name for pairs of notes standing in these ratios is 'concordant'. But this will hardly do, if only because rather few of the relevant ratios are those of concordant intervals. On either interpretation, the subsequent argument would, I suggest, be absurd. It depends on the thesis that in each concordant interval the paired sounds blend to form a unity, and that each multiple and epimoric ratio is, in a comparable sense, unified, as is indicated by the fact that each has a single, unified name. Then the point is not that there is a single name to cover all multiples and epimorics taken together, but that each such ratio has a single name of its own. The reference of 'these' must then be neither to ratios nor to notes, but to numbers: multiple and epimoric numbers – that is, numbers in a ratio of either of these kinds – are related to one another, not to other ratios, under a single name. This interpretation has the advantage of making the author say something true. In Greek each multiple ratio has a one-word name (*diplasios*, 'double', *triplasios*, 'triple', and so on), and so does each epimoric (*hēmiolios*, 'half-and-whole' for 3:2, *epitritos*,'third-in-addition' for 4:3, and similar formations with *epi-* for the rest), whereas the epimerics do not. (Epimerics have to be specified by phrases that mention the two terms of the ratio separately instead of uniting them, phrases like 'five to three', 'seven to four', etc.) It also fits with the echo of the present lines in the last sentence of the introduction, where it is explicitly said to be numbers that are linked by a single name. Finally, it makes reasonable sense of the argument of the next paragraph (see n. 8 below), where the other readings do not.

[7] That two notes in a concordant relation blend together to form a single, unified percept, whereas those in discordant relation do not, is a commonplace of Greek musicology. The notion appears, for example, at 2.5 Plato *Tim.* 80b, 3.12, 3.13 Arist. *De Sensu* 447a–b, 448a, and is used in the definitions of concord given by many later writers (for references see n. 56 to 12 Arist. Quint. *De Mus.* 10.1–5).

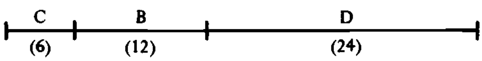
[8] See n. 6 above. I construe the argument as follows. (a) The relation between two notes differing in pitch is to be expressed as a ratio of numbers. (b) Two notes differing in pitch but forming a concord are in such a relation that a single unity is constituted out of them. (c) The ratio between two numbers is expressed as a unity, called by a single name, if and only if they stand to one another in a multiple or an epimoric ratio. Then (d) 'it is to be expected' that the ratio expressing the relation between two notes that form a concord will be either multiple or epimoric. The argument obviously turns on the assumption that the Greek usage is not fortuitous, but reflects an intrinsic feature of the ratios in question. The conclusion stated in (d) is crucial to the arguments of the treatise, and became an entrenched doctrine of 'Pythagorean' harmonics despite its generation of some awkward consequences (see especially 11 Ptol. *Harm.* Book I ch. 6). The present argument clearly does nothing to explain why only some multiple and epimoric ratios, and not others, represent concordant intervals. For other approaches to the question why multiple and epimoric ratios should be considered superior (*kreittōn*, n. 6 above), with some hints at an answer to the further problem just mentioned, see 1.21 Ptol. *Harm.* 30.9ff. with the Appendix to ch. 1, Adrastus ap. Theon Smyrn. 107.23ff: compare 11 Ptol. *Harm.* Book I ch. 7.

150 Proposition 1 *If a multiple interval put together twice makes some interval, this interval too will be multiple*[9]



Let there be an interval BC, and let B be a multiple of C, and let B be to D as is C to B. I assert then that D is a multiple of C. For since B is a multiple of C, C therefore measures B.[10] But B was to D as C was to B, so that C measures D too. Therefore D is a multiple of C.

151 Proposition 2 *If an interval put together twice makes a whole that is multiple, then that interval will also be multiple*



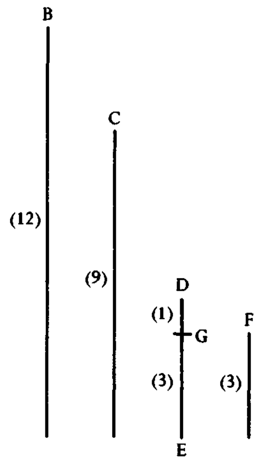
Let there be an interval BC, let B be to D as is C to B, and let D be a multiple of C. I assert that B is also a multiple of C. For since D is a multiple of C, C therefore measures D. But we have learned[11] that where there are numbers in proportion – however many of them – and where the first measures the last, it will also measure those in between. Therefore C measures B, and B is therefore a multiple of C.

[9] The first nine propositions import no musical data, and can be treated as theorems in pure mathematics. Hence the expression 'interval' (*diastēma*) should not here be construed as a specifically musical term: it means 'distance' or 'separation', and applies equally to a relation between two lengths or two numbers. An interval is 'put together twice' when we have three terms, A, B, C, and A:B = B:C. Then the interval AC is the result of moving from A through an interval of a certain ratio to B, and then moving from B through an interval of the same ratio again. It is clear that the author is thinking, for the most part, geometrically, but the diagrams are not certainly those of the original. The manuscript tradition gives several versions, often inaccurately drawn, and the most plausible are usually not those of the oldest MSS. In the interests of clarity I have followed Jan (in *MSG*) in adopting the more helpful versions of the later MSS, but it should not be assumed that they correspond to those drawn by the author, if indeed he gave any.

[10] That is, the length of B can be divided, or 'measured', a whole number of times by that of C; or if B and C are treated as numbers, C is a factor of B.

[11] This refers to Eucl. *Elements* VIII. 7; it is one of several places at which the theorems rest on propositions proved, or techniques established, outside the *Sect. Can.* itself.

152 Proposition 3 *In the case of an epimoric interval, no mean number, neither one nor more than one, will fall within it proportionally*[12]

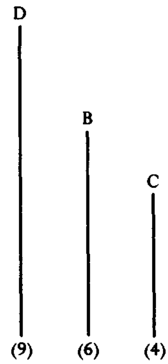


Let BC be an epimoric interval. Let DE and F be the smallest numbers in the same ratio as are B and C. These then are measured only by the unit as a common measure. Take away GE, which is equal to F. Since DE is the epimoric of F, the remainder DG is a common measure of DE and F. DG is therefore a unit. Therefore no mean will fall between DE and F. For the intervening number will be less than DE and greater than F, and will thus divide the unit, which is impossible. Therefore no mean will fall between DE and F.
153 And however many means fall in proportion between the smallest numbers, there will fall in proportion exactly as many between any others which have the same ratio.[13] But none will fall between DE and F; nor will one fall between B and C.

[12] Boethius, *Inst. Mus.* III.11, records a proof of this proposition, similar to the present one, which he attributes to Archytas. The sense of the proposition is that if two terms, B and C, are in a ratio of the form $n+1:n$, then there is no term D between B and C such that $B:D = D:C$, and no series of terms D, E, F, ... N, such that $B:D = D:E = E:F = \dots = N:C$. The proofs are essentially sound, though both versions leave formally necessary steps unstated. They are of obvious importance in harmonics, where all the fundamental concordant and scalar intervals are assigned ratios of epimoric form. The upshot will be that no such interval can be divided into two or more equal intervals (cf. propositions 10, 16, 18). For a lucid discussion of the theorem see Knorr (1975) ch. 7. An informal alternative proof is as follows. Let B and C be the smallest integers in an epimoric ratio. Then $B-C = 1$, and B and C are consecutive integers. Suppose that there is a rational mean proportional, X, between them. Then $B:X = X:C$, and $\frac{B}{X} = \frac{X}{C}$. Hence $BC = X^2$, and X is the product of the square roots of B and C. Then both B and C must have rational square roots, but they are consecutive integers, and no consecutive integers both have rational square roots. Similar reasoning referring to cube roots and so on will rule out greater numbers of proportional divisions.

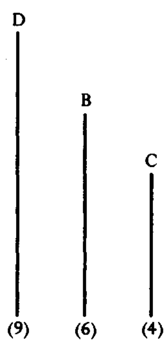
[13] Proved at Eucl. *Elements* VIII.8.

Proposition 4 *If an interval which is not multiple is put together twice, the whole will be neither multiple nor epimoric*



Let BC be an interval which is not multiple, and let B be to D as C is to B. I say that D is neither a multiple nor an epimoric of C. First let D be a multiple of C. Now we have learned that if an interval put together twice makes a whole that is multiple, that interval is also multiple.[14] Then B will be a multiple of C:
154 but it was not. Hence it is impossible for D to be a multiple of C. Nor is it an epimoric: for within an epimoric interval there falls no mean in proportion.[15] But B falls within DC. Therefore it is impossible for D to be either a multiple or an epimoric of C.

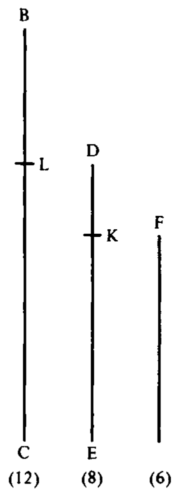
Proposition 5 *If an interval put together twice does not make a whole that is multiple, that interval itself will not be multiple either*



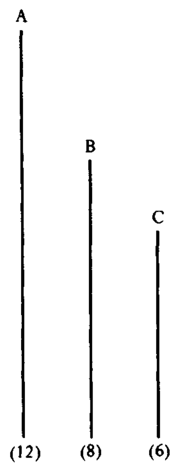
Let BC be an interval; let B be to D as C is to B; and let D not be a multiple of C. I say that B will not be a multiple of C either. For if B is a multiple of C, D will therefore be a multiple of C.[16] But it is not. Therefore B will not be a multiple of C.

[14] See proposition 2.
[15] See proposition 3.
[16] The reasoning is very abbreviated, but perfectly cogent given the proof of proposition 1.

Proposition 6 *The duple interval is composed of the two greatest epimoric intervals, the hemiolic and the epitritic*[17]



Let BC be the hemiolic of DE, and let DE be the epitritic of F. I say that BC is double F. I take away EK, equal to
155 F, and CL, equal to DE. Then since BC is the hemiolic of DE, BL is a third part of BC, and half of DE. Again, since DE is the epitritic of F, DK is a fourth part of DE, and a third part of F. Then since DK is a fourth part of DE, and BL is half of DE, DK will therefore be a half of BL. Now BL was a third part of BC: therefore DK is a sixth part of BC. But DK was a third part of F: therefore BC is double F.



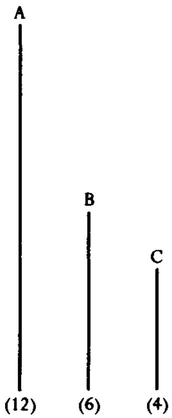
Alternatively: let A be the hemiolic of B, and let B be the epitritic of C. I say that A is double C. Since A is the hemiolic of B, A contains B and half of B. Then two A's are equal to three B's. Again, since B is the epitritic of C, B contains C and a third of C. Therefore three B's are equal to four C's. Now three B's are equal to two A's.[18] Therefore two A's are equal to four C's. Therefore A is equal to two C's: therefore A is double C.[19]

[17] Throughout the book I have retained terms such as 'hemiolic' and 'epitritic' (for the ratios 3:2 and 4:3) wherever possible, partly in order to keep the 'single word' formulations mentioned in the introductory passage of this treatise, and partly because in some of their occurrences a representation in terms of a pair of numbers would be quite inappropriate. The sense of the present proposition, then, is that an interval in the ratio 2:1 results from the 'putting together' of a 3:2 interval and a 4:3 interval (for the notion of 'putting together' see n. 9 above). The first of the two proofs that follow is omitted in Porphyry's version. The first is conceived geometrically, the second arithmetically.

[18] Porphyry omits this sentence and the next.

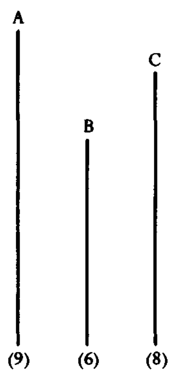
[19] Porphyry's text (*Comm.* 100.26ff.) here adds a further proposition. 'No multiple ratio is composed of epimoric ratios, except the duple ratio alone. For if it were possible, let

156 Proposition 7 *From the duple interval and the hemiolic, a triple interval is generated*



Let A be double B, and let B be the hemiolic of C. I say that A is triple C. Since A is double B, A is therefore equal to two B's. Again, since B is the hemiolic of C, B therefore contains C and half of C. Hence two B's are equal to three C's. But two B's are equal to A. Therefore A is equal to three C's: therefore A is triple C.

Proposition 8 *If from a hemiolic interval an epitritic interval is subtracted, the remainder left is epogdoic*[20]



Let A be the hemiolic of B, and let C be the epitritic of B. I say that A is the epogdoic of C. Since A is the hemiolic of B, A therefore contains B and a half of B. Therefore eight A's are equal to twelve B's. Again, since C is the epitritic of B, C therefore contains B and a third of B.
157 Therefore nine C's are equal to twelve B's. But twelve B's are equal to eight A's, and therefore eight A's are equal to nine C's. A is therefore equal to C and an eighth of C, and A is therefore the epogdoic of C.

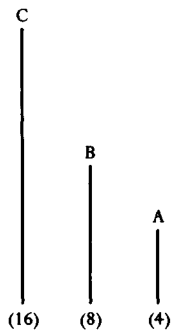
another multiple ratio AC consist of the two epimoric ratios AB and BC. Let D be the hemiolic of E, and E the epitritic of Z: then D is double Z. And since the hemiolic ratio is the greatest of the epimorics, and the epitritic is the second, either one of the ratios DE and EZ is equal to one of AB and BC, or else one or both of the former is greater than one or both of the latter. But in either case, the ratio of D to Z is greater than that of A to C. But this is impossible, for the duple is the smallest of the multiple ratios. Then no multiple ratio consists of two epimoric ratios, except the duple ratio alone.' Almost certainly the proposition should not stand in the text. No other proposition consistently treats 'ratios' as its subject: they all speak of 'intervals'. Further, in proposition 12 where the theorem might conveniently have been used, it is not referred to, nor is it drawn on elsewhere in the treatise.

[20] That is, if an interval of ratio 4:3 is taken from one of ratio 3:2, the remainder is in the ratio 9:8.

Proposition 9 *Six epogdoic intervals are greater than one duple interval*

Let A be one number. Let B be the epogdoic of A, let C be the epogdoic of B, let D be the epogdoic of C, let E be the epogdoic of D, let F be the epogdoic of E, and let G be the epogdoic of F. I say that G is more than double A. Since we have learned how to find seven numbers that are epogdoics of one another,[21] let the numbers A, B, C, D, E, F, G have been found. A is 262,144,
158 B is 294,912, C is 331,776, D is 373,248, E is 419,904, F is 472,392, G is 531,441; and G is more than double A.[22]

Proposition 10 *The octave interval is multiple*



Let A be *nētē hyperbolaiōn*, let B be *mesē* and let C be *proslambanomenos*. Then the interval AC, being a double octave, is concordant.[23] It is therefore either epimoric or multiple.[24] It is not epimoric, since no mean falls proportionally within an epimoric interval.[25] Therefore it is multiple. Thus since the two equal intervals AB and BC put together make a whole that is multiple, AB is therefore multiple too.[26]

[21] See Eucl. *Elements* VII.2.

[22] Each number must be such that one ninth of it is a whole number. The greatest is 9^6, the next $9^6–9^5$ (that is, 8×9^5), and so on. This completes the purely mathematical theorems. The author now proceeds to apply their conclusions to the data of harmonics, to show which musical intervals correspond to which ratios, and what mathematical properties they must therefore possess.

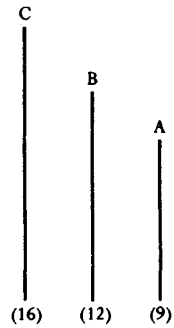
[23] *Nētē hyperbolaiōn* is the highest note of the two-octave system within which Greek harmonic analysis was standardly set. *Mesē* is an octave lower, and an octave above *proslambanomenos*, the lowest note of the system. The author assumes familiarity with the named notes; he assumes that we know in advance of mathematical analysis the sizes of the intervals between them (in terms of octaves, fifths, fourths, tones, etc.); and he assumes that we know which of these intervals are concordant and which are not.

[24] From the principle established at the end of the introduction.

[25] From proposition 3. The note *mesē* divides the double-octave interval in half. That is, the relation of *proslambanomenos* to *mesē* is the same as that of *mesē* to *nētē*. What proposition 3 establishes is that no note can divide an epimoric interval in that way.

[26] From proposition 2.

Proposition 11 *The interval of the fourth and that of the fifth are each epimoric*



159 Let A be *nētē synēmmenōn*, let B be *mesē*, and let C be *hypatē mesōn*.[27] The interval AC, being a double fourth, is therefore discordant: it is therefore not multiple.[28] Thus since the two equal intervals AB and BC when put together make a whole which is not multiple, neither is AB multiple.[29] And it is concordant: therefore it is epimoric.[30] The same demonstration applies also to the fifth.

Proposition 12 *The octave interval is duple*

We have shown that it is multiple: it is thus either duple or greater than duple. But since we showed that the duple interval is composed of the two greatest epimoric intervals,[31] it follows that if the octave is greater than duple it will not

[27] *Nētē synēmmenōn* is a fourth above *mesē*, which is in turn a fourth above *hypatē mesōn*. On *nētē synēmmenōn* see n. 77 below.

[28] That it is discordant is a fact taken from musical experience. The claim that it is therefore not multiple cannot be justified on grounds that the author has established. The principle recommended in the introduction is that all concords are either multiple or epimoric. It does not follow that no discord is multiple, any more than it follows that none are epimoric (and within the usual system of ratios, many are, for example, the 9:8 tone). In fact some discords are of multiple ratio. The smallest is the interval corresponding to the ratio 5:1, which lies between the double octave (4:1) and the double octave plus a fourth (16:3, cf. n. 38 below), and cannot therefore be concordant in the Greek sense. It might be argued that the author is interested only in intervals within the double octave, and there all multiple intervals are indeed concordant. But that interpretation robs his arguments of the mathematical generality they evidently seek. Worse, it begs the question. The fact that 2:1, 3:1 and 4:1 are the ratios of octave, octave plus fifth, and double octave respectively has not yet been established, and the proofs (in proposition 12) depend on the conclusion of the present argument. We cannot use the thesis that all multiples within the double octave are concordant as an assumption in the proof of a proposition needed to establish that very thesis. The author's procedure cannot be rescued: there is no principle to which he is entitled that would enable him to construct an adequate proof of proposition 11, and the proposition is essential to what follows. (The conclusion of the present argument is of course true, but the point is that it has not been demonstrated, within the framework of mathematical proofs that the author is using.)

[29] From proposition 5.

[30] On the introduction's principle that all concords are either multiple or epimoric.

[31] In proposition 6.

be made up of just two epimoric intervals, but of more. But it is made up of two concordant intervals, the fifth and the fourth.[32] Therefore the octave will not be greater than duple: therefore it is duple.[33]

But since the octave is duple, and the duple is made up of the two greatest epimorics, it follows that the octave is made up of the hemiolic and the epitritic,
160 since these are the greatest.[34] But it is made up of the fifth and the fourth, and these are epimoric. The fifth, therefore, since it is greater, must be hemiolic, and the fourth epitritic.[35]

It is clear, then, that the octave and a fifth is a triple interval. For we showed that the triple interval is generated from a duple and a hemiolic interval,[36] so that the octave and a fifth is also a triple interval.

The double octave is a quadruple interval.[37]

We have demonstrated, therefore, for each of the concords, in what ratios their bounding notes stand to one another.[38]

Proposition 13 *It remains to consider the interval of a tone, to show that it is epogdoic*[39]

We have learned that if an epitritic interval is subtracted from a hemiolic interval, the remainder left is epogdoic.[40] And if the fourth is taken from the fifth, the remainder is the interval of a tone.[41] Therefore the interval of a tone is epogdoic.[42]

Proposition 14 *The octave is less than six tones*

161 It has been shown that the octave is duple, and that the tone is epogdoic.[43] Six epogdoic intervals are greater than a duple interval.[44] Therefore the octave is less than six tones.

[32] This again is an assumption drawn from musical experience.

[33] The reasoning is compressed: it hangs on the thesis that the fifth and the fourth are not merely concordant, but epimoric. (If they were multiple, the octave would not be in duple ratio.) That they are epimoric is claimed in proposition 11, whose proof, I have suggested, is inadequate.

[34] Porphyry's version (*Comm.* 102.30) omits the clause from 'it follows' to 'the greatest'.

[35] That is, they are 3:2 and 4:3 respectively. The argument again turns on the thesis of proposition 11.

[36] In proposition 7.

[37] This sentence is missing from Porphyry's version (*Comm.* 103.4–5).

[38] No mention is made of the interval of an octave plus a fourth, which seems to be a concord, but will have the embarrassing ratio 8:3, neither multiple nor epimoric. See 11 Ptol. *Harm.* Book 1 ch. 5.

[39] That is, it is in the 'eighth-in-addition' ratio, 9:8.

[40] See proposition 8.

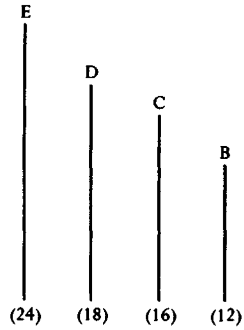
[41] This may be considered either a datum of experience, or a matter of definition (see, for example, 7 Aristox. *El. Harm.* 21.22–3, 45.34–46.1).

[42] In view of the ratios assigned to the fifth and the fourth in proposition 12.

[43] In propositions 12 and 13. Aristoxenian theory assumes that six tones are exactly equal to an octave.

[44] This is shown in proposition 9.

Proposition 15 *The fourth is less than two and a half tones, and the fifth is less than three and a half tones*[45]



Let B be *nētē diezeugmenōn*, let C be *paramesē*, let D be *mesē*, and let E be *hypatē mesōn*.[46] Then the interval CD is a tone,[47] and BE, which is an octave, is less than six tones.[48] Therefore the remainders, that is BC and DE, which are equal,[49] are less than five tones. Hence the interval in BC,[50] which is a fourth, is less than two and a half tones, and BD, which is a fifth, less than three and a half tones.

Proposition 16 *The tone will not be divided into two or more equal intervals*[51]

It has been shown that it is epimoric.[52] Within an epimoric interval there falls neither one nor more than one mean in proportion.[53] Therefore the tone will not be divided into equal intervals.

[45] Another proposition that challenges Aristoxenus: see particularly 7 Aristox. *El. Harm.* 56.13ff. The contrast between this passage and that one encapsulates neatly some important aspects of the difference between the assumptions and procedures, as well as the doctrines, of the two schools.

[46] *Nētē diezeugmenōn* is a fourth above *paramesē*; *paramesē* is a tone above *mesē*; *hypatē mesōn* is a fourth below *mesē*, and an octave below *nētē diezeugmenōn*. The fifths in this framework are those from *hypatē* up to *paramesē* and from *mesē* up to *nētē*.

[47] This is either a straightforward musical datum, or an inference from the fact that the interval *paramesē–mesē* is the difference between a fifth and a fourth (see proposition 13).

[48] That BE is an octave is a musical datum. That it is less than six tones is shown in proposition 14.

[49] This too is based on a musical datum (that each is a fourth), not something mathematically inferred.

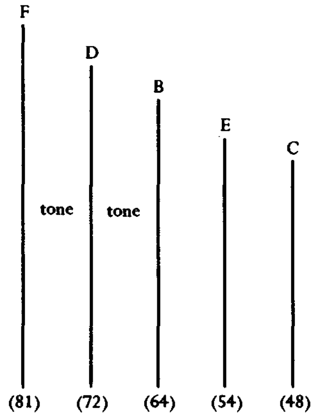
[50] The words 'are less than...interval in BC' are omitted in Porphyry's text (*Comm.* 103.20–1).

[51] This proposition, like propositions 14 and 15, is flatly incompatible with Aristoxenus' procedure, which divides tones into halves, thirds, quarters and so on.

[52] In proposition 13 its ratio was shown to be 9:8, which is epimoric.

[53] See proposition 3.

162 Proposition 17 *The* paranētai *and* lichanoi *will be found by means of concords, as follows*[54]



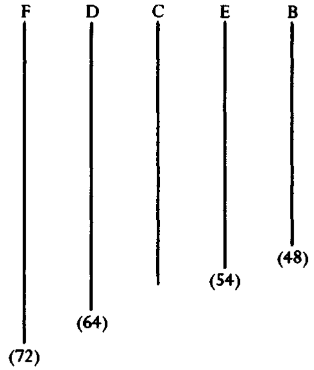
Let B be *mesē*. Let a fourth be constructed upwards to C, and from C let a fifth be constructed downwards to D. BD is therefore a tone.[55] Again, from D let a fourth be constructed upwards to E, and from E let a fifth be constructed downwards to F. FD is therefore a tone. Therefore FB is a ditone, and F is therefore *lichanos*.[56] The *paranētai* will also be found in the same way.

[54] Porphyry omits this and the remaining propositions. For Aristoxenus' version of the 'method of concordance' see 7 *El. Harm.* 55.3ff. It was undoubtedly a practical musician's method of tuning, as well as a theorist's device. The *paranētai* and *lichanoi* are the second-highest notes of their respective tetrachords. The construction given here assumes that each stands at a ditone below the top note of its tetrachord, which is true (according to, for example, 7 Aristox. *El. Harm.* 22.27–23.22, 50.22–25) of the enharmonic genus, but of no other. In the present author's analysis, the interval is a true ditone of $9:8 \times 9:8 = 81:64$, unlike that of, for example, Archytas in 1.21 Ptol. *Harm.* 30.9ff., which is $5:4 = 80:64$. The pair of intervals between *paranētē* or *lichanos* and the bottom of the tetrachord (the *pyknon*) will thus amount to the *leimma* of 256:243, which appears in the analyses of diatonic systems implied by Philolaus (1.12 frag. 6), given by Plato (2.3 *Tim.* 35b–36b) and again implied in proposition 20 below. Only propositions 17 and 18 are concerned with the enharmonic. For a suggestion about their role in the *Sect. Can.* see n. 57 below. Other 'Pythagorean' authors who give a similar account of the enharmonic include 9.5 Thrasyllus ap. Theon Smyrn. 93, 10 Nicomachus *Ench.* ch. 12.

[55] On the assumption articulated in proposition 13.

[56] On the assumption that *lichanos* is two tones below *mesē*: see n. 54 above.

Proposition 18 *The* parhypatai *and* tritai *do not divide the* pyknon *into equal intervals*[57]



163 Let B be *mesē*, C *lichanos*, and D *hypatē*. Let a fifth be constructed downwards from B to F. FD is therefore a tone.[58] Then let a fourth be constructed upwards from F to E. The interval FD is therefore a tone, and so is CE.[59] Let DC be added to both of them. FC is therefore equal to DE.[60] But FE is a fourth, and therefore no mean falls proportionally within FE, since the interval is

[57] The *pyknon* is what remains in the tetrachord below the highest interval, where this remainder adds up to less than half the total interval of the fourth (see 7 Aristox. *El. Harm.* 24.11–14, 48.26–31, 50.15–19). There is a *pyknon* only in a chromatic or an enharmonic tetrachord, since what is left below the highest interval in diatonic is always equal to or greater than half the span of the fourth (see 7 Aristox. *El. Harm.* 51.19–21). In the present proposition, like proposition 17, the author is dealing with the enharmonic. The *pyknon* is divided into two intervals by an intermediate note (*parhypatē* in the lower tetrachords, *tritē* in those above *mesē*). In Aristoxenian theory the two are commonly equal, but the *Sectio* argues here that this is impossible. It has sometimes been suggested that propositions 17 and 18 cannot originally have been part of the same treatise as propositions 19 and 20, since (though neither mentions a genus by name) the former are concerned with the enharmonic, the latter with the diatonic. More probably, I think, the point is that the diatonic analysed in propositions 19–20 is the canonical form of scale in the Pythagorean tradition transmitted through Philolaus and Plato, but that theorists of a more empirical tradition had concentrated on the enharmonic (see 7 Aristox. *El. Harm.* 2.7ff.), and their analyses, like those of Aristoxenus, had divided the *pyknon* into equal quarter-tones. The task of proposition 18, then, like that of proposition 16, is mainly polemical rather than constructive; it shows that the work of a rival school had been based on a mistake. Proposition 17 is required to introduce this argument. Both are incidental to the main project, which culminates in propositions 19–20, but that is no reason for rejecting them as spurious.

[58] The note labelled F is involved only as part of the theoretical construction. No note stands at a tone below *hypatē* in the standard forms of the enharmonic (though there is one there in diatonic). But it is possible that the author has in mind the note *hyperhypatē*, which may have had a special role in older systems. See the Appendix to ch. 1, and cf. n. 65 below.

[59] In editing the treatise for *MSG*, Jan emended the MSS text here and in several places below, but quite unnecessarily. At this point he reads BE for CE. BE is indeed a tone, but so is CE if the genus is enharmonic, as in proposition 17, and CB is therefore a ditone.

[60] Jan reads FE for FC and DB for DE.

epimoric.[61] Now DF is equal to CE:[62] hence no mean falls proportionally within DC, which is the interval from *hypatē* to *lichanos*.[63] Therefore the *parhypatē* does not divide the *pyknon* into equal intervals.[64] And in the same way, neither does the *tritē*.

Proposition 19 *To mark out the* kanōn *according to the so-called immutable* systēma[65]

Let there be a length of the *kanōn* which is also the length AB of the string, and let it be divided into four equal parts, at C, D and E. Therefore BA, being the lowest, will be the bass note.[66] Now this AB is the epitritic of CB, so that CB
164 will be concordant with AB at the fourth above it.[67] And AB is *proslambanomenos*: therefore CB is *diatonos hypatōn*.[68] Again, since AB is double BD, BD will be concordant with AB at the octave, and BD will be *mesē*. Again, since AB is quadruple EB, EB will be *nētē hyperbolaiōn*.

I cut CB in half at F. CB will be double FB, so that CB is concordant with

[61] That the fourth is epimoric is argued in proposition 11. The proof that there is no mean proportional in an epimoric interval is given in proposition 3.

[62] Jan emends to 'DB is equal to FE'.

[63] That is, it is the interval comprising the *pyknon*, *hypatē* being the lowest note of a tetrachord and *lichanos* the lower boundary of its upper interval.

[64] The gist of the argument is that if there were a note that divided the *pyknon* DC equally, it would also divide equally the fourth FE. But since a fourth is epimoric, that is impossible.

[65] The heading poses problems. In later usage (e.g., 9.5 Thrasyllus ap. Theon Smyrn. 92–3) the expression 'perfect immutable *systēma*' (where 'immutable' renders *ametabolon*) referred to the combination of the LPS with the GPS. If a comparable sense were intended here, the heading would have to refer to proposition 20 as well as to proposition 19, and its promise would not be fulfilled, since proposition 20 does not complete the LPS. More probably 'immutable' is contrasted with 'moveable' in the heading of proposition 20, and the system referred to here is that of the 'fixed notes', which do not vary from genus to genus. But in the present author's treatment (as in that of Thrasyllus, though his procedure is different in some respects: see 9.4 Theon Smyrn. 87.7ff.), this first phase of the division of the *kanōn* includes the locating of one specifically diatonic note, *diatonos hypatōn* (otherwise known as diatonic *lichanos hypatōn*). This may be merely a matter of mathematical convenience: on the other hand it might add weight to the suggestion that a note in this position, sometimes called *hyperhypatē* (as by Thrasyllus at 9.4 Theon Smyrn. 88.18), had a structural role not limited to the diatonic genus, at least in some older systems. See n. 58 above and n. 71 below. The *kanōn* is a ruler or measuring rod placed lengthwise under the string of the monochord. For a careful description of the instrument see 11 Ptol. *Harm.* Book I ch. 8, cf. Book II chs. 12 and 13, Book III chs. 1–2.

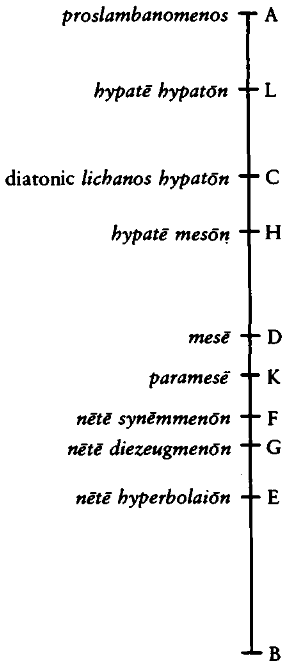
[66] 'Bass' translates *bombyx*, a word of various meanings related to depth of pitch. Thus for Pollux the *bombyx aulos* is a deep-pitched instrument (*Onomastikon* IV.82). More closely related to the present usage is Arist. *Metaph.* 1093b, where the *bombyx* note of the *aulos* is its lowest note. For fuller discussion see *GMW* vol. 1, p. 187, n.4.

[67] The associations of epitritic ratio (4:3) with the fourth, of hemiolic ratio (3:2) with the fifth, duple ratio (2:1) with the octave and quadruple ratio (4:1) with the double octave are all argued for in proposition 12. No other ratios are involved in the present phase of the division.

[68] It will be the note a tone below *hypatē mesōn*, and not standardly a fixed note; see n. 65 above.

FB at the octave: hence FB is *nētē synēmmenōn*.[69] From DB I subtracted DG, a third part of DB. DB will be the hemiolic of GB, so that DB will be concordant with GB at the fifth. Therefore GB will be *nētē diezeugmenōn*.

I then constructed GH, equal to GB, so that HB will be concordant with GB at the octave, making HB *hypatē mesōn*. From HB I subtracted HK, a third part of HB. HB will be the hemiolic of KB, so that KB is *paramesē*. I marked off LK, equal to KB, and LB will be the lower *hypatē*.[70] Thus we shall have found on the *kanōn* all the fixed notes of the immutable *systēma*.[71]



Proposition 20 *It remains to find the moveable notes*[72]

I divided EB into eight parts, and I constructed EM, equal to one of the parts,

[69] On the oddity of the reference to this note see n. 77 below.

[70] That is, *hypatē hypatōn*, a tone above *proslambanomenos*.

[71] See again n. 65 above, for the point that *diatonos hypatōn* is not one of the normal 'fixed notes'. This deserves emphasis, since it is odd to construct the fundamental fixed notes through manoeuvres that involve a moveable one, and in the present construction *diatonos hypatōn* is essential from the start. The word 'fixed' in this sentence is in fact an addition to the text, made by Jan. But even without it the issue remains. The present proposition would claim to have established all the notes of the 'immutable' *systēma*, which would have to mean 'the system of fixed notes'. It would again seem to put *diatonos hypatōn* on an equal footing with the others identified here, as belonging to a more fundamental level than those of proposition 20, which are explicitly called 'moveable'.

[72] The construction locates these notes in their diatonic positions, though their description as 'moveable' indicates the author's recognition that others are also possible (cf. propositions 17 and 18).

so that MB is the epogdoic of EB.[73] Next, I divided MB into eight parts, and constructed NM, equal to one of these parts. Thus NB is a tone lower than BM, and MB is a tone lower than EB, so that NB will be *tritē hyperbolaiōn*, and MB will be *diatonos hyperbolaiōn*.[74] I took a third part of NB and constructed NX, so that XB is the epitritic of NB, and is concordant with it at the fourth below: XB is *tritē diezeugmenōn*. Again, I took a half of XB and constructed XO, so that OB is concordant at the fifth with XB:[75] therefore OB will be *parhypatē mesōn*. And I constructed OP, equal to XO, so that PB becomes *parhypatē hypatōn*.[76] Finally I found CR, a fourth part of BC, so that RB becomes *diatonos mesōn*.[77]

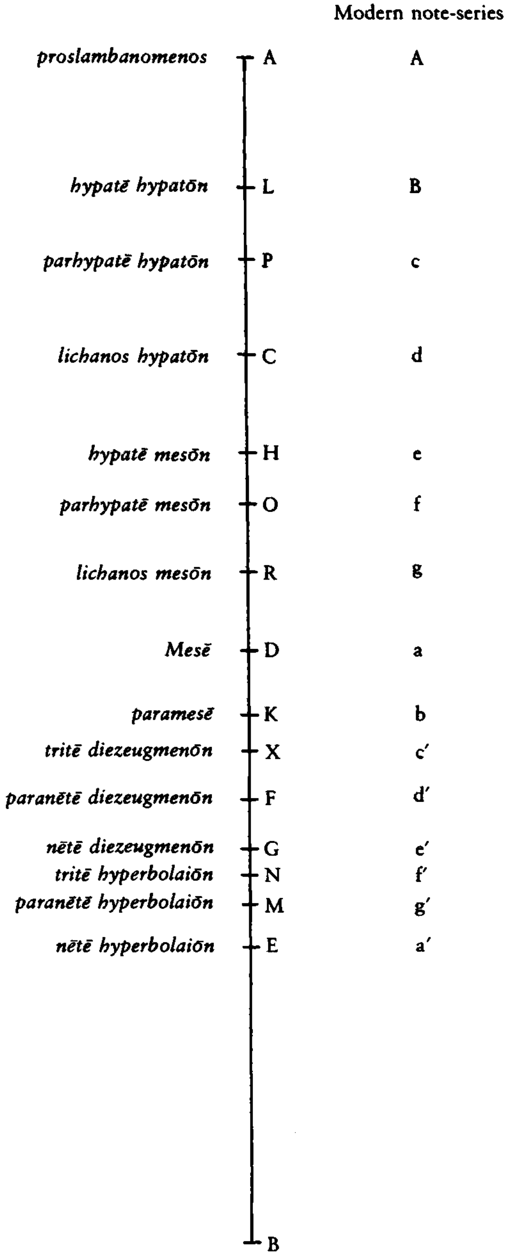
[73] It is interesting that the author (like Plato at 2.3 *Tim.* 36b) adopts this method of constructing a tone downwards, when it was open to him to do it without recourse to divisions into eight, through moves of a fifth downwards and a fourth upwards, since the fifth exceeds the fourth by a tone (proposition 13, cf. proposition 17). That the tone is epogdoic (in the ratio 9:8) is established in proposition 13.

[74] Otherwise known as diatonic *paranētē hyperbolaiōn*. The location of *tritē* is also specific to the diatonic genus, as are those of the other notes mentioned below.

[75] Because OB is XB plus half XB, and therefore stands to XB in the ratio 3:2.

[76] Because PB is OP plus XO plus XB, and each of OP and XO is equal to half XB. Then PB is double XB, and *parhypatē hypatōn* (PB) is an octave below *tritē diezeugmenōn* (XB).

[77] That is, diatonic *lichanos mesōn*. BC is *diatonos hypatōn* (see proposition 19): it stands to RB in the ratio 4:3, and hence RB is a fourth higher. It is to be noticed that despite the reference to *nētē synēmmenōn* in proposition 19, the moveable notes of the tetrachord *synēmmenōn* are not mentioned in proposition 20. Now diatonic *paranētē synēmmenōn* is at the same pitch as *tritē diezeugmenōn*, just as diatonic *paranētē diezeugmenōn* is at the same pitch as *nētē synēmmenōn*; and this might account for the fact that neither the *paranētē synēmmenōn* nor the *paranētē diezeugmenōn* is mentioned as such in propositions 19–20. But *tritē synēmmenōn*, two tones below *nētē synēmmenōn*, is not at the same pitch as any note in the tetrachord *diezeugmenōn*, and its location is not established here at all. In the accompanying diagram (which appears in various forms in the MSS, but has no great authority) the reference to the tetrachord *synēmmenōn* has wholly disappeared, the place of *nētē synēmmenōn* being taken by its pitch-equivalent *paranētē diezeugmenōn*. But the MSS are unanimous in naming the former in proposition 19 (it is mentioned also in proposition 11), and unless the text is incomplete the author must be convicted of a certain carelessness. Either the tetrachord *synēmmenōn* is within the scope of his analysis, in which case it should have been completed, or it is not, in which case the references to *nētē synemmēnōn* are anomalous. It might be argued that the author did not wish to construe this tetrachord as part of the fundamental system (cf. 11 Ptol. *Harm.* Book II ch. 6), but found it necessary to construct the pitch corresponding to *paranētē diezeugmenōn* as part of the set of fixed notes, which it can be only if it is treated as *nētē synēmmenōn*. But this suggestion fails, since once that pitch is established in proposition 19, it is not used at all in the construction of the others. It could perfectly well have been omitted there, and constructed in proposition 20 as a moveable note.


Modern note-series
proslambanomenos A A
hypatē hypatōn L B
parhypatē hypatōn P c
lichanos hypatōn C d
hypatē mesōn H e
parhypatē mesōn O f
lichanos mesōn R g
Mesē D a
paramesē K b
tritē diezeugmenōn X c′
paranētē diezeugmenōn F d′
nētē diezeugmenōn G e′
tritē hyperbolaiōn N f′
paranētē hyperbolaiōn M g′
nētē hyperbolaiōn E a′
B

CHAPTER 9

Minor authors quoted by Theon and Porphyry

Virtually nothing remains of works in the musical sciences written between the time of Euclid and the end of the first century B.C. We know that the subjects continued to be studied, and a few individual contributions can be reconstructed from references in Ptolemy and later writers, but for the most part the authors of the early Christian era say little directly about their immediate predecessors, leaving us to infer the character of the contents of the gap from that of their own work. Even from the first century A.D. no complete treatises survive (unless 10 Nicomachus *Enchiridion* belongs to its last years), but substantial passages are preserved as quotations in later essays. In this chapter I have put together a selection of them, some from writers of the first century, others a little later; in a few cases the dates are quite uncertain.

The excerpts are drawn from two authors, Theon of Smyrna and Porphyrius of Tyre, more commonly known as Porphyry. Neither made major contributions to the subject in his own right, though Porphyry certainly had ideas of his own to convey, and his intellectual powers are much more impressive than Theon's. For a modern student the main value of both treatises lies in their quotations and paraphrases of previous writers.

Part 1 Passages from Theon of Smyrna

Theon wrote during the second century A.D. He devoted himself to the works of Plato, and is known to have written scholarly essays about him, including a commentary on the *Republic*. Nothing survives from his pen except two long sections of a book called *Mathematics Useful for Reading Plato*.

It is not a commentary on specific passages of the dialogues, but a fairly elementary introduction to certain aspects of mathematics. It was designed (Theon 1.15ff.) to cover all five of the mathematical disciplines discussed by Plato in *Republic* Book VII, which are number theory, plane geometry, solid geometry, astronomy and harmonics (see the introduction to chapter 2). The sections on the two kinds of geometry are lost. Theon explains in 16.24ff. his intention to take the subjects in a different order from Plato's, though the latter is the more 'natural'. Number theory is discussed first. But the appropriate sort of harmonics is the study of '*harmonia* in the cosmos', and this presupposes a grasp of 'music in numbers'. Since 'music in numbers' is close to the study of numbers as such, it will be taken second, after number theory. The first surviving section of his work pursues this two-stage programme. The study of '*harmonia* in the cosmos' does not appear till later, as an aspect of Theon's account of astronomy (it is not included in this volume). There is an additional complication. We are told that another kind of music, 'music in instruments', is not strictly a part of the subject in hand (16.26ff.). But some understanding of it is useful as a preliminary to the study of 'music in numbers' (cf. 47.6–8). Thus 'music in instruments' is the topic considered from 47.18 to 72.20, directly after the passage on number theory, and the promised account of 'music in numbers' follows.

In the passages translated below, Theon quotes material from two authors, Thrasyllus and Adrastus. Thrasyllus was an astrologer, a serious and well educated student of the subject, by no means a charlatan. He was a close adviser to the emperor

Tiberius from about 5 B.C. until his death in A.D. 36. Like Theon, he paid much attention to the works of Plato, and wrote an essay on the principles of Pythagorean and Platonist philosophy. His work on music, quoted here, may well have been related to that project. Of Adrastus rather little is known, and his writings, like those of Thrasyllus, survive only as fragments quoted by others. He was born in Aphrodisias, probably towards the end of the first century A.D. Theon and Porphyry call him 'the Peripatetic' because of his studies of Aristotle. But he also wrote on Plato, and Porphyry describes an excerpt which is also quoted by Theon as coming from a commentary on the *Timaeus*. This is probably the source of all Theon's citations, which include passages on astronomy and on arithmetic as well as those translated here.

The quotations from Thrasyllus (9.1, 9.4–9.5) need little introduction. The first presents some of the basic terms and concepts of harmonic analysis, but its grasp of the subject is feeble, as Theon himself implies at the beginning of 9.2. The others give Thrasyllus' procedure for the division of the *kanōn*, and have genuine points of interest, especially when compared with 8 Eucl. *Sect. Can.* propositions 19–20. The division is mentioned in a caustic aside by Nicomachus at 10 *Ench.* 260.16.

Not everything presented here as a quotation from Adrastus is certainly from his hand. Theon is careless about indicating beginnings and endings of quotations, and some parts of 9.2–9.3 may be his own, or derived from other sources. It is not likely, however, that he contributed much independently to his section on 'music in instruments', from which all this material is taken. The issue is made all the harder to resolve by the tattered and corrupt condition of the text.

It is plain that Theon has not preserved the musicological section of Adrastus' *Commentary* intact, and the overall plan of the whole is not immediately clear from the excerpts we have. The key lies, I think, in a statement made by Adrastus with some emphasis at 61.20ff.: 'When these instruments designed for the discovery of the concords have been previously prepared in accordance with the ratios, perception agrees with their testimony, while when perception has been taken first, reason is in attunement with it.' The thesis is echoed with much greater sophistication by Ptolemy (11 *Harm.* Book I chs. 1–2: cf., for example, the opening paragraph of Book II), but it is not a routine commonplace. As is shown by 9.10–9.14 (Ptolemaïs and Didymus ap. Porph. *Comm.* 22.22ff.), it was generally supposed that the 'rational' procedures of the Pythagoreans and the 'perception-based' approach of theorists closer to Aristoxenus led to different and incompatible results.

Adrastus' intention is not precisely to reconcile the two traditions. There are real conflicts between them, to which he draws attention, and in these cases he sides with the Pythagoreans and Platonists. But he evidently recognises that from a musical point of view the Aristoxenian analyses are far richer, and he draws on them extensively. His project seems to be that of transposing the Aristoxenian accounts of certain elementary musical structures into Pythagorean terms, with suitable 'corrections' where necessary. The repertoire of structures he considers is relatively small, and of the possible Pythagorean representations of them he mentions none but those consistent with Plato's scalar divisions in the *Timaeus* (2.3 *Tim.* 35b–36b). The same is true of 10 Nicomachus *Enchiridion*. Neither discusses any aspect of harmonics beyond those needed to provide a bridge between the perceptible music anatomised by Aristoxenus, and the music of numbers (underlying the harmony of the cosmos) to which Plato's *Timaeus* was thought to hold the key. No doubt this is why Adrastus draws attention at the opening of 9.3 to the different purposes for which Aristoxenus' and Plato's investigations were undertaken. He is at pains to show that even on its own ground, where the phenomena given to perception are considered by themselves, the Aristoxenian system is wrong where it conflicts with that of Plato and the Pythagoreans (see especially 9.3 Theon 71–2).

His project presents a serious difficulty, for it is not obvious that Aristoxenus' account of what is perceived and Pythagorean calculations about the ratios of physical variables are even attempts at describing the same thing. Adrastus makes a valiant attempt to resolve the problem. Like Theophrastus, he seems to have felt that the crucial weakness of Aristoxenian harmonics lies in its conception of a note as a mere 'point' on a continuum of pitch, a conception which apparently gives no distinguishing features to the note itself. (See 6 Theophrastus ap. Porph. *Comm.* 64.24–65.9; the criticism is more lucidly set out by Ptolemy, 11 *Harm.* Book I ch. 9.) The entities that the Pythagoreans describe, on the other hand, have distinct quantifiable features, but in so far as they are treated, for example, as movements of the air, they seem not to be notes at all, or even sounds, conceived in their character as things heard, but only some of their causes. Like the Pythagoreans and Ptolemy, Adrastus emphasises the role of the note, rather than the interval, as the basic element of melody (9.2 Theon 49–50). But in a difficult and original argument designed to show that it is to lower notes that greater numbers should be assigned (9.3 Theon 65–6), he tries to develop the notion of perceived pitch as a quantitative characteristic of sound itself, belonging to the note as such, and distinct from the properties of the non-auditory physical events that underlie it. Armed with this conception of perceived pitch as itself varying quantitatively (in sharp contrast to the views of Theophrastus, and of 9.9 Panaetius ap. Porph. *Comm.* 65.21ff.), he can proceed with his project of 'translation', while still using Pythagorean arguments against Aristoxenus wherever the two systems are at odds.

This reconstruction of Adrastus' intentions gives him an intelligible programme. I should make it clear that the reconstruction is speculative, and the detailed study that would be necessary to establish its credentials cannot be given here. At a more general level one would need to investigate the role of such a project in a commentary on the *Timaeus*. Some progress may be made by comparing the work of Adrastus with that of Nicomachus. The scope of their reflections is similar, in so far as each restricts the subject matter of his harmonics to what are, from an Aristoxenian point of view, the same rather elementary structures. Both seek to redescribe these structures in Pythagorean terms, on the basis of the quantitative findings of physical acoustics. Neither even hints at the existence of controversies among mathematical theorists about the correct way of quantifying the tetrachordal divisions in each genus; the attempts of Archytas, Eratosthenes and Didymus to give better approximations to systems in actual use are simply ignored. (For these divisions see 11 Ptol. *Harm.* Book II ch. 14.) The framework in which Adrastus' and Nicomachus' accounts are given, and that assumed by Thrasyllus' division of the *kanōn* too, is consistently that of the 'Pythagorean diatonic' excogitated on theoretical grounds by Philolaus, Plato and Euclid, together with representations of the chromatic and the enharmonic that are directly derived from it. Through this simplification of the data they are able to connect their musical and mathematical metaphysic with the perceptible music whose organisation Aristoxenus had revealed. Theon's own tactics and intentions are very similar, and it seems likely that Nicomachus put his work together out of ideas already current in contemporary commentaries on Plato (see the introduction to chapter 10).

9.1 Theon's preface, and Thrasyllus on the basic concepts of harmonics: Theon Smyrn. 46.20–49.4

46.20 Since people also call some numbers 'concordant', and without the science of
number [*arithmētikē*] the proper account of concordance could not be
47 discovered, and since this concordance possesses the greatest power, in speech
being truth, in life happiness, and in nature *harmonia*, one could also not

discover what this *harmonia* is in the cosmos if one had not first discovered fully what it is in numbers.[1] This *harmonia* is intelligible, but the intelligible kind is more easily grasped from a beginning in the perceptible kind.[2] We must now speak, therefore, about the two *harmoniai*, that which is perceptible in instruments and that which is intelligible in numbers. After giving an account of all the mathematical *harmoniai* we shall add, finally, that of the *harmonia* in the cosmos. We shall not hesitate to take the discoveries of our predecessors and write them down again ourselves, just as we have passed on what was previously handed down by the Pythagoreans,[3] setting it out again to make it better known, without claiming to have discovered any of these things ourselves. In offering examples of the things handed down by our predecessors, we made a collection of these things too, containing what is necessary for anyone who is to understand what Plato says.

Thrasyllus, then, when discussing the *harmonia* perceptible in an instrument, says that a note [*phthongos*] is a pitch of an attuned sound.[4] It is called 'attuned' if there can be found a note higher than the high one and lower than the low one: the same note is also 'intermediate'. Thus if we conceived a note that exceeded all height of pitch, it would not be 'attuned', for we shall not call
48 the noise of an immense thunder-clap 'attuned' – one that is often deadly because of its excessive greatness: 'The bloodless wound of a thunder-clap has slain many', as someone says [Euripides frag. 972]. And again, if there were some note so low that there was none lower, it would not even be a note, since it would not possess attunement.

For this reason, then, not every sound nor every pitch of a sound is called a note, but only that which is attuned, as for instance that of *mesē*, *neatē* and *hypatē*. An interval [*diastēma*], he says, is a specifically qualified relation that notes have to one another, such as the fourth, the fifth and the octave;[5] and a *systēma* is a specifically qualified container of intervals, such as the tetrachord, the pentachord and the octachord.[6] A *harmonia* is an ordering [*syntaxis*] of

[1] 'Concordant' may be intended here in a sense extending to all proper melodic relations, as in 2.1 Plato *Rep.* 531c, where the germ of these ideas is concisely stated.

[2] The thesis that perception has its (humble) part to play in initiating the ascent to an understanding of intelligible principles is also Platonic: see, for example, *Phaedo* 74a–c, *Rep.* 523a–525a.

[3] Theon has expounded a good deal of Pythagorean number theory in the earlier part of his work. On the ordering of the musical subjects to be addressed see the introduction to this chapter.

[4] The definition has some relation to that of 7 Aristox. *El. Harm.* 15.15–16. Those of later Aristoxenian writers (e.g., Anon. Bell. 48, Cleonides *Eisagoge* 179.9, Bacchius *Eisagoge* 292.15–16) are a little closer: 'the incidence of melodic sound (or 'the melodic incidence of sound') on a single pitch'. But no other source gives an account of what is meant by 'attuned' (here *enharmonios*) that is comparable with the one offered below by Thrasyllus.

[5] Aristoxenian sources offer 'that which is bounded by two notes that are unlike in height and depth' (Cleonides *Eisagoge* 179.11–12, cf. Anon. Bell. 50, 12 Arist. Quint. *De Mus.* 10.18–19), or 'the difference between two notes that are unlike...' (Bacchius *Eisagoge* 292.20–1, cf. Anon. Bell. 50, 9.7 Aelianus ap. Porph. *Comm.* 35.21–2).

[6] Aristoxenian writers give 'that which is put together from more than one interval' (Cleonides *Eisagoge* 180.2–3, cf. 12 Arist. Quint. *De Mus.* 13.4–5). As an alternative, Anon. Bell. offers 'an ordering of several notes in the range of the voice, possessing some definite kind of position' (cf. Bacchius *Eisagoge* 292.18–19).

systēmata, such as Lydian, Phrygian and Dorian.[7] Of the notes some are high, some low, some intermediate, the high being those of the *nēta*, the low those of the *hypata*, and the intermediate those of the ones between.[8] Of the intervals some are concordant, some discordant. Concordant ones are those sounding in correspondence, such as the octave and the double octave, and those sounding in parallel, such as the fifth and the fourth.[9] Those sounding in correspondence are concordant because the depth of pitch, by lying opposite to the height, is in concord with it, and those sounding in parallel are also concordant, because 49 one note sounds neither at the same pitch as another nor in discord with it, but similarly to it at the distance of some recognisable interval.[10] Notes are discordant and not concordant if the interval between them is a tone or a diesis: for the tone and the diesis are an origin of concord, but are not yet concord.[11]

9.2 The treatise of Adrastus, first part: Theon Smyrn. 49.6–62.4

49.6 Adrastus the Peripatetic, who discusses *harmonia* and concordance more intelligibly [than Thrasyllus], speaks as follows. Just as the wholes constituting the primary parts of written sound and of all speech are the verbs and nouns, and their parts are the syllables, which themselves arise from letters, while the letters are the primary sounds, elementary, indivisible and smallest (for speech is put together out of letters as its first constituents and is resolved into them as its last), so also the wholes that are parts of melodic and attuned sound and of all melody are what are called *systēmata* (tetrachords, pentachords and octachords): they arise out of intervals [*diastēmata*], and the intervals out of notes [*phthongoi*], which once again are primary and indivisible and elementary sounds, out of which, as its first constituents, all melody is put together, and 50 into which, as its last, it is resolved.[12] The notes differ from one another in their

[7] In most treatments, a *harmonia* is an arrangement of intervals over the span of an octave: see, for example, 7 Aristox. *El. Harm.* 36.30–2, 12 Arist. Quint. *De Mus.* 15.9ff. Even in Aristoxenus, this use of the word reflects the classifications of earlier theorists, not his own. In later writers it almost disappears, overtaken by theories of the *tonoi*, except as a conscious piece of antiquarianism. Thrasyllus' account is inept: one may suspect that he has succumbed to the allure of a neat progression of definitions without troubling too much about the sense. (In general, it is hard to dissent from the judgement on Thrasyllus implied at the opening of 9.2.)

[8] With this feeble classification compare 12 Arist. Quint. *De Mus.* 28.11–12, 30.2–4, 30.10.

[9] Octaves are distinguished from other concords by means of a different terminology at 11 Ptol. *Harm.* 15.6ff. For this use of the words 'in correspondence' (*kat' antiphōnon*), and other reflections on the distinction, see 4.24 ps.-Ar. *Probs* XIX.39.

[10] This definition is largely empty. The notion 'recognisable interval' is unhelpful. In origin it is probably Aristoxenian, and the claims made at 7 *El. Harm.* 55.3ff. may have some relevance, but see also n. 63 to *El. Harm.* 16.30.

[11] What is called a 'diesis' by Thrasyllus at 9.5 Theon 91–2 is the *leimma* of ratio 256:243 (cf. particularly Adrastus at 9.2 Theon 56, 1.12 Philolaus frag. 6). Thrasyllus in 9.5 also implicitly recognises the existence of another small interval, the difference between a 9:8 tone and a *leimma*, but he does not allude to the fact that it is not itself a *leimma*. Probably the sense here is quite vague, 'scalar interval smaller than a tone'. These intervals are an 'origin' of concord in the sense, presumably, that each concordant interval can be constructed from a combination of these elements.

[12] Analogies between analyses of speech and of melody became common: they seem to have their origins in Plato *Philebus* 17a ff. (quoted in part at 2.6) and 7 Aristox. *El.*

pitches, since some of them are higher and some lower; and their pitches are determined according to certain ratios.

He says also that the Pythagoreans gave a technical account of these things in the following way. Since all melody and every note is sound [*phōnē*], and all sound is noise [*psophos*], and noise is an impact of air that has been prevented from dispersing, it is clear that when there is stillness in the air there can be no noise nor sound, and hence no note.[13] But when impact and movement occurs in the air, if it is swift a high note is produced, if it is slow a low one, and if it is vigorous a great sound[*ēchos*] is produced, if it is gentle a small one.[14] The speeds of the movements, and their vigours, are produced either in some ratios or irrationally with respect to one another. Under irrational relations noises are irrational and unmelodic, and should not strictly even be called notes, but only sounds; but under relations that place them in certain ratios to one another, the multiple or the epimoric or simply that of number to number, they are melodic, and are strictly and properly notes.[15] Of these, some are merely in attunement [*hērmosmenoi*], but those in the primary and best known and most important ratios, both multiples and epimorics, are also concordant.[16]

51 Notes are in concord with one another if, when one or the other is struck on a stringed instrument the other one also sounds with it, through some sort of kinship and sympathy: under the same conditions, if both are struck together, a sweet and agreeable sound arising from the mixture is heard.[17] Of notes attuned in a continuous series, those fourth in order from one another, firstly,

Harm. 27.18ff. With this description of notes as 'indivisible' compare the definition of 'note' mentioned in n. 29 to 10 Nicomachus *Enchiridion.*

[13] On the effects of dispersion see especially 5 *De Audib.* 800b–801a, cf. 3.15 Arist. *De Anima* 419b. But this sentence and the next belong primarily to the tradition that stems from 1.19 Archytas frag. 1, and which includes the introductory passage of 8 Eucl. *Sect. Can.*

[14] This reflects Peripatetic refinements on Archytas' theory (see especially 3.17 Arist. *De Gen. An.* 786b–787a) and is distinct from the doctrines of 8 *Sect. Can.*

[15] Since he includes 'number to number' ratios among rational relations, Adrastus is apparently using the notion of irrationality in its strict sense (not as a negation of the 'commensurability' discussed at 11 Ptol. *Harm.* 16.13ff.): cf. 3.11 Arist. *De Sensu* 439b, 9.7 Aelianus ap. Porph. *Comm.* 36–7. But the conditions under which such irrational relations could exist are left unclear. The thesis that to call something a note is to treat it as 'melodic' is common to many sources (cf., for example, n. 4 above), but few bring out so clearly the important implication that its status depends on its relation to others, not merely on its own intrinsic features.

[16] This remark is evidently addressed to the vexed question of what mathematical properties distinguish concordant multiple and epimoric ratios from others. It is perhaps not as vague as it seems. At Theon Smyrn. 107.24ff. Adrastus is credited with an ingenious procedure for deriving the different forms of ratio mathematically from an initial equality. The method yields an appropriate order of precedence among types of ratio (multiples first, then epimorics, then epimerics; a number of subspecies are also distinguished), and an order of precedence within each type, such that ratios involving smaller numbers have priority. The scheme can give some content to the present passage, with the qualification that it allows for a gradation, rather than a sharp distinction, between concordant and discordant ratios. Cf. 11 Ptol. *Harm.* 13.23ff. with n. 59.

[17] On sympathetic vibration see particularly 4.20 and 4.26 ps.-Ar. *Probs* XIX. 24 and 42, cf. 12 Arist. Quint. *De Mus.* 90.1–6, and in a context close to the present one, 9.9 Panaetius ap. Porph. *Comm.* 66.16ff. On the second form of description see 8 Eucl. *Sect. Can.* 149.18–20 with n. 7.

are in concord with one another, and form the concord which for that very reason is called the fourth [*dia tessarōn*, lit. 'through four']; secondly, those fifth in order form the concord of a fifth [*dia pente*, 'through five']; and next, the notes bounding both those concords, and being eighth in order from one another, form the concord of an octave [*dia pasōn*, 'through all'],[18] so called because, to begin with, the first and lowest note of the eight-stringed *lyra*, called *hypatē*, in relation to the last and highest, which is *nētē*, was found to contain this same concord, in correspondence. And when music had been augmented and instruments had become many-stringed and many-noted,[19] by the addition of several other notes, both lower and higher, to the original eight, nevertheless the names of the first concords were preserved, 'through four', 'through five', and 'through all'.

52 Several other concords were discovered in addition to these. For when any other concord, smaller, greater or equal, is added to the octave, another concord arises from the two,[20] as for instance the octave and a fourth, the octave and a fifth, and the double octave; and the same would hold if any of these were again added to the octave, as for instance the double octave and a fourth, and similarly for the others, to the limit of our capacity to make sounds or to judge them by ear. For there is what is called the 'range' of the voice,[21] which it traverses beginning from some lowest note and progressing in a continuous series upwards, or in the opposite direction. The process of advancing in a continuous series and in a melodic way, however, does not occur either randomly or in just one unique manner, but according to certain determinate methods [*tropoi*], in accordance with which the differences between what are called the 'genera' of melody are understood. For as in speech and in written sound not every letter when joined to just any other produces a syllable or a word, so also in melody, in respect of attuned sound, not every note placed after just any other within the range of attuned sound makes a melodic interval, but only, as we have said, when placed according to certain determinate methods.[22]

53 The most clearly recognisable part, and the measure of the range we have mentioned and of every interval within it, is what is called the interval of a tone [*toniaion diastēma*], just as the foot length is the measure of the strictly 'spatial' distance [*topikon diastēma*] which bodies in movement traverse. The interval of a tone is most recognisable because it is the difference between the first and most recognisable concords; for the fifth exceeds the fourth by a tone.[23] The semitone, however, is not so called as being half of a tone, as

[18] For the earliest enunciations of this rule as it applies to fourths and fifths see 7 Aristox. *El. Harm.* 29.6ff., 53.33ff., where it is stated more accurately than here. For qualifications see n. 144 to 11 Ptol. *Harm.* 39.5.

[19] The reference is to the musical changes of the later fifth century B.C., associated with Phrynis, Timotheus and others: see *GMW* vol. 1 ch. 7.

[20] See 7 Aristox. *El. Harm.* 20.17ff., 11 Ptol. *Harm.* 13.3ff.

[21] See 7 Aristox. *El. Harm.* 13.30ff., cf. 10 Nicomachus *Ench.* ch. 2.

[22] Cf. 7 Aristox. *El. Harm.* 28.18ff.

[23] Cf., for example, 7 Aristox. *El. Harm.* 21.21–3. On the tone's recognisability see 11 Ptol. *Harm.* 20.9–18 with n. 82.

Aristoxenus thought, in the way that a half-foot is half of a foot, but as being a melodic interval smaller than the tone, in the way that a letter is called a 'semivowel' not as being half of a vowel, but as not being in its own right an autonomous sound. For it is demonstrated that the tone cannot possibly be divided into two equal parts, when conceived as being in epogdoic ratio [9:8], just as is the case for every other epimoric interval. For 9 is not capable of being divided into equal parts.[24]

Now when the voice, as it sings within its range, the one previously mentioned, moves from some lower note to the next note in sequence above it
54 by making the interval of the so-called semitone, and when after that it arrives first at another note after traversing a tone from that one, then, if it seeks to progress melodically in a continuous sequence, it can make no other interval, and produce no other note that is melodic and in attunement, than the interval of a tone, and the note which bounds that interval at the top and stands in the concord of a fourth with the original note.[25] The *systēma* that is sung in this way is called a tetrachord, and is constituted out of three intervals, semitone, tone and tone, and four notes, of which the extremes, the lowest and the highest, are directly concordant in the concord which we said is called the fourth, and which is of two tones and a semitone. This sort of genus of melody is called 'diatonic', either because it travels for the most part through tones [*dia tōn tonōn*], or because it displays a character that is somewhat solemn and powerful and well-tensioned [*eutonon*].[26]

If however the voice, after determining the first and original note and moving a semitone upwards, comes to the same second note as before, and then again traverses a semitone from that one and determines a different third note, then when it tries to advance melodically and in a succession from there, it can neither make any interval other than what is left of the tetrachord that was first formed, which is an incomposite triple semitone [*trihēmitoniaion*], nor
55 determine any note other than the one that bounds the first tetrachord at the top, and which is in concord with the lowest note at the fourth. Hence this sort of melody arises through semitone, semitone and an incomposite triple semitone. The genus of this sort of melody is, in its turn, called 'chromatic', because it is turned aside and altered and displays a character more mournful and emotional than the previous one.[27]

[24] On the impropriety of the designation 'half-tone' see especially 9.9 Panaetius ap. Porph. *Comm.* 65.21ff. For the demonstration see 8 Eucl. *Sect. Can.* proposition 3. The explanation given here is nonsense, which is further compounded at 9.3 Theon 70.14ff.: see nn. 62, 65 below.

[25] With this way of putting the matter compare 7 Aristox. *El. Harm.* 28.10ff.

[26] The terms in which the genera are described here and in the next two paragraphs betray their Aristoxenian origins. Though the author is officially committed to the view that the 'semitone' is not an Aristoxenian half-tone, his pervasive use of the word is significant, as is especially his description of the fourth as two tones and a semitone (see 7 *El. Harm.* 46.1–2). It is so in Pythagorean theory too if 'semitone' means '*leimma*', but such authors more commonly quantify it by its ratio. See also the next two notes.

[27] The treatment of 'semitones' in this paragraph makes sense on Pythagorean assumptions only if they are assigned different 'sizes' or ratios ('major' and 'minor'

There is also a third genus of melody, called 'enharmonic', occurring whenever the voice, beginning from the lowest note, sings the tetrachord by progressing through diesis, diesis and ditone. The followers of Aristoxenus give the name 'smallest diesis' to the quarter of the tone, half of a semitone, as being the smallest melodic interval, whereas the Pythagoreans give the name 'diesis' to what we are calling the semitone.[28] Aristoxenus says that this genus we have referred to is called *harmonia* because it is best, taking this title away from
56 attunement as a whole.[29] But it is very hard to sing, and, as Aristoxenus himself says, it requires skill and demands much habituation; and hence it does not come into practical use easily, whereas the diatonic genus is rather simple and noble and more natural.[30] That is why Plato prefers to adopt the diatonic.[31]

It seems that Pythagoras was the first[32] to have identified the concordant notes in their ratios to one another, those at the fourth in epitritic ratio [4:3], those at a fifth in hemiolic [3:2], those at an octave in duple [2:1]; those at an octave and a fourth in a ratio of 8 to 3, which is multiple-epimeric, since it is duple and double-epitritic;[33] those at an octave and a fifth in triple ratio [3:1], those at a double octave in quadruple [4:1]; and of the other attuned notes, those bounding the tone in epogdoic ratio [9:8], those bounding what is now called the semitone but was then called the diesis[34] in a ratio of number to
57 number, that of 256 to 243. He investigated the ratios through both the lengths and the thicknesses of the strings, and again through the tension arising from the turning of the *kollaboi*, or in a more clearly discernible way from the attachment of weights; and also in wind instruments through the width of the bores or through the tension and relaxation of the breath; or again through solid bodies and weights such as discs and vessels. For whichever of these is

semitones: cf., for example, 12 Arist. Quint. *De Mus.* 95.19ff.). There is no sign of this complication here; the passage is again Aristoxenian.

[28] Cf. n. 11 above. Despite his comment, Adrastus attempts no 'Pythagorean' characterisation of the small enharmonic intervals (no more does 8 Eucl. *Sect. Can.*, though it makes a relevant point in proposition 18; at 9.5 Theon 93, Thrasyllus ducks the issue completely: see n. 91 on this passage, and cf. 11 Ptol. *Harm.* 40.6–8). For analyses of the enharmonic (and the other genera) in terms of ratios see the tables in 11 Ptol. *Harm.* Book II ch. 14. For a simple division of the 'semitone' see 12 Arist. Quint. *De Mus.* 96.4ff.

[29] '*Harmonia*' is often used by Aristoxenus as the name of the enharmonic genus: see particularly 7 *El. Harm.* 2.7ff. For his estimate of its excellence see 23.3ff.

[30] See 7 Aristox. *El. Harm.* 19.10ff. On the 'characters' of the genera see also the interpolated passage at 12 Arist. Quint. *De Mus.* 92.19ff. with n. 212.

[31] Plato's real reasons (in connection with 2.3 *Tim.* 35b–36b) have little to do with the powers of performers or the tastes of audiences. His 'diatonic' is chosen as the system that most economically emerges from applications of Archytan proportion theory.

[32] Cf., for example, 1.1, 1.2, and 10 Nicomachus *Ench.* ch. 5.

[33] The 'multiple-epimeric' (*pollaplasiepimerēs*) appears as a category of ratios at Theon Smyrn. 79.15ff.; the source there is still Adrastus. It is defined as the case where the larger term is a multiple of the smaller plus a multiple of some factor of the smaller. Thus, for example, 11:3 is multiple-epimeric because 11 is 3 × 3 plus two times one third of 3, and in the case relevant here 8 is 2 × 3 plus two times one third of three. Hence, it is called 'duple and double-epitritic', that is 'twice the lower term and twice its third'. On the problems associated with the ratio 8:3 see especially 11 Ptol. *Harm.* 13.1ff., 16.8–12.

[34] The interval is more commonly called the *leimma*: see n. 11 above.

taken according to one of the ratios mentioned, other factors being equal, it will produce the concord that corresponds to the ratio.[35]

For the present let us be content to give a demonstration through the length of a string on what is called the *kanōn*. When the single string on the instrument is measured off into four equal parts, the note from the whole
58 string, in relation to that from the three parts, will sound in concord at the fourth, being in epitritic ratio; in relation to that from the two parts, that is, from the half string, it will sound in concord at the octave, being in duple ratio; in relation to that from the fourth part it will sound in concord at the double octave, being in quadruple ratio. The note from the three parts in relation to that from the two parts will sound in concord at the fifth, being in hemiolic ratio; and in relation to the note from the fourth part it will sound in concord at the octave and a fifth, being in triple ratio. If the string is divided by measure into nine parts, the note from the whole string, in relation to that from the eight parts, will bound the interval of a tone, in epogdoic ratio.[36]

All the concords are contained by the *tetraktys*, since the latter is composed
59 of 1, 2, 3 and 4. In these numbers there is the concord of a fourth, the fifth and the octave, the epitritic ratio, the hemiolic, the duple, the triple and the quadruple.[37]

Some people thought it proper to derive these concords from weights, some from magnitudes, some from movements and numbers, and some from vessels. Lasus of Hermione, so they say, and the followers of Hippasus of Metapontum, a Pythagorean, pursued the speeds and slownesses of the movements, through which the concords arise... Thinking that ... in numbers, he constructed ratios of these sorts in vessels. All the vessels were equal and alike. Leaving one empty and filling the other up to halfway with liquid, he made a sound on each, and the concord of the octave was given out for him. Then, again leaving one of the vessels empty, he poured into the other one part out of the four, and when he struck it the concord of the fourth was given out for him, as was the fifth when he filled up one part out of the three. The one empty space stood to the other in the octave as 2 to 1, in the fifth as 3 to 2, and in the fourth as 4 to 3.[38]

He studied the ratios in a similar way also in accordance with the divisions of strings, as was said above, not however on one string, as on the *kanōn*, but

[35] For accounts of alleged experiments of these various sorts see, for example, 1.3 Schol. to *Phaedo* 108d, 1.4 (= Theon Smyrn. 59.4ff., below), 4.19, 4.28 ps.-Ar. *Probs* XIX. 23 and 50, 9.7 Aelianus ap. Porph. *Comm.* 33.16ff., 9.9 Panaetius ap. Porph. *Comm.* 65.21ff., 10 Nicomachus *Ench.* ch. 6, 12 Arist. Quint. *De Mus.* Book III ch. 1. Not all the experiments will work. For comment see also 11 Ptol. *Harm.* 16.32ff.

[36] Adrastus' way of demonstrating the ratios of the concords and the tone has the merit of simplicity. It differs from, for example, those of 10 Nicomachus *Ench.* ch. 10 and 11 Ptol. *Harm.* Book I ch. 8 in using just one division of the string, that into quarters, to display every one of the concords. Its practical disadvantage is that it requires the bridge to be moved, in most cases, between the sounding of one note of a concord and the other.

[37] These numbers form the '*tetraktys* of the decad', summing to the number 10. It had great significance in early Pythagorean numerology: see 1.2 Sextus Emp. *Adv. Math.* VII. 94–5 with n. 3.

[38] This paragraph has already been quoted as 1.4; for commentary see the notes.

on two. For having made two strings equal in tension, when he divided one of
60 them by pressing on it in the middle, the half of it made the concord of an octave in relation to the other string: and when he subtracted a third part, the remaining parts made the concord of a fifth in relation to the other string. Similarly too with the concord of a fourth: in this case he subtracted a quarter of one of the strings and related the remaining parts to the other string.[39] This he did also on the *syrinx* according to the same method. Some people took the concordances from weights, attaching to two strings weights in the ratios specified; others did so from lengths, and pressed down ⟨on various parts⟩ of the strings, so revealing the concordances in the strings.[40]

...⟨says that⟩ a note is the incidence of the voice on a single pitch. For he says that the note must be similar to itself and at variance to the least possible degree, not consisting of differing pitches, of depth and of height.[41] Of sounds some are high-pitched, some low, and hence this is true also of notes, of which the high is swift, the low slow.[42] Then if one were to blow into two pipes of equal thickness and equal bores, drilled in the manner of a *syrinx*,[43] where the length of one is twice that of the other, the breath would burst out from the half length at double the speed, and there would be the concord of the octave, the low note being that coming through the larger, the higher that through the
61 smaller. The cause is the speed and slowness of the movement. They also displayed the concords on the basis of the spacings of the finger-holes [*trēmata*] on *auloi*, even on one *aulos* alone. For if the *aulos* is blown when divided in half and also as a whole, from the hole at the halfway point the concord of an octave will be produced. If it is divided into three, with the two parts towards the mouthpiece [*glōssis*] and the one below them, when the whole is blown along with the two parts, there arises the concord of a fifth. When there are four divisions, three above and one below, when the three are blown together in relation to the whole there arises the concord of a fourth.[44] And those in the circle of Eudoxus and Archytas thought that the ratio of the concords is in numbers, they too agreeing that the ratios are in movements and that the swift movement is high-pitched [*oxys*, 'sharp'], in that it strikes continuously and jabs the air more swiftly [*ōkyteron*, 'more sharply', also

[39] For more complex constructions involving several strings see 11 Ptol. *Harm.* Book I ch. 11, Book II chs. 1–2, cf. chs. 12–13, Book III chs. 1–2.

[40] The text is in some disorder in this passage, and can hardly be complete as it stands.

[41] One's first instinct is to fill the gap in the first sentence of the paragraph with the name 'Aristoxenus': see 7 *El. Harm.* 15.15–16. But the second sentence reflects nothing in *El. Harm.*, and could hardly be expressed by Aristoxenus in quite these terms. (The remark has affinities with the interpolated sentence quoted in n. 29 to 10 Nicomachus *Ench.* ch. 4.) The sequel is plainly not Aristoxenian. It seems likeliest, despite the words that begin the next paragraph, that Theon is simply re-identifying his source (we do not know how much has dropped out of the text here), and that the missing name is 'Adrastus'.

[42] This is the simplest and commonest of the ancient theories about the determinants of pitch. See n. 9 to 1.4 Theon Smyrn. 59.4ff.

[43] Here the word refers to the familiar Pan-pipe, the second type of *syrinx* mentioned in a similar context in 4.19 ps.-Ar. *Probs* XIX.23.

[44] On all these tests with instruments see n. 35 above. The sceptical comments of Ptolemy, cited in that note, are particularly apposite in connection with these claims about *auloi.*

'more shrilly'], while the slow is low-pitched [*barys*, 'heavy'], in that it is more sluggish.[45]

So much for the discovery of the concords: let us return to the things handed down by Adrastus.[46] He says that when these instruments designed for the discovery of the concords have been previously prepared in accordance with the ratios, perception agrees with their testimony, while when perception has been taken first, reason is in attunement with it.[47] How it is that the notes bounding the so called semitone are in the ratio of 256:243 to one another will
62 become clear a little later.[48] But it is plain that the combinations and divisions of the concords are found to be in agreement and in consonance with the combinations and divisions of the ratios that correspond to them, the combinations and divisions that we mentioned previously.[49]

9.3 Adrastus' treatise continued: Theon Smyrn. 63.25–72.20

63.25 Plato extended the diatonic genus and the size of its *systēma* to the quadruple
64 octave plus a fifth and a tone [*Timaeus* 35b–36b]. If one were to say, says Adrastus, that one ought not to stretch it out so far (since Aristoxenus made his diagram of the many *tropoi* with a magnitude of the double octave and a fourth, while more recent authorities make their fifteen-stringed *tropos* extend at the most to the triple octave and a tone[50]), one must reply, he says, that these latter act in this way with a view to our own usage, believing that anything greater than these can neither be uttered by competitive performers
65 nor accurately assessed by the hearers; whereas Plato has an eye to nature, since it is necessary that the soul, being constituted in accordance with *harmonia*, should advance as far as the solid numbers and be attuned through

[45] Eudoxus was an eminent fourth-century mathematician and astronomer, an associate of Plato. On this aspect of Archytas' theories see 1.19 Archytas frag. 1.

[46] This suggests that at least some of what precedes was not recorded by Adrastus; Theon may have added information from other sources. But the next sentence shows that material of the sort reviewed there was, after all, in Adrastus' treatise. Perhaps the sense is only that Theon is returning from what Adrastus reports of others to what he says in his own right.

[47] This sentiment is echoed by Ptolemy, who generalises it into a major methodological principle. See particularly 11 Ptol. *Harm.* Book I chs. 1–2 and, for example, Book II ch. 1, Book III ch. 1. On its role in Adrastus' work see the introduction to this chapter.

[48] See 9.3 Adrastus ap. Theon Smyrn. 67–8.

[49] The passage between the end of this excerpt and the beginning of the next calculates the ratios of combinations of concords, and of their differences.

[50] These constructions evidently concern the 'Aristoxenian' system of thirteen *tonoi* or *tropoi* and the fifteen *tonoi* of some of his successors. See 12 Arist. Quint. *De Mus.* 20.5ff. According to Aristides, the *proslambanomenoi* of Aristoxenus' *tonoi* were arranged over an octave, those of his successors over an octave and a tone. In that case Aristoxenus' diagram may have displayed each *tonos* over the range of an octave and a fourth, and those of the later theorists over that of the double octave. This would account for Adrastus' figures, but the matter remains uncertain. (As Adrastus' next sentence shows, there may also be some relevance in the remark at 7 Aristox. *El. Harm.* 20.23–31 that the largest concord available to any voice or instrument is two octaves plus a fifth.) For criticisms of these proliferations of *tonoi* see 11 Ptol. *Harm.* Book II chs. 7–11.

two means, so that it can pass through the whole of the complete, solid, cosmic body, and grasp all existing things;[51] and hence he extended its *harmonia* to that point, even though in one way, and in respect of its own nature, *harmonia* is capable of extending without limit.[52]

He [Adrastus] says also that it is appropriate to assign the greater numbers to the lower notes, even if this seems in discord with the cases of certain tensions, for example in the case of the tension that arises through the attachment of weights. For given two strings equal in length and thickness, and similar in other respects, the greater weight through its greater tension will make the higher note. Now since the greater weight makes a greater tension, the power that it adds from outside to the higher note is greater, and therefore the note has less intrinsic strength than does the attached object. And it is plain that the lower note, conversely, possessing a power proper to itself that is greater than that of the attached object, is strong enough to preserve its own proper *harmonia* and concordance. The other cases agree with this too. For the
66 lengths and the thicknesses again give the strings a resistance to movement, and so produce weakness, so that they do not easily move or strike quickly and impose form on the greater quantity of surrounding air. It is clear, then, that the lower notes possess the force proper to themselves in correspondence with the greater number. Similar things can be found in the case of wind instruments too. For the lower of their notes impose form on a greater quantity of air because of the length and the width of the bores, or indeed are less tense and weaker because of the relaxation of the breath, in cases like the *salpinx* or the windpipe, and so possess a power proper to themselves which is by nature greater.[53]

The most important of all the concords, he says, is that of the fourth, for from this all the rest are discovered. The fifth differs from the fourth by a tone: this is of course how they define the tone – the interval from the fifth to the fourth.[54] From the fourth and the fifth the octave is discovered, for it is composed of the fourth and the fifth.

The ancients took the first interval of the voice to be the tone, for they did

[51] See especially Plato *Tim.* 32a–b: cf. also the use made of the notion of 'solid proportion' at 12 Arist. Quint. *De Mus.* 101.12–23, 'solid number' at 102.5–9.

[52] Cf. 7 Aristox. *El. Harm.* 20.12–23.

[53] This obscure and fascinating argument is unparalleled elsewhere: it has affinities with 6 Theophrastus ap. Porph. *Comm.* 63.1ff., but the conclusions reached there are quite different. It was common practice among ancient theorists to assign larger numbers to lower notes, if only for convenience when dealing with string-lengths. Nevertheless, they often thought of the larger number as attaching properly to higher notes (on the grounds of their greater velocity or more frequent impacts). Hence the notion of 'converse qualification' elaborated at 9.4 Thrasyllus ap. Theon Smyrn. 87. Adrastus' conception is based on the idea that a pitched sound has a power or force distinct from that of the physical agency that produces it. Then where two sounds are of equal strength (cf. 6 Theophrastus ap. Porph. *Comm.* 63.1ff.), but the higher is produced by greater physical force, the intrinsic strength of the lower must be greater. The argument has roots in 1.19 Archytas frag. 1, but subtly reverses Archytas' conclusions. On the importance of the argument for Adrastus' programme see the introduction to this chapter.

[54] E.g., 7 Aristox. *El. Harm.* 21.21–3.

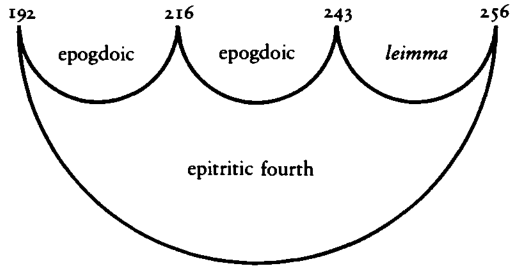
not recognise the semitone and diesis.[55] The tone was found to be in epogdoic ratio [9:8] in contrivances involving discs, vessels, strings, *auloi*, weights, and several other things: for nine items in relation to eight made one hear the
67 interval of a tone. The reason why the tone is the first interval is that down as far as this interval the voice in its movement keeps the hearing free from error, but after that the hearing is not able to grasp the interval accurately.[56] After all, people dispute about the interval next in order, the so called semitone, some saying that it is a complete half-tone, others that it is a *leimma*. Now the fourth, which is epitritic [4:3], is filled up by the tone, that is by the epogdoic interval, as follows. It is agreed by everyone that the fourth is greater than a ditone but smaller than a tritone. But Aristoxenus says that it consists of two and a half complete tones, while Plato says that it consists of two tones and what is called the *leimma*.[57] He says that this *leimma* is not incapable of being expressed, and that it is in the ratio of number to number in which 256 stands to 243. That is the interval, and the difference [*hyperochē*, 'excess'] is 13. It is found as follows. The number 6 could not be the first term, since it has no eighth, through which its epogdoic could come into being. Nor indeed could it be 8, for while it has an epogdoic, 9, the 9 in its turn has no epogdoic. But one must take an epogdoic of an epogdoic, since the epitritic interval of a fourth is greater than a ditone. So we take the fundamental epogdoic, 8 and 9. Taking 8 with
68 itself we find 64; then we take 8 with 9 and 72 is produced; then 9 with itself and 81 is produced. Then again let each of these be taken three times: three times 64 will be 192, three times 72 will be 216, and three times 81 is 243. Thus: 8, 9, 64, 72, 81, 192, 216, 243. Then we add, beyond 243, the epitritic based upon 192, which is 256. Thus the setting-out [*ekthesis*] is as follows: the fundamental epogdoic, 8, 9; the second epogdoics, 64, 72, 81; the two that are third epogdoics of one another, 192, 216, 243; and let there be also the epitritic of 192, which is 256. This last will be the epitritic filled out by two tones and the so called *leimma*.[58]

[55] This probably means only that the earliest exponents of Pythagorean theory found no reason to pursue their investigations beyond the fundamental intervals of the concords and the tone. Our sources clearly reflect a tradition to this effect. (Cf. 1.1–1.4, 1.8: 10 Nicomachus *Ench*. ch. 6; and note how in 12 Arist. Quint. *De Mus*. Book III ch. 1 the procedure followed changes at the point where reference to the 'semitone' is introduced, 95.19.) Pythagoreans of the earlier fifth century may have thought that only these were needed for the purposes of their metaphysical speculations. Philolaus (1.12 frag. 6) was perhaps the first to take the matter further, and it was not until Archytas (1.20, 1.21) that complete musical systems were treated as subjects for mathematical investigation in their own right. Before Philolaus and Archytas, then, there may have been no attempt to assign ratios to intervals smaller than the tone.

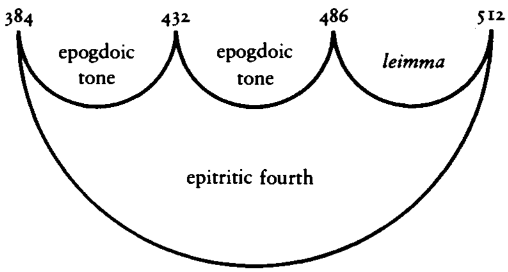
[56] See n. 63 to 7 Aristox. *El. Harm*. 16.30.

[57] See 7 Aristox. *El. Harm*. 24.4ff., 46.1–2, 56.13ff., and 2.3 Plato *Tim*. 36b.

[58] The aim is to find four whole numbers, A, B, C, D, such that D:A = 4:3, B:A = 9:8, C:B = 9:8, and thus to identify the ratio of D:C. A and B could be 8 and 9, but no whole number stands to 9 in the ratio 9:8. A, B, C could be 64, 72, 81, but no whole number stands to 64 in the ratio 4:3, since 3 is not a factor of 64. Hence these terms are multiplied by 3, after which the 'epitritic' of the first can be calculated: A, B, C, D are 192, 216, 243, 256. The procedure is echoed at 12 Arist. Quint. *De Mus*. 96.18ff.: compare 11 Ptol. *Harm*. Book I ch. 10.



Some people, however, take 384 as the first term. For in order to take two epogdoics, they multiply the first term, 6, by 8, making 48, and take this again
69 eight times, making 384, whose epitritic is 512, and between them two epogdoics, that of 384, which is 432, and that of 432, which is 486; and from this to 512 is the ratio of a *leimma* [*leimmatios logos*].



But other people say that these numbers are not correctly taken. For the excess of the fourth term over the third is not 13, which Plato said the *leimma* must contain. Yet nothing prevents us from finding in different numbers too the same ratio as 256 has to 243, for Plato did not take a determinate number, but a determinate ratio of number;[59] and the ratio which 256 has to 243 is also that which 512 has to 486. For 512 is double 256, and 486 is double 243.

That this interval [*diastēma*] between 256 and 243, that is, 13, is less than a half-tone is plain. For since the tone is epogdoic, the half-tone will be a double epogdoic, that is, an ephekkaidekatic.[60] But 13 is in a ratio of more than an eighteenth to 243, which is a smaller part than a sixteenth.[61] For the epogdoic

[59] It is not uncommon for ancient theorists to confuse the roles of ratios between terms and of differences between terms, but, as Adrastus says, Plato is not guilty of this blunder. On the other hand, the author of the remainder of this part of the passage (69–70) seems sadly muddled about this and other matters.

[60] That is, the ratio 17:16. But of course this is not the ratio of a half-tone, which would be the square root of 9:8. Adrastus was a competent mathematician, as many passages in Theon testify. He is unlikely to be the author of the paragraphs from here to the end of 70, though it is possible that their naiveties are due to Theon's own abbreviation and misunderstanding of his source.

[61] If the half-tone were indeed 17:16, then the greater term would exceed the less by one sixteenth of the latter. But 13, the difference between 256 and 243, is less than an eighteenth of 243 ('in a ratio of more than an eighteenth' means that the ratio 243:13 is more than 18:1).

is not susceptible of division, even if those who judge these things not by reason
70 but by ear think that it is. After all, the unit does not divide the interval of the fundamental epogdoic, that is of 9 to 8.[62] And if one asks of what the so-called *leimma* is a *leimma*, he should realise that it is of the fourth, for it falls short [*leipei*] of making the fourth two and a half complete tones.[63]

The tone was found as follows. Since the fourth was evidently in epitritic ratio and the fifth in hemiolic, there was taken the first number that has a half and a third; and this is 6. Its epitritic is 8 and its hemiolic 9. Thus: 6, 8, 9. Then the interval from the hemiolic to the epitritic was found to be in epogdoic ratio, for 9 is the epogdoic of 8. (And pitch is also called 'tone'.[64])

The fact that the tone is not divided in half is shown as follows. First, the fundamental epogdoic has as its interval a unit, which is indivisible. Secondly, the epogdoic interval is never divided into parts equal in respect of number. For even in the case of 216 to 243 the difference [*hyperochē*], 27, is not divided into equal parts, but into 13 and 14, since the unit is not divided.[65]

71 Now given that the tone is variously grasped, one tone being grasped by thought [*noēsis*], one in numbers, one in intervals, one through the hearing in sounds, the one that is in numbers is never divided into equal parts, as has been shown, and neither is the one in perceptible and visible intervals.[66] For on the *kanōn* the bridge, being perceptible, will always have some width, and in the division it will not, by being without width, avoid occupying some part of the tone – both a part of the last boundary of the first section and a part of the

[62] The writer seems to be acquainted with the Archytan proof that epimoric ratios have no mean proportional (see 8 Eucl. *Sect. Can.* proposition 3), but has plainly not understood it. See also the last paragraph of 70.

[63] A misleading derivation; the relevant sense of *leimma* is 'remainder': it is 'what is left' in the fourth after two whole tones.

[64] The parenthesis is probably a marginal jotting that has crept into the text, but the whole section is so confused and so poorly related to its context that certainty about the origins and relations of its parts is hard to come by. On the various senses of *tonos* see 12 Arist. Quint. *De Mus.* 20.1ff.

[65] Here again there are distant echoes of the Archytan proof (see n. 62 above). But the assertion that the difference between terms in epogdoic (or other epimoric) ratio is never equally divisible is both false and irrelevant. Thus 18:16 is as good an example of epogdoic ratio as is 243:216, and the difference is equally divisible, though of course the ratio is not, in the sense required; and the difference between, for example, 25 and 16 is not divisible into two equal whole numbers, but the ratio 25:16 is equally divisible in the sense intended (i.e., $25:16 = 5:4 \times 5:4$). With these confusions about the proof cf. 9.2 Theon Smyrn. 53.15–16.

[66] The contrast between what is conceived by reason and what is perceived by the senses is familiar, but this classification of kinds of tone is unusually complex. The 'tone grasped by thought' corresponds roughly to our notion of the 'abstract conception'. Adrastus probably intends a reference to a Platonic Form, and claims at the end of this passage that this tone is equally divisible. The tone as mathematically defined consists in a ratio of numbers, and has already been discussed. In the third category, 'intervals' turns out to mean 'visible distance', not differences in pitch between heard sounds. As the sequel shows, they are the distances between points on the string of a monochord. Hence this 'tone' consists in a relation between visible entities, lengths that the eye perceives and judges. Finally there is the 'tone grasped through hearing', which exists only as a relation between sounds. The rest of the passage seeks to show that neither of the two kinds of perceptible tone is divisible into exactly equal parts.

initial origin of the second – and hence some part of the tone will have been used up. Again, in the divisions there are three things, the two things divided off and thirdly the thing taken out; and some part of the things divided off is expended in the division itself, just as when one uses a saw something is expended in the cutting, that which is taken out by the cutting itself. Thus just as in various perceptible cases something is taken out, so it is in all cases that even if it escapes perception still something is always expended in cutting. For instance, if you measure a stick or a reed or any other perceptible length before dividing it, and then divide it into many parts, you will find that the joint measurement of all the parts divided off is smaller than that of the whole before it was divided. Again, if you divide a string and then cut it up, after the cutting the extension has contracted again; and if you again stretch the pieces that were
72 cut up, it is necessary to take away something of the magnitude in order to fasten the stretched piece at its points of contact at each end.[67] Hence there will not be two complete half-tones.

Nor indeed does one find the division of the tone into equal parts in the case of voices. For suppose that after singing tone, tone, I next sing the two semitones of the one tone, returning to the pitch through three notes and two intervals. Now the third note is higher than the second: it is at the distance of a tone from the first, and appears to be at the distance of a half-tone from the second; but it is not an identical half-tone, exactly like the distance of the second note from the first, since the lower cannot be just like the higher.[68] For even if we want to sing twice on the same note, we shall not produce the same sound after interrupting the voice, but there must necessarily be some difference which escapes the hearing: it is not even possible to prick exactly the same point twice, nor to strike the same string twice in the same way – we shall do it either more slackly or more vigorously – nor to dip a finger twice in the same way into the same liquid, nor, having dipped it, to take out with one's finger the same amount of ink or honey or pitch. The tone that is grasped by thought [*noēsis*], however, can be thought of also as divided into equal parts.[69]

[67] This last point is a little naive. The others are most easily read as assuming that this tone is just a certain length of string, and that any (visible) division of it into two will produce sections that sum to less than the whole, since the visible divider occupies some of the length. This will not do: the 'visible' tone is presumably a ratio of lengths, not a length. But the argument could be restated. If the ratio of OB to OA is a tone, then the division of this tone into halves involves placing a visible divider, C, between A and B, such that OC:OA = OB:OC, and such that AC+CB = AB. But according to the argument this latter condition cannot be met, since a part of AB is occupied by C, and does not fall within either AC or CB.

[68] The explanatory phrase seems to hint at some special reason why higher and lower intervals must differ, perhaps one akin to 11 Ptol. *Harm.* 20.23ff. But if so it is not explored. The supporting claims that follow are grounded in an entirely general intuition, to the effect that nothing whatever can be done twice in exactly the same way. For the thesis that there is no sense in the notion of 'halving' the tone presented to the hearing see 9.9 Panaetius ap. Porph. *Comm.* 65.21ff.

[69] The thesis seems to be that there is nothing incoherent or self-contradictory about the conception 'half of a tone', and indeed there is not. (Further, the conception can nowadays be paired off with a mathematical expression, $3:\sqrt{8}$. Greek mathematical procedures would not have ruled out a comparable approach, couched geometrically:

9.4 Thrasyllus' division of the *kanōn*, first part: Theon Smyrn. 87.4–90.1

87.4 The division of the *kanōn* is done through the *tetraktys* of the decad,[70] which consists of monad, dyad, triad and tetrad, 1, 2, 3, 4. For it contains epitritic, hemiolic, duple, triple and quadruple ratio. Thrasyllus divides it as follows.[71]

When they divide the magnitude in half, the octave in duple ratio makes *mesē*,[72] which in its movements is qualified in the converse way, having double the pitch upwards.[73] 'Converse qualification' is like this: however much you subtract of the magnitude of the whole string on the *kanōn*, the same amount is added to the tension [*tonos*], and however much you add to the magnitude of the string, the same amount of the tension is taken away. For the half magnitude contains double the pitch [*tasis*] upwards, and the double magnitude contains half the pitch downwards.

88 When the division is into three, *hypatē mesōn* and *nētē diezeugmenōn* are produced.[74] *Nētē diezeugmenōn* is at the fifth in relation to *mesē*, since there are two intervals in relation to three, and at the octave in relation to *hypatē*, since there is one interval in relation to two. It is at the octave and a fifth in relation to *proslambanomenos*, since *proslambanomenos* is at an octave from *mesē*, and the interval as far as *nētē* that is, a fifth in relation to *mesē*, has been taken in addition. *Mesē* is at the fourth in relation to *hypatē* and at the octave in relation to *proslambanomenos*: *hypatē* is at the fifth in relation to *proslambanomenos*. The magnitude from *hypatē* to *mesē*, a fourth, is equal to that from *mesē* to *nētē*, a fifth. And the numbers of movements are again qualified conversely to the division of the magnitudes.

When the division is into four, there are added both the note called *hyperhypatē*, also known as *diatonos hypatōn*, and *nētē hyperbolaiōn*.[75] *Nētē hyperbolaiōn* is at the fourth in relation to *nētē diezeugmenōn*, at the octave in relation to *mesē*, at the octave and a fourth in relation to *hypatē*, at the octave and a fifth in relation to *hyperhypatē*, and at the double octave downwards to
89 *proslambanomenos*. The ratio for *hyperhypatē* is at the fourth downwards to *proslambanomenos*, at the fifth upwards to *mesē*, and exceeds *hypatē* by a tone

the ratio of the half-tone is the ratio between the sides of two squares whose areas are 9 and 8 square units respectively. But this ratio is not of course a ratio of numbers, where 'numbers' means 'positive integers'.)

[70] Cf. 9.2 Theon Smyrn. 58–9 with n. 37.

[71] Thrasyllus' method of division should be compared with that of 8 Eucl. *Sect. Can.* propositions 19–20. There are many similarities; salient differences will be noted below. See also 10 Nicomachus *Ench.* 260.16 for a scathing reference to Thrasyllus' procedure.

[72] 8 *Sect. Can.* proposition 19 begins with a division into four, which includes the construction of *mesē*.

[73] On the notion of 'converse qualification' see n. 53 above. The *Sectio* does not explicitly invoke it, though it could appropriately have done so.

[74] In 8 *Sect. Can.*, *nētē diezeugmenōn* is found by subtracting a third from the half-length, *hypatē mesōn* by doubling the length of *nētē diezeugmenōn*.

[75] These are constructed in the first move of 8 *Sect. Can.* In the standard schemata *diatonos* (diatonic *lichanos*) *hypatōn* is not a fixed note, unlike all the others constructed at this stage of the division, and the name *hyperhypatē* is unusual. See nn. 65 and 71 to 8 *Sect. Can.* proposition 19.

downwards. And the magnitude of the tone between *hyperhypatē* and *hypatē* is equal to that of the fourth from *nētē diezeugmenōn* to *nētē hyperbolaiōn*. Once again the numbers of the movements are qualified conversely to the magnitudes of the intervals.[76]

What has been said can be made clear by means of numbers.[77] For if the *kanōn* has a magnitude of 12 units of whatever sort, *mesē* will arise when the whole string is divided in half, and will be at a distance of 6 units from each end. *Hypatē mesōn* will be 4 units from the origin, and *nētē diezeugmenōn* 4 units from the end: the distance between them is 4. *Hyperhypatē* will be at a distance of 3 magnitudes from the origin and 1 from *hypatē*. *Hyperbolaia* will be 3 units from the end, and 1 from *diezeugmenē*.[78] Between them will be 6, so that each is 3 from *mesē*. The whole division is then as follows: from the origin to *hyperhypatē*, 3; then 1 to *hypatē*; then 2 to *mesē*; then from *mesē* to *diezeugmenē* 2; then 1 to *hyperbolaia*, and from it to the end, 3: in all there are 12.

Then the ratio of *nētē diezeugmenōn* to *hyperbolaia* is 4 to 3, the epitritic
90 ratio of a fourth...[79]

9.5 Thrasyllus' division of the *kanōn*, concluded: Theon Smyrn. 90.22–93.9

90.22 The remaining movements are qualified conversely [to the distances] when both the epitritic fourth and the hemiolic fifth on the *kanōn* are closely filled
91 up with the epogdoic tone.[80] Since the hemiolic fifth exceeds the epitritic fourth by an epogdoic tone (for instance, if we take a number, 6, which has both a half and a third, its epitritic is 8 and its hemiolic is 9, and 9 is the epogdoic of 8: thus 6, 8, 9: then the excess of the hemiolic over the epitritic is in epogdoic ratio), and since the epitritic fourth consists of two tones and the *leimma* that is a diesis, one must fill them up closely with epogdoic tones and *leimmata* that are dieses.[81] They can be closely filled up if we begin from *nētē hyperbolaiōn*. For if we go further [i.e., increase the length] by an eighth of the distance between there and the end, we shall have *diatonos hyperbolaiōn*, a tone lower

76 The first stage of the division is now complete. It is more geometrically systematic than that of 8 *Sect. Can.* proposition 19, but unlike the *Sectio* it fails to capture three of the fixed notes, *nētē synēmmenōn*, *paramesē* and *hypatē hypatōn* (these are found in the course of 9.5 below). On the ambiguous status of *nētē synēmmenōn* cf. 11 Ptol. *Harm.* Book II ch. 6.

77 8 *Sect. Can.* has no equivalent to this paragraph.

78 The notes mentioned in this sentence are the two *nētai*, *hyperbolaiōn* and *diezeugmenōn*.

79 From here to 90.21 the author sets out the ratios, and the familiar names of the intervals, between the notes he has constructed.

80 The text of this sentence is uncertain. It is clear, however, that the notion of 'close filling up' was expressed either by means of the adjective *pyknos*, 'compressed', familiar from the use of its neuter form, *pyknon*, as a technical term, or by the passive participle of the verb *katapyknoein*, which is related to the pre-Aristoxenian noun *katapyknōsis* (the noun appears here in a marginal heading). For the verb see 7 Aristox. *El. Harm.* 7.31, and on *katapyknōsis* see 7.24, 28.2, 38.3, 53.4.

81 On this use of the term 'diesis' see n. 11 above. There are none of these preliminaries in 8 *Sect. Can.*

than *nētē*. If we go further by an eighth of the distance from *diatonos* to the end, we shall have *tritē hyperbolaiōn*, a tone lower than *diatonos*. And the remaining distance to *nētē diezeugmenōn* will be the *leimma* that is a diesis, filling up the fourth between there and *nētē hyperbolaiōn*. Again, if we take a ninth of the distance from *nētē diezeugmenōn* to the end and go back [i.e., reduce the length] by that amount, we shall have *chrōmatikē hyperbolaiōn*, a tone higher than *nētē diezeugmenōn*.[82] If we go further [from *nētē diezeugmenōn*] by an eighth, we shall have *paranētē diezeugmenōn*: this is both *diatonos* and *nētē synēmmenōn*, a tone lower than *nētē diezeugmenōn*.[83] If we take an eighth of the distance from *nētē* [*synēmmenōn*], to the end and go
92 further by this amount, we shall have *tritē diezeugmenōn*, a tone lower, which is also *diatonos synēmmenōn*.[84] Similarly if we go further by an eighth of the distance from here to the end we shall have *tritē synēmmenōn*, a tone lower.[85] The remainder down to *mesē* will be the *leimma* that is a diesis, completing the octave. If we go back by a ninth from *mesē* in the same way as before, we shall have *paramesē* or *chrōmatikē synēmmenōn*, a tone higher than *mesē*.[86] And if we go back by a ninth of this, we shall have *chrōmatikē diezeugmenōn*. If we go further from *mesē* by an eighth of it, we shall have *diatonos mesōn*, a tone lower than *mesē*, and then by going further by an eighth from here we shall have *parhypatē mesōn*, a tone below *diatonos*. And the remainder down to *hypatē mesōn* is the *leimma* that is a diesis, completing the fourth in relation to *mesē*. If we go back from *hypatē* by a ninth there will be *chrōmatikē mesōn*, a tone higher.[87] If we go further [from *hypatē mesōn*] by an eighth we shall find that we have *hyperhypatē*:[88] if we go further than this by an eighth there will be *parhypatē hypatōn*. By inversion of this process, if you begin from *proslambanomenos*, cutting the whole distance into nine and leaving one part behind at the opposite end to the *nētai*, there will be *hypatē hypatōn*, a tone higher than the whole string, closing off the tetrachord *hypatōn* with the *leimma* in relation to *parhypatē*.[89]

[82] The author alters his procedure here (constructing the upper boundary of a tone by shortening the length assigned to its lower boundary by a ninth, instead of constructing its lower boundary by first finding the upper one, and lengthening it by an eighth). This is necessary because no note of the system lies a tone above *chrōmatikē* (chromatic *paranētē*) *hyperbolaiōn*. In 8 *Sect. Can.* proposition 20 only diatonic notes are located, and there is therefore no reference to the chromatic ones mentioned here and below.

[83] The note is not mentioned under the name *diatonos* (or diatonic *paranētē diezeugmenōn*) in 8 *Sect. Can.* propositions 19–20, but the note *nētē synēmmenōn*, whose pitch is the same, is found in proposition 19. See n. 77 to *Sect. Can.* proposition 20, and cf. n. 76 above.

[84] That is, diatonic *paranētē synēmmenōn*. Here and in the rest of its construction, the procedure of 8 *Sect. Can.* is different from Thrasyllus'. Where Thrasyllus continues to work by building tones, the *Sectio* adopts a sequence of moves through concords (fourths and fifths).

[85] This note is not included in the division of 8 *Sect. Can.*

[86] This has already been found in proposition 19 of 8 *Sect. Can.* among the fixed notes, and by a different procedure.

[87] On this note and *chrōmatikē diezeugmenōn* see n. 82 above.

[88] See n. 75 above.

[89] See n. 76 above. It is odd that the author does not go on to construct *chrōmatikē hypatōn*, in the same way that he has found the other chromatic notes.

In this way there will be filled out the entire immutable [*ametabolon*]
93 *systēma* according to the diatonic and the chromatic genus.[90] The enharmonic arises when the *diatonoi* that recur in each tetrachord are removed.[91] We could also find these things in numbers, beginning from *nētē hyperbolaiōn*, if we assume that it is 10,368. The successive epogdoics and the remainders are taken according to the ratios previously stated, which it is unnecessary to set out: it is an easy task for anyone who has followed what we have said.[92] That, then, is the method of dividing the *kanōn* which has been handed down by Thrasyllus.

Part 2 Passages from Porphyry

Porphyry was a Syrian, born in Tyre in about A.D. 232. At about the age of thirty, after studying philosophy in Athens, he joined the disciples of the great Neoplatonist Plotinus in Rome. He became head of the school at some time after Plotinus' death, perhaps in the 270's, and died shortly after A.D. 300. It is he who was responsible for editing and publishing Plotinus' work, and for transforming Neoplatonism into a systematic school of thought that became influential throughout the Roman Empire. He produced philosophical writings of his own, and commentaries on Plato and Aristotle, whose views he probably sought to reconcile. He was a man of wide learning, with a special interest in religious thought and practice: he portrayed as an ideal the submersion of the individual soul in the Deity. Quotations from his treatise *Against the Christians* (which was suppressed in the reign of Theodosius II), and passages of his *On Abstinence*, indicate that he agreed with some early Christians in attacking secular music, on the grounds that its sensual allure distracted people from their true spiritual objective. The Platonic origins of this attitude are evident enough.

Porphyry's *Commentary on the Harmonics of Ptolemy* concentrates on Ptolemy's introductory discussions, on matters to do with the basic conceptions of the science and its methods. He has less to say about the technical details of Ptolemy's harmonic analyses themselves, and his work breaks off in the course of a discussion of Book II ch. 7.

Among the writings that Porphyry quotes at length are those already presented in chapters 5 and 6 above, as well as the bulk of chapter 8. Those translated below fall into

[90] See the previous note. The lower of the two moveable chromatic notes in each tetrachord is at the same pitch as its diatonic counterpart, and therefore is not constructed separately. The 'immutable *systēma*' here is the combination of the GPS and the LPS in their standard or 'Dorian' arrangement, a usage different from that apparently intended in 8 *Sect. Can.* proposition 19 (and see notes to that passage).

[91] This gives an enharmonic system with no note between the second-highest and the lowest notes of each tetrachord. No attempt is made to find equivalents to the 'quarter-tones' of Aristoxenus' enharmonic *pykna*. See n. 28 above. Though it is likely that Thrasyllus has no better motive than the avoidance of mathematical complexities, there is evidence of an early form of the enharmonic in which the *pyknon* was undivided: see ps.-Plut. *De Mus.* 1134f ff. (in *GMW* vol. 1, pp. 215–17).

[92] If integers are to be found for all diatonic and chromatic notes in the upper octave, the number attached to the highest note, *nētē hyperbolaiōn*, must be divisible by 64 and by 81. Hence it must be at least $64 \times 81 = 5{,}184$. To allow those in the lower octave to be constructed too, the number must be doubled, giving 10,368. (In fact if 5,184 is used, one note in the higher part of the system, *tritē synēmmenōn*, cannot be constructed, since it requires the original number to be divisible by 128. The number 10,368 satisfies this condition. Since *tritē synēmmenōn* has no counterpart in the lower octave, the number does not have to be doubled again.)

two groups. In 9.6–9.9 three authors discuss issues connected with the causation of sound and the nature and determinants of pitch. Passages 9.10–9.14 are the reflections of two scholars on the methods and presuppositions of different schools of harmonic thought, distinguishing them under a finer-grained system of classification than is commonly found elsewhere. Our independent information about the five authors is limited.

(i) The writer whom Porphyry calls 'Aelianus the Platonist' (9.6, 9.7) is probably, but not certainly, Claudius Aelianus, a teacher of rhetoric in Rome in the late second and early third centuries A.D. His extant works include moralising reflections on animal and human life, and stylistic exercises in the form of letters; there are also fragments on divine providence and justice. In philosophy his ideas were of a Stoic colour, and he mounted vitriolic attacks on the Epicureans.

(ii) The Heraclides of 9.8 is almost certainly not the famous Heraclides of Pontus who was a follower of Plato in the fourth century B.C. Much more probably he is a person named elsewhere as Heraclides Ponticus the Younger, who studied in Alexandria and was in Rome in the time of Claudius and Nero. He wrote poetry, and scholarly works on literature, grammar and the history of culture. For his relationship with another of our five authors see under (v) below.

(iii) Panaetius (9.9) is called 'The Younger' probably to distinguish him from Panaetius of Rhodes, who was head of the Stoa from about 129 B.C. The later Panaetius was evidently a mathematician, in view of the title of the work that Porphyry quotes, but nothing further is known about him.

(iv) About Ptolemaïs of Cyrene (9.10–9.12) we have no information at all outside Porphyry's work, and he says nothing about her, not even remarking on the fact that she was a woman. (The fact is striking: few female scholars, and no other female musicologists, are known to us from classical antiquity.) Even her date is a matter of conjecture, and might conceivably lie anywhere between the third century B.C. and the first century A.D.; a time near the end of this span seems to me the most probable (see also n. 133 below). Her work is a 'catechism', in the form of question and answer, and was perhaps intended as a school text (the catechism form is paralleled in the *Eisagoge* of Bacchius). Though Porphyry gives its title as *Pythagorean Elements of Music*, it is not at all clear that Ptolemaïs preferred the Pythagorean approach to others. If anything it is Aristoxenus' view that is most favourably presented.

(v) Didymus (9.13, 9.14) is also Porphyry's source for the passage quoted as 1.8, and possibly for the excerpts from Ptolemaïs as well. His modifications to the *kanōn* or monochord are discussed by Ptolemy (11 *Harm.* Book II ch. 13), who also records his scalar divisions for each of the genera (Book II ch. 14). Ptolemy mentions rather few of his predecessors by name, and the sum of the evidence suggests that Didymus' work in harmonics was substantial and well thought of. Ptolemy calls him 'Didymus the musician', almost certainly to distinguish him from the famous Didymus of Alexandria, an extraordinarily prolific scholar of the first century B.C. (This learned Didymus was nicknamed *Chalkenteros*, 'Brazen-guts', for his omnivorous scholarship, and *Bibliolathas*, 'Book-forgetter', on the grounds that his output was far too large for him to remember what he had previously written.) Our Didymus is probably the one said in the *Suda* to have lived in the time of Nero. He was a grammarian and a fine musician (the Didymus whose writings on Pythagorean philosophy are mentioned by Clement of Alexandria may be the same). According to the *Suda*, his father's name was Heraclides. It is an intriguing and plausible guess that this is the younger Heraclides Ponticus discussed in (ii) above. It is known that this Heraclides Ponticus studied in Alexandria with 'Brazen-guts' Didymus, and his son's name may be the pupil's little tribute to his master.

9.6 Aelianus the Platonist on concords: Porph. *Comm.* 96.7–15

Aelianus the Platonist, writing in his *On the Timaeus*, says the following, in exactly these words.

'Concord is the simultaneous incidence and blending [*krasis*] of two notes that differ in height and depth of pitch.[93] Of the concords, which are six in number,[94] the ancients called the fourth and the fifth "simple" and the remainder "composite". They are called "simple" because the others are constituted out of concords, but these ones are not.'[95]

9.7 Aelianus on the determinants of pitch: Porph. *Comm.* 33.16–37.5

Aelianus, in the second book of his *Commentary on the Timaeus*, attempted to defend this sort of view:[96] his words, which we shall transcribe exactly, are as follows.

'Sounds [*phōnai*] differ from one another in height and depth of pitch. Let us see, then, what are the principal causes of the difference between notes [*phthongoi*]. The principal cause of all sound is movement.[97] For if sound is air that has been struck,[98] the impact is a movement, and if it is the organ of hearing [being struck], as the Epicureans argue (when a quasi-sound [*paraphōnē*] comes from the sounds to the organ of hearing through the agency of certain fluxes), in this case too movement is the cause of the experience.[99] We shall therefore consider what difference there is in respect of movement, and what sort of movement is the cause of one sort of sound, and what sort is the cause of another. Our predecessors paid attention, first of all, to the phenomena, and taking their starting point from them they provided us with the answer we are seeking. For they found that swift movement is the cause of high-pitched sounds and slowness that of low-pitched: and this can be confirmed from the phenomena by anybody, by the use of the senses.[100] For if one takes two *auloi*, equal in the width of their bores, and blows into them

[93] Cf. 8 Eucl. *Sect. Can.* 149.20 with n. 7, 12 Arist. Quint. *De Mus.* 10.1–5 with n. 56.

[94] Aelianus is thinking of those within the two-octave system. Porphyry adds a parenthesis here, noting that Ptolemy too restricts them to six (11 *Harm.* 11.1ff.), while Aristoxenus (7 *El. Harm.* 45.6–7, cf. 20.23ff.) and others recognise eight.

[95] Cf., for example, 7 Aristox. *El. Harm.* 45.6–33. Like many other authors, Aristoxenus refers to the fourth and the fifth as the 'first' concords, the others as 'composite'.

[96] That is, the view that high and low pitch are associated with swift and slow movement respectively.

[97] Cf. especially 1.19 Archytas frag. 1, 8 Eucl. *Sect. Can.* 148–9.

[98] Cf., for example, 2.4 Plato *Tim.* 67, 5 *De Audib.* 800a. For this definition and a variant see 12 Arist. Quint. *De Mus.* 5.20–2.

[99] The Epicurean thesis is that since all that exists objectively is void and atoms in motion, 'sound' is a subjective or psychological category, and a movement is also a sound only in so far as it is heard. Hence the movements that cause the hearing of sounds are only 'quasi-sounds'. For an account of sensation which includes reference to sound and hearing see Epicurus *Letter to Herodotus* 46–53, translated in Long and Sedley (1987), pp. 72–4.

[100] Some references to passages for comparison with the discussion that follows are given at n. 35 above.

34 using the same breath, from a single power of breath there will be heard through the larger *aulos* a lower sound, and through the smaller a higher one. Since the breath runs through the smaller and strikes the surrounding air more swiftly, while it runs through the greater one more slowly, pushing forward the air contained in the larger *aulos*, it is evident that the sound through the *aulos* that is smaller in size is higher in proportion [*kata logon*], and that through the larger is slower. *Syringes*, too, show this fact clearly, when the little pipes [*auliskoi*] are made in sizes that are unequal in their lengths but equal in the widths of their bores. For the pipe that is smaller in length produces a very high note, and the greatest a lower one, while those in between sound proportionately [*analogountōs*]. Again, if you take two *auloi* equal in their lengths but differing in the widths of their bores, as are Phrygian *auloi* in comparison with Greek, you will find in a similar way that the wide-bored one emits a higher pitched note than the narrow-bored. We see, at any rate, that the Phrygian *auloi* are narrow in their bores, and project much lower sounds than the Greek ones.[101] Here again, then, the cause is in the speed of the movement, for in those with narrow channels the breath has a difficult passage, and is impeded by the smallness of the channel, so that its movement is slower, while in the one with a wider bore, since there is no obstruction, the passage of the breath through to the outside is swifter. The same thing can be observed with just one *aulos*. For the finger-holes [*trēmata*] have been devised to produce high and low notes. Through the finger-holes nearest the mouthpiece [*glōssis*], that is, those furthest up, the breath comes out more quickly into the air outside, and hence the note is higher, while the note produced through the finger-holes that are further away is lower, and hence lowest of all through the nethermost finger-holes. Thus if people want to produce a higher note, they open the upper finger-holes and close those further down,[102] while if they want to produce a lower note, they do the opposite.

The same thing will be observed on stringed instruments too. For the ancients made the *trigōnon*, which is also called the *sambykē*,[103] from strings unequal in length, with the longest of all on the outside, the one smaller than it next to it, while those still closer to the inside, lying near the angle of the

[101] Instruments called 'Phrygian' or '*elymoi*' *auloi* were distinguished by their low pitch, and by the fact that the left-hand pipe was considerably longer than the right, ending in a bell that was normally made of horn. See Athenaeus *Deipn.* 185a, Pollux *Onomastikon* IV.74 (*GMW* vol. 1, p. 272 and p. 267 n. 31). But Phrygia was commonly treated as the birthplace of the *aulos*, and the designation is often less specific. No other author, so far as I know, mentions this feature of the instruments' bores. The observation about their pitches may reflect the fact that a narrow cylindrical pipe, blown with a reed, will give the effect of a 'stopped pipe' (sounding a lower harmonic than an 'open pipe') more readily than one with a wide bore. The latter is inclined to behave more like the conical bore of a modern reed instrument.

[102] Closing the lower holes is of course usually unnecessary. Aelianus may merely be looking for symmetry of expression. Just possibly his remark is an inept reference to cross-fingering, or to the practice of opening a small hole near the top of the pipe to facilitate access to the higher harmonics (see 5 *De Audib.* 804a with n. 42).

[103] The *trigōnon* or *trigōnos* is a triangular harp. On problems about the identity of the *sambykē* see Athenaeus *Deipn.* 633f–634b (*GMW* vol. 1, pp. 292–3 with nn. 164, 166).

instrument, were still more curtailed in length. They made the strings of equal thickness, for differences in thickness were as yet not understood. The result was that when the shorter strings were struck they produced a higher note, and the longer ones a lower one. For in the longer strings the forward movement [*antistasis*] is slow, and so, similarly, is the recoil [*apokatastasis*] after the impact: hence the air, being struck slowly by the string, produces a low note. In the shorter strings the striking [*plēxis*] and the recoil are swift.[104] Later, people discovered that on strings of equal length the speed of the movement through the thicker strings is slower, and that through the finer ones is swifter. Although I could support the same point by many other demonstrations, I shall rest content with those mentioned, so as not to make my essay too long; for we have explained all these things accurately in more appropriate places.

Since swift movement is the cause of the note's being made high, and slow movement that of its being made low, it is apparent that the high note stands apart from the lower; and what distinguishes a higher note by comparison with a lower, or a lower by comparison with a higher, is called an interval. Not every high and low note struck simultaneously produce a concord, but in some pairs one or the other predominates,[105] so that our hearing apprehends both discordant and concordant mixtures: this is why we call what distinguishes a higher note from a lower an interval. Thus "interval" is defined as the difference between two sounds that are dissimilar in height and depth;[106] and an interval does not always have concordance too. It is of course possible for something to be both an interval and concordant at the same time, so that if something is concordant, it is also contained by an interval, but if something is an interval it is not always concordant. Concordance is the coincidence and blending of two notes that differ in height and depth of pitch.[107] For the notes, when they are plucked together, must produce some other species of note,[108] beside the notes from which the concord arose. It is like when someone wants to make oxymel, taking so much honey and so much wine: when he mixes them in such a way that neither the wine nor the honey predominates, but they are mixed in balanced proportion [*symmetria*], a third thing, a blend, arises, which is neither wine nor honey. In the same way, when a high and a low note are plucked, and present to the hearing one single blend, where neither of the two notes displays its own special character [*dynamis*], but a third thing resounds to the hearing beside the low and the high note, then it is called

[104] The notions of 'striking' and 'recoil' clearly belong to a theory in which an apparently continuous sound is made by a series of discrete impacts (though Aelianus makes nothing of it). See particularly 4.8, 4.23, 4.24 ps.-Ar. *Probs* XI.19, XIX.37, 39, 5 *De Audib.* 803b–804a, 8 Eucl. *Sect. Can.* 148–9, 9.8 Heraclides ap. Porph. *Comm.* 31.

[105] For the thesis that in a concord neither note predominates see 12 Arist. Quint. *De Mus.* 10.2–3, cf. 3.12, 3.13 Aristotle *De Sensu* 447a–b, 448a, 6 Theophrastus ap. Porph. *Comm.* 63.15ff.

[106] See n. 5 above.

[107] This sentence appears also in 9.6 above.

[108] Most authors are careful not to use the word 'note' (*phthongos*) of the special sound produced jointly by the two notes of a concord. But compare the analogy with colour at 3.11 Aristotle *De Sensu* 439b.

"concordant". But if the hearing apprehends the lower note, or the higher, to a greater degree, this sort of interval is non-concordant [*asymphōnon*].'

That is what Aelianus says.

Since the Pythagoreans located the concords in numerical ratios (epitritics, hemiolics, duples and others of that sort), in a way that we shall describe accurately in our discussion about the concords, when he [Aelianus] is explaining how the movement that makes the high or the low note can be measured, he writes as follows.

'Since we have demonstrated that a swift movement produces a high note and a slow movement a low one, it is clear that the movement, or the speed of the movement from which the high note arises is in epitritic ratio with the movement, or the speed of the movement from which the low note arises.[109] But to ensure that nothing has been passed over, we shall also clarify what is meant by saying that the speed of one movement is in epitritic or duple or any other ratio to that of another. For if there are two things moving at unequal rates, and one of them, during the same period of time, travels at double the speed of the other, the distance [*diastēma*] traversed by the one that moves more swiftly will be double that of the other, so that the distance traversed by the one moving more swiftly is ten feet, for instance, while the other distance is five feet. That is what is meant by its having double the speed.[110] The exposition of a comparison of speeds can be understood in another way too. Suppose that the same distance, say ten stades, is traversed by the thing moving more swiftly in two hours, and by the one moving more slowly in four hours. Then the ratio of the time in which the one moving more slowly traverses the ten stades to the time in which the one moving more swiftly traverses the same distance – that is, the ratio of four hours to two – will be the ratio, conversely, of the speed of movement of the one moving more swiftly to the speed of movement of the one moving slowly. Now since it is of the nature of times [i.e., durations of time] to be continuous, and since the distances traversed by the things in motion, that is, the magnitudes, also belong to the class of continuous things, it is plain both that the times compared with one another are things of the same kind [*homogeneis*], and that the distances traversed are of the same kind too – just as are straight lines in relation to circumferences of circles.[111] And since the division of continuous things can proceed to infinity, some of them are commensurable, others incommensurable: the commensurable are grasped through a ratio of numbers, while the incommensurable are

[109] Aelianus has evidently been discussing the concord of a fourth in the passage before this quotation begins.

[110] For difficulties invited by this theory see 3.14 Arist. *De Sensu* 448a, 6 Theophrastus ap. Porph. *Comm.* 63.20ff.

[111] The requirement of 'homogeneity' arises from the Euclidean definition of ratio (*Elements* Book V Def. 3), commonly echoed in later sources: cf., for example, Theon Smyrn. 73.16–18, 'a ratio is...the character of the relation that two terms of the same kind (*homogeneis*) have to one another, such as double or triple'. A similar point is discussed in relation to 'blending' or 'mixture' at 3.12 Arist. *De Sensu* 447a–b.

not in ratios of numbers. The same fact must be understood in relation to speeds, and also that some of them, too, are commensurable while others are not. Where the comparison of speeds is expressed in a commensurate relation, the speeds have a ratio to one another, the ratio of a number to a number.'[112]

9.8 Heraclides: Porph. *Comm.* 30.1–31.21

Heraclides also writes about these things in his *Musical Introduction*, as follows.

'Pythagoras, as Xenocrates says, discovered that the intervals in music, too, do not arise in separation from number: for they are a blending [*synkrisis*] of quantity with quantity.[113] He therefore investigated the conditions under which there arise concordant and discordant intervals, and everything that is well-attuned and ill-attuned [*hērmosmenon* and *anharmoston*]. And turning to the way in which sound comes about, he said that if something concordant is to be heard, arising from an equality, some movement must have occurred. Now movement does not arise without number, and neither does number without quantity.[114] He says that there are two forms of movement, one being alteration, the other locomotion. There are also two forms of locomotion, circular and rectilinear.[115] Of circular movement, one kind travels from place to place, as do the sun and the moon and the rest of the stars, while the other does so in a fixed place, as do cones and spheres that move around their own axis. Of rectilinear locomotion there are several forms, about which it is unnecessary to speak now.

Let us postulate, then, he says, that the motion to do with notes is a kind of motion from place to place, travelling in a straight line as far as the organ of hearing. But when an impact has occurred outside, a sound travels from the impact until it comes to the organ of hearing. When it arrives it moves the hearing and makes in it a perception.[116] The impact, he says, is in no [duration of] time, but in a boundary between time past and time to come. For it is not when someone is setting himself to strike the blow that the impact has occurred, nor when he has finished; rather, the impact is in what is between the time to come and the time past, and is as it were a cutting and demarcation of time. Just as when a line cuts a plane, he says, the line is in neither of the two

[112] This implies that for Aelianus a relation is 'commensurate' if it is expressible as a ratio of integers, no matter what form (multiple, epimoric, epimeric) the ratio takes: see n. 15 above.

[113] It is likely (but uncertain) that only the first sentence of this passage is taken from Xenocrates, who was the third head of the Academy (from 339 to 314 B.C.). Reports of his doctrines make clear his interest in and development of a form of Pythagoreanism: see also n. 20 to 10 Nicomachus *Enchiridion*. Some of what follows may have its source in fourth-century reports about Pythagoreans, but there are no good grounds for thinking that it goes back to Pythagoras himself.

[114] Cf. especially 1.19 Archytas frag. 1, 8 Eucl. *Sect. Can.* 148–9.

[115] Both distinctions are characteristic of Aristotle; for the latter see also Plato *Laws* 839c.

[116] Cf., for example, 3.15 Arist. *De Anima* 419b–420a, 5 *De Audib.* 800a, 802a.

planes but is a boundary of both of them, so too the impact is at the "now", and is in neither of the times, neither the time past nor the time to come.[117] But the impact appears to occur, he says, in some time that is imperceptible because of the weakness of the hearing, like things that we see to be the case with respect to sight.[118] For often when a cone is in motion, and there is on the cone one white or black spot, it appears that there is a circle on the cone, of the same
31 colour as the spot. And again, if there is a single white or black line on the moving cone, the whole surface appears to be of the same colour as is the line. Similarly, the spot does not appear to be even one of the parts of the circle, nor the line of the surface: the sense of sight cannot achieve accuracy in such a case.

The same sort of thing happens, he says, in connection with hearing as well: indeed the hearing is in even greater confusion than the sight. For if one stretches a string, he says, and plucks it, and allows it to resound, the result will be that notes are heard, while the string continues in its vibratory movement, bending back and forth in the same place, in such a way that its movement is more clearly perceived by sight than by hearing. With every impact in which the air is struck by it, it is inevitable that a sound impinges again and again on the hearing. But if this is so, he says, it is plain that each of the strings emits several notes [*phthongoi*]. Then if each note occurs in the impact, and if it is the case that an impact occurs not in a time but in a boundary of time, it is clear that in between the impacts that correspond to the notes there must be silences, which exist in a time [i.e., a duration]. But the hearing does not perceive the silences because silences are not capable of moving the hearing, and also because the gaps [*diastēmata*] are small and cannot be apprehended. The notes, because they are closely successive, create the appearance [*phantasia*] of a single sound stretched out over some amount of time.'[119]

[117] On the notion of a 'now' that is a limit and not a duration of time see Aristotle *Physics* 218a, cf. 239b.

[118] In the sequel (explicitly at 31.15–16) Heraclides reverts to the thesis of the preceding lines, which looks incompatible with that of this sentence. But we are told that the example of the spinning cone is to be understood in the light of the second theory. Heraclides appears to vacillate, but possibly relies on the contrast between 'being' and 'seeming'. The impact seems to occupy an imperceptible duration of time, but really occurs at a 'now' which has no duration. But then it is unclear in what an 'imperceptible seeming' could consist. With the analogy of the spinning cone compare 5 *De Audib.* 803b.

[119] Of the Greek authors who rely on the thesis that sounds are created by discrete impacts, several allude to the fact that higher pitches are associated with more frequent impacts (see n. 104 above). Only one (8 Eucl. *Sect. Can.* 148–9) plainly argues that it is greater frequency of impact that causes or constitutes higher apparent pitch (see also nn. 39–40 to 5 *De Audib.* 803b–804a). Heraclides does not explicitly make even the first of these claims, though he presumably accepts it. His language is incompatible with the second, since he refers to each impact as a 'note', whereas the theory of the *Sectio* would entail that more impacts than one are required to give a sound a determinate pitch, and so constitute it as a note. Heraclides is concerned here only with the relation between the fact of discrete impacts and the phenomenon of continuous sound, not with issues to do with pitch.

9.9 Panaetius: Porph. *Comm.* 65.21–67.10

Panaetius the Younger, in his *Concerning the Ratios and Intervals in Geometry and Music*, has spoken concisely about these matters, giving a well reasoned defence on behalf of the older writers, and instruction for the procedure that is based on numbers.[120] He writes as follows.

'In music the designation "half-tone" is a misuse of language. For someone who thinks that the interval between high and low pitch is divided in half by a note in the middle is like one who says that the interval between white and black or between hot and cold is divided in half. For the investigation that deals with concordances is not based on the magnitudes of the notes, but on their qualities.[121] When specialists in mathematics say that the octave is in duple ratio, they do not mean that the magnitude of the note *nētē* is double the magnitude of *hypatē*, or vice versa. There is evidence for this in the fact that
66 if they strike the strings vigorously, or if they strike one string more powerfully and another less, the interval is still the same, but the string struck more powerfully produces a greater sound: hence it appears wrong to say that the interval depends on magnitude.[122] If then it depends on qualities, what is meant by saying that the octave is in duple ratio, the fourth in epitritic [4:3], the fifth in hemiolic [3:2], the octave and a fifth in triple, and the double octave in quadruple ratio? Since sight is not strong enough to assess the relative dimensions of magnitude unless some measure is found, by which the relative dimensions are naturally suited to being measured and assessed, and neither is the sense of touch strong enough to assess a comparison in respect of weights, unless a weighing device is found by which the weights are assessed, it seems absurd to suppose that hearing, which is much feebler than sight, can assess concordant intervals without some *kanōn* as a measure.[123] For those who

[120] This passage is quoted by Porphyry immediately after the fragment of Theophrastus translated in chapter 6 of this volume. Porphyry takes Panaetius, like Theophrastus, to be attacking ('instructing' or 'correcting') quantitative theories of pitch, and this is indeed part of his intention, but he is prepared to allow some point to 'canonic' procedures. His main concern in this excerpt is to argue that there is no sense in the notion of a 'half-tone', either from a purely auditory point of view or from that of 'canonics'. The latter point is familiar from, for example, 8 Eucl. *Sect. Can.* propositions 3 and 16; with the former compare 9.3 Adrastus ap. Theon Smyrn. 72.

[121] The claim seems to be that in hearing something as a concord we do not hear its components in the character of quantities, or as standing to one another in a quantitative relation. Rather, we are struck by an aesthetic relation between perceived qualities. The approach is in keeping with Aristoxenian theory, but even writers of the mathematical or 'Pythagorean' tradition had to use the qualitative perceptual 'given' as their starting point, in order to identify the cases to be subjected to quantitative analysis. See, for example, 8 Eucl. *Sect. Can.* introduction and propositions 10–12. For an attempt to attach a quantitative characterisation to the perceptual 'given' as such, see 9.3 Adrastus ap. Theon Smyrn. 65–6.

[122] For subtler theorists this point presents no obstacle (cf., for example, 11 Ptol. *Harm.* Book 1 ch. 3), but some, beginning from 1.19 Archytas frag. 1, had certainly found difficulty in distinguishing the determinants of pitch from those of volume: cf. 3.17 Arist. *De Gen. An.* 786b–787a.

[123] Cf. 11 Ptol. *Harm.* Book 1 ch. 1. Panaetius does not explain clearly how he is using the point. One might expect the claim that since concords can be identified by ear and these

concentrate on perception, as though listening to a sound from next door,[124] seem like people who make assertions about the relative dimensions of magnitudes without the aid of any measure, and they miss the truth by a long way.

There was a great deal of investigation,[125] beginning initially with the Pythagoreans, and subsequently among specialists in mathematics, as to which ratios they are, in the concordant intervals, in which from notes differing in quality there arises a single blend even when just one of the two strings is plucked, the one which is naturally moved in concordance.[126] They enquired whether these intervals too correspond to the smallest ratios.[127] Thus, with different people among our predecessors investigating the problem by different methods on what is called the *kanōn* – which I take to have acquired this name because it is the criterion for the hearing of the quantity involved in the concords[128] – they discovered that when a string has been stretched and the bridge is placed under it at the halfway point, the whole is concordant with the half at the octave: when it is placed at a quarter of the string the whole is in concord with the three parts at the fourth, and with the quarter at the double octave: when it is placed at the third of the string the whole is in concord with the two parts at the fifth, and with the third at the octave and a fifth: and the tone is in epogdoic ratio [9:8], because the whole makes its special interval in relation to the eight parts.[129] Hence when they say that the octave is in duple
67 ratio, they do not mean that the one note is double the other, but that the strings from which there arise the notes making the octave have this ratio; and so on for the others.[130] In the case of the epogdoic interval there is no mean proportional in numbers, and in canonic theory they deny that the tone can be

quantitative relations cannot, the relation of concord cannot be quantitative. But the next sentence seems to deny that concords can be accurately identified by ear. His thesis is rather that concord, as experienced through the hearing, is essentially a vague, inexact relation, but that there are precise quantitative relations underlying it (not constituting it), which the ear cannot detect alone. It is the task of 'canonics' to demonstrate them (see the next paragraph). Panaetius' view and Ptolemy's differ in emphasis, and Ptolemy never concedes that pitch-relations as heard are purely qualitative, but their positions on the respective roles of hearing and scientific measurement (using the sense of sight) are not dissimilar.

[124] A reminiscence of 2.1 Plato *Rep.* 531a.

[125] I would judge that this paragraph too comes from Panaetius. Düring (1932 and 1934) assigns it to Porphyry himself.

[126] On the notions of sympathetic vibration and of 'blending' in the context of concordance see n. 17 above.

[127] The word 'too' is mysterious. Possibly Panaetius had previously discussed small-number ratios in the context of geometry, as the title of his work suggests. An explicit emphasis on small-number ratios, as such, is unusual in this connection (though of course it is these that are in fact used), but compare n. 16 above and 11 Ptol. *Harm.* Book 1 ch. 7.

[128] Cf. 9.10 Ptolemaïs ap. Porph. *Comm.* 22, and the opening sentence of 11 Ptol. *Harm.* Book 1 ch. 2.

[129] Cf., for example, 9.2 Adrastus ap. Theon Smyrn. 57, 59–60.

[130] Panaetius sharply distinguishes the causal determinant of pitch from the pitch itself; something a little similar is in view at 6 Theophrastus ap. Porph. *Comm.* 63.1ff. But the distinction is not peculiar to those who deny that pitch relations are quantitative: see 9.3 Adrastus ap. Theon Smyrn. 65–6, and cf. 9.4 Thrasyllus ap. Theon Smyrn. 87 on the notion of 'converse qualification'.

divided in half.[131] Hence neither when people refer the matter to qualities nor in canonic theory is the "half-tone" half of the tone, but the term is merely a misuse of language, as in the case of the "semivowel" [lit. "half-vowel"] and the "half-ass" [mule]. For here too there is not a half of a vowel or a half of an ass.[132] So as to the fact that talk of the double and the triple ratio is also a misuse of language, I think that what we have said is sufficient.'

9.10 Ptolemaïs on schools of harmonic theory, first extract: Porph. *Comm.* 22.22–23.22

Ptolemaïs of Cyrene also writes about this matter [i.e., the use of the *kanōn*] in her *Pythagorean Elements of Music*, as follows.[133]

'The science of *kanonikē* ["canonics"] – of whom is it mainly characteristic? In general, of the Pythagoreans; for what we now call *harmonikē* ["harmonics"] they used to name *kanonikē*.[134] From what do we derive the term "*kanonikē*"? Not, as some people think, by transference from the instrument called the *kanōn*, but from straightness, on the grounds that it is through this science that reason [*logos*] discovers what is correct, and discovers the *parapēgmata* of what is well attuned.[135]

23 They also call "*kanonikē*" the investigation that employs *syringes* and *auloi* and the rest, though these are not strictly canonic;[136] but they call them "canonic" too, because the ratios and the theorems fit them. Hence it is rather that the instrument was named "*kanōn*" by derivation from the science of *kanonikē*. A *kanonikos*, in general, is a harmonic theorist [*harmonikos*] who constructs ratios in connection with attunement [*to hērmosmenon*]. *Mousikoi*

131 See, for example, 8 Eucl. *Sect. Can.* propositions 3 and 16.

132 Cf. 9.2 Adrastus ap. Theon Smyrn. 53.

133 The passages from Ptolemaïs and Didymus which complete this chapter should be considered as a group. Düring (1934) thought it likely that Porphyry found the quotations from Ptolemaïs in Didymus' work. Didymus himself takes over the distinctions between schools of musical theory identified by Ptolemaïs and elaborates them further. If Düring is right, and if this Didymus has been correctly identified in the introduction to this chapter, Ptolemaïs can be no later than the early first century A.D., and is perhaps a generation or two earlier. Didymus may have been introduced to her writings through his links with the scholars of Alexandria. On the assumption that readers can easily make their own comparisons, I have not provided detailed cross-references between their discussions in the notes that follow. In this excerpt Porphyry is commenting on the first sentence of 11 Ptol. *Harm.* Book I ch. 2.

134 We do not know how old this use of the term is; Ptolemaïs is probably the earliest extant source to use it in this way.

135 The instrument is of course the monochord. A *kanōn* was originally a ruler, used to judge straightness and to measure lengths: cf. 9.9 Panaetius ap. Porph. *Comm.* 66.22–3, and the first sentence of 11 Ptol. *Harm.* Book I ch. 2. The adjective *kanonikos* was used to mean 'regular', 'according to rule' (the Epicureans used its neuter form, *kanonikon*, to mean 'logic'). A *parapēgma* is literally something 'fixed beside'. The word sometimes referred to a kind of calendar on which indications of astronomical and meteorological events were inscribed. Beside these indications moveable pegs were placed, marked with the days of the month. Ptolemaïs seems to be referring to the marks that would be made on the ruler or *kanōn* that was placed alongside the string, to show the points of division (cf., for example, 11 Ptol. *Harm.* 18.22–19.1). The word does not occur in this connection elsewhere.

136 Cf., for example, the opening of 11 Ptol. *Harm.* Book I ch. 8.

and *kanonikoi* are different; for "*mousikoi*" is the name given to the harmonic theorists who begin from perceptions, while "*kanonikoi*" is that given to the Pythagorean harmonic theorists. But each are in the generic sense *mousikoi*.'[137]

To this she adds, in the form of question and answer once again:

'The theory that uses the *kanōn* – of what does it consist? Of the things postulated by the *mousikoi* and those adopted by the *mathēmatikoi*. The things postulated by the *mousikoi* are all those adopted by the *kanonikoi* on the basis of perceptions, for instance that there are concordant and discordant intervals, and that the octave is compounded from the fourth and the fifth, and that the excess of a fifth over a fourth is a tone, and similar things.[138] Those adopted by the *mathēmatikoi* are all those which the *kanonikoi* study theoretically in their own special way, only beginning from the starting points given by perception, for instance that the intervals are in ratios of numbers, and that a note consists of numbers of collisions, and other things of the same sort.[139] Hence one might define the postulates of *kanonikē* as lying both within the science concerned with music, and within that concerned with numbers and geometry.'

9.11 Ptolemaïs, second extract: Porph. *Comm.* 23.24–24.6

Concerning these matters Ptolemaïs writes as follows in the introductory treatise [*eisagōgē*] mentioned above.[140]

'Pythagoras and his successors wish to accept perception as a guide for reason at the outset, to provide reason with a spark, as it were; but they treat reason, when it has set out from these beginnings, as working on its own in separation from perception. Hence if the *systēma* discovered by reason in its investigation no longer accords [*synaidei*] with perception, they do not retrace their steps, but level accusations, saying that perception is going astray, while reason by itself has discovered what is correct, and refutes perception.[141]

[137] That is, the adjective *mousikos* has both the general sense 'musical theorist' and the specific sense in which it picks out those who 'begin from perception', by contrast with those of a 'Pythagorean' persuasion. The former group includes Aristoxenians, but the distinction is not altogether straightforward. It is elaborated and complicated in the sequel and in 9.11 and 9.12.

[138] Some such postulates, extrinsic to mathematics and grounded in perception, are plainly required in any form of canonics. In 8 Eucl. *Sect. Can.*, for instance, the first of those mentioned here is used in the introduction (149.17ff.) and frequently in propositions 10ff., the second implicitly in proposition 11 and explicitly in proposition 12, the third in proposition 13. This sentence and the remainder of the paragraph may be from Porphyry's pen, rather than that of Ptolemaïs.

[139] Both these principles are set out in 8 Eucl. *Sect. Can.* 148–9. The special focus of the *Sectio* on the thesis that 'numbers of collisions' are determinants of pitch (cf. n. 119 above) suggests that it may be this treatise that the author has in mind. The 'theoretical studies' characteristic of the *mathēmatikoi* are exemplified in propositions 1–9.

[140] Porphyry is commenting on the second and third sentences of 11 Ptol. *Harm.* Book 1 ch. 2. What he quotes here from Ptolemaïs is repeated almost word for word in the latter part of 9.12.

[141] On conflicts between perception and the conclusions of Pythagorean canonics see especially 11 Ptol. *Harm.* Book 1 ch. 6.

An opposite position to this is held by some of the *mousikoi* who follow Aristoxenus, those who applied themselves to a theoretical science based in thought, while nevertheless setting out from expertise on instruments. For they treated perception as authoritative, and reason as attending on it, for use only when needed. According to these people, to be sure, it is only to be expected that the rational postulates of the *kanōn* are not always concordant with the perceptions.'[142]

9.12 Ptolemaïs, third extract: Porph. *Comm.* 25.3–26.5

Concerning these matters[143] Ptolemaïs of Cyrene wrote briefly in her introductory treatise, and Didymus entered on them at greater length in his *On the Difference Between the Aristoxenians and the Pythagoreans*. We shall write out what each of them says, altering a few things for the sake of brevity. Ptolemaïs, then, writes as follows.

'What is the difference between those who are distinguished in the field of music? Some preferred reason by itself, some perception, some both together. Reason was preferred by those of the Pythagoreans who were especially keen on disputing with the *mousikoi*, arguing that perception should be thrown out completely, and that reason should be brought in as an autonomous criterion in itself. These people are wholly refuted by their practice of accepting something perceptible at the beginning, and then forgetting that they have done so.[144] The instumentalists [*organikoi*], on the other hand, preferred perception: they gave no thought at all, or only feeble thought, to theory.[145]

What is the distinction between those who prefer the combination of both? Some accepted both perception and reason in the same way, as being of equal power, while others accepted the one as the leader, the other as the follower. Aristoxenus of Tarentum accepted both in the same way. For what is perceived cannot be constituted by itself apart from reason, and neither is reason strong enough to establish anything without taking its starting points [*archai*] from perception, and delivering the conclusion of its theorising [*theorēma*] in agreement with perception once again.[146] In what way does he want perception to be in the lead of reason? In order [*taxis*], not in power [*dynamis*]. For when

[142] The identity of these people is unclear and the description does not fit with that of Aristoxenus in 9.12. Ptolemaïs may be referring to those who are there called *organikoi* (see n. 145 below). See also the beginning of Didymus' discussion in 9.13.

[143] Here Porphyry's commentary deals with what is left of 11 Ptol. *Harm.* Book 1 ch. 2 after the sentences to which 9.10 and 9.11 are directed.

[144] Cf. notes 138 and 141 above.

[145] The comment is reminiscent of things said by Aristoxenus about the predecessors he sometimes calls *harmonikoi*: see, for example, 7 *El. Harm.* 32.28–31, cf. 2.1 Plato *Rep.* 531a–b. (This whole paragraph of Ptolemaïs may be based on 7 *El. Harm.* 32.19–31.) But if their description as *organikoi* reflects anything in Aristoxenus, it is probably *El. Harm.* 41.25ff., and it is not clear that Aristoxenus is referring to the same people there as in 32.28; see also n. 151 below.

[146] The theme of the primacy of perception is pervasive throughout 7 Aristox. *El. Harm.* On *archai* ('starting points' or 'principles') and their dependence on perception see especially 43.30–44.20, and on the roles of reason see 33.1–34.31, 38.27–39.3.

the perceptible thing, whatever it may be, has been reviewed by perception, then, he says, we must put reason in the lead, for the theoretical study of this percept.[147] Who are those who treat both together alike?[148] Pythagoras and his successors.[149] For they wish to accept perception as a guide for reason at the outset, to provide reason with a spark, as it were; but they treat reason, when it has set out from these beginnings, as working on its own in separation from perception. Hence if the *systēma* discovered by reason in its investigation no longer accords with perception, they do not retrace their steps, but level accusations, saying that perception is going astray, while reason by itself has
26 discovered what is correct, and refutes perception. Who are in opposition to these? Some of the *mousikoi* who follow Aristoxenus, those who applied themselves to a theoretical science based in thought, while nevertheless setting out from expertise on instruments. For they treated perception as authoritative, and reason as attending on it, for use only when needed.'

9.13 Didymus on schools of harmonic theory, first extract: Porph. *Comm.* 26.6–29

Didymus, working out these subjects in detail, writes as follows.[150]

'In general, then, of those who entered upon the study of music, some concentrated on perception alone, ignoring reason completely. I do not mean that they made their perceptual judgement altogether in separation from reason, or without conforming to certain rational principles [*logoi*] inherent in things, but that as far as was at all possible they offered no demonstration [*apodeixis*][151] and referred nothing to reason, and had no thought of a coherent theory, but were satisfied to rely on nothing but the perceptual procedure [*tribē*] they had acquired through habituation. This approach was characteristic of the instrumentalists [*organikoi*] and the vocal trainers [*phōnaskikoi*], and in a word all those who even now are commonly said to make use of a non-rational procedure [*tribē*]. Those who set out in the opposite way to them esteemed reason most highly as a judge, and no longer attended to perception in the same manner, but only to the extent that it suffices to give a starting point – that derived from perceptibles – so that reason might keep watch from there onwards. These are the Pythagoreans. For they adopt certain kindling sparks in the case of each matter, and construct the theorems that are put together out of these through reason on its own, taking no further notice of perception. Hence on occasions when only what follows rationally is carefully preserved,

147 This may reflect 7 Aristox. *El. Harm.* 33.6–9.

148 The text may have suffered some garbling here. It may be due to Porphyry himself, who has said that he will 'alter a few things for the sake of brevity' in these quotations. Ptolemaïs has stated that some accept perception and reason as equal in power, and has identified this view explicitly with Aristoxenus. He was described as 'accepting both in the same way'. Here we would expect a reference to those who use both but not in the same way, treating 'one as leader, the other as follower'.

149 The passage from here to the end is repeated almost verbatim from 9.11.

150 The quotation comes in Porphyry immediately after 9.12.

151 This is certainly an echo of 7 Aristox. *El. Harm.* 32.28–31 (see n. 145 above).

and perception bears witness against it, it is possible for them to be not in the least embarrassed by this sort of discord, but to pin their faith to reason and dismiss perception as going astray. And they accept the things that find favour with people who draw upon practical experience only when they do not bear witness against reason.'[152]

After supporting what he says with several more pieces of evidence, which we shall use more appropriately later, he adds: 'And there are others who give a place to both perception and reason, but who give some sort of precedence to reason: one of these is Archestratus.'[153]

9.14 Didymus, second extract: Porph. *Comm.* 27.17–28.26

'There remains the approach characteristic of those who give an equal place
to both criteria, perception and reason: the same approach belongs to those
who sometimes give precedence to perception over reason, which is the
20 approach of Aristoxenus. For he brings to the judgement of perception those
of his theoretical propositions that are evident [to perception], but to that of
reason those theoretical propositions that are demonstrated; and he says that
perception alone is the criterion of the former kind, reason that of the latter.[154]
He says that there is never any interchange between them, and that each of
these criteria has equal power, within its own proper domain. But when that
which is constituted from both is being considered, then, he says, perception
takes the lead and reason follows it in order. For he says that we begin from
28 the things that are evident to perception [*ta phainomena*], and attach to them
their consequences [*ta symbebēkota*] according to what follows by reason,
these consequences being in agreement with the things that are evident to
perception and never contradicting them.[155] For here, he says, it is impossible
for reason to give reasons [*aitiologēsai*] for what is evident to perception:[156]
hence we must pretty well rely on that, in its own right. But we investigate the
consequences in accordance with what agrees with perception, and adopt a
conclusion such that it is once again harmonious [*synaidon*] with perception.

152 See n. 141 above.

153 Not much is known of Archestratus. A characteristic sneer by the waspish Philodemus (first century B.C.) gives a little information: 'The followers of Archestratus, who say that issues about the nature of sound and notes and intervals and similar things are philosophical aspects of music, were quite intolerable, not only because they set off into a thoroughly extraneous domain of theory, and babbled childishly about these things with no benefit to knowledge, but also because they represented music as no more than the study of these matters' (Philodemus *De Mus.* 91). These remarks show plainly that Archestratus was a devotee of musical theory rather than practice, and that he was no Aristoxenian. Taken with what Didymus says, they suggest that his approach to harmonics may have been closer to those of Archytas, Eratosthenes and Didymus himself than to the Platonising branch of Pythagoreanism: see the Appendix to chapter 1.

154 Nothing in Aristoxenus' extant writings quite corresponds to this. The nearest is perhaps 7 *El. Harm.* 43.30ff.: cf. n. 146 above. The 'theoretical propositions that are demonstrated' will include those argued for in the theorems of *El. Harm.* Book III.

155 Once again, 7 *El. Harm.* 43.30ff. (especially 44.2ff.) is the closest parallel. Cf. also 11 Ptol. *Harm.* 5.13–15.

156 Cf. 7 *El. Harm.* 44.14–15.

This is why he instructs us to make each of the criteria as accurate as possible.[157] For he says that each thing he will say is such as is evident to him through perception, and he never thinks it proper for reason to make any postulate with which perception will not agree. For he says that music is not only a rational [*logikos*] branch of learning, but is perceptual and rational at the same time, and hence that it is necessary for the genuine student not to neglect either of the two, while putting what is evident to perception in first place, since it is from there that reason must begin. For a geometer can take a curved line on his drawing-board and postulate that it is straight, and can complete his theorem without any hindrance, since he is not concerned to persuade the eye about what is straight, since the subject matter which he investigates is in the domain of reason. But a student of music who postulates that something which is not a fourth is a fourth cannot consider anything correctly, since it [i.e., the supposed fourth] ought to be made to agree with perception too, and reason should attach, in addition, that which follows in accordance with what is perceptually evident; and hence when this [the supposed fourth] is incorrectly viewed by perception, reason too will go astray from the truth.[158] This, then, is the character [*tropos*] of the Aristoxenian criteria, as is clear to those who have studied his work, and especially from the points he puts forward in person in the introduction to the first book of his *Harmonic Elements*.[159] So let that complete our account of the difference between the criterion of the Pythagoreans and that of Aristoxenus, an account that has also related, in a more general way, the postulates of other musical experts about these matters.'

157 See 7 *El. Harm.* 33.9–10.

158 The geometrical comparison and its sequel are loosely based on 7 *El. Harm.* 33.10–26.

159 The references given above are all to passages in what we know as the second Book of *El. Harm.* It is likely that this is what Didymus (or Porphyry, for these sentences may well be his) is calling the first book, though there are remarks in our Book 1, for example 9.2ff., that have a bearing on these matters. On problems concerning the relations between the parts of Aristoxenus' extant work see the introduction to chapter 7.

CHAPTER 10

Nicomachus

Nicomachus belongs to roughly the same period as Adrastus and Theon. His exact dates are not known, but he was probably active around the beginning of the second century A.D. About his life the only clear fact we have is that he was born in Gerasa (even that is more problematic than it might seem, since there were several cities with that name). He drops a few autobiographical hints in the essay translated here, but they add up to very little. The work for which he is best known is his *Introduction to Arithmetic*. Though modern commentators agree that it shows him as no more than a second-rate mathematician, the book is a useful compendium of Greek studies in number theory. Through a Latin translation by Apuleius in the second century A.D., it became the Romans' principal source of knowledge about Greek achievements in this field. Among Nicomachus' other writings were an introduction to geometry and a biography of Pythagoras; both are lost. He also wrote a treatise called *Theologoumena Arithmeticae* ('Theology of Number') of which we have only a cursory summary by Photius, but an anonymous work with the same title survives, much of which is certainly derived from Nicomachus. His essay was evidently an elaborate presentation of Pythagorean theories about the symbolic and mystical significance of numbers, such as are set out much more briefly in 12 Arist. Quint. *De Mus.* Book III ch. 6.

The *Enchiridion* or 'handbook' of harmonics has the honour of being the only work on the subject to have survived complete from the period between Euclid and Ptolemy. Like the *Introduction to Arithmetic*, it is not an intellectually distinguished or original piece, but it had an important influence on later writers (notably Boethius and Bryennius, who quote it extensively). It is, by its own statement, a mere 'introduction' to harmonics, and promises a fuller treatment at a later time. We do not know whether the larger work was completed: some extracts that may be drawn from it exist in a collection of MSS pages printed by Jan in *MSG* under the title *Excerpta ex Nicomacho*.

The programme of the *Enchiridion* differs markedly from that of any Aristoxenian harmonic treatise, and has little in common with our earliest intact Pythagorising specimen, the *Sectio Canonis*. By the standards of either, it is eccentric and incomplete, and though it includes what are now valuable if obscure accounts of some non-standard scalar systems, it makes no significant original contributions to harmonic theory. For these and other reasons one modern scholar (Levin 1975) has concluded that it is not properly considered as a treatise in harmonics at all, but rather as a piece of propaganda for the Pythagorean world-view in general. She takes Nicomachus' presentation of it in the guise of an elementary manual of harmonics to be little more than a pretext.

This interpretation has something to be said for it, but I think it is exaggerated. The work has a clear pattern of development, different from those mentioned above, but perfectly appropriate to an essay of the kind it purports to be. It is true that the material is given a thoroughly Pythagorean slant, though not one that is rigorously mathematical in Euclid's style. It appears not only in the author's insistence on basing harmonics in physical acoustics, in his use of the language of ratios, and in his emphatic references to Pythagoras himself, but also in his attempts to link the foundations of musical 'harmony' to those underlying the workings of the universe at large. Such an approach is to be expected from the writer of the *Theologoumena Arithmeticae*. Nicomachus'

reverence towards number and his belief in its role as a divine principle are evident, even though he makes no use here of the numerological symbolism found in the 'theological' treatise. But like the writings of Adrastus (9.2–9.3), the *Enchiridion* draws heavily (without acknowledgement) on Aristoxenian forms of analysis too, 'correcting' them in certain instances, and unrolls quite systematically as an exposition of the simplest and most basic structures that Aristoxenus himself identifies, re-expressed and explicated in Pythagorean terms. There are admittedly many concepts and structures of which it says nothing. These, it can fairly be assumed, are reserved for the fuller treatment which it promises, and some of them are hinted at in the final chapter. The topics it does pursue do not differ much in scope from those addressed by Adrastus.

A brief summary of the subjects considered will give the gist of its design. After a short introduction (chapter 1), Nicomachus first discusses the distinction between continuous and intervallic vocal movement, and the kinds of 'space' within which each occurs. His material is drawn from Aristoxenus (7 *El. Harm.* 8.13ff.), with modifications that import little but confusion (chapter 2). Chapter 3 introduces the thesis that music as we know it is derived from that of the heavens, and sketches a planetary scale of seven notes, consisting of two tetrachords in conjunction. Chapter 4 gives an account of the causes of sound and pitch, designed to justify the doctrine (already touched on in the celestial context in chapter 3) that notes differ and are related to one another 'in accordance with number', adducing in evidence the acoustic properties of instruments, which it distinguishes into three groups.

These early chapters have set the scene for the development of harmonic investigations in the Pythagorean style. In the next five, Nicomachus gradually introduces and confirms an analysis of the simplest and most important extended harmonic structure, the diatonic octave system. He begins by explaining (chapter 5) that the system to be analysed is not the seven-note scale of chapter 3, but an eight-note scale whose invention he attributes to Pythagoras, and he sketches its general shape, adding a reference to the ratios fundamental to its construction (those of the concords and the tone). Chapter 6 tells the story of Pythagoras' discovery of these basic ratios, and speaks of their confirmation in experiments with various instruments. He has now prepared us for a full analysis of the diatonic octave (in the form familiar from Philolaus, Plato and the *Sectio Canonis*). This is given in chapter 7, together with a parenthetical reference to the other two genera (more fully treated in chapter 12). The seven-note and eight-note scales are again compared. This stage of Nicomachus' account is completed in chapters 8–9, in which the evidence of Plato (misquoted and misinterpreted) and of Philolaus is called on to confirm the analysis. (Nicomachus therefore finds it necessary to add an explication of the system of Philolaus, a seven-note scale differing from those previously mentioned.) The two chapters can be construed as an attempt to underwrite the credentials of the 'Pythagorean diatonic octachord', first from the point of view of mathematics and metaphysics, and secondly from the facts of musical history, as presented in the earliest Pythagorean account that Nicomachus knew.

Chapter 10 returns to the properties of instruments, expanding what has been said in chapter 4, and discussing the relation of 'converse qualification' that holds between tensions and pitches on the one hand and lengths of string or pipe on the other (cf. 9.4 Thrasyllus ap. Theon Smyrn. 87). The chapter is best treated as a digression, but in view of the sequel is a relevant reminder of the conceptual apparatus involved in the Pythagorean approach.

The last two chapters pick up the programme of analysis where chapter 9 had left it. Chapter 11 sets out the form of the diatonic double octave, giving the names of the notes and tetrachords, the sizes of the intervals involved, and an account of the conjunctions and disjunctions between tetrachords. The terminology here is mainly Aristoxenian, but

among the further details that the last sentences of the chapter mention and postpone for a fuller treatment elsewhere, is a 'division of the *kanōn*' in proper Pythagorean style. The task of chapter 12 is to complete the elementary division of tonal space by an account of the progressions proper to the chromatic and enharmonic genera. But it begins with a brief account of some basic harmonic conceptions (note, interval, relation, difference, *systēma*, concord, discord). These remarks are said to be 'reminders', though in fact they correspond to nothing earlier in the text (but see n. 29 below), and apart from the distinction between 'difference' and 'relation' the sketches they offer are drawn from Aristoxenian models. Little use is made of them in the remainder of the chapter. Perhaps Nicomachus was merely uneasy at having so far said nothing to explicate these apparently important notions, and fitted them in here as best he could. The descriptions of the chromatic and enharmonic systems follow, again in thoroughly Aristoxenian terms (though only their simplest forms are considered, as in Adrastus and Thrasyllus). Nicomachus covers his tracks with a paragraph explaining, in effect, that when he spoke of 'semitones' and 'quarter-tones' in giving these descriptions he was not to be taken literally. He subscribes to the Pythagorean doctrine that tones cannot be divided into equal parts, though he does not explain why. The work ends with a list of the names of all the notes of the double octave system in all three genera, and a valedictory paragraph that once again promises a fuller treatment at a later date.

The details of Nicomachus' treatise are sometimes muddled, often naive. But its overall programme makes good sense as an elementary introduction to harmonics, interpreted to fit a particular form of Pythagoreanism. From a harmonic point of view, this 'Pythagoreanism' maintains a standard of metaphysical purity (or oversimplification) whose source is Plato, in that its principal systems are all direct derivatives of the 'cosmic scale' in 2.3 *Tim.* 35b–36b. No allowance is made for the existence of scales whose analysis involves more complex manoeuvres in the theory of ratio and proportion. Here the Platonising branch of the tradition differs sharply from that which leads from Archytas through Eratosthenes and Didymus to Ptolemy (see the Appendix to chapter 1). In this respect, as in its scope and method, the *Enchiridion* has clear affinities with the writings of Adrastus, Theon and perhaps Thrasyllus (see 9.1–9.5), and is perhaps scarcely more original in general conception than in its details. Its greater influence on musical writers of later antiquity is due less to its scientific merits than to Nicomachus' admirable talents as a populariser. Non-specialists, curious about the musical aspects of Pythagorean philosophy, could find in Nicomachus a straightforward and quite charming exposition of ideas which others had surrounded with mathematical complexities, or had hidden in technical commentaries on the dialogues of Plato.

On the life and works of Nicomachus see especially D'Ooge (1926). The most substantial study of the *Enchiridion* in English is Levin (1975).

10 The *Enchiridion*

237 ### *Chapter 1 That this book is a manual serving as a memorandum of the elementary teachings of Harmonics*

Though the description of the intervals and relationships involved in the elements of harmonics is in itself complex and difficult to pull together in a single memorandum, and though I especially, because of the restlessness and hurry of a traveller's life, am unable to devote myself with unruffled attention and mental concentration to the teaching of these matters, whose proper clarification demands above all both an opportune moment and careful

reasoning, with leisure and without distraction, nevertheless, best and noblest of women, I must arouse my greatest efforts, since it is you who have bidden
238 me at least to set out the major propositions for you in simple form, without elaboration or complex demonstrations, and to do so at once, so that by having these propositions collected in a single synopsis, and by using these brief notes as a manual, you may remind yourself of what is stated and taught there in rough outline under each heading.[1] If the gods are willing, as soon as I have some leisure and a break from my travels, I will put together for you a longer and more accurate introduction to these matters, connected together with a fully reasoned argument, as the saying goes: it will be in several books, and I shall send it to you at the first opportunity, wherever I am told that you are living.[2] But now, to make my exposition easy to follow, I shall begin from the same place where I began my instruction when I was expounding these things to you in person.

Chapter 2 Concerning the two forms of vocal sound, the intervallic and the continuous, and the spaces occupied by each

Those belonging to the Pythagorean school used to say that there are two types of human vocal sound, standing as species of a single genus.[3] Their technical word for the one was 'continuous', for the other 'intervallic', terms which they assigned to each on the basis of its special qualities. They took the intervallic to be that used in song, which comes to rest at each pitch, and makes plain the changes between each of its parts, which does not confusedly run together, and
239 which is articulated and divided by the magnitudes which go with each note, where the parts of the vocal sound lie beside one another as it were in an aggregation, not in a blended mixture, being readily separated and distinguished, and in no way melted into one another.[4] For sound in song is such as to display all the notes clearly to those who understand the subject, showing

[1] The identity of the lady to whom the treatise is addressed is unknown. The vast, sprawling sentence with which it opens is typical of Nicomachus. I have not always translated his sentences in a way that preserves their original shape and length, but have kept this one in roughly its original form as a specimen. For another extraordinary example see the beginning of chapter 5.

[2] On the promised major work see the introduction to this chapter. The opening address, giving the *Enchiridion* the form of a letter, tells us virtually all that is known of Nicomachus' life.

[3] The claim that this distinction is due to the Pythagoreans is disingenuous. The fundamental source for the whole passage is Aristoxenus, as its language plainly shows (see 7 Aristox. *El. Harm.* 3.5ff., 8.13ff., and compare, for example, 12 Arist. Quint. *De Mus.* 5.24ff.). Some attempt has been made to reshape the ideas to fit a Pythagorean framework, and Nicomachus thereby succeeds in introducing a fair measure of confusion. For a masterly transformation of the essential material into a form that Pythagoreans could have used see 11 Ptol. *Harm.* Book 1 ch. 4.

[4] The first part of this sentence paraphrases Aristoxenus (see 7 *El. Harm.* 8.25–9.1, 9.24ff.). The latter part (from 'and which is articulated') presents the idea in Pythagorean terms, in which notes are items with magnitude, placed side by side, not Aristoxenian points on a linear continuum of pitch. The next sentence is also 'Pythagorean', in the same way.

how great is the magnitude possessed by each. Anyone who employs sound otherwise is said not to sing, but to speak. The other kind, the continuous, is that by which we converse with one another or read aloud, when we have no need to make clear the pitches of the notes and to distinguish them from one another, but string out our speech continuously until we have completed what we are saying.[5] Anyone who, while conversing, or recounting something, or reading aloud, makes clear distinctions between the magnitudes associated with each note, dividing and shifting the vocal sound from one to the next, is said not to be speaking or reading, but chanting.[6]

Since then human vocal sound is of two kinds, they believed quite reasonably that there are also two spaces, each occupied by one of them in the course of its motion.[7] The space occupied by the continuous species is by nature unlimited in magnitude, and takes its own particular limit from the point at which the speaker starts to that at which he stops; that is, it is the space between the first utterance and the final silence.[8] Hence the greater part of it is dependent on us. That of the intervallic species, however, is not dependent
240 on us, but is determined by nature, though it too is bounded by distinct operations.[9] Its starting point is the first thing that can be heard, its final point the last which can be sounded.[10] For we begin to be able to grasp and to compare the magnitudes of sounds and the differences between them from the point at which our hearing is first found to operate, though it is possible for there to be produced in nature still fainter sounds, not yet perceptible by us, and for these to escape our notice.[11] In the same way, for instance, there are

[5] This paraphrases the content of 7 Aristox. *El. Harm.* 8.19–25, 9.21–24.

[6] This sentence may refer to the third, 'intermediate' sort of vocal movement introduced at 12 Arist. Quint. *De Mus.* 6.3–5, said there to be used in declaiming poetry. Compare 7 Aristox. *El. Harm.* 9.26–33.

[7] Aristoxenus also speaks frequently of the 'space' (*topos*) in which the voice moves (e.g., 7 *El. Harm.* 14.5, in the passage which is Nicomachus' main source here), but he does not distinguish two different 'spaces'. There is just one, the dimension of pitch, in which both intervallic and continuous vocal movements take place, though in different ways. Nicomachus' conception is not altogether clear. Perhaps he means only that the boundaries of the space are different for the speaking and for the singing voice. Compare 9.2 Theon Smyrn. 52.1ff.

[8] The last phrase echoes 7 Aristox. *El. Harm.* 8.24–5.

[9] The 'nature' that determines this limit cannot be that of intervallic motion abstractly conceived, since it will be said to depend on our powers of hearing and utterance (cf. 7 Aristox. *El. Harm.* 15.2–11). It is then unclear why it is not 'dependent on us', but perhaps this phrase means merely 'not dependent on the individual's whim'. Contrast *El. Harm.* 21.11–17.

[10] This is roughly parallel to Aristoxenus' contention that the smallest usable interval is the smallest that the ear can judge and the voice produce (and these coincide, 7 *El. Harm.* 14.15–24), while the greatest is a little larger for the ear than for the voice (*El. Harm.* 14.25–8). But it turns out that Nicomachus' 'starting point' and 'final point' are not the smallest and largest usable intervals, after all: see notes 11–13 below.

[11] The starting point is then determined by the capacities of our hearing. But Nicomachus now seems to be speaking of the faintness of a sound, rather than its pitch, still less of the size of an interval. Probably there is some confusion here between pitch and volume (cf. **1.19** Archytas frag. 1 with n. 48), though Nicomachus does make the distinction at the beginning of chapter 4. Then his intention may be to speak of the lowest pitched sound that is melodically usable (cf. the opening of chapter 11, and *Exc. ex Nicom.*

bodies which reveal no weight on a balance – bits of chaff or bran or other such things – but when under the addition of a number of such things the beginning of movement is detected, we say that we have the first candidate for the science of weighing. In the same way, when faintness in sound is gradually augmented we make the first thing detectable by the hearing the starting point of the space belonging to the sound proper to song.[12] However, it is not our hearing but the human voice that determines its final limit. The point which it can reach while remaining melodic, and to which it can advance in song, is the one that we identify as the final limit of the space belonging to this kind of vocal sound.[13] But it need make no difference for the present whether we relate this account to the sound we make with our own windpipes, or to that made by instruments, whether stringed, blown or percussive, which are constructed by way of imitation of the sound we make ourselves.[14] Let us pass over the distinction between them for the time being, to avoid fragmenting our exposition right at the outset.

241 *Chapter 3 That the first music among sensible things is studied in connection with the planets, and that the music that exists among us is studied as an imitation of that one*

That the names given to the notes have been taken from the seven stars which move in the heavens and go round about the earth is a view that carries conviction.[15] For they assert that all bodies that rush through a yielding

274.11–20). The 'imperceptible' sounds may not be literally so, but only such that their pitch cannot be assessed by ear. Thus chapter 11 speaks of a 'hoarse whisper' at the bottom of the vocal compass and squeaks at the top, and *Exc. ex Nicom.* of 'croaks and coughs, utterances that are indistinct and unarticulated and unmelodic' as lying beyond the bass range, 'crowings and sounds like the howlings of wolves' beyond the treble. Compare 9.1 Thrasyllus ap. Theon Smyrn. 47.20–48.8, and for a different use of these images 11 Ptol. *Harm.* 10.8–11.

[12] Gradual augmentation of volume brings sounds to the threshold of audibility. The analogy is not obvious in the case of pitch. But pitches are associated with magnitudes, and Nicomachus links greater magnitudes with higher pitches: see especially chapter 10, and the penultimate sentence of chapter 6. Then it is by augmentation in magnitude that a sound is brought up to a pitch that we can assess (compare the analogous but distinct ideas of 8 Eucl. *Sect. Can.* 148–9).

[13] This final limit is evidently the upper boundary of the range of pitch that is melodically usable.

[14] What Nicomachus would say on this issue is not clear, given that some instruments can reach beyond the range of the human voice, nor is it obvious why he insists that the voice and not the ear is the determinant in this case. Possibly the latter thesis is based on the fact that both voices and instruments tail off into 'squeaks' (see n. 11 above) at a point before the ear's capacity to judge pitch gives out. Then the upper limit of melody falls lower in the range than the upper threshold of the ear's judgements. Aristoxenus states explicitly that the ear can judge somewhat larger intervals than the voice can produce (see n. 10 above). On the question whether voice or instruments should provide the criterion, Nicomachus may be relying on an imperfect memory of 7 Aristox. *El. Harm.* 20.28–21.17.

[15] This is of course false: names were assigned to notes quite independently of any astronomical hypothesis, mainly in the context of practice, not of theory (though the origin of some of the names is in doubt).

medium that is readily stirred into waves must make noises differing from one another in their magnitude and in the range of their sound,[16] in relation either to their respective masses or to their particular velocities or to the positions in which the swinging movement of each is completed, such positions being either quite readily disturbed, or the opposite, resistant to being shifted.[17] These same three differences are clearly seen in relation to the planets, which differ from one another in size and in speed and in location,[18] and which rush through the expanse of the aether unceasingly and without rest. That is why each of them has been given the name 'star' [*astēr*], since it is without rest [*staseōs esterēmenos*] and always in swift motion [*aei theōn*]; and for this reason each has been entitled a 'god' [*theos*] and 'aether'.[19]

Now from the course of Saturn, which is highest in relation to us, the deepest note in the octave was named '*hypatē*' ['uppermost'], for what is highest is uppermost.[20] From that of the Moon, which is furthest down of all and circles

[16] 'They' are the Pythagoreans: cf., for example, 1.6 Arist. *De Caelo* 290b. 'Range' translates *topos*, the word rendered as 'space' in the previous chapter. Aristoxenus also uses it both for the 'space' or dimension of pitch in which the voice moves, and for the range within that space that a given note can occupy. The latter seems to be the sense here: see n. 17 below.

[17] The first of these clauses is clear: the pitch ('magnitude') of the note emitted by a planet may depend either on its mass or on its velocity. In translating the latter part of the sentence I follow Levin (1975), pp. 35–9, particularly in her interpretation of *epochai* as 'positions' (for other suggestions see her discussion). The thesis is then that a planet's pitch may depend on its position relative to the other bodies, varying as does the distance between its orbit and those of the others. The irregular apparent movements of the planets encourage the view that the distances between the orbits is not constant. The paths of sun and moon are more stable (compare particularly Arist. *De Caelo* 291b). By assigning stability also to Saturn, Nicomachus is able to associate the three fixed notes of his system with bodies in stable orbits, and the four moveable notes with ones whose orbits shift, sometimes coming closer to an adjacent stable orbit, sometimes moving further away (see the next paragraph). Sun, Moon and Saturn, then, are those whose orbital 'positions' are resistant to being shifted, and the other planets are those whose positions are 'quite readily disturbed'. It will thus be the size of each planet's orbital variation that determines the range, *topos*, through which the pitch of its note can move in changes of genus. Aristoxenus uses the term *topos* in just this way (e.g., 7 *El. Harm.* 22.24ff.), to refer to the range of variation in pitch of such notes as *lichanos* and *parhypatē*. If this account is on the right lines, there are similarities between Nicomachus' theory and that of 11 Ptol. *Harm.* Book III ch. 11; cf. 12 Arist. Quint. *De Mus.* Book III ch. 21 with n. 176.

[18] 'Location' is another rendering of *topos*, which colloquially means merely 'place'. I offer the translation with some hesitation. If it is right, *topos* here is simply equivalent to *epochē*, 'position', in the previous sentence. Alternatively, it still means 'range', in which case the sense is not that the planets differ from one another in the positions of their orbits in space (i.e., in the distances of their orbits from the earth), but that their orbits have different ranges of variation.

[19] These etymological suggestions are entirely fanciful.

[20] Nicomachus now seeks to associate individual bodies with specific notes. Unusually, his system links lower notes with bodies further from the earth. Most authors take the opposite view (e.g., 12 Arist. Quint. *De Mus.* Book III ch. 21, cf. 11 Ptol. *Harm.* Book III ch. 16, though his system is not strictly comparable; Aristotle frag. 203; 2.2 Plato *Rep.* 617b with n. 9; even *Exc. ex Nicom.* 272.9 says that the earliest writers linked the most distant body with the highest note. Boethius *Inst. Mus.* 1.20, 27 follows Nicomachus). For discussions see Levin (1975), pp. 38–42, Burkert (1972), pp. 352–5. Nicomachus' determination to derive the notes' names from their planetary associations may have

the earth most closely, there was taken the name '*neatē*' ['lowest']:[21] for what is furthest down is lowest. From the courses of those which are next to [*para*]
242 each of these there is named on the one hand *parhypatē* (from the one below Saturn, which is that of Jupiter), and on the other *paraneatē* (from the one above the Moon, which is that of Venus).[22] From the one which is in the middle, that of the Sun, which lies fourth in order from each end, there is named *mesē* ['middle'], which is set – at least according to the ancient practice, within the heptachord[23] – at the interval of a fourth from both extremes, just as the Sun also is fourth from each end among the seven planets, and lies in the middle. As to those on each side of the Sun, from the course of Mars, which has been allotted the sphere between the Sun and Jupiter, there is named *hypermesē* ['above *mesē*'], which is also known as *lichanos*;[24] and from that of Mercury, which holds the region between the Sun and Venus, there is named *paramesē* ['beside *mesē*'].[25] We shall fully confirm all these points for you, with

helped to incline him to his heterodox view. Though notions of pitch were rarely expressed, in classical Greek, in terms of 'up' and 'down', this terminology was sometimes used from the fourth century onwards. Hence it could seem odd that a low note was given the name *hypatē*, which in non-musical contexts often means 'highest', as *nētē* means 'lowest'. Nicomachus explains it by linking the lowest note in his system with the planet that is highest in space, and supposing that the note was given its name because of this connection. This is certainly wrong. Probably the notes were named from the positions of the strings or finger-holes used to sound them, *hypatē* being 'furthest away' and *nētē* 'nearest' (again, quite familiar uses of the words). For another conjecture see Sachs (1940), p. 135. Nicomachus may have been influenced by the resonances of the adjectives *hypatos* and *neatos* in religious discourse, where this has astronomical associations too. Thus the Pythagorising Platonist Xenocrates (fourth century B.C.) is said to have distinguished a Zeus *hypatos* (a 'higher' Zeus) who rules in the region of eternal self-sameness, and a Zeus *neatos*, whose province is the world of change below the moon (Plut. *Quaest. Plat.* IX. 1). Xenocrates also spoke of the planets as 'seven gods', and it is possible that Nicomachus drew the ideas of this chapter from a tradition linked to his writings.

[21] *Neatē* is a Doric variant for *nētē*, used probably to give the flavour of antique Pythagoreanism (compare the language of Philolaus, quoted by Nicomachus in chapter 9).

[22] Musically speaking, *parhypatē* is next 'above' *hypatē*, and *paraneatē* (or *paranētē*) next 'below' *nētē*; both are moveable notes. On the relative positions of Venus and Mercury see n. 8 to 2.2 Plato *Rep.* 617a.

[23] Nicomachus reduces the cosmic system to a heptachord by giving no role to the sphere of the fixed stars. His claim for the antiquity of a musical seven-note system falling short of the octave is repeated in chapter 5. The heptachord discussed in the last paragraph of chapter 9 is different. Whether or not he is right (cf. particularly 4.27 ps.-Ar. *Probs* XIX.47), he is plainly motivated at least in part by a wish to assign the invention of the complete octave structure to Pythagoras (in chapter 5).

[24] The usual name for this note has to be treated as a mere variant because *lichanos* means 'forefinger' (the finger used to touch the relevant string, see the table in chapter 11), and quite obviously has no planetary significance. The term *hypermesē* is most unusual: it is formed in the same way as *hyperhypatē* (see nn. 58 and 65 to 8 Eucl. *Sect. Can.*), and the note it names is musically 'below' *mesē*.

[25] This cannot be the note usually called *paramesē*, the upper boundary of the disjunctive tone above *mesē*, since that has no place in the conjunct system of this heptachord. It is true that at the end of chapter 9 Nicomachus describes a bizarre 'ancient heptachord' that spanned an octave, and in which there was a note placed in the usual location of *paramesē*, a whole tone above *mesē*. But he has said that the one intended here treats *mesē* as the common boundary of two fourths, and we are therefore dealing with a

more precision and with diagrammatic and numerical demonstrations, in the treatise which we promised you,[26] noblest of women and best lover of beauty; and we shall explain the reasons why we do not hear this cosmic concord,[27] which utters, as our account sketchily indicates, a sound full to repletion, containing all *harmonia*. But now we must hurry on, for our time goes swiftly, and consider the subjects next in succession.[28]

Chapter 4 That things to do with notes are ordered in accordance with number

20 In general, we say that sound is an impact of air which is unbroken as far as
243 the hearing.[29] A powerful impact or exhalation falling on the surrounding air and striking it in many of its parts results in a large sound, a slight impact makes a small sound, an even one a smooth sound, an uneven one a rough sound; and if it is impelled slowly the sound it makes is deep, if swiftly, the

system spanning a seventh, not an octave (cf. chapter 5). In chapter 7 we are told that each note of the lower tetrachord in this system was at a fourth below its equivalent in the upper, and that the sequence of intervals in the tetrachord, from the bottom upwards, is (in diatonic) semitone, tone, tone. Then the present so-called *paramesē* is a semitone above *mesē*, and is in effect what was usually called *tritē synēmmenōn*.

[26] See the second sentence of chapter 1.

[27] See 1.6 Arist. *De Caelo* 290b with n. 23.

[28] A table of the system sketched in this chapter may be helpful. I have placed the series in astronomical order from highest to lowest, corresponding to a musical order from lowest to highest. The system is equivalent to the pair of tetrachords usually called *mesōn* and *synēmmenōn*. The intervals given presuppose the diatonic genus.

	Saturn	—	*hypatē (mesōn)*	Tetrachord *mesōn*
semitone				
	Jupiter	—	*parhypatē (mesōn)*	
tone				
	Mars	—	*hypermesē (= lichanos mesōn)*	
tone				
	Sun	—	*mesē*	
semitone				Tetrachord *synēmmenōn*
	Mercury	—	*paramesē* (here = *tritē synēmmenōn)*	
tone				
	Venus	—	*paraneatē (= paranētē synēmmenōn)*	
tone				
	Moon	—	*neatē (= nētē synēmmenōn)*	

[29] Cf., for example, 3.15 Arist. *De Anima* 420a, 5 *De Audib.* 800a, and 9.2 Adrastus ap. Theon Smyrn. 50.7, which uses a similar phrase in repeating doctrines attributed to the Pythagoreans. The account has affinities with 1.19 Archytas frag. 1 and the introduction to 8 Eucl. *Sect. Can.* At this point the MSS continue with a passage that is clearly interpolated – the jottings of a reader. 'A note is a breadthless pitch of a melodic sound; and a pitch is a sort of immobility and self-sameness in respect of magnitude of a continuous note. An interval is a sort of route from depth to height or the reverse, and a *systēma* is a combination of more intervals than one.' Cf. nn. 4–6 to 9.1 Thrasyllus ap. Theon Smyrn. 47–8, nn. 12 and 41 to 9.2 Adrastus ap. Theon Smyrn. 49 and 60. Though these sentences can hardly stand where the MSS put them, they may summarise something that was elsewhere in the original text and is now lost, since chapter 12 includes a discussion of these and other conceptions that is said to 'remind' us of things that have already been said.

sound it makes is shrill.[30] Wind instruments, such as *auloi*, *salpinges*, *syringes*, *hydrauloi*, and others like them, are necessarily affected in the opposite way to stringed instruments like the *kithara*, the *lyra*, the *spadix*, and others of that sort. Between these groups, and as it were common to both and affected similarly to both, are found the monochords, which most people call *phandourai* and the Pythagoreans call *kanones*, and also the *trigōnoi* among stringed instruments, as are the *plagiauloi* along with the *phōtinges*, as our account will show later on.[31] In the case of stringed instruments, greater and tauter tensions produce larger and higher sounds, while lesser tensions produce duller and lower ones.[32] For when the plectrum moves the strings, they are displaced from their normal positions, and the tenser ones recoil with a powerful agitation and strike the air about them in many places, as though impelled by the vigorous tension itself, while the slacker ones recoil gently and
244 without agitation, like a carpenter's plumbline. By contrast, in the case of wind instruments the larger cavities and dimensions produce a sluggish and relaxed sound.[33] For if the breath comes out into the surrounding air lacking in tension because of its long journey, and strikes and moves it lethargically, in this way too the sound generated is deep.[34] And it must then be noted that the greater

[30] These remarks can be paralleled in many sources, notably in 5 *De Audib.* Compare 3.17 Arist. *De Gen. An.* 786b ff., particularly for the distinction between a sound's volume and its pitch (see n. 11 above); cf. also the first passage of Adrastus cited in n. 29 above.

[31] For discussions of most of the instruments mentioned here see the index to *GMW* vol. 1. Here I give only a brief indication of their main relevant features. *Auloi* are pipes sounded with a reed, equipped with finger-holes. *Salpinges* are the ancient equivalent of trumpets, though they may occasionally have had a double-reed mouthpiece. The *syrinx*, here as often, is probably the familiar Pan-pipe. The *hydraulos* or *hydraulis* is an organ powered by water-pressure. *Kithara* and *lyra* are the commonest stringed instruments, with strings of equal length. The *spadix* is obscure. Its name means 'branch', specifically 'branch of a palm tree (*phoenix*)'. It is then probably related to the instrument called the *phoenix*, which had a structure like the *lyra* but with palm-branches for its arms (in the list of stringed instruments at Pollux IV. 59, *spadix* appears between *phoenix* and *lyrophoenix*). Quintilian (*Inst. Orat.* I.10.31) brands the *spadix* as 'effeminate'. That is about the sum of our knowledge of it. The monochord has a single string, stopped at different points to make the notes; normally a tool of theorists, but occasionally used for music making (see 11 Ptol. *Harm.* Book II ch. 12). The *phandoura* or *pandoura* is treated here as identical with the monochord, but sometimes had three strings, and was played rather like a guitar. *Trigōnoi* are triangular harps, with strings graded in length. *Plagiauloi* and *phōtinges* are pipes played transversely, like a modern flute. In some cases they may have had a laterally inserted reed, in others a plain hole, again like a flute.

[32] Nicomachus now begins his explanation of how the three groups of instruments are 'affected' in different ways. The distinctions seem to turn on whether a higher pitch is produced by a greater or a smaller amount of a relevant variable. In the first group discussed (*kithara*, *lyra*, *spadix*) the factor involved is tension, since the strings are equal in length: greater tension goes with higher pitch.

[33] In *auloi*, *syringes* and *hydrauloi* the important variable is length of pipe: greater length goes with lower pitch. But if *salpinges* are trumpets without finger-holes (as they usually were), their different notes are not produced in this way, and it is odd that Nicomachus places them in this group.

[34] Comparable reflections on the causes of pitch-variation in instruments are found in many authors: for an elaborate account see 9.7 Aelianus ap. Porph. *Comm.* 33.16ff. See also 6 Theophrastus ap. Porph. *Comm.* 63.1ff., who like Nicomachus (but for different reasons) points to the effect of several distinct variables, some of which diminish in quantity while others increase. Cf. 9.3 Adrastus ap. Theon Smyrn. 65.10–66.11.

and the less come about in relation to quantity, which we ourselves impose by stretching and slackening or by making tubes long or short. Hence it is clear that all these things are ordered by number, since it is of nothing but number that quantity is a proper characteristic.[35]

Chapter 5 That Pythagoras, by adding the eighth string to the seven-stringed lyra, *put together the* harmonia *of an octave*[36]

Pythagoras, first of all men, forming the intention that the middle note should not stand, by conjunction, in the same relation to each of the extremes, and thus display distinctly only the concord of a fourth, in relation both to *hypatē* and to *nētē*,[37] and that we should have a more complex object for study,[38] where the extremes would produce in relation to one another the completest concord, that is the octave, which is in duple ratio, something that cannot be constructed out of two tetrachords, with this aim in view he, first of all men,
245 added an eighth note, fitting it in between *mesē* and *paramesē*, and making it stand at a whole tone from *mesē* and at a semitone from *paramesē*,[39] so that the note which formerly, in the heptachord, was *paramesē*, and which is still third from *nētē*, is called '*tritē*' ('third') and lies in that position no less than before,[40] while that which was inserted is fourth from *nētē*, and forms with it the concord of a fourth, the same concord as that which the original *mesē* made

[35] This is precisely the thesis that Theophrastus (see n. 34) seeks to undermine. Nicomachus has said nothing to explain the 'intermediate' character of his third group of instruments. In the case of *trigōnoi*, monochords, etc., the point seems clear: they are stringed instruments, like the *kithara* and *lyra*, but unlike them they move to a higher pitch by decreasing the relevant variable (length of string). The intention is less plain in the case of *plagiauloi* and *phōtinges*. The idea should be that these, like *auloi*, are wind instruments, but that their pitch is raised by increasing the quantity of a suitable variable. The only likely candidate is pressure ('tension') of breath, which is often a relevant factor in pitching notes on a wind instrument. But these instruments had finger-holes too, and breath-pressure is also influential in the playing of *auloi* (see especially 7 Aristox. *El. Harm.* 41.32–42.14, 43.10–15); it is fundamental in the case of the *salpinx* (cf. 9.3 Adrastus ap. Theon Smyrn. 66.9). Nicomachus has at any rate not said enough to make his claims convincing. Though some further details are offered in chapter 10, he does not make good his promise to explain later his thesis about *plagiauloi* and *phōtinges*.

[36] For the thesis that the Pythagoreans called the octave *harmonia* see chapter 9. Chapters 5–9 are concerned entirely with the structure of the octave. The topics of chapter 4 are revisited in chapter 10.

[37] On the nature of the old structure that Pythagoras is said to have found unsatisfactory see chapter 3. On the shape of this astonishing sentence see n. 1 above.

[38] The claim that the eight-note octave structure was invented for purely intellectual purposes is of course fantastic. It is designed to fit with the attribution of this invention to Pythagoras, whom Nicomachus is inclined to treat as the *fons et origo* of harmonic truth.

[39] The original '*paramesē*' stood a semitone above *mesē* (see n. 25 above). The model that Nicomachus has in mind is that of a new note being inserted, as he says, between the *mesē* and the old *paramesē*, displacing the latter (with the remaining notes in its tetrachord) upwards by a tone. It is thought of as the 'same' note displaced upwards because it is still the third-highest note in the tetrachord to which it belongs (see the next sentence).

[40] On Nicomachus' account, a change of name was essential, since the note was no longer 'next to *mesē*' as the term *paramesē* implies.

with *hypatē*. The tone between *mesē* and the newly inserted note (named *paramesē* in place of its predecessor in that position), no matter to which tetrachord it is attached, whether considered as lying higher than the tetrachord involving *hypatē* or as deeper than that involving *nētē*, will display the concord of a fifth, which is the *systēma* made up of the conjunction of the tetrachord itself and the added tone.[41] Now the ratio of the fifth, which is hemiolic [3:2], is found to be a *systēma* of an epitritic [4:3] together with an epogdoic [9:8]; therefore the tone is an epogdoic.[42]

Chapter 6 How the numerical ratios of the notes were discovered

The intervals of a fourth and a fifth, of that which is formed by the combination of the two, known as the octave, and of the tone which lies additionally between the two tetrachords, were established as having this numerical quantity by Pythagoras.[43] The method he adopted was like this. He
246 was plunged one day in thought and intense reasoning, to see if he could devise some instrumental aid for the hearing which would be consistent and not prone to error,[44] in the way that sight is assisted by the compasses, the measuring rod [*kanōn*] and the *dioptra*,[45] and touch by the balance and by the devising of measures; and happening by some heaven-sent chance to walk by a blacksmith's workshop, he heard the hammers beating iron on the anvil and giving out sounds fully concordant in combination with one another, with the exception of one pairing: and he recognised among them the consonance [*synōidia*] of the octave and those of the fifth and the fourth. He noticed that what lay in between the fourth and the fifth[46] was in itself discordant, but was essential in filling out the greater of these intervals. Overjoyed at the way his project had come, with god's help, to fulfilment, he ran into the smithy, and through a great variety of experiments he discovered that what stood in direct relation to the difference in the sound was the weight of the hammers, not the force of the strikers or the shapes of the hammer-heads or the alteration of the

[41] That is, the tone between *mesē* and the new note (now called *paramesē* in accordance with standard usage) is the tone of disjunction between two tetrachords (see particularly 7 Aristox. *El. Harm.* 58.14ff.). A combination of it with either tetrachord will span a fifth.

[42] Cf., for example 8 Eucl. *Sect. Can.* propositions 12 and 13.

[43] The tradition that Pythagoras discovered the fundamental intervallic ratios is very persistent (see, for example, 1.1, 1.2) though others attributed the discovery to his disciples (e.g., 1.3, 1.4). Many of the stories told in this connection (including the present one and 12 Arist. Quint. *De Mus.* Book III ch. 1) are plainly fictions. We cannot be sure even that the discovery was first made by Pythagoreans, though they made notable use of it. It may have been due to practical musicians or instrument-makers, who would have found it useful in the construction of harps (these became popular in Greece during the sixth century B.C., but were widespread in eastern Mediterranean cultures from much earlier times). Most scholars are prepared to give Pythagoras the benefit of the doubt, at least so far as the dissemination of these ideas among the Greeks is concerned.

[44] Compare 11 Ptol. *Harm.* Book I chs. 1–2.

[45] A rod used as an instrument for measuring indirectly the height of tall objects.

[46] That is, their difference, the tone.

iron which was being beaten.[47] He weighed them accurately, and took away for his own use pieces of metal exactly equal in weight to the hammers. Then he fixed a single rod from corner to corner under his roof, so that no variation should arise or even be suspected of arising from the peculiarities of different
247 rods, and hung from it four strings, each of the same material, and consisting of an equal number of strands, and each of equal thickness and twisted to the same extent as each of the others.[48] He then attached a weight to the lower part of each string. And having so contrived it that the length of every string was in all respects absolutely equal, he then plucked strings two at a time in turn, and found the concords previously mentioned, a different concord for each pairing.[49] He perceived that the string under tension from the biggest object attached sounded at an octave in relation to the one under tension from the smallest. The former was of twelve units of weight, the latter of six. Hence he showed that the octave is in duple ratio, as the weights themselves implied. He found that the biggest sounded at a fifth in relation to the smallest but one (which had eight units of weight), and revealed from this that the fifth is in hemiolic ratio, the ratio in which these weights stood to each other. In relation to the one second in weight to itself and greater than the others, which was of nine units, it sounded at the interval of a fourth, in conformity with the relations of the weights. And he at once perceived that this ratio was epitritic, and that this same string was in hemiolic ratio to the smallest (since that is the ratio of 9 to 6): and in the same way the smallest but one, carrying eight units, stood in epitritic ratio to the one that carried six, and in hemiolic ratio to that which carried twelve. And hence he established that what lies between the fourth and the fifth, that is, that by which the fifth exceeds the fourth, is in
248 epogdoic ratio, that in which nine units stand to eight. It was also proved that the octave can be constructed in each of two ways, either in the conjunction of the fifth and the fourth, since duple ratio consists in a conjunction of hemiolic and epitritic – as in the numbers 12, 8, 6 – or the other way round, in the conjunction of the fourth and the fifth, since duple ratio consists in a conjunction of epitritic and hemiolic – as in the numbers 12, 9, 6, which are ordered in that sort of way.[50] Then, having worked on the weights until both hand and hearing were sore, and having established with reference to them the ratios appropriate to their relative positions, he skilfully transferred the

[47] This betrays the legendary character of the tale of this harmonious blacksmith. It is not true that the ratios between the pitches will correspond to those between the weights of the hammer-heads. (See 1.3 with n. 6, 1.4 with n. 11, Burkert (1972), pp. 375ff.)

[48] The matter is hardly so simple. Compare the elaborate procedures described by Ptolemy in 11 *Harm.* Book I ch. 8 and Book III chs. 1–2 for determining whether a string is true along its whole length, and whether two or more have identical properties.

[49] The desired results will again fail to be produced. The pitch-ratios are not directly related to the ratios of the weights, but to those of their square roots. Ptolemy (11 *Harm.* 17.7ff.) knew that the experiment would not work, but his explanation, though it has some truth in it, is beside the point.

[50] The numbers 12, 9, 8, 6 are often used in discussions of this sort, since they are the smallest whole numbers in which this set of ratios can be expressed. An early example is Aristotle frag. 47, quoted in ps.-Plut. *De Mus.* 1139c.

common tying-point of the strings, where they were all suspended together from the diagonal rod, to a stick attached to his instrument, a stick which he called a *chordotonon*, and he transferred the amounts of tension, in the same ratios that were produced by the weights, to a proportionate degree of twist in the *kollaboi* at the upper end.[51] Using this as a foundation and as it were an indubitable indicator, he went on to extend his researches to various kinds of instrument, including beaten pots, *auloi*, *syringes*, monochords, *trigōna* and others like them, and he found the conception arrived at through number to be concordant and immutable in all of them.[52] He named the note characterised by the number 6 *hypatē*, the one characterised by 8 *mesē*, which stands to *hypatē* in epitritic ratio, the one characterised by 9 *paramesē*, which is a tone higher than *mesē* and thus stands to it in epogdoic ratio, and the one characterised by 12 he called *nētē*.[53] And at the same time he filled in the gaps between them according to the diatonic genus, with notes in their proper ratios, and thus made the octachord subservient to concordant numbers, that is, to duple, hemiolic and epitritic ratio, and to the difference between the latter two, the epogdoic ratio.[54]

249 *Chapter 7 Concerning the division of the octave according to the diatonic genus*

It was in the following way that he discovered the natural and necessary progression of the scale in the diatonic genus from the lowest note to the highest. (The chromatic and enharmonic genera he later articulated by derivation from the diatonic, as I shall explain to you another time.[55])

[51] On *lyrai* and similar instruments, the *chordotonon* was at the bottom of the sound-box. The *kollaboi* or *kollopes* were devices for adjusting tension (originally strips of raw hide, later pegs or gadgets of various other sorts) fitted to the cross-bar. No Greek musician, of course, had a reliable way of creating the same degree of tension with his *kollaboi* as he had previously produced with weights – except by begging the question and simply adjusting his strings to the same pitches.

[52] Compare, for example, the passages cited in n. 43 above, and 4.19, 4.28 ps.-Ar. *Probs* XIX.23, 50.

[53] Thus Nicomachus (on Pythagoras' behalf) links larger numbers with higher pitches: cf. chapter 11. Some authors, notably Ptolemy, adopt the opposite strategy. For other examples of both approaches see Burkert (1972), p. 380, n. 47 (though I think he is wrong about Archytas). Authors rarely argue for the position they adopt on this issue; an exception is 9.3 Adrastus ap. Theon Smyrn. 65.10ff., whose intriguing argument has some affinities with Nicomachus ch. 10 (but his conclusion is the opposite).

[54] This implies that the tetrachords were filled out with two intervals of a tone each, leaving the ratio of the *leimma* (256:243) for the lowest interval. This is the system of Philolaus, Plato, Euclid, Adrastus, Thrasyllus, and many others in the Pythagorean tradition, but not of the branch that stems from Archytas (see the Appendix to chapter 1).

[55] The 'derivations' of the chromatic and enharmonic that Nicomachus has in mind are comparable to those of 9.5 Thrasyllus ap. Theon Smyrn. 90.22–93.2. They reflect Aristoxenian accounts of a chromatic tetrachord of semitone, semitone, trihemitone, and an enharmonic one of quarter-tone, quarter-tone, ditone (see Nicomachus ch. 12). The more complex analyses recorded in 11 Ptol. *Harm.* Book II ch. 14 cannot be 'derived' from Nicomachus' diatonic in any straightforward way.

However, it appears that it is of the nature of this diatonic genus to involve the following kinds of steps and progressions: semitone, then tone, then tone. That is the *systēma* of the fourth, consisting of two tones and the so-called semitone.[56] Then by the addition of another tone, the one which was inserted in the middle, there is generated the fifth, which is a *systēma* of three tones and a semitone. Following on from this come a semitone, a tone and a tone, making another fourth, that is, another epitritic interval. Hence in the older arrangement, the heptachord,[57] all the notes throughout, from the lowest up, formed the concord of a fourth with the notes fourth in order from themselves, with the semitone changing its position in the tetrachord from first to middle to third in the course of the progression.[58] But in the Pythagorean octachord, whether it is a *systēma* constituted by the conjunction of a tetrachord and a pentachord, or one constituted by the disjunction of two tetrachords separated from one another by a tone, the progression from the lowest string will be of such a kind that all the notes form the concord of a fifth with the notes fifth in order from themselves, with the semitone shifting progressively into the four available positions, first, second, third and fourth.[59]

Chapter 8 Explication of what is said about harmonics in the Timaeus

Now that we have got this far, it will be helpful to explain Plato's opportune remark which he makes during his account of the creation of the soul. He says: 'Thus in each interval there are two means, the one exceeding and exceeded by the same part of its extremes, the other exceeding and exceeded equally in respect of number. And the difference between hemiolic and epitritic intervals is filled up by an epogdoic remainder.'[60]

[56] 'So-called', because it is not an exact semitone (e.g., 8 Eucl. *Sect. Can.* proposition 16); Nicomachus himself makes the point in chapter 12.

[57] See Nicomachus' discussions in chapters 3 and 5.

[58] Nicomachus is probably not thinking of three 'species of the heptachord' analogous to the seven 'species of the octave' generated by the cyclic reordering of intervals (7 Aristox. *El. Harm.* 6.21–31, 36.30–1, 12 Arist. Quint. *De Mus.* 15.9–20). He means only that the interval of a fourth between a note in one tetrachord and its equivalent in the next may take the form semitone, tone, tone; or tone, semitone, tone; or tone, tone, semitone.

[59] The sense is parallel to that explained in the previous note. With the rules of progression mentioned here compare 7 Aristox. *El. Harm.* 29.6ff., 53.33ff., 58.14ff., and 9.2 Adrastus ap. Theon Smyrn. 51.4ff.

[60] This is 2.3 Plato *Tim.* 36a–b, but the second sentence is garbled. Taken as it stands, it reduces Plato's construction of a complete diatonic scale to that of the mere octave framework, fourth – tone – fourth (or fifth plus fourth, where their difference is the 9:8 tone). It also misuses the term *leimma* (here translated 'remainder'), making it refer to the tone. Levin (1975), pp. 87ff. argues that the misquotation is deliberate, designed to suggest that Plato's analysis was less complete than that of the Pythagoreans. She also doubts the authenticity of the material attributed to Philolaus in chapter 9, believing it to be dependent on Plato, whereas Nicomachus wishes to imply that the relation is the other way round, again diminishing Plato's originality. But the Philolaus material differs from Plato's account in important ways. It also involves a complete analysis of a diatonic scale spanning an octave. If Nicomachus meant that Plato's version is based on

A duple interval is that of 12 to 6; and it has two means, the numbers 9 and 8. Now the number 8 is a mean in harmonic proportion between 6 and 12, exceeding 6 by one third of that 6, and exceeded by 12 by one third of that 12. Thus because it is understood to exceed and be exceeded by the extremes by the same part of themselves, Plato called 8 a mean, in the sense that it stands to them in harmonic proportion.[61] For just as the largest bounding number is
251 double the smallest, so the difference between the largest and the middle term, which is 4, is double the difference between the middle term and the smallest, which is 2; and here again the 4 stands to the 2 in duple ratio. A peculiar feature of this sort of mean is that if the extremes are added together and multiplied by the middle term, the result is twice the product of the extremes. Thus eight times the sum of the extremes – that is 18 – makes 144, and that is double the product of the extremes, which is 72.[62]

The other mean, which is 9, and which is so placed as to correspond to *paramesē*, is reckoned to stand as an arithmetical mean in relation to the extremes, exceeding 6 by the same number, 3, as that by which it is exceeded by 12.[63] A peculiar feature of this mean is that the sum of the extremes is twice the middle term, and also that the square of the middle term, in this case 81, is greater than the product of the extremes, in this case 72, by exactly the square of the difference between terms, in this case three times 3, which is 9, since the difference in this example is 3.[64] One can also display a third mean, in what is known as ruling proportion, consisting in both these middle terms, 9 and 8. For
252 12 stands to 8 as does 9 to 6, the ratio being hemiolic in both cases. And the product of the extremes is equal to that of the middle terms, twelve times 6 being equal to nine times 8.[65]

his, he could hardly deny that Plato knew of the whole structure. Again, Nicomachus says later (chapter 11) that he will set out elsewhere a complete 'division of the *kanōn*' according to the system of Timaeus of Locri, 'whom Plato also followed'. This can only mean that Plato's analysis in the *Timaeus* (whether plagiarised or not) located every note of the scale. Levin's diagnosis, I think, must fail (I would also accept the authenticity of the Philolaus passage). So, far from seeking to diminish Plato, Nicomachus like many others is enrolling him as an honorary Pythagorean. But I do not know why the *Timaeus* quotation appears here in this truncated and distorted form.

61 This represents accurately Plato's conception of this mean, which he almost certainly derived from Archytas.

62 These additional calculations are correct, and help to give the sense of mathematical coordination. Other authors add different characterisations of this mean, e.g., Theon Smyrn. 114.14ff. Like Theon, Nicomachus discusses the forms of mean and proportion mentioned here, and a series of others, in his *Intr. Arithm.* Book II chs. 27–9. On harmonic proportion see 27.5. On Plato's three forms of proportion see also Proclus *In Timaeum* III, 171.20–174.10 (Diehl).

63 This again represents Plato's intentions accurately.

64 The first point in this sentence is also made, for example, at Theon Smyrn. 113.22–5. A string of properties of terms in arithmetic proportion is listed at Nicomachus *Intr. Arithm.* Book II. 27.3.

65 'Ruling' proportion is discussed at length in Nicomachus *Intr. Arithm.* Book II ch. 29, where it is called 'the most perfect' proportion. Iamblichus calls it 'musical'.

Chapter 9 Evidence for what we have said, derived from Philolaus

Even the most ancient writers show agreement with what we have explained. Their name for the octave is '*harmonia*', for the fourth '*syllaba*' (since it is the first concordant combination [*syllēpsis*] of notes), and for the fifth '*di' oxeian*' (since the fifth is continuous with the concord first generated and goes on upwards);[66] and the combination of both *syllaba* and *di' oxeian* together is the *dia pasōn*, and was given the name '*harmonia*' because it is the first concord to be fitted together out of concords.[67] Their agreement with what we have said is made clear by Philolaus, the disciple of Pythagoras,[68] who writes roughly as follows in the first book of his *Physics*: pressure of time demands that we rest content with just one witness, though many people say similar things in various ways about the same subject. Philolaus' statement goes like this.

'The magnitude of *harmonia* is *syllaba* and *di' oxeian*. The *di' oxeian* is greater than the *syllaba* in epogdoic ratio. From *hypatē* to *mesē* is a *syllaba*, from *mesē* to *neatē* is a *di' oxeian*, from *neatē* to *tritē* is a *syllaba*, and from *tritē* to *hypatē* is a *di' oxeian*. The interval between *tritē* and *mesē* is epogdoic, the
253 *syllaba* is epitritic, the *di' oxeian* hemiolic, and the *dia pasōn* is duple. Thus *harmonia* consists of five epogdoics and two dieses; *di' oxeian* is three epogdoics and a diesis, and *syllaba* is two epogdoics and a diesis.'[69]

It must be remembered that what he here calls '*tritē*' is the *paramesē* of the heptachord which there was before the additional insertion of the disjunctive tone in the octachord.[70] For this note used to stand at an undivided interval of three semitones from *paraneatē*,[71] from which interval the string inserted next to it cut off a tone, and there was left a semitone as the remainder between *tritē* and *paramesē* in the disjunction.[72] It is then correct to say that the old *tritē*

[66] On these names see 12 Arist. Quint. *De Mus.* 15.8–10 with n. 99.

[67] *Dia pasōn*, 'through all (the strings)' is the usual expression for 'octave'. This explanation of the old use of *harmonia* derives from the fact that it means a 'fitting together', not necessarily a musical one.

[68] Levin (1975), p. 85 takes this expression to imply that Philolaus was a direct pupil of Pythagoras (so emphasising Philolaus' antiquity and authority). In that case Nicomachus would be quite wrong (Philolaus lived in the late fifth century). But it need only mean that he was a 'follower' of Pythagoras' doctrines.

[69] For commentary on this passage see notes to 1.12 Philolaus frag. 6.

[70] This seems to suggest that Philolaus' system is the 'old heptachord' of chapters 3 and 5. But that was a pair of conjoined tetrachords spanning a seventh, in which the so-called *paramesē* stood a semitone above *mesē* (in the place of the note usually called *tritē synēmmenōn*). In Philolaus' system, the *tritē* is a tone above *mesē*, at the site of the *paramesē* of the normal octachord that was formed by two tetrachords disjoined by a tone. This system spans an octave.

[71] Thus the upper tetrachord, beginning at the so-called *tritē*, a tone above *mesē*, was defective, containing only three notes: *tritē* was three semitones below *paraneatē*, and *paraneatē* a tone below *neatē*. Hence, the octave was spanned by an arrangement of only seven strings: see 1.12 with n. 34.

[72] The new, post-Philolaan note is treated as being inserted below the old '*tritē*', and as displacing it upwards by a semitone. The diagnosis is parallel to that given for a similar case in chapter 5 (see n. 39 above). The *tritē* then comes to stand where diatonic *tritē diezeugmenōn* normally does, a tone below *paranētē*, and *paramesē* takes its place a tone above *mesē*.

stood at an interval of a fourth from *nētē*, the interval that *paramesē* now marks off, instead of *tritē*. But people who do not understand this make the criticism that it is impossible for *tritē* to be in epitritic ratio with *nētē*. And others quite plausibly say that the additional note was inserted not between *mesē* and *tritē*, but between *tritē* and *paraneatē*: and they say that this note was called *tritē* in place of the other, and that the old *tritē* becomes *paramesē* in the disjunction.[73] And they say that Philolaus calls *paramesē* by its former name,
254 *tritē*, even though it stands at a fourth from *nētē*.

Chapter 10 Concerning the fitting together [harmosis] of the notes through the numerical ratios

Let us go back to our earlier discussion[74] and add to it the next step, which is to assert that by contrast with the proportionality [*symmetria*] taken in relation to tension, which makes the notes high through large quantity and low through small, when we consider the lengths or thicknesses of strings, or of the bores of *auloi*, we do so according to the reverse proportion [*analogia*]. For in these cases it is the other way round, small dimensions yielding high pitch and greater ones low pitch. For if you take a long string at a single uniform tension lying above a measuring rod [*kanōn*], and fixed away from the rod so as not to touch it, and if you compare the note from the whole string when it is plucked with that from half of it, the string being divided off exactly in the middle by a bridge or something of the sort, so that the vibration from the blow does not extend further than half-way, you will find that the sound from the half string stands at an octave to the larger sound from the whole – that is, it is double that sound, being qualified in the opposite way to the relationships of the lengths.[75] And if you exclude the vibration from a third part of the length, accurately measured, the sound uttered by two thirds of the string will necessarily stand in hemiolic ratio to that from the whole, the opposite way round to the length. And if you cut off a fourth part of the string from the blow,
255 not allowing the vibration to extend further, the sound from three parts of the string will stand in epitritic ratio to that from the whole, the opposite way round to the relationship of length. In the same way in the case of an *aulos*

[73] This way of viewing the matter may seem more natural. A new note is inserted a semitone above the old *tritē*, and because it is now the third note from *nētē* (inclusive), it takes over the name *tritē* ('third'). The old *tritē* stays where it was, and changes its name to *paramesē*.

[74] Chapter 5–9 are the core of the treatise, and have been presented as a historical narrative. This chapter returns to the analytic, non-historical mode, and its subject is continuous with that of chapter 4.

[75] This explains why monochords are treated in chapter 4 as having affinities with *auloi* etc. The notion of 'reversing the relationships' is made much of in Thrasyllus' account of the division of the *kanōn*, 9.4 Theon Smyrn. 87.9ff. Nicomachus' phrase 'double that sound' reveals that he does not regard the two kinds of relation as equally valid expressions of the sounds' pitch-ratios: greater numbers belong properly to higher notes (see n. 53 above). Nicomachus probably rests his view on the familiar association of higher pitch with greater speed: for a detailed discussion see 9.7 Aelianus ap. Porph. *Comm.* 36.9ff., and cf. 11 Ptol. *Harm.* Book 1 ch. 3.

divided into four equal lengths by three holes, if the holes are sealed up by covering them with the fingers, and we compare the note from the whole *aulos* with that given out from the middle hole when the finger covering it is lifted, the latter will be found to be double the former, and the sound from the middle hole will be at an octave from that of the whole *aulos*. And this middle note stands in hemiolic ratio to the note from the hole below it, which lies lowest down, next to the bottom of the instrument; while this one is in epitritic ratio to the note from the whole. The sound from the hole nearest the mouthpiece [*glōssis*] is double that from the middle hole, and quadruple that from the whole, the reverse of the proportions of the lengths. And in *syringes* the lengths produce a similar result, and the widths of the pipes function in the same way as the thicknesses of strings: for the sound of a pipe in two sections is double that of one in four.[76]

Chapter 11 Concerning the double octave in the diatonic genus

Now the scope of the diagram in the diatonic genus is a double octave of quadruple breadth.[77] And this is the maximum that a voice trained for
256 competition can traverse without some kind of risk or slippage, since it becomes difficult to pitch at each of the extremes, tending towards a mere squeak in the area of *nētē* and to a hoarse whisper around the deeper of the *hypatai*.[78]

To the ancient style of *lyra*, that is the seven-stringed one, put together by the conjunction of two tetrachords (in which *mesē* formed a boundary for both the concordant intervals, the upper boundary for the lower tetrachord, which goes down to *hypatē*, and the lower boundary for the upper tetrachord, which goes up to *nētē*),[79] they added two more tetrachords, one at each end. Next to the original *nētē* they added what is called the tetrachord *hyperbolaiōn*, since it consists of higher and additional sound, taking it in conjunction, and beginning from the old *nētē* itself. Then the added tetrachord reached its limit by the attachment of only three further notes, which were given, reasonably

[76] Compare particularly 9.2 Theon Smyrn. 61.2ff., and, for example, 4.19 ps.-Ar. *Probs* XIX.23. Nicomachus' statement about the widths of pipes is misleading. Pitch does not vary with pipe-width or string-thickness in the same way as it does with pipe-length or string-length, nor do pipe-width and string-thickness affect pitch in equivalent ways, but compare 9.2 Theon Smyrn. 57.5. The reference to 'sections' suggests that the pipes of the *syrinx* were sometimes made up of different numbers of pieces, each of the same size, but the inference is insecure. The wing-shaped *syrinx* common in the Roman period seems usually to have had continuous, not 'modular' pipes. In the older Greek form the pipes were all the same length, blocked to different sounding lengths with wax (see *GMW* vol. 1, p. 196, n. 46).

[77] That is, the ratio between notes bounding an interval whose breadth is a double octave is 4:1.

[78] See chapter 2 above with n. 11. The deeper of the *hypatai* is *hypatē hypatōn*. For a rather larger estimate of the compass of the human voice see 7 Aristox. *El. Harm.* 20.28–31. Compare also 9.3 Adrastus ap. Theon Smyrn. 64.1ff.

[79] This describes the heptachord of chapters 3 and 5, not that attributed to Philolaus in chapter 9.

enough, the following names: *tritē hyperbolaiōn*, then *paranētē hyperbolaiōn*, then *nētē* of the same tetrachord. In order to make the distinction, the notes of the previous tetrachord – the one conjoined with *mesē* – should be given the following names: after *mesē*, *tritē synēmmenōn*, then *paranētē synēmmenōn*, then *nētē synēmmenōn*. And when the whole construction upwards from *mesē*
257 is enumerated, it too necessarily completes a heptachord. Going down from the original *hypatē* they added the second of the tetrachords mentioned, again in conjunction, with the old *hypatē* being incorporated in it as well, as the highest of the notes in it. In the same way as before, in order to distinguish this from the previous arrangement, it also was given clearer names. To each was added the term *hypatōn*, giving names like *hypatē hypatōn*, *parhypatē hypatōn*, and *diatonos hypatōn* or *lichanos hypatōn*, for it makes no difference which of these expressions is used. And this whole *systēma* from *mesē* to *hypatē hypatōn* also came out as a heptachord composed of two conjoined tetrachords, employing as their one common note the old *hypatē*. Thus from *hypatē hypatōn* to *nētē hyperbolaiōn* there were four conjunct tetrachords. And so a thirteen-string *systēma* was discovered, with seven strings arranged diatonically in both directions.[80] Later, as we said before, those who elaborated the *harmonia* inserted the eighth note, the one which stands at the interval of a tone, between *mesē* and the old *tritē* (or, as some say, between *tritē* and *paranētē*), thus displaying the character of the fifth.[81] And *mesē* was then found
258 to be no longer really *mesē*. For where there is an even number of strings, there can be no single middle one, but there must be two, the seventh and the eighth. Further, they took in addition below *hypatē* one outermost note, the lowest of all – which for that reason they called *proslambanomenos* – this note too being at a distance of a tone, below *hypatē hypatōn*, so that there should be octachord *systēmata* in both directions from *mesē*, while *mesē* itself became genuinely the middle note, since it lay eighth from each end in a series of fifteen notes, and so that the complete magnitude of the diagram should become two octaves, that is, the double taken twice, which is quadruple.[82] The arrangement of the names, taken in sequence upwards, was thus as follows.

[80] Nicomachus' description is admirably clear, but the thirteen-note system that he describes is otherwise unknown (it reappears in the thirteenth-century treatise of Pachymeres, *Harm.* ch. 11, 127.10–12 (Tannery)). Its oddity is the addition of a fourth tetrachord in conjunction above *nētē synēmmenōn*, forming a complete system of four conjoined tetrachords spanning altogether two sevenths, not two octaves. The tetrachord *hyperbolaiōn* is thus placed a tone lower than usual. It was in any case not standard practice to conceive this tetrachord as continuing a scale from the tetrachord *synēmmenōn* at all, even through a tone of disjunction (see particularly Cleonides *Eisagoge* 199.11–201.7), but this disjunction is recognised at Bacchius *Eisagoge* 301.10–16, and see chapter 11 below. For a discussion of Nicomachus' scale see Chailley (1956), but the system may be pure invention.

[81] This refers to the 'invention' made by Pythagoras, chapter 5 above.

[82] The result is the perfect system spanning two octaves, as described by many authors. Nicomachus has not explained that the insertion of the new note (above *mesē*, or above the old *tritē*) has transposed the tetrachord above *mesē* upwards by a tone to form the tetrachord *diezeugmenōn*, and that this change has also displaced the tetrachord *hyperbolaiōn* upwards through a tone. *Nētē hyperbolaiōn* is now in its usual position, an octave above *mesē*. The name *proslambanomenos* means 'taken in addition'.

(1) *Proslambanomenos*

(2) then, after a distance of a whole tone, *hypatē hypatōn*

(3) then after a semitone, *parhypatē hypatōn*

(4) then after a tone, *lichanos hypatōn*, so named because the finger of the left hand next to the thumb, which is called by this name, '*lichanos*', is always placed on this string[83]

(5) then after another tone, *hypatē mesōn*

(6) next in succession [*hexēs*], after a semitone, *parhypatē mesōn*

(7) after a tone, *lichanos mesōn*, which they also call '*diatonos*' after the diatonic genus itself

(8) then after another tone, *mesē*

(9) then *paramesos*, after a whole tone[84]

(10) then *tritē diezeugmenōn*, after a semitone

259 (11) then after a tone, *paranētē diezeugmenōn*

(12) and after another, *nētē diezeugmenōn*

(13) next in succession [*hexēs*] to this, after a semitone, *tritē hyperbolaiōn*

(14) then after a tone, *paranētē hyperbolaiōn*

(15) and above them all, after a tone, *nētē hyperbolaiōn*

In a way that is reminiscent of the conjunction involved in the original heptachord, there was also inserted, between the tetrachord *mesōn* and the tetrachord *diezeugmenōn*, another, called the tetrachord *synēmmenōn*, which begins with its own *tritē* a semitone away from *mesē*, then, after a tone, has a *paranētē* peculiar to itself, and then, after another tone, has the *nētē synēmmenē*, which is in all respects of the same tension and sound as *paranētē diezeugmenē*.[85] Thus there are in all five tetrachords, the tetrachords *hypatōn*, *mesōn*, *synēmmenōn*, *diezeugmenōn* and *hyperbolaiōn*: and these involve two disjunctions and three conjunctions. The disjunctions are those between the tetrachords *synēmmenōn* and *hyperbolaiōn*,[86] and between *mesōn* and *diezeugmenōn*, each disjunction making a separation of the magnitude of a
260 tone: and the three conjunctions are the one conjoining *hypatōn* with *mesōn*, the one conjoining *mesōn* again with *synēmmenōn*, and lastly the one conjoining *diezeugmenōn* with *hyperbolaiōn*. In our broader treatment of the subject we shall set out for you, for each note of these tetrachords, the ways they were discovered and their causes and their progressions, how these discoveries came about, and by whom, and when, and on what basis, beginning from the tetrachord and going on to the most complete filling out of the octave,

[83] Compare chapter 3 above and n. 24; the same explanation is given, for example, at 12 Arist. Quint. *De Mus.* 8.17–19.

[84] The usage *paramesē* is more common, but see, for example, 12 Arist. Quint. *De Mus.* 7.24, 8.7.

[85] More commonly *synēmmenōn*, *diezeugmenōn*.

[86] See n. 80 above.

not only in this diatonic genus, but also in the chromatic and enharmonic, using the ancient evidence of the most trustworthy and eminent men. In addition we shall set out the division of the so-called Pythagorean *kanōn*, worked out accurately and completely according to the doctrine of that school, not in the manner of Eratosthenes or Thrasyllus, who misunderstood it,[87] but in that of Timaeus of Locri, whom Plato also followed, right up to the twenty-seven-fold ratio.[88]

Chapter 12 Concerning the progression and division of notes according to the three genera[89]

So that you may have in good order the progressions according to the three
261 genera, extending from *proslambanomenos* to *nētē hyperbolaiōn*, it will be useful for the sake of clarity to begin by reminding you first of a few points which have already been mentioned above.

A note [*phthongos*] is an indivisible sound, as it were a unit in respect of hearing: or, according to more recent writers, it is the incidence of sound on a single simple pitch: or, as some say, it is a sound with no breadth, having no extension in space.[90]

An interval [*diastēma*] is what lies between two notes.[91] A relation [*schesis*] is the ratio which measures the distance in any interval: and a difference [*diaphora*] is the excess or deficiency of one note with respect to another. Those

[87] The complaint against Thrasyllus may be that he took his divisions only through the compass of the musical octave or double octave, unlike Plato, whose cosmic scale runs, as Nicomachus says, up to the ratio 27:1 (four octaves and a sixth). For a defence of this aspect of Plato's project see 9.3 Adrastus ap. Theon Smyrn. 64.1–65.9. Alternatively, or additionally, Nicomachus may be recommending the principle that division should be accomplished on the basis of a scheme of proportions (see chapter 8 above), rather than by the mere application of ratios, even if the methods produce the same result. Possibly he takes a critical view of some of the details of Thrasyllus' procedure, for example, those in which it differs from that of 8 Eucl. *Sect. Can.* propositions 19–20. For Thrasyllus' divisions see 9.4–9.5 Theon Smyrn. 87.9ff. Those of Eratosthenes are tabulated in 11 Ptol. *Harm.* Book II ch. 14, and are certainly open to other objections from Nicomachus' point of view: see n. 117. Nicomachus would find fault with the actual ratios of his divisions, unlike those of Thrasyllus, since they are not direct derivatives from the Platonic system, and his general procedure is arguably incoherent.

[88] The character in Plato's dialogue who sets out the division of the world-soul is of course Timaeus. There were insinuations even in the fourth century (retailed by Aristoxenus) that the dialogue was plagiarised from Pythagorean sources. But Nicomachus may be alluding to a treatise that passed in later antiquity for the work of Timaeus of Locri. It is in fact a pseudo-Pythagorean forgery, whose date is not much earlier than that of Nicomachus himself, loosely based on Plato's *Timaeus*. In its scalar constructions, however, it does not even use the number 27. (For comments and references see Levin (1975), pp. 91–2. The spurious treatise is printed in Thesleff (1965), pp. 205–25.)

[89] This will fulfil the promise made at the opening of chapter 7: see n. 55 above.

[90] For a definition of the first sort see 12 Arist. Quint. *De Mus.* 7.15–16. The second is derived ultimately from 7 Aristox. *El. Harm.* 15.15–16, the third from *El. Harm.* 3.21–4. For other references see n. 37 to Arist. Quint. *De Mus.* 7.15–16. This 'reminder' and those that follow echo nothing earlier in the text, but see n. 29 above.

[91] Though the immediate sequel reintroduces Pythagorean conceptions, this definition, like most of the chapter, belongs to the Aristoxenian tradition. See 7 *El. Harm.* 15.24–32 and, for example, 12 Arist. Quint. *De Mus.* 10.18–19.

who think that relation and difference are the same are wrong: for as you will understand, the difference between 2 and 1 is the same as that between 1 and 2, whereas their relation is not the same. For 2 is double 1, while 1 is half of 2. Or again, the difference between three or more terms in arithmetic proportion is the same in each case, but the relation is different in each. But you will learn more fully about this in our fuller treatment.[92]

A *systēma* is an assemblage of two or more intervals.[93] Now while no note in these intervals is concordant with its successor, but always discordant, some
262 *systēmata* are concordant and others discordant. They are concordant when the notes which bound them are different in magnitude, but when struck or sounded simultaneously, mingle with one another in such a way that the sound they produce is single in form, and becomes as it were one sound. They are discordant when the sound from the two of them is heard as divided and unblended.[94]

Since the first and most elementary concord is the fourth in a continuous tetrachord and in epitritic ratio, it is naturally here that the differences between the three genera of melody are to be found.[95] The diatonic, about which we spoke earlier, proceeds as follows: a semitone, then a tone, then a tone, three intervals between four numbers, that is, four notes. And this is why it is called 'diatonic', because it is the only one of them to proceed through tones [*dia tōn tonōn*].[96] The chromatic progresses like this: a semitone, then another semitone, then above them an undivided interval of three semitones, so that it too, even though it is not straightforwardly composed of two tones and a semitone, nevertheless evidently has intervals equal to two tones and a semitone. It is in the nature of the enharmonic to have the following division: a diesis – that is, half of a semitone – and then another diesis, together equal to a semitone, and then the remainder of the tetrachord, a whole, incomposite ditone. Thus this is also equal to two tones and semitone: for no note can be concordant with another inside these limits. So it is clear that the distinctions

[92] This simple distinction between 'difference' and 'relation' is not the same as that made by some authors between 'interval' (*diastēma*) and 'ratio' (*logos*). See, for example, Theon Smyrn. 81.6–82.5, where the essential points are that the interval between A and B is the same as that between B and A, while the ratios A:B and B:A are different, and that there can be a ratio, but no interval, between two items that are equal. The former claim sounds like the first of those made by Nicomachus, but in Theon it is clear that 'interval' is not equivalent to 'arithmetical difference', while this is plainly the sense of 'difference' in Nicomachus' point about arithmetical proportion.

[93] This is equivalent to 7 Aristox. *El. Harm.* 15.34–16.1. For other references and alternative descriptions see 12 Arist. Quint. *De Mus.* 13.4–5 with n. 81.

[94] On these conceptions of concord and discord see 8 Eucl. *Sect. Can.* 149.17–20 with n. 7. For the notion of *systēmata* as concordant or discordant see 7 Aristox. *El. Harm.* 17.5–7, 12 Arist. Quint. *De Mus.* 14.7–10.

[95] Compare 7 Aristox. *El. Harm.* 21.34ff., 46.19ff. The analyses that follow are based entirely on Aristoxenus, corresponding to his tense diatonic, his tonic chromatic and his enharmonic: cf. 9.2 Adrastus ap. Theon Smyrn. 53.17ff. It is only after the analyses have been set out that Nicomachus alludes to the Pythagorean view that divisions into half-tones and quarter-tones are impossible.

[96] Compare 9.2 Adrastus ap. Theon Smyrn. 54.12–15, and the interpolated passage at 12 Arist. Quint. *De Mus.* 92.22–3.

263 between the genera do not arise from the four notes of the tetrachord, but only from the two central ones.[97] Thus in the chromatic the third note is different from that in the diatonic, while the second remains the same as that of the diatonic while being of the same pitch as the third note of the enharmonic. In the enharmonic the two middle notes are different from those of the diatonic; and thus the enharmonic lies at the opposite pole to the diatonic, and the chromatic is between them. For it involves only a slight alteration, by just one semitone, from the diatonic. (Hence we say of changeable people that they have 'colour' [*chrōma*].)[98] The extremes of the tetrachord are called fixed notes, since they do not change in any of the genera. The notes in the middle are moveable – or at any rate they are in the enharmonic. In the chromatic the second note is both moveable and not moveable: it does not change with respect to diatonic, but with respect to enharmonic it does.

The octave, which is the *systēma* in eight strings from *mesē* down to *proslambanomenos*, or from *mesē* up to *nētē hyperbolaia*, granted that the fourth is two tones and a semitone, and the fifth three tones and a semitone, does not straightforwardly add up to six tones, as more recent writers think, but to five tones and two so-called semitones. If these were really halves of tones, what would prevent them making up a tone together, and the octave
264 being six tones?[99] But we shall give a clear and detailed demonstration of this point in our fuller account. Philolaus is in agreement with us, since in the work previously mentioned he says: '*Harmonia* is five epogdoics and two dieses', that is, two semitones: and these together would have made one tone, if they were really half a tone each.[100]

When the three genera are mixed together with one another in the same diagram, their names will be as follows.[101]

Proslambanomenos	A
Hypatē hypatōn	B
Enharmonic *parhypatē hypatōn*	B+
Chromatic and diatonic *parhypatē hypatōn*	c
Enharmonios hypatōn	c

[97] See the references to Aristoxenus in n. 95 above, and for a formal argument 7 *El. Harm.* 61.5ff.

[98] Compare 9.2 Adrastus ap. Theon Smyrn. 55.4–7, 12 Arist. Quint. *De Mus.* 92.24–5.

[99] Nicomachus does nothing to prove that these addition sums are wrong, and his expression is a little confused. But the general point is clear enough: the relevant doctrines are those set out in 8 Eucl. *Sect. Can.*, particularly propositions 3, 9, 14–16. Compare also 9.2 Adrastus ap. Theon Smyrn. 53.8–16, 9.9 Panaetius ap. Porph. *Comm.* 65.21ff.

[100] See the Philolaus passage quoted in chapter 9.

[101] The table is unusual only in listing enharmonic *parhypatai* and *tritai* separately from those of diatonic and chromatic. Most writers do not treat them as distinct entities for these purposes. The fact may ultimately be traceable to the early existence of systems in which the second-lowest note of the tetrachord was not moveable (see 1.21 Archytas ap. Ptol. *Harm.* 30.9ff.). The names *enharmonios, chrōmatikē, diatonos* are common variants for 'enharmonic (etc.) *lichanos*' and 'enharmonic (etc.) *paranētē*'.

Chrōmatikē hypatōn	$c^{\sharp}$
Diatonos hypatōn	d
Hypatē mesōn	e
Enharmonic *parhypatē mesōn*	e+
Chromatic and diatonic *parhypatē mesōn*	f
Enharmonios mesōn	f
Chrōmatikē mesōn	$f^{\sharp}$
Diatonos mesōn	g
Mesē	a
Enharmonic *tritē synēmmenōn*	a+
Chromatic and diatonic *tritē synēmmenōn*	$b^{\flat}$
Enharmonios synēmmenōn	$b^{\flat}$
Chrōmatikē synēmmenōn	b
Diatonos synēmmenōn	c'
Nētē synēmmenōn	d'
Paramesē	b
Enharmonic *tritē diezeugmenōn*	b+
Chromatic and diatonic *tritē diezeugmenōn*	c'
Enharmonios diezeugmenōn	c'
Chrōmatikē diezeugmenōn	$c'^{\sharp}$
Diatonos diezeugmenōn	d'
Nētē diezeugmenōn	e'
Enharmonic *tritē hyperbolaiōn*	e'+
Chromatic and diatonic *tritē hyperbolaiōn*	f'
Enharmonios hyperbolaiōn	f'
Chrōmatikē hyperbolaiōn	$f'^{\sharp}$
Diatonos hyperbolaiōn	g'
Nētē hyperbolaiōn	a'

265 Forgive the hasty nature of this essay: as you are aware, you set me this task just as I was poised to go off on my journey: and with your accustomed great kindness and thoughtfulness for your friends,[102] accept it as a beginning and a friendly offering. You can expect, if the gods are favourable, a most thorough and altogether a most complete technical treatise on these matters, which I will send you just as soon as the first opportunity arises.

[102] The phrase is difficult, and might possibly mean 'intellectual concern about common matters' (i.e., about matters of general or abstract interest). This conclusion picks up the remarks of the first chapter, and rounds the work off as a letter to the lady it addresses.

CHAPTER II

Ptolemy

Claudius Ptolemaeus worked in Egypt during the second century A.D. References in his own writings indicate that he was active as a scientist from the 120's, and he may have lived well into the second half of the century. He seems to have lived in Alexandria, and probably also at Canopus. His reputation as one of antiquity's finest scientists rests principally on his astronomical writings, and especially on the magnificent treatise whose original Greek title was *Mathematike Syntaxis* ('Mathematical Treatise'). This work seems later to have been nicknamed *Megiste* ('Greatest') *Syntaxis*: a transmutation of the latter expression through Arabic (perhaps first through Persian) and into mediaeval Latin gave it the name *Almagest*, by which it is now best known. Hand in hand with astronomy went astrology, a field to which Ptolemy's so-called *Tetrabiblos* brought an intriguing mixture of rationally grounded conjecture and scientific caution. He was an original and influential geographer. He wrote essays in mechanics and in optics, and a monograph called *On the Criterion*, which discusses the foundations of scientific knowledge and the methods by which it is to be pursued.

He is also the author of a substantial and systematic treatise on harmonics, certainly written at a time when his main researches in astronomy were already well advanced or complete. Though musicologists acknowledge its merits and originality, and though the influence of some of its ideas on Byzantine, mediaeval and Renaissance theorists is widely recognised, no English translation of the whole work has previously been published, and students of the ancient sciences have seldom given it the attention it deserves. Yet Ptolemy's sophisticated views about the aims and methods of science are nowhere better stated and exemplified, and the *Harmonics* is perhaps as important a document in the history of scientific method as in musicology. My translation is bound to be imperfect, and my notes do not approach the status of an adequate commentary, but I hope they are good enough to open the work to a wider readership, and to provide a basis on which other scholars can build.

Ptolemy was well read, and his work is deeply indebted to his predecessors, but he seldom mentions earlier musicologists explicitly except to criticise them, and his criticisms do not spare theorists of any doctrinal persuasion. (In this respect he resembles Aristoxenus, though his polemics are more closely argued, and he indulges less in mere abuse.) But while he attacks Aristoxenus and his followers at the level of their most basic conceptions, the faults he finds with the Pythagoreans are either in points of detail, or else arise from their failure to apply essentially sound principles in the right way. If we adopt a crude classification of theorists into Aristoxenians on the one hand and Pythagoreans and Platonists on the other, Ptolemy falls squarely into the Pythagorean camp. Like them, he ties his quantification of harmonic relations to an account of the physical basis of sound and pitch. He expresses intervals between notes as ratios of numbers, rejecting the alternative, Aristoxenian mode of representation outright. More fundamentally, he holds that the principles of harmonic order are mathematical: their nature and credentials are to be discerned by reason. Though reason's findings must then be checked against perceptual experience, it is through mathematical reasoning, not by induction or abstraction from experience, that we can come to articulate the laws to which the fundamental harmonic structures conform, and

can explain their special modes of coherence. Because the harmonic structures are expressions in sound of a mathematical order that is not peculiar to music or sound as such, but is the rational order underlying formal perfections throughout the universe, Ptolemy can argue that the same principles are at work in the soul and in the heavens. Here again the spirit of his work is plainly Pythagorean, quite foreign to the 'autonomous' harmonics of the Aristoxenians.

The 'Pythagorean' tradition, however, was not monolithic, and Ptolemy's method diverges in important respects from one branch of it at least, that most closely allied to Platonism. Though it is through reason that the form of harmonic structures is discerned, its findings are subject to the test of perception. If the analyses generated out of rational principles cannot handle the systems of musical practice, or if the findings of reason do not square with those of perception, this cannot, on Ptolemy's view, be merely because practice and perception are imperfect: it must be because the wrong principles have been chosen. The 'appearances' are not to be rejected. The task of harmonics is to explicate the mathematical foundations of systems whose beauty and excellence is evident to the ear, not those of some other, purely theoretical constructions. It seeks, in fact, to show that it is on rationally coherent mathematical patterns of order that the perceived beauty of real music rests.

Ptolemy was not the first 'Pythagorean' to adopt this approach. It lies behind the analyses of Archytas, Eratosthenes and Didymus, and is paid lip-service at least by Adrastus, despite the Platonism of his harmonic divisions. Ptolemy's originality lies in the subtlety and sophistication of his account of the roles of reason and perception, in the diligence with which he pursues its consequences, and in his development of mathematical principles that avoid the difficulties into which previous theorists had fallen. (There are also, of course, many specific points of musicological doctrine on which Ptolemy has new views to present.)

Perception provides the harmonic theorist with his raw data, identifying sonorous relations as concordant or discordant, melodic or unmelodic, and classifying systems in terms of genus, *tonos* and the like. By itself it cannot discern the 'forms', or patterns of order, which the qualitative intuitions expressed in these distinctions reflect. A concord, for example, is for perception a complex of sounds with a certain audible quality. Its 'form', the underlying cause of this quality, is a mathematical relation between quantitative variables, and is not as such perceptible, but must be discovered by reason. As we have seen, however, the claims of reason must in turn be submitted to the judgements of perception. If 'rational' principles seem to show that what perception accepts as concordant is not, or that aesthetically satisfying scalar systems are rationally indefensible, then the principles must be wrongly conceived or inappropriately applied. But the need for perceptual tests creates a further difficulty, for the unaided ear cannot make reliable judgements about fine-grained musical distinctions. We cannot construct intervals, just by ear, in exactly the ratios we wish to test, nor can hearing alone assure us that the intervals we have constructed are the best possible representations of the aesthetic category we intend. Hence it is essential to equip ourselves with experimental aids, technical devices of the greatest possible accuracy, through whose use we can confidently construct notes in exactly the pattern of numerical relations that theory proposes, and bring them to the judgement of hearing. If this is done, Ptolemy believes, and if the theory is right, our hearing will recognise the perfection of the resulting systems and the inadequacy of lesser approximations, even though it cannot generate them itself without help.

The experimental instruments that are needed (the monochord and its more complicated derivatives) must themselves be built on principles that reason discovers and underwrites. Their basis lies in physics and mathematics, most obviously in the laws

that link the relations between pitches to those between lengths of a sounding string. But such devices are useless unless they conform to these 'rational' principles in every detail, not merely in general conception. Ptolemy's meticulous accounts of their construction, of the problems that may arise in their use, and of such minor but crucial matters as the procedure for testing whether a string is 'true', are quite unparalleled in other ancient sources.

It is a consequence of Ptolemy's stance that the systems correctly derived from mathematical theory must also be acceptable to perception. But this 'acceptability' itself turns out to involve an unexpected complication. The fact that perception recognises a certain division of the tetrachord as harmonically admirable and proper does not mean, necessarily, that it is one current in actual musical practice. This implies that perception can recognise the formal credentials of structures which find no place in the familiar contemporary repertoire (though it represents them, of course, under aesthetic categories, and not in their mathematical essence). Those that are actually used, however, must be among those approved by reason, or be built in intelligible ways out of components that are themselves rationally accredited. Hence Ptolemy devotes several chapters, once the 'rational' divisions have been constructed and perceptually confirmed, to a study of systems actually employed in his day by players of the *lyra* and the *kithara*, and to their analysis in terms of the divisions previously set out. The care with which he undertakes this investigation, and his willingness to modify theory in the light of the evidence of practice, give us good reason to trust the accounts he gives. They are precious evidence, unique in their combination of mathematical precision with empirical good faith, about systems of attunement that Greek musicians really used.

On the mathematical side, there were two major challenges that any writer in this tradition had to face. Granted that physical theory underwrites a conception of pitches as differing quantitatively, and that the 'form' of a relation between pitches is properly expressed as a numerical ratio, the central questions are these. First, is there a mathematically coordinated class of ratios to which all fundamental harmonic relations belong? Are there, alternatively, several such classes, different ones perhaps for concords and for other melodic intervals? In either case, what principle underlies the status of these ratios, and not others, as legitimately harmonic? Secondly, once we have identified the general class into which the elementary ratios of a scalar system must fall, what principles determine that certain sequences of such ratios and not others constitute properly harmonic divisions?

As to the first question, previous theorists had often given special status to ratios whose form is multiple ($mn:n$) or epimoric ($n+1:n$). Most Pythagoreans insisted that all concords must have one or other of these forms. Some had sought to ensure that all the small scalar intervals that subdivided the concords were also epimoric. The former thesis led to notorious difficulties about the octave plus fourth, and had no answer to the question why just some multiples and epimorics, and not others, correspond to concords (see Ptolemy Book 1, chapter 6). The latter principle is one that Ptolemy will accept, but for which no earlier writer seems to have found adequate justification (a possible and partial exception is Archytas: see the Appendix to chapter 1).

Ptolemy's solution to the problem is outlined in Book 1, chapter 7. He first distinguishes relations between notes of different pitch into three sorts: homophones (i.e., octaves and their multiples), concords and melodic intervals (i.e., legitimate elementary scalar steps into which concords are divided). But each of these classes, he says, is 'close' to its neighbours; they are not three radically distinct sorts of thing. Hence the kinds of ratio proper to each are not sharply different, but form a gradation between the more and the less perfect.

Notes of equal pitch correspond to equal numbers. The homophone of the octave,

since it is evidently most closely related to this equality, must have a ratio which is itself as near to equality as possible. This does not mean that the terms of its ratio must be nearly equal: the equality holds between the smaller term on the one hand, and the difference between the terms on the other. Where this equality holds, the smaller term will be half the greater, and their ratio is 2:1. By the same principle, the next most perfect intervals, the concords of a fifth and a fourth, must have ratios that approximate most closely to the equality exhibited by that of the octave. They must also, as perception demands, be such that they combine to form an octave; and a different application of the guiding principle requires that their ratios must be those that most nearly divide the octave in half. But here a further constraint is imposed on the ratios to be considered. In every simple concord, and every elementary melodic interval, the difference between the terms of the ratio must be 'commensurable' with the terms themselves. That is, it must be a 'simple part' or factor of each. It is a consequence of this principle that all such ratios will be epimoric. Hence the ratios of the primary concords must be the two epimorics that most nearly bisect the octave, and these are 3:2 and 4:3. A further extension of these principles provides conditions to be met by the ratios of the 'melodic' intervals. The difference between their terms must be commensurable with the terms, and hence they are epimoric. The 'more melodic' among them are those that 'most nearly make divisions into halves' – i.e., those in which the difference between the terms is most nearly equal to the smaller term. By these principles, the more melodic will be those whose terms are smaller numbers, and the ratios fundamental to harmonics will form an orderly series, 2:1, 3:2, 4:3, 5:4, and so on, graded from the more perfect to the less.

Ptolemy's answer to the second question is presented in Book 1, chapter 15, and it turns out to be a development of the first (supplemented by further guidelines drawn directly from perceptual experience). I shall not pursue it in detail here. It depends on an application of the principle of 'approximately equal division' to the task of subdividing the fourth to form a tetrachord. The process has two stages. First, the fourth is divided in as many ways as possible into two intervals of epimoric ratio. Secondly, the interval assigned (on other grounds) to the lower part of the tetrachord is in turn divided in accordance with the law of near-equality (Ptolemy's procedure at this point becomes rather complex). The divisions of the tetrachord that emerge from this process are then assigned to the harmonic genera distinguished by musical perception, and must submit themselves to perceptual testing.

Ptolemy's mathematical solution to the problems of harmonic division is elegant and economical, and allows him to escape the pitfalls that had trapped his predecessors. In particular, since his principles apply only to simple homophones, concords and melodic intervals, not to compound ones, he can evade the puzzles connected with the octave plus fourth. And his procedure of grading perceived intervals on a scale of 'excellence', and of grading ratios by their closeness to equality, abolishes the insoluble problem of identifying clear differences in kind between classes of harmonic ratios, to correspond to radically distinct classes of interval. It must be emphasised that though some of his scalar divisions found favour with Renaissance theorists, they were not formulated with an eye to the issues about vocal and instrumental intonation that exercised those later writers. They sprang directly from his search for a mathematical theory under which all harmonic systems could be coordinated and their credentials rationally explained. Musical perception, we are told, will accept these systems as ideally excellent. But the reasoning in which their construction is rooted arises more from Ptolemy's metaphysical and meta-scientific convictions than from reflection on questions specific to the domain of practical music.

The plan of the *Harmonics* is relatively straightforward, and I shall not try to

summarise it in detail. An outline of the topics considered will be found in the tables of contents that preface each of the three books. Book I, after a review of methods and aims, and of the principles fundamental to harmonics, proceeds to analyse just those elementary structures that were discussed in the writings of lesser Platonists and Pythagoreans such as Adrastus and Nicomachus, though its procedures and conclusions are quite different from theirs. It deals with the ratios of concords and simple melodic intervals, and gives an account of the tetrachordal divisions constituting the various genera and their species. Ptolemy's critiques of previous analyses, and the means by which he develops and justifies his own quite original ones, are vastly more sophisticated than his predecessors', but the pattern of topics is the same. More complex concepts and structures, largely ignored by those writers, are discussed in Book II and the first two chapters of Book III. The focus of this part of the work is on the *tonoi*, of whose nature and intermodulations Ptolemy gives a subtle and original account (see also the General Introduction, pp. 19–22). His analysis demands a prior grasp of various other important topics. He discusses, in particular, the structure of the 'complete' or 'perfect' system of the double octave and its relation to the so-called Lesser Perfect System (which Ptolemy treats as derivative); the distinction between 'position' (*thesis*) and 'function' (*dynamis*) in their application to the naming of notes (his account is probably dependent on existing practice, but the details of its articulation are certainly his own); and the distinctions between various forms of modulation. The climax of this phase of the treatise is in the fourteenth and fifteenth chapters of Book II, where Ptolemy is in a position to offer complete quantitative descriptions of the two-octave 'changeless' system in each genus, and of the 'simple' and 'mixed' generic divisions that musical practice makes familiar in each of the seven *tonoi*. These are then ready to be brought to the test of perceptual judgement through the use of carefully prepared experimental instruments. Throughout the two books, Ptolemy is faithful to his declared methodological principles, and cuts no corners in describing the details of these devices – different ones being introduced from time to time, as the complexity of the structures under investigation demands. For reasons touched on above, four chapters have also been given to the discussion of forms of attunement actually employed by contemporary musicians.

In the remainder of Book III, after some general reflections on the nature of *harmonia* and its relation to the 'matter' in which it is embodied, Ptolemy advances into new territory. He traces the ways in which the mathematical relations underlying the structures of audible music also constitute the 'forms' that are the essence and cause of perfection in other domains, in the human soul and in the movements and configurations of the stars. He does not suppose, like some earlier Pythagoreans, that the heavenly bodies emit sounds, or that the soul makes music in a literal sense. He holds rather that it is the same patterns of organisation, which mathematics can grasp and describe, that constitute the forms of perfection imposed on matter of all three sorts. Hence, musical structures have their analogues in the soul and in the heavens. In different material matrixes, the same formal relations create musical beauty, excellences of character and intellect, and the perfect celestial geometry of the skies. It is these intelligible relations that underlie perceptible beauties wherever they are found, and it is only these formal aspects of things, susceptible to mathematical analysis and understanding, that can be completely and scientifically known. This conception of science, and of the relation between intelligible form and unstable but perceptible matter, is also evident in the *Almagest* (see especially the first two chapters of Book I). The *Almagest*'s general theory of planetary motion, and some traces of its detailed observations, are reflected in the astronomical passages of the *Harmonics*. In the final chapter we also find hints of the astrological ideas of the *Tetrabiblos*. Unfortunately the work as we have it is not quite

complete. Byzantine commentators conjectured that Ptolemy died before it was finished, but this is rather unlikely (see notes 86 and 87 to Book III, chapters 14 and 16).

Ptolemy's *Almagest* is translated in Toomer (1984), with a valuable introduction, notes and bibliography. The *Tetrabiblos* is translated in Robbins (1940). The Greek text of the *Harmonics* is published in Düring (1930), German translation and commentary in Düring (1934). Further notes on the text are provided by Alexanderson (1969). There are no full-scale studies of the work in English, but discussions of various specific issues will be found in Monro (1894), Mountford (1926), Winnington-Ingram (1932 and 1936), Barbour (1953), Gombosi (1951), Henderson (1957).

11 The *Harmonics*

2 **Book I**

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3 **Book I**

1 *Concerning the criteria in Harmonics*

Harmonic knowledge is the power that grasps the distinctions related to high and low pitch in sounds [*psophoi*]:[1] sound is a modification [*pathos*] of air that has been struck (this is the first and most fundamental of things heard):[2] and the criteria of *harmonia* are hearing and reason, not however in the same way.[3] Rather, hearing is concerned with the matter and the modification, reason with the form and the cause,[4] since it is in general characteristic of the senses to discover what is approximate and to adopt from elsewhere what is accurate, and of reason to adopt from elsewhere what is approximate, and to discover what is accurate.[5] For since matter is determined and bounded only by form, and modifications only by the causes of movements, and since of these the former [i.e., matter and modifications] belong to sense perception, the latter to reason, it follows naturally that the apprehensions of the senses are determined and bounded by those of reason, first submitting to them the distinctions that they have grasped in rough outline – at least in the case of the things that can be detected through sensation – and being guided by them towards distinctions that are accurate and accepted.[6] This is because it is a feature of reason that it is simple and unmixed, and is therefore autonomous and ordered,[7] and is

[1] Ptolemy strikes an un-Aristoxenian attitude from the start. His preliminary definition is simple and general where theirs tend to complexity, and informal where theirs are technical. It points to connections with physical acoustics, where theirs are linked to the autonomous terminology of harmonics itself (later Aristoxenians, for example, Cleonides *Eisagoge* 179.3–8, often incorporate a list of the parts of the science into their definition). It defines the subject by reference to sounds and their pitches, without mentioning intervals, as the Aristoxenians always do. For the archetypal Aristoxenian definitions see 7 Aristox. *El. Harm.* 1.18–21, 32.10–14; for an attempt to place Aristoxenian harmonics within an altogether more ambitious framework see 12 Arist. Quint. *De Mus.* Book I ch. 5.

[2] The style of definition goes back to Archytas and to the development of 'Pythagorean' ideas in the Lyceum. (Compare, for example, 1.19 Archytas frag. 1, 2.4 Plato *Tim.* 67a–c, 3.15 Arist. *De Anima* 420a, 5 *De Audib.* 800a, 8 Eucl. *Sect. Can.* 148, 9.2 Adrastus ap. Theon Smyrn. 50, 10 Nicomachus *Ench.* ch. 4, 12 Arist. Quint. *De Mus.* 5.20ff., the last giving a formulation very close to Ptolemy's.) It confirms Ptolemy's position in a tradition more Pythagorean than Aristoxenian: see ch. 9 below, and contrast, for example, 7 *El. Harm.* 9.3ff., 12.4ff.

[3] For differences of opinion about the 'criteria' to be used in harmonics see especially 9.10–9.14 Ptolemaïs and Didymus ap. Porph. *Comm.* 22.22ff.

[4] The contrast between hearing and reason is native to controversies in the history of harmonics, particularly prevalent in Pythagorean writings. That between form and matter is Aristotelian; that between modification and cause is prominent in Stoic thought. See Düring (1934), pp. 142–3. But Ptolemy's starting point is probably Plato's *Timaeus* (see particularly 42e–47e, 51d–52c). On the roles of reason and the senses in science see especially 3.7 Arist. *Post. An.* 78b34ff., cf. Ptolemaïs and Didymus, cited in n. 3 above.

[5] This approach, in the form worked out below, seems to be original to Ptolemy, despite the multiplicity of sources he draws on.

[6] Compare particularly 2.6 Plato *Philebus* 17a–e (with 16c ff.), Aristotle *Post. An.* Book II ch. 19, *Metaph.* Book I ch. 1.

[7] Ptolemy will have found this idea in Stoic sources. Its roots go back at least to Anaxagoras (frag. 12); compare, for example, Arist. *De Anima* 429a.

always the same in relation to the same things, while perception is always involved with multifariously mixed and changeable matter, so that because of the instability of this matter, neither the perception of all people, nor even that of the very same people, remains the same when directed repeatedly to objects in the same condition;[8] but it needs, as it were as a crutch, the additional
4 teaching of reason. Thus just as a circle constructed by eye alone often appears to be accurate, until the circle formed by means of reason brings the eye to a recognition of the one that is really accurate, so if some specified difference between sounds is constructed by hearing alone, it will commonly seem at first to be neither less nor more than what is proper: but when there is tuned against it the one that is constructed according to its proper ratio, it will often be proved not to be so, when the hearing, through the comparison, recognises the more accurate as legitimate, as it were, beside the bastardy of the other; for judging is in general easier than doing the same thing (as it is easier to judge wrestling than to wrestle, to judge dancing than to dance, to judge *aulos*-playing than to play the *aulos*, and to judge singing than to sing).[9] This sort of deficiency in perceptions does not miss the truth by much when it is simply a question of recognising whether there is or is not a difference between them, nor does it in detecting the amounts by which differing things exceed one another, so long as the amounts in question consist in larger parts of the things to which they belong.[10] But in the case of comparisons concerned with lesser parts the deficiency accumulates and becomes greater, and in these comparisons it is plainly evident, the more so as the things compared have finer parts. The reason is that the deviation from truth, being very small when taken just once, cannot yet make the accumulation of this small amount perceptible when only a few comparisons have been made, but when more have been made it is obvious and altogether easy to detect.[11] Thus given a straight line it is very easy to construct a smaller or a greater than it by eye, not just because this is a broad sort of distinction, but because it also involves only one comparison. Dividing it in half, too, or doubling it is still easy, if not to the same extent, since only

[8] See Plato *Theaetetus* 151e–152e, 159b–160c, 181a–183b, etc., and compare, for example, *Phaedo* 65a–66c. The doctrine that matter is in continual flux is attributed by Aristotle (frag. 207) to Pythagoras, and by many ancient writers to Heraclitus; the idea was common throughout antiquity.

[9] Ptolemy's main point is that the ear is often unreliable in assessing intervals. But its unreliability can itself only be detected by ear, through its comparison of the sound of an accurately, 'rationally' tuned interval with that of the inaccurate one which it had previously constructed on its own. This is why Ptolemy needs his second thesis, that our senses are better equipped to judge such things than to construct them. Hence we can detect our mistakes through the same channels that we relied on when we made them.

[10] That is, when the difference in quantity between A and B is a large fraction of each of them.

[11] For instance, if we try to judge by eye the relative lengths on the *kanōn* that correspond to scalar intervals, inaccuracies may be hard to detect in individual cases taken alone. But if we construct a whole series of such intervals through, for example, an octave, the errors will accumulate, and the inexactness of the resulting relation between the boundaries of the supposed octave will be obvious. See Ptolemy's criticisms of Aristoxenian procedure in the last paragraph of chapter 10 and the beginning of chapter 11.

two comparisons take place. To construct a third of it or to triple it is harder, since in this case three adjustments [*harmogai*, 'fittings together'] are made, and it becomes continually and proportionately harder to achieve in the case of things assessed through greater numbers of measuring operations. This is so when we construct the thing we are looking for simply as itself, the seventh or the seven-times multiple, for instance, and not through easier stages, as when we construct an eighth by first constructing a half, then the half of that, and then again the half of that, or the eight-times multiple by first constructing the double, then the double of that, and then again the double of that. For here it will no longer be the eighth of the one, or its eight-times multiple, that has been grasped, but the halves or the doubles of several unequal things. Since similar things occur in relation to sounds and to the hearing, there is needed to help them, just as there is for the eyes, some rational criterion working through appropriate instruments, as the ruler is needed to deal with straightness, for instance, and the compasses for the circle and the measurement of its parts.[12] For the ears, similarly, which with the eyes are most especially the servants of the theoretical and rational part of the soul, there is needed some method derived from reason, to deal with the things that they are not naturally capable of judging accurately, a method against which they will not bear witness, but which they will agree is correct.[13]

2 *What the aim of the student of Harmonics is*

The instrument of this kind of method is called the harmonic *kanōn*, a term adopted out of common usage, and from its straightening [*kanonizein*] those things in sense perception that are inadequate to reveal the truth.[14] The aim of the student of Harmonics must be to preserve in all respects the rational postulates [*hypotheseis*] of the *kanōn*, as never in any way conflicting with the perceptions that correspond to most people's estimation, just as the astronomer's aim is to preserve the postulates concerning the movements of the heavenly bodies in concord with their carefully observed courses, these postulates themselves having been taken from the obvious and rough and ready phenomena, but finding the points of detail as accurately as is possible through reason.[15] For in everything it is the proper task of the theoretical scientist to

[12] Similar examples are used to the same purpose at the beginning of 10 Nicomachus *Ench.* ch. 6.

[13] Compare 9.2 Adrastus ap. Theon Smyrn. 61.2, 9.11–9.14 Ptolemaïs and Didymus ap. Porph. *Comm.* 23.24–28.26. The thesis that hearing needs some technical aid for the accurate assessment of intervals is stated also at 9.9 Panaetius ap. Porph. *Comm.* 66.10ff. The similarity of some of his remarks suggest that he is a source on whom Ptolemy drew.

[14] On the etymology see also 9.10 Ptolemaïs, cited by Porph. (*Comm.* 22.22ff.) in connection with this sentence.

[15] The objective realities are to be thought of as matter determined in accordance with some form. Our senses grasp the qualitative character of this matter, and of the modifications that arise through its formal orderings. These are the 'phenomena'. They are at two removes from the reality constituted by the formal ordering itself, since it is not identical with the modifications it creates in a specific material medium, and these

show that the works of nature are crafted with reason and with an orderly cause, and that nothing is produced by nature at random or just anyhow, especially in its most beautiful constructions, the kinds that belong to the more rational of the senses, sight and hearing.[16] To this aim some people seem to have given no thought at all, devoting themselves to nothing but the use of manual techniques and the unadorned and irrational exercise of perception, while others have approached the objective too theoretically.[17] These are, in
6 particular, the Pythagoreans and the Aristoxenians, and both are wrong. For the Pythagoreans did not follow the impressions of hearing even in those things where it is necessary for everyone to do so, and to the differences between sounds they attached ratios that were often inappropriate to the phenomena, so that they provided a slander to be directed at this sort of criterion by those whose opinions differed.[18] The Aristoxenians, by contrast, gave most weight to things grasped by perception, and misused reason as if it were incidental to the route, contrary both to reason itself and to the perceptual evidence [*to phainomenon*] – contrary to reason [*logos*] in that it is not to the distinguishing features of sounds that they fit the numbers, that is, the images of the ratios [*logoi*], but to the intervals between them, and contrary to the perceptual
10 evidence in that they also associate these numbers with divisions that are inconsistent with the submissions of the senses.[19] Each of these things will become clear from the points I shall introduce, if the matters which contribute to an understanding of what follows are first given some analysis.

3 *How the height and depth that relates to sounds is constituted*

Among sounds, as among all other things, there are differences in respect both of quality and quantity, and to which of the two classes mentioned difference

modifications are themselves only roughly and imperfectly grasped by the senses. But we must cling to the hypothesis of perfect, rationally intelligible order underlying the phenomena, and learn how the appearances can be interpreted consistently with it. Ptolemy's principle of 'saving the hypotheses' is the reverse side of the coin whose more familiar face is the Platonic 'saving the phenomena'. It differs both from the modern notion that scientific descriptions are idealisations of the facts, and from the Platonist view that perceptible things are imperfect copies of the higher realities that science seeks to discover. For Ptolemy, as perhaps for Plato's Pythagorean precursors and contemporaries, the ideal realities are identical with the knowable aspects of the physical ones, but they have to be reached by a reinterpretation of the vague data of perception. (The term *hypothesis* may be better construed, here and elsewhere in this work, as referring to the principles that 'underlie' or 'support' the order of phenomena, principles conceived as existing in external reality, not just as propositions in the scientist's mind.)

[16] Compare particularly Book III ch. 1, 85.15ff.

[17] Compare the contrasts made in 2.1 Plato *Rep.* 530c–531c, 7 Aristox. *El. Harm.* 32.18ff., and in 9.10–9.14 (Ptolemaïs and Didymus); it is here that 9.11 is quoted by Porphyry. In the sequel Ptolemy turns these points against both Aristoxenian and Pythagorean approaches.

[18] These remarks are elaborated in chapters 5–6 below.

[19] For detailed versions of these criticisms see chapters 9–10 below. Related analyses of the rival approaches are given by Ptolemaïs and Didymus: 9.12–9.14 are quoted in reference to this passage by Porphyry.

relating to heights and depths belongs is not something that can be shown off-hand, before the causes of this kind of occurrence have been investigated, causes which seem to me to be shared in some way with variations in other sorts of impact. For the modifications [*pathē*] arising from them become different in accordance with the force of the striker, and with the bodily constitutions of both the thing struck and the thing with which the impact is made, and again in accordance with the distance of the thing struck from the origin of the movement.[20] For it is clear that if the other factors involved remain the same, each of the things mentioned, when it is varied in one way or another, has its own specific effect on the modification [*pathos*]. In the case of sounds the difference related to the constitution of the thing struck either does not occur at all, or is not perceptible, because variations in the air are also imperceptible to the senses,[21] while difference related to the force of the striker
7 is the cause only of magnitude [i.e., loudness], and not of height or depth. For if other factors are the same, we find no alteration of this sort arising in sounds when, for instance, people make utterances more gently or more loudly, or again when they blow in or pluck more mildly or more vigorously or strongly, but only that the greater follows upon the more forceful, the lesser upon the weaker. The variation related to the things with which the impacts are made is found here in correspondence with the primary constitution of their body, the constitution, that is, which makes a thing diffuse or dense, thin or thick, smooth or rough, and again in correspondence with their shapes.[22] (As for their more affective qualities, smells, I mean, and tastes and colours, what connection have they with impacts?)[23] Through shape, in the case of things that admit such a variation, such as tongues and mouths, it makes configurations – modes [*nomoi*], as it were – for the sounds, in correspondence with which names are coined such as clatters, thuds, voices, clangs, and a thousand like them; and we ourselves imitate each of the configurations through man's possession of the most rational and skilful ruling principle. Through smoothness and roughness, again, it creates only a quality in accordance with which sounds are described by the same words, smooth or rough, since the qualities are essentially the same.[24] Through diffuseness or density and thickness or fineness it makes qualities in accordance with which we again call sounds by the same words, dense or flabby, thick or thin; and from here it also

[20] It is possible that *pathē* here and in the sequel means 'experiences' rather than 'modifications', but see chapter 1 above. The acoustic ideas of this chapter draw on Peripatetic elaborations of work begun by Archytas, as exemplified in Aristotle, the *Problems*, the *De Audib.*, etc. In his opening comments on the chapter Porphyry quotes 9.8 Heraclides, 9.7 Aelianus, a few lines of Aelius Dionysius, and a fragment each (both probably spurious) of Archytas and Democritus; on the present passage specifically, see 3.15 Arist. *De Anima* 419b11.

[21] Here the 'thing struck' is always the air external to the instrument, or to the singer's body. The irrelevance of its constitution to the quality of a sound is suggested also at 5 *De Audib.* 800a.

[22] That variations in the qualities of sounds are correlated with similar variations in the qualities of the agents that cause them is a cardinal principle of 5 *De Audib.*

[23] Here Porphyry (*Comm.* 40.24ff.) quotes Arist. *Categ.* 9a28ff.

[24] Porphyry quotes Arist. *Categ.* 10a16ff. (*Comm.* 42.25ff.).

makes heavinesses and sharpnesses,[25] since each of these, being a quality of the kinds of composition mentioned, comes about in correspondence with the quantity of substance. For a denser thing is one that has more substance in an equal bulk, and a thing that is thicker than things of the same constitution is one having more substance in an equal length. The denser and the finer are creative of the sharper, the more diffuse and the thicker of the heavier. In all other things too, the sharper is described as such because it is finer, just as is the blunter because it is thicker. For finer things strike more compactly because they can penetrate more quickly, and denser things because they penetrate further.[26] It is for this reason that bronze makes a sharper sound than wood, or gut-string than flax (for they are denser): of pieces of bronze with the same density and equal length it is the finer that is sharper in tone, of strings with the same density and equal length it is the more slender, while hollow things
8 have a sharper tone than solids, and of windpipes, again, it is the denser and finer that do so. In each case this happens not through the density or fineness as such, but through the high tension, since it is an attribute of things like this that they are tenser, while what is tenser is more vigorous in its impacts: the more vigorous is more compacted, and the more compacted is sharper.[27] Hence if a thing is tenser in some other way, for instance by being harder to a greater degree than it is larger overall, it makes a sharper sound; and where there exists in both of two things something that has the same effect, victory goes to the excess of the one ratio over the other – as when bronze makes a sharper sound than lead, since it is harder than lead in a greater degree than lead is denser than it. And again, any larger and thicker piece of bronze makes a sharper sound than the smaller and finer, whenever the ratio in respect of magnitude is greater than that in respect of thickness.[28] For sound is a sort of continuous tensing [*tasis*] of the air, penetrating to the outer air from the air that immediately surrounds the things making the impacts, and for this reason, to whatever degree each of the things making the impacts is tenser, the sound is smaller and sharper to the same degree.[29]

For these reasons it seems that difference of sounds in respect of sharpness and heaviness is a form of quantity,[30] and arises more particularly from

25 I have retained the primary senses of the Greek terms in this part of the translation, since the associations of 'heavy' and 'sharp' are required by the context. But of course 'heavy', *barys*, is the usual term for 'low-pitched', and 'sharp', *oxys*, for 'high-pitched'. It is here that Ptolemy addresses the main theme of the chapter.

26 Here Porphyry quotes 2.4 Plato *Tim.* 67b, and a series of segments from 3.15 Arist. *De Anima* 419b19ff.

27 Porphyry (*Comm.* 50.14ff.) quotes 5 *De Audib.* 803b18ff.

28 Compare 3.15 Arist. *De Anima* 420a–b, 6 Theophrastus ap. Porph. *Comm.* 63.1–14. Porphyry (*Comm.* 51.16ff.) quotes Arist. *Hist. An.* 538b14.

29 Ptolemy's theory of sound does not then involve the transmission of air from one place to another, either as a missile (as apparently in Archytas and Plato) or by a process of 'shunting', where one parcel of air pushes another forward (as perhaps in 4.2 ps.-Ar. *Probs* XI.6). It is rather the transmission of a condition of 'tension' through a stationary medium (cf. 5 *De Audib.* 800a). Porphyry here (*Comm.* 52.5ff.) quotes 3.15 Arist. *De Anima* 419b25–7.

30 For other arguments to this crucial thesis see, for example, 1.19 Archytas frag. 1, 8 Eucl. *Sect. Can.* 148–9, 10 Nicomachus *Ench.* ch. 4; for an attempt to deny it, 6 Theophrastus ap. Porph. *Comm.* 61.16ff.

inequality in the distances between the striker and the thing struck. For it is in
the quantity of these that they are most clearly constituted, sharpness following
upon the smaller distances because of the vigour arising from proximity,
heaviness upon the greater because of the relaxation that goes with being
further away, so that the sounds are modified in the opposite way to the
distances.[31] For as the greater distance from the origin is to the less, so is the
sound from the smaller distance to that from the greater,[32] just as with weights,
as the greater distance of the weight [from the fulcrum] is to the lesser, so is the
downward thrust from the lesser distance to that from the greater.[33] That this
is true is obvious from sounds that come about as the consequence of some
length, like those of strings and *auloi* and windpipes: for so long as other
9 factors remain the same, in strings those produced with smaller distances
between the bridges are invariably sharper than those produced with greater
ones, in *auloi* those coming out through the holes nearer the *hypholmion*[34]
(nearer the striker, that is) are sharper than those coming through the holes that
are further away, and in windpipes those in which the source of the impact is
higher up and near the thing struck are sharper than those coming from the
depths. For the windpipe is like a sort of natural *aulos*, differing in only this one
respect, that in *auloi* the location of the striker remains constant while that of
the thing struck shifts closer to or further from the striker, by the device of the
10 finger-holes, whereas in windpipes it is the other way round, the location of the
thing struck remaining constant, while that of the striker shifts closer to or
further from the thing struck, as our ruling principles, with their inborn music,
find and grasp marvellously and easily, in the manner of a bridge, the places on
the windpipe from which the distances to the outside air will produce
differences of sounds in proportion to the amounts by which the distances
exceed one another.[35]

4 *Concerning notes and their differences*

Let this outline suffice to indicate how height and depth of sound are constituted, and that their form is a kind of quantity. Let it also be understood

[31] Compare, for example, Thrasyllus in 9.4 Theon Smyrn. 87, 10 Nicomachus *Ench.* ch. 10.

[32] This implies that greater numbers go with higher notes (contrast Adrastus in 9.3 Theon Smyrn. 65.6). In his accounts of harmonic divisions Ptolemy regularly assigns greater numbers to lower notes, but that is because he is thinking in terms of lengths of string on the *Kanōn*.

[33] Ptolemy presumably refers to the case where the two weights balance.

[34] That is, the bulbous segment of the *aulos* immediately above the main pipe, below the *holmos* which held the reed. Since all the finger holes were drilled in the main pipe, those nearest the *hypholmion* are nearest the blown end.

[35] The 'thing struck' is still the external air. In the cases of wind instruments and voice, the 'striker' is the missile of breath. In *auloi* it travels from the lips to the nearest open finger hole, where it impinges on the 'thing struck'; in vocalisation it travels, so Ptolemy supposes, from any of a series of positions on the windpipe from which it is bounced to strike the air outside the mouth. In each case the pitch is determined by the distance from the point of transmission to the external air. This special theory of vocal pitching is unusual, but in other respects the passage is comparable to, for example, 9.2 Theon Smyrn. 60–1, 9.7 Aelianus ap. Porph. *Comm.* 33.16ff., 10 Nicomachus *Ench.* chs. 4 and 10. On the marvellous accuracy of our power to utter the pitches we wish, compare the opening of 6 Theophrastus ap. Porph. *Comm.* 61.16ff.

that their increases are potentially unlimited, but in actuality are limited in the same way as are those of magnitudes, that they have two boundaries, the one proper to the sounds themselves, the other to the hearing, and that the latter is greater than the former.[36] When the things that make the sounds alter progressively in their constitutions, even if the distances at each step from the lowest in pitch to the highest differ by no significant amount, still their two limits will often differ greatly, in some cases towards the lower, in some towards the higher. But the hearing grasps sounds still lower than the lowest and higher than the highest, to the extent that we can devise ways of increasing such distances in the manufacture of instruments.[37]

These things being so, then, it must next be explained that of sounds some are equal-toned, some unequal-toned. Equal-toned sounds are those that do not change in respect of tone [*tonos*], unequal-toned ones those that do change. What is called 'tone' in this sense must be a genus common to both height and depth, understood in relation to one form, that of pitch [*tasis*], as limit is common to both end and beginning.[38] Of unequal-toned sounds some are continuous, some divided, the continuous being those the locations of whose movements in each direction are not clearly apparent, or of which no single part is equal-toned over a perceptible interval [of time], as with what happens to the colours of the rainbow.[39] Of this sort are the sounds that accompany the actual movements of tensing or relaxing [in tuning strings], or again, in a downwards direction, the lowing of cattle as it dies away, and in an upwards direction the howling of wolves.[40] Divided sounds are those the locations of whose movements are clearly apparent, when their parts remain equal-toned over a perceptible interval [of time], as in the juxtaposition of different colours that are unmixed and have not run together. But the former are foreign to harmonics, never laying down anything that is one and the same, so that – contrary to what is proper to the sciences – they cannot be encompassed by a definition or a ratio; while the latter are at home in harmonics, being defined by the limits constituted by the even-toned sounds, and measured in relation to

[36] Cf. 7 Aristox. *El. Harm.* 13.30–15.12, 10 Nicomachus *Ench.* ch. 2.

[37] At 7 *El. Harm.* 14.25–8 Aristoxenus agrees that the ear can discriminate pitch and interval over a wider compass than the voice can produce them, but not by much. Ptolemy seems to credit it with considerably more extensive powers, since the compass of instruments, taken overall, is wider than that of the voice: cf. 7 Aristox. *El. Harm.* 20.22–21.19.

[38] Compare 7 Aristox. *El. Harm.* 13.1ff. 'Height and depth' represents '*oxytēs* and *barytēs*', lit. 'sharpness and heaviness', one form or mode of which is pitch. The noun *tonos*, and the adjectives based on the same stem, e.g., *isotonos*, 'equal-toned', are used by Ptolemy primarily with reference to pitch, but there is a close connection with the idea of tension (an alternative sense for both *tonos* and *tasis*); see particularly 8.12 above.

[39] With this sentence and the subsequent discussion cf. 7 Aristox. *El. Harm.* 8.13ff.; related passages include 10 Nicomachus *Ench.* ch. 2, 12 Arist. Quint. *De Mus.* 5.24ff. While Aristoxenus' discussion is couched in terms of two styles of movement of a sound (the voice) across intervals from pitch to pitch, Ptolemy's refers to two kinds of sound. In his conception, a sound that shifts in pitch does not move from place to place on a continuum, but alters in its own quantitative properties, either continuously or by 'quantum-jumps'. The notion of vocal 'movement' is at best metaphorical.

[40] Cf. 10 Nicomachus *Ench.* ch. 2 with n. 11.

each other by the orderings of their differences.[41] And indeed we could now call sounds of this sort 'notes' [*phthongoi*], since a note is a sound that retains one and the same tone [*tonos*].[42] Hence each taken alone has no ratio, for it is one and undifferentiated in relation to itself, whereas ratio is a relation and occurs first in two terms. In a comparison between two notes, when they are unequal-toned, it makes a ratio from the quantity by which one exceeds the other, and it is in these that the melodic and the unmelodic appear.[43] Melodic notes are those which, when joined together with one another, are acceptable to the hearing, unmelodic notes those that are not. Again, people give the name 'concordant' [*symphōnoi*], which they take from that most beautiful of sounds, the voice [*phōnē*], to those which create a homogeneous impression on the hearing, 'discordant' [*diaphōnoi*] to those which do not.[44]

11 5 *Concerning the principles adopted by the Pythagoreans in their postulates about the concords*

Perception accepts as concords the fourth, as it is called, and the fifth, the difference between which is called a tone [*tonos*], and the octave; and also the octave and a fourth, the octave and a fifth, and the double octave. Let us ignore the concords greater than these for present purposes.[45] The theory of the Pythagoreans rules out one of these, the octave and a fourth,[46] by following its own special assumptions, ones which the leaders of the school put forward on the basis of ideas such as the following. They laid down a first principle of their method that was entirely appropriate, according to which equal numbers should be associated with equal-toned notes, and unequal numbers with unequal-toned; and from this they argued that just as there are two primary classes of unequal-toned notes, that of the concords and that of the discords, and that of the concords is finer, so there are also two primary distinct classes

[41] This makes it clear that harmonics is concerned with relations between pitches, not with features of sounds taken individually. On the theme (common to all schools of thought) that science can deal only with what is determinate see, for example, 2.6 Plato *Philebus* 17a–e, 7 Aristox. *El. Harm.* 69.1ff.

[42] Corresponding to the first definition offered by Aristoxenus, 7 *El. Harm.* 15.15–16. See the next note.

[43] There is thus a distinction between 'note', as 'sound of constant pitch', and 'melodic note', as 'note standing in appropriate relations to other notes'. There is a corresponding difference between Aristoxenus' first definition and his later treatment of notes as *dynameis* (7 *El. Harm.* 15.15–16, for example, 34.5, 36.6, etc.). Compare also the definition of Thrasyllus (9.1 Theon Smyrn. 47–8) with the discussion of Adrastus (9.2 Theon Smyrn. 50). The 'joining together' of notes referred to by Ptolemy in the next sentence is of course sequential, not simultaneous.

[44] For reference to other definitions of concord and discord see n. 56 to 12 Arist. Quint. *De Mus.* 10.1–5.

[45] Cf. 7 Aristox. *El. Harm.* 20.1–21.19 and, for example, 9.3 Adrastus ap. Theon Smyrn. 64–5. But Ptolemy is not being dogmatic about the size of the compass usable by voice or instruments. He restricts himself to the double octave because a scale of this compass is the smallest that contains all the structures to be described in the course of his theoretical analyses. See particularly Book II ch. 4.

[46] For a discussion of this problematic concord see chapter 6 below.

of ratio between unequal numbers, one being that of what are called 'epimeric' or 'number to number' ratios, the other being that of the epimorics and multiples;[47] and of these the latter is better than the former on account of the simplicity of the comparison, since in this class the difference, in the case of epimorics, is a simple part, while in the multiples the smaller is a simple part of the greater.[48] For this reason they fit the epimorics and multiple ratios to the concords, and link the octave to duple ratio [2:1], the fifth to hemiolic [3:2], the fourth to epitritic [4:3].

Their procedure here is very rational, since the octave is the finest of the concords, and the duple is the best of the ratios, the former because it is nearest to the equal-toned, the latter because it alone makes an excess equal to that which is exceeded;[49] and again, because the octave consists of the two first concords taken successively,[50] and the duple consists of the two first epimorics taken successively, the hemiolic and the epitritic; and while in the latter case the hemiolic ratio is greater than the epitritic, in the former the concord of the fifth is greater than that of the fourth, so that the difference between them –
12 that is, the tone – is assigned to the epogdoic ratio [9:8], by which the hemiolic is greater than the epitritic; and in accordance with these points they also adopt among the concords the magnitude put together from the octave and the fifth, and again that put together from two octaves – that is, the double octave – since it follows that the ratio of the latter is constituted as quadruple, and that of the former as triple. But they do not adopt the magnitude put together from the octave and the fourth, because it makes the ratio of 8 to 3, which is neither epimoric nor multiple.[51]

They argue to the same conclusion in a more geometrical [lit. 'linear'] way, as follows. Let AB, they say, be a fifth, and let BC be another fifth, continuous

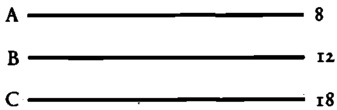
[47] See especially the introductory passage of 8 Eucl. *Sect. Can.* Much of the detail that follows echoes the *Sectio*: Porphyry quotes the bulk of that treatise in his commentary on the present chapter, believing (probably rightly) that it was one of Ptolemy's principal sources for Pythagorean ideas.

[48] The arithmetical difference between two terms, A and B, in epimoric ratio (n+1:n) is a unit fraction of each term ($\frac{1}{n+1} \times A$, $\frac{1}{n} \times B$). Where two terms are in multiple ratio (mn:n), the smaller is a unit fraction or factor of the greater. Neither proposition holds of terms in epimeric ratio (n+m:n where m is greater than 1). The three kinds of ratio and their subspecies are discussed in great detail at Theon Smyrn. 76.1ff. Ptolemy himself gives priority to ratios where the difference between the terms is a simple part of each term (see particularly 16.12–21 and Book 1 ch. 15), but it is a different sort of priority from that postulated by the Pythagoreans (see 15.18–16.12).

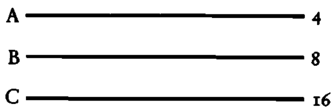
[49] That is, if A:B = 2:1, it follows that A − B = B. In Ptolemy's theory, equality of 'tone' corresponds to equality between the terms of the ratio. Approximation to equality of tone (exemplified not in notes that are adjacent in pitch, but in ones whose combined sounds are most like a unison – first octaves, then concords, and so on) corresponds to equality, or nearness to equality, between each of the terms and their difference. Then one (epimoric) ratio is 'nearer to equality' than another where the difference between the terms is a larger simple part of each. See especially ch. 7 below.

[50] As was unanimously accepted by all schools of theory. Cf., for example, 7 Aristox. *El. Harm.* 45.14–16, 8 Eucl. *Sect. Can.* proposition 12.

[51] All the correspondences recorded in this paragraph are routinely accepted by Pythagorean and Platonist writers; the rogue interval of the octave plus fourth is rarely mentioned by them (as it is not in 8 Eucl. *Sect. Can.*).



with the first, so that AC is a double fifth. Since the double fifth is not concordant, it follows that AC is not multiple, so that neither is AB multiple: but it is concordant, and hence the fifth is superparticular. In the same way they show that the fourth is also an epimoric, a smaller one than the fifth.[52] Again, they say, let AB be an octave and let BC be another octave continuous with the first, so that AC is a double octave. Then since the double octave is concordant,



it follows that AC is either epimoric or multiple: but it is not epimoric – since then no mean would fall within it proportionately – and hence AC is multiple, so that AB is also multiple: therefore the octave is multiple.[53] From these things it is plain to them that the octave is duple, and that of the others the fifth is hemiolic and the fourth epitritic, since of the multiples only the duple ratio is constituted by the two greatest epimorics, so that ratios put together from two of the other epimorics are together smaller than the duple, and there is no multiple smaller than the duple.[54] Since the tone is accordingly shown to be epogdoic, they reveal that the half-tone is unmelodic, because no epimoric ratio divides another proportionately as a mean, and melodic magnitudes must be in epimoric ratios.[55]

13 6 *That the Pythagoreans reason incorrectly about the causes of things to do with the concords*

This being the postulate of the Pythagoreans concerning the concords, the octave and a fourth, which is quite plainly a concord, is an embarrassment to

[52] This follows the reasoning of 8 *Sect. Can.* proposition 11; and on its flaws see n. 28. It is curious that Ptolemy makes no comment.
[53] See 8 *Sect. Can.* proposition 10.
[54] See 8 *Sect. Can.* proposition 12, and n. 19 to prop. 6. Throughout this passage (and in the *Sectio*) ratio A:B is 'smaller' than ratio C:D if $A = B + \frac{B}{n}$ and $C = D + \frac{D}{m}$, and n is greater than m.
[55] Compare 8 *Sect. Can.* propositions 3 and 16. But the proofs in the *Sectio* are designed to show that there is no mean whatever in an epimoric ratio, not merely none that is melodic, and there is nothing in the *Sectio* to suggest (as Ptolemy holds) that melodic ratios, like the non-multiple concords, must be epimoric; the contrary is plainly implied in propositions 17–20. This principle was accepted, however, with certain qualifications, by many theorists from Archytas onwards: see chapters 13 and 14 below, and the tables in Book II ch. 14.

the ratio fitted to it by them. For it is always true of the concord of the octave, whose constituent notes do not differ in their function [*dynamis*] from a single note, that when it is attached to one of the others it keeps the form of the latter unaltered, just as does the number 10, for example, in relation to numbers smaller than itself.[56] And if one takes a note that lies in the same direction from both the extremes of the octave, downwards from both of them, or again upwards, as it is to the nearer of them so it appears to be to the further, and it has the same function [*dynamis*] as does that one.[57] The concords of the fourth and the fifth are sung by themselves in relation to the nearer note of the octave, and the fourth with an octave, or again the fifth with an octave, in relation to the further, so that it is to be expected that the same impression comes to the ears from the fourth and an octave as there does from the fourth alone, and that the impression of the fifth and an octave is the same as that of the fifth alone; and that it plainly follows from the fact that the fifth is concordant that the octave and a fifth is concordant too, and from the fact that the fourth is concordant that the octave and a fourth is concordant too; and that the impression of the fifth and an octave is related in the same way to that of the fourth and an octave as is that of the fifth alone to that of the fourth alone, which accords with what is found by plain perceptual experiment.[58]

Another crucial problem is made for them by the fact that they associate the concords only with those epimoric and multiple ratios and not with others – I mean such ratios as the epitetartics [5:4] and the five-times multiple [5:1], though when they are compared with the others there is a single form[59] – and

[56] Cf. 7 Aristox. *El. Harm.* 20.17–21, and, for example, 9.2 Adrastus ap. Theon Smyrn. 52.

[57] That is, suppose that note P is an octave above note Q, and that note X is higher in pitch than both of them, or lower than both. Then if X is a given interval above P, it will also seem to be at that interval above Q: if it is at a given interval below Q it will also seem to be at that interval below P.

[58] This is slightly disingenuous, if Ptolemy means us to construe it as a case where perception confirms the findings of pure reason, since the whole argument has been based on premisses derived from perception. But it is perfectly fair to appeal to perception in this context. The Pythagoreans might wish to claim that the octave plus fourth cannot really (metaphysically) be a concord, even though our hearing perceives it as one, but since their initial identification of octaves, fifths, etc. as genuine concords depends on the fact that they strike the ear in a certain way (see, for example, 8 *Sect. Can.* 149.17–20 and propositions 10, 11), it is arguable that their position is incoherent. Compare the comments on Pythagorean method made by Ptolemaïs and Didymus in 9.10–9.14. For a different treatment of the octave plus fourth see 9.2 Adrastus ap. Theon Smyrn. 56.

[59] This is a genuine difficulty, already hinted at in 2.1 Plato *Rep.* 531c. It seems to have no satisfactory mathematical solution, so long as a sharp separation between concords and discords is assumed. An approach like Ptolemy's in chapter 7 below allows intervals to be graded for their closeness to the quality of a unison, and ratios to be correspondingly graded for their closeness to equality (cf. n. 49 above). Hence the question why only certain multiple and epimoric ratios are concordant can be eliminated in favour of the answerable question why the intervals corresponding to certain ratios are 'more concordant' than others. But Ptolemy still operates, for the most part, with a clear distinction between concords and discords, since this is a distinction that perception seemed to demand; and his mathematical theory can do little to explicate it. Cf. n. 16 to 9.2 Adrastus ap. Theon Smyrn. 50.

further by the fact that they make their selection of the concords in whatever way suits their fancy.[60] From each of the first numbers that make up their ratios they subtract a unit, on behalf of the similarity arising from both, and the remaining numbers they posit as belonging to the dissimilarities; and the smaller these turn out to be, the more concordant they say they are. This procedure is utterly ludicrous. For the ratio is not characteristic only of the first numbers that constitute it, but belongs absolutely to all those that are related to one another in the same way; and hence the same result will occur in their case too, the combined 'dissimilarities' of the very same ratios turning out sometimes to be the least, sometimes the greatest. For if, as would seem more appropriate to their enquiry, we assign the same number to all the lesser boundaries, the number 6, for example, and if, after taking away from the greater terms numbers equal to this, to correspond to the 'similarity', we reckon the combined remainders as comprising the 'dissimilarities', this will be 6 in the case of the duple and 3 in that of the hemiolic, and the 'dissimilarities' of the more concordant will be greater.[61] And in any case, by their method the octave and a fifth is proved to be more concordant than the rest, after the octave, since the 'dissimilarities' remaining in it amount to 2, and are more in all the others, being 3, for example, in the fifth and in the double octave, each of which is quite patently more concordant in its constitution than is the octave and a fifth, as one would surely expect. That is because the fifth is simpler and less complex than the fifth and an octave, and its concordance is purer; and the relation of the double octave to the fifth and an octave – that is, of the quadruple ratio to the triple – is the same as that of the octave alone to the fifth alone – that is, of the duple ratio to the hemiolic. For if the triple and the quadruple, and again the hemiolic and the double of a single number are taken, the quadruple will make an epitritic ratio with the triple, and so will the duple with the hemiolic; so that to the degree that the octave is more concordant than the fifth, to the same degree the double octave is more concordant than the octave and a fifth.

7 *How the ratios of the concords may be more correctly defined*

It would not be right to attribute these errors to the power of reason, but to those who ground reason in faulty assumptions,[62] and we should try to grasp reason in its true and more natural form, first dividing distinct and unequal-

[60] The notion of 'making a selection' of concords evidently involves grading them for greater or lesser degrees of concordance, as the sequel shows (cf. previous note). For notes on the procedures discussed in the rest of this chapter, and on Ptolemy's sources, see 1.8 Porph. *Comm.* 107.15ff.

[61] In Porphyry's account, the ratios are taken in their lowest terms, 2:1, 3:2, 4:3, etc. Subtracting a unit from each term of each ratio (the 'similarity'), and summing the remaining numbers, we get 1, 3, 5 (the 'dissimilarities'). Smaller dissimilarity goes with greater concordance. Ptolemy suggests taking 6 as the smaller term in each case, giving 12:6, 9:6, 8:6, and then treating 6 as the 'similarity'. The dissimilarities will then be respectively 6, 3, 2, giving the opposite order of precedence. But if the interpretation offered in the notes to 1.8 is near the mark, this criticism misses the point.

[62] Compare chapter 2 above, and the passages cited in n. 17.

toned notes into three classes. Preeminent in excellence is the class of homophones, second that of concords, and third that of the melodic. For the octave and the double octave plainly differ from the other concords as do the latter from the melodic, so that it would be more appropriate for them to be called 'homophones'.[63] Let us define as homophones those which, when played together, create for the ear the impression of a single note,[64] as do octaves and those that are composed of octaves; as concordant those closest to the homophones, like fifths and fourths and those composed of these and the homophones; and as melodic those closest to the concords, like tones [*toniaioi*] and the others of that sort. Thus in a way the homophones go together with the concords, and the concords with the melodic.

Given these preliminary distinctions we must move on into the discussion that follows from them, adopting the same initial principle as the Pythagoreans – the principle, that is, according to which we assign equal numbers to equal-toned notes, and unequal numbers to unequal-toned notes, since that sort of thing is self-evident. Then since it is in accordance with this principle that we should measure and compare the differences that have been set out between unequal-toned notes by their closeness to the equalities, it is at once clear that the duple ratio is closest to this equality, since it has an excess equal to and the same as the number that is exceeded;[65] and of the homophones the most unitary and finest is the octave; so that we should fit to it the duple ratio, and to the double octave, obviously, the double duple ratio, that is, the quadruple, and so on for any others that are measured by the octave and by duple ratio. Again, after the duple ratios, the nearer to equality are those that most nearly
16 divide that one [i.e., duple ratio] in half, that is, the hemiolic and the epitritic.[66] For what divides most nearly into halves approximates to dividing into two equals. After the homophones the first of the concords are those that divide the octave most nearly into halves, that is, the fifth and the fourth, so that we can again posit the fifth as being in hemiolic ratio and the fourth in epitritic: and

[63] 'Homophone' as a term designating the quality of the octave relation may be Ptolemy's own coinage: at 12 Arist. Quint. *De Mus.* 10.5–6, for example, it implies 'unison'. Similar distinctions between three classes of musical interval are adumbrated in 9.1 Thrasyllus ap. Theon Smyrn. 48–9. 'Melodic' intervals, in Ptolemy's usage, are those that can constitute legitimate simple scalar steps; cf. also n. 59 above.

[64] This description is obviously related to familiar definitions of concord (e.g., 8 *Sect. Can.* 149.17–20). It differs by speaking of the impression of a single 'note' (cf. ch. 6, 13.3–5), where they are careful not to do so, since the 'single blended sound' of a non-homophonic concord is plainly not that of a single note. As chapter 6 makes clear, 'single note' involves 'single melodic function', a harmonic and not merely an acoustic relation. Compare 4.24 ps.-Ar. *Probs* XIX.39. At 12 Arist. Quint. *De Mus.* 10.2–3 the definition of concord perhaps suggests a certain ambiguation of function between the notes of concords, not restricted to notes at the octave.

[65] On the motion of 'closeness to equality' see n. 49 above.

[66] That is, the ratios 3:2 and 4:3. There are of course other pairs of ratios that divide 2:1 more nearly in half (e.g., 17:12 × 24:17), but they are not epimorics. Ptolemy does not explicitly adopt here the doctrine that they must be epimorics (in his comment on the octave plus fourth, below, he almost seems to reject it, but see n. 67). But he lays down in the next paragraph that melodic ratios must be epimoric, since a ratio is more melodic if the difference between its terms is a larger simple part of each term, and it cannot be a simple part of each at all unless the ratio is epimoric. See also especially chapter 15.

second are those formed by putting each of the first concords with the first of the homophones, the octave and a fifth in the ratio put together from the duple and the hemiolic, which is the triple, and the octave and a fourth in the ratio put together from the duple and the epitritic, which is that of 8 to 3. For the fact that this ratio is neither epimoric nor multiple will now be no embarrassment to us, since we have adopted no preliminary postulate of that sort.[67]

Next after the epitritic ratio, those closer to equality will be those that come together to compose it and whose excesses are commensurable,[68] that is, the epimoric ratios that are smaller than these, and following the concords in excellence come the melodics, such as the tone and all those that come together to compose the smallest of the concords; so that to these we should fit the epimoric ratios that are smaller than the epitritic. Of these, too, those that make divisions most nearly into halves must be more melodic, for the same reason, as are all those whose differences contain larger simple parts of those that are exceeded; for these, too, are nearer to the equal, just as the half is nearest of all, then the third, and then each of the others in succession.[69]

From these points we may say in summary that the first multiple and those measured by it are homophones, that the two first epimorics and those composed from them and the homophones are concordant, and that those of the epimorics that come after the epitritic are melodic. The ratio peculiar to each of the homophones and concords has been stated; and of the melodic class the tone has thus simultaneously been shown to be epogdoic [9:8], because of the difference between the two first epimorics and concords.[70] The ratios of the remainder will receive their appropriate definition in the proper places.[71] But now it would be a good thing to demonstrate the clear truth of those that have already been set out, so that we may have their agreement with perception established beyond dispute, as a basis for discussion.[72]

[67] As explained in n. 66, Ptolemy does adopt a principle that gives priority to epimorics. His point, presumably, is that while the difference between terms in the ratio 8:3 is not a simple part of each (nor is it in the cases of the ratios 3:1, 4:1), nevertheless the ratio is compounded from ratios that do have this property (4:3 × 2:1). Since each constituent ratio corresponds to a legitimate harmonic interval, so does their product, and since one ratio is that of a concord, the other that of the octave, their product is a concord (see chapter 6). Ptolemy has not accepted the principle that every legitimate interval and every concord, whether simple or compound, must be epimoric or multiple. The Pythagoreans did adopt that principle in the case of concords, and that is why their theory breaks down at this point.

[68] That is, the difference between the terms of the ratio is commensurable with the terms (i.e., a factor of each). Such ratios, reduced to their lowest terms, are bound to be of the form $n+1:n$. For a different, mathematically elegant way of ordering ratios by their closeness to equality (cf. n. 49 above) see Adrastus ap. Theon Smyrn. 107.24ff. Adrastus' procedure (perhaps derived from Eratosthenes) also gives the order multiples, epimorics, epimerics, and finds good reasons for giving priority to ratios between smaller numbers.

[69] See notes 49 and 66 above.

[70] This reflects the standard definition of the tone as the difference between a fourth and a fifth, e.g., 7 Aristox. *El. Harm.* 21.20–3, 8 Eucl. *Sect. Can.* proposition 13.

[71] See chapters 15 and 16 below.

[72] Ptolemy is firmly committed to the principle that the findings of reason must be confirmed through empirical demonstrations. Special technical devices are required for

8 *In what way the ratios of the concords will be demonstrated beyond dispute by means of the single-stringed* kanōn

Let us reject the attempt to base the proof we are seeking on *auloi* and *syringes*, or on weights suspended from strings, since such demonstrations cannot reach the peak of precision, but serve rather as a source of controversy for those who undertake them. For in *auloi* and *syringes* it is difficult to find a way of correcting unevenness, and also the limits in relation to which the lengths must be compared are established only approximately. This is to do with the presence of some irregularity in the majority of wind instruments, and also with the blowings-in of breath.[73] In the case of weights attached to strings,[74] where the strings are not kept in all respects identical with one another – since it is a hard job to find strings of which each is in this condition even with respect to itself – it will no longer be possible to fit the ratios of the weights to the sounds that arise through them, since denser and finer strings under the same tension make higher notes. Much more important even than that is the fact that even if one assumes that these things are possible, and again that the lengths of the strings are equal, the bigger weight by its greater tension will increase the length of the string attached to it, and will make it denser, so that from this too will arise a difference in the sounds that is not in accordance with the ratio of the weights.[75] Similar things happen, too, with sounds arising from percussion [*synkrousis*], such as people contrive with spheres or discs of unequal weight, and with bowls, empty or full, since it is a very hard task to maintain identity of materials and shapes in all these things.[76] But the string stretched over what is called the *kanōn* will show us the ratios of the concords more accurately and readily.[77] It does not acquire its pitch in any random way, but in the first place it is equipped with a way of assessing any unevenness that might arise from the apparatus, and secondly its limits are appropriately placed so that the limits of the plucked sections between them, into which the whole length is divided, have suitable and clearly perceptible points of origin.

this purpose, several of which are described in the course of his work. His procedures arise directly out of the methodological considerations presented in chapters 1 and 2 above.

[73] For attempts to base demonstrations of ratios on effective sounding-lengths of wind instruments see, for example, 4.19 ps.-Ar. *Probs* xix.23, 9.7 Aelianus ap. Porph. *Comm.* 33.16ff., 10 Nicomachus *Ench.* ch. 10. The 'unevenness' of which Ptolemy complains is primarily irregularity of bore. The problem of 'limits' is especially acute on a reed instrument, since the point on the reed to which measurements should be made is hard to establish. On vagaries of pitching due to variations of 'blowing in' see 7 Aristox. *El. Harm.* 42.6–14.

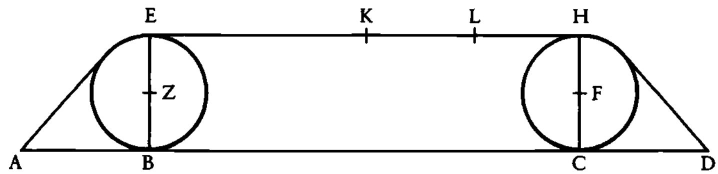
[74] Reports of demonstrations in this style tend to be associated with Pythagoras, and are invariably suspect: see for instance 10 Nicomachus *Ench.* ch. 6, 12 Arist. Quint. *De Mus.* Book III ch. 1 (where Pythagoras is not named).

[75] In fact the method would not straightforwardly yield the required results even if these difficulties were avoided: see notes to 10 Nicomachus *Ench.* ch. 6.

[76] Cf., for example, 1.3 Schol. to *Phaedo* 108d, 1.4 Theon Smyrn. 59.4ff.

[77] A *kanōn* is a ruler: the word refers here to one placed under the string of the monochord, divided in appropriate ratios of lengths. Hence the common phrase 'division of the *kanōn*', as in the title of 8 Eucl. *Sect. Can.* and, for example, at 9.4 Theon Smyrn. 87.4.

Let us think of a *kanōn* on the straight line ABCD, and at its limits, bridges [*magades*] that are in all respects equal and similar, with the surfaces that lie under the string spherical, as nearly as is possible. Let one bridge, BE, have Z as the centre of the surface mentioned, and let the other, CH, have F, similarly,
18 as the centre, where points E and H are found by bisection of the convex surfaces. Let the bridges be so placed that the lines drawn through the points of bisection E and H and through the centres Z and F are perpendicular to ABCD. If then from A and D we stretch a string of appropriate length, AEHD,



The harmonic *kanōn*

it will be parallel to ABCD, because the bridges have equal height; and at points E and H it will have the beginnings of its plucked sections. For it is at these points that it will touch the convex surfaces, since EZB and HFC are perpendicular to it too. To the string we shall now fit a measuring-rod [*kanonion*], and use it to divide up the length EH, so that we may make the comparative measurements more easily. First at the bisection of the whole length, K, and then at the bisection of the half, L, we shall place blades, very thin and smooth, or indeed other bridges [*magadia*], a little higher than the others, but no different from them in respect of their position, equality or similarity about the line through the middle of the convexity, which will be under the exact bisection of the measuring-rod or again under the bisection of the half. Then if part EK of the string is found to be of equal pitch to KH, and again KL to LH, the string's evenness of constitution will be evident to us. If they are not so, let us transfer the test to another part, or to another string, until the required consequence is preserved – that is, sameness of pitch in parts that are similar, corresponding, equal in length, and of a single tension.[78] When something of this kind has been found, and the measuring-rod has been divided in the ratios of the concords that have been set out, by shifting the bridge to
19 each point of division we shall find that the differences of the appropriate notes agree most accurately with the hearing. For if distance EK is constructed of four such parts as those of which KH is three, the notes corresponding to each of them will make the concord of a fourth through the epitritic ratio. If EK is constructed of three such parts as those of which KH is two, the notes

[78] Ptolemy is the only ancient theorist who describes the construction of the monochord and related devices in such detail, and who suggests tests for ensuring that the string is true. Associated issues are discussed in the latter part of chapter 11. For descriptions of other instruments available to the theorist see Book II ch. 2 (cf. ch. 16), Book III chs. 1–2; and for further reflections on the monochord see Book II chs. 12–13.

corresponding to each will make the concord of a fifth through the hemiolic ratio. And again, if the whole length is so divided that EK is made up of two sections and KH of one that is the same, there will be the homophone of the octave, in accordance with the duple ratio. If it is divided so that EK is put together from eight sections and KH from three of the same, there will be the concord of an octave and a fourth, corresponding to the ratio of 8 to 3. If it is divided so that EK is of three sections and KH of one that is the same, there will be the concord of an octave and a fifth, corresponding to the triple ratio. And if it is divided so that EK is put together from four sections and KH from one that is the same, there will be the homophone of the double octave, in accordance with the quadruple ratio.[79]

9 *That the Aristoxenians are wrong in measuring the concords by the intervals and not by the notes*

This shows, then, that we should not find fault with the Pythagoreans in the matter of the discovery of the ratios in the concords, for here they are right, but in that of the investigation of their causes, which has led them astray from the objective; but we should find fault with the Aristoxenians, since they neither accepted these ratios as clearly established, nor, if they really lacked confidence in them, did they seek more satisfactory ones – assuming that they were genuinely committed to the theoretical study of music. For they must necessarily agree that such experiences come to the hearing from a relation that the notes have to one another, and further that where the impressions are the same, the differences are determinate and the same.[80] Yet in what relation, for each species [of concord], the two notes that make it stand, they neither say nor enquire, but as if the notes themselves were bodiless and what lie between them were bodies, they compare only the intervals [*diastaseis*] belonging to the species, so as to appear to be doing something with number and reason.[81] But the truth is precisely the opposite. For in the first place they do not define in this way what each of the species is in itself (as when people ask what a tone is, we say that it is the difference between two notes that comprise an epogdoic

[79] This is not a 'division of the *kanōn*' conceived as a procedure for locating the ratios of notes in a scale, as are, for example, 8 Eucl. *Sect. Can.* propositions 19–20, 9.4 Thrasyllus ap. Theon Smyrn. 87.4ff., 12 Arist. Quint. *De Mus.* Book III ch. 2: it merely provides a way of confirming empirically the ratios assigned to the concords: cf., for example, 9.2 Adrastus ap. Theon Smyrn. 56–58, 10 Nicomachus *Ench.* ch. 6.

[80] That is, where intervals AB and CD are in different levels of pitch, but each creates the same harmonic impression in that both are, for example, fourths, there must be some definite kind of difference between the notes bounding AB which is repeated, relevantly unchanged, in the case of CD.

[81] The point is elaborated in the third paragraph of this chapter. Aristoxenus conceives pitches as dimensionless points on a quasi-linear continuum. Ptolemy insists that the difference between two pitches is a difference in features intrinsic to the sounds themselves. Aristoxenus seems to allow no room for pitched sounds, as such, to possess 'features' at all. Their differences lie only in their positions on the continuum and in the sizes of the intervals between them, which are mere 'gaps', and cannot be heard. For a comparable line of criticism see 6 Theophrastus ap. Porph. *Comm.* 64.24ff.

ratio [9:8]); instead, there is an immediate shift to yet another undefined item, as when they say that the tone is the difference between the fourth and the fifth – despite the fact that if perception wishes to tune a tone, it has no prior need of the fourth or of any of the others, but is capable of constructing each interval [*diaphora*, lit. 'difference'] of this size simply as such.[82] And if we enquire after the magnitude of the difference in question, they do not explain even this
20 without reference to another, but would only say, perhaps, that it is two of those of which the fourth is five, and that this again is five of those of which the octave is twelve, and similarly for the rest, until they come back round to saying '...of which the tone is two'.[83]

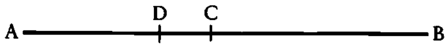
Secondly, they do not, in this way, even define the differences, because they do not relate them to the things to which they belong [i.e., the notes]; for there will turn out to be infinitely many of them in each ratio if the things that make them are not defined first, just as in instrument-making not even the distances that make the octave, for instance, are kept the same, but at the higher pitches they are made shorter. Thus if you compare with one another equal concords
21 based on different boundaries, the length [*diastasis*] of the difference [*hyperochē*] will not be equal in all cases, but if one attunes to one another those of the notes that are higher it will be smaller, and if one attunes those that are lower it will be greater.[84] For if we assume that the distance AB is an octave, A being thought of as the higher limit, and take two fifths, one downwards from A – call it AC – and the other upwards from B – call it BD – the distance AC will be smaller than BD because it falls between higher pitches, and the difference BC will be greater than AD.[85]

[82] Aristoxenus himself might dispute this (see 7 *El. Harm.* 55.3ff.), though he sometimes treats the tone as directly recognisable (see n. 59 to *El. Harm.* 16.30). Cf. also 9.2 Adrastus ap. Theon Smyrn. 53. Note, however, that Ptolemy directs his attack against 'Aristoxenians', and may be thinking principally of later and simplified versions of the theory.

[83] Aristoxenus, for his part, treats an approach comparable to Ptolemy's as unacceptable and irrelevant: see 7 *El. Harm.* 9.1ff., 12.4ff., 32.18ff. He would find nothing objectionable in the apparent circularity to which Ptolemy points: all terms in the science must be defined ultimately by reference to one another and to what is recognised through perception, not to something extrinsic to the harmonic domain. See particularly *El. Harm.* 44.15ff.

[84] Düring follows MSS that read 'greater...smaller' for 'smaller...greater'. But some MSS have the text translated here, which puts the matter in a way that fits the sequel better.

[85] Whichever textual variant is adopted, the argument is a fantastic muddle. Ptolemy is enjoying himself, but can hardly be allowed to get away with it. He assumes first (a) that notes differ quantitatively, and can be represented by different numbers, corresponding to lengths of string, and (b) that equal intervals are properly represented by pairs of numbers in the same ratio. These premisses are adopted because he thinks them true. He then grafts on the Aristoxenian thesis (c) that equal intervals are equal 'distances' between notes, and in the light of (a) and (b) he interprets it as meaning (d) that equal intervals are equal arithmetical differences between the numbers mentioned in (a), or equal linear distances between points bounding corresponding lengths of string. But the Aristoxenians, while accepting a version of (c), would have no inclination to interpret it under the rubric of (a) and (b), which they reject. Even if the serious considerations in the next paragraph induced them to admit that notes must be characterised by quantities, they have every reason to resist the identification of those quantities with string-lengths (or with velocities of movement, etc.): the quantities would have to be



In general, it would seem an absurdity to think that the differences possess a ratio that is not exhibited through the magnitudes that make the differences, and to suppose that the magnitudes have none – the magnitudes from which it is possible immediately to derive the ratio of the differences. And if they were to deny that their comparisons are of the differences between the notes, they would be unable to say of what other things they are. For the concordant or the melodic is not just some empty distance or mere length, nor is it bodily, and predicated of one single thing, the magnitude: rather, it is predicated of two things at least, these being unequal[86] – that is, the sounds that make them – so that it is not possible to say that the comparisons in respect of quantity are of anything but the notes and the differences between them, neither of which do they [the Aristoxenians] make known or provide with a common definition, a definition, that is, that is one and the same, and through which it is shown how the sounds are related both to one another and to the difference [between them].

10 *That they [the Aristoxenians] are wrong in assuming that the concord of a fourth consists of two and a half tones*

They are mistaken, furthermore, about the measurement of the first and smallest concord, composing it as they do from two tones and a half, so that the fifth is put together from three and a half tones, the octave from six tones, and each of the other concords in the way that follows from this one. For
22 reason, being more worthy of trust than perception in the case of differences as small as these,[87] proves that this is not so, as will be clear.

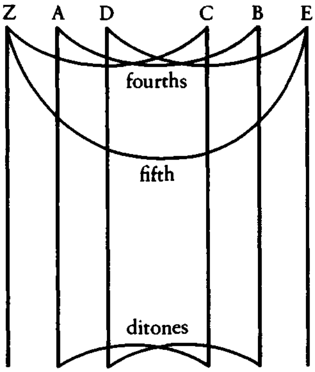
They attempt to prove their proposition as follows.[88] Let there be two notes, A and B, concordant at the fourth, and from A let a ditone, AC, be taken upwards; and from B, similarly, let a ditone BD be taken downwards. Then AD and CB are equal, and each is as great as is that by which the ditone is less

conceived as perceptible 'amounts' of sharpness or heaviness (height and depth), rather as things whose different temperatures can be plotted as points on a linear scale may be thought of as exhibiting different 'amounts' of heat. Hence they cannot fairly be manoeuvred into accepting (d), and need not be worried either by its inconsistency with (b), or by the fact that it is inherently ridiculous. It is not clear whether Ptolemy is obfuscating deliberately, or is confused himself: there are passages in Book II that suggest the latter (see notes 112, 116, 117 to Book II).

86 Perhaps Ptolemy's best point against Aristoxenus. It is relations between features of notes that are concordant, discordant or melodic; these are not properties of single items ('lengths' or 'magnitudes'). Two notes forming, for example, a concord must therefore have distinctive properties (their 'quantities') between which the relation holds. They cannot be featureless points, and the concordance cannot be a property inhering in the empty distance between them.

87 The theoretical background to this claim is provided in chapter 1 above.

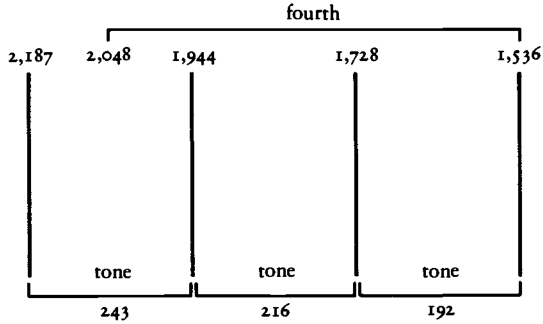
88 See 7 Aristox. *El. Harm.* 56.13ff. Ptolemy paraphrases the passage well enough (with the qualifications mentioned in the next note) to suggest that he had it in front of him as he wrote.



than the fourth. Again, from D let a fourth be taken upwards, DE, and from C, similarly, let a fourth be taken downwards, CZ. Then since each of BA and CZ is a fourth, BC is equal to AZ, and on the same grounds AD is also equal to BE. The four intervals are therefore equal to one another. But, they say, the whole of ZE will make the concord of a fifth, so that since AB is a fourth and ZE is a fifth, and the difference between them is ZA plus BE, these differences taken together make up a tone, and each of them – that is [in effect], each of AD and CB – is a half-tone; and since AC is a ditone, the fourth AB is put together from two and a half tones.[89]

But as soon as the tone has been shown to be epogdoic and the fourth
23 epitritic, it is obvious from this very fact that reason [*logos*] entails that the difference by which the fourth exceeds the ditone, called the *leimma*, is smaller than a half-tone. For if the first number is taken which is capable of displaying the proposition, which is of 1,536 units, its epogdoic is the number of 1,728 units, and the epogdoic of that is the number of 1,944 units, which will obviously have the ratio of a ditone to that of 1,536 units. Now the number of 2,048 units is the epitritic of that of 1,536: hence the *leimma* is in the ratio of 2,048 units to 1,944. But if we also take the epogdoic of 1,944, we shall have the number of 2,187 units, and the ratio of 2,187 to 2,048 is greater than that of 2,048 to 1,944. For 2,187 exceeds 2,048 by more than a fifteenth part of 2,048 and less than a fourteenth. But 2,048 exceeds 1,944 by more than a nineteenth part of 1,944 and less than an eighteenth. Hence the smaller segment of the third tone is included within the fourth in addition to the ditone, so that the

[89] Ptolemy does not allude to the fact that the initial ditones are themselves to be constructed through concords: see 7 Aristox. *El. Harm.* 55.13–23. The omission leads to some slightly misleading comments in the closing sentences of the chapter. It is also misleading to say flatly that according to Aristoxenus the whole of ZE will make the concord of a fifth. Rather, he invites us to perform the construction in practice and to assess ZE by ear, to judge whether it is a fifth or not. Certainly he believed that it would be, if the construction had been carried out properly, but the difference is methodologically important. As Ptolemy recognises, Aristoxenus rests his thesis on the ear's judgement, not simply on the mathematics of the procedure.



magnitude of the *leimma* comes to less than a half-tone, and the whole fourth is less than two and a half tones. And the ratio of 2,048 to 1,944 is in fact the same as that of 256 to 243.[90]

One should not suppose that this sort of conflict is between reason and perception, but that it is the fault of those who adopt erroneous premises, the more recent of them having employed a combination based on both sets of criteria.[91] For perception virtually shrieks its clear and unmistakable recognition of the concord of the fifth, when in the exposition of the monochord that has been set out it is constructed according to the hemiolic ratio, and of that of the fourth, when it is constructed according to the epitritic.[92] But they do not hold to its agreements, from which it follows absolutely that the difference between these concords, which is a tone, is in epogdoic ratio, and that the concord of a fourth is constituted from less than two and a half tones; but in the cases where perception is naturally competent to judge, that is, in respect of the greater distinctions, they are altogether distrustful of it, while in those where it is not by itself sufficient, that is, in cases where the differences are less, they trust it, or rather they attach additional judgements opposed to those that are primary and more authoritative.[93] We would get a further view of the naivety of their demonstration if we calculated the magnitude of the deviation of the *leimma* from the half-tone. For since

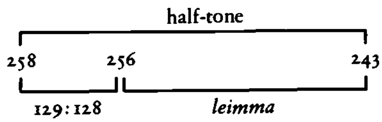
[90] This is of course the ratio standardly assigned to the *leimma* (2.3 Plato *Tim.* 36b and many other sources). The argument is clear and to the point, once Ptolemy's initial assumptions are granted. It is one of a family of arguments of which the Euclidean proof that six tones exceed the octave is another straightforward example (8 Eucl. *Sect. Can.* propositions 9 and 14). It is independent of the theorem, alluded to in the next paragraph, that epimoric intervals such as the 9:8 tone cannot be divided equally.

[91] Compare the opening of chapter 7 above. On different ways in which theorists had tried to combine the criteria (reason and perception) see particularly 9.10–9.14. The 'more recent' theorists are presumably contemporary Aristoxenians: cf. 58.3, and see n. 93 below.

[92] Compare Ptolemy's remarks in chapter 1 about the recognition of a 'rationally' constructed circle, 3.20ff.

[93] It is not clear whether Ptolemy is directing these remarks against Aristoxenians who straightforwardly refuse to countenance an approach based on ratio theory, or against theorists who inconsistently combine elements of both systems. Cf., for example, 10 Nicomachus *Ench.* ch. 7 with n. 55; in 12 Arist. Quint. *De Mus.*, the bulk of Book I is 'Aristoxenian' (but contrast, for example 12.9–11), while much of Book III is 'Pythagorean' (but contrast, for example, III.12–22).

neither the epogdoic ratio nor any other of the epimorics is divided into two equal ratios,[94] and the most nearly equal ratios that make the epogdoic are 17:16 and 18:17, the half-tone would be in the ratio that lies somehow between these, that is, greater than 18:17 and smaller than 17:16.[95] Now 15 is greater than a seventeenth part of 243 and less than a sixteenth part, so that when these, 243 and 15, are added together, the half-tone is in a ratio very close to that of 258 to 243. The ratio of the *leimma* was shown to be that of 256 to 243; and hence the half-tone will differ from the *leimma* in the ratio of 258 to 256, that is, 129:128.



That so slight a variation is capable of being judged by the hearing not even they would say. If then it is possible for perception to mis-hear something of this size in one instance, it must be much more possible in the addition of several instances,[96] something that their proposed demonstration involves them in, with the fourth being taken three times and the ditone twice, in different positions – when it is not easy for them to construct a ditone accurately even once.[97] For they could more readily construct a tone than a ditone, given that the tone itself is melodic and in epogdoic ratio, whereas the incomposite ditone is unmelodic, as being in the ratio of 81 to 64, and for perception the more commensurate intervals are the more easily grasped.[98]

11 *How the octave can be shown, through perception, to be less than six tones, by the use of the eight-stringed* kanōn

Their proposition can be more clearly refuted, and the incapacity of the hearing in relation to things of this size more clearly shown, from consideration of the homophone of the octave. For they claim that it consists of six tones, in

[94] See 8 Eucl. *Sect. Can.* propositions 3 and 16.

[95] Compare 12 Arist. Quint. *De Mus.* 95.19ff. Throughout Ptolemy's treatise I have abandoned the practice of representing epimoric ratios by their one-word Greek names (see notes 6, 8, 17 to 8 Eucl. *Sect. Can.*) on the grounds that it would be too obstructive to the reader (but I have generally retained 'hemiolic' for 3:2 and 'epitritic' for 4:3). Here, for instance, the ratio 17:16 is 'ephekkaidekatic' ratio.

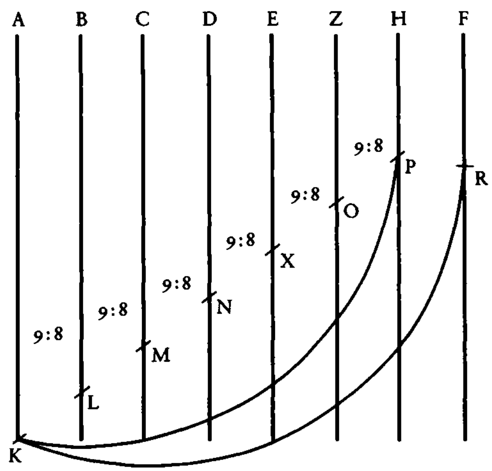
[96] Compare Ptolemy's general discussion of this issue at 4.13ff.

[97] See n. 89 above, and compare 8 Eucl. *Sect. Can.* proposition 17.

[98] The ditone is unmelodic only when it is incomposite; successions of two tones do appear in Ptolemy's systems. (But, compare the use of 'unmelodic' in Book III ch. 9, 104.4ff.) An interval is 'commensurate' when the difference between the terms of its ratio is a factor of each of the terms, and 'more commensurate' when it is a higher factor: see chapter 7 above. The immediate consequence will be that the highest interval of an enharmonic tetrachord cannot be a true ditone (81:64). Like several other Pythagorizing writers, Ptolemy uses the major third of 5:4 (= 80:64) instead: see ch. 15 and Book II ch. 14. But some such writers imply that a ditone is required in this position (e.g., 8 Eucl. *Sect. Can.* proposition 17, 9.5 Thrasyllus ap. Theon Smyrn. 91–3), and Ptolemy himself believed that it was sometimes so used by practising musicians (40.2–8).

accordance with the thesis that the concord of a fourth consists of two and a half tones, since the octave contains the fourth twice and a tone in addition. If we instruct the most expert musician to construct six tones in succession, just by themselves, and without the aid of other notes attuned beforehand, so that he cannot refer to some other of the concords, the first note will not make an octave with the seventh. Now if this sort of result is not due to the weakness of perception, the claim that the concord of the octave consists of six tones would be shown to be false; but if it is because perception cannot construct the tones accurately, it will be much less reliable in the construction of ditones, from which he supposes that he can discover that the fourth consists of two and a half tones. But the following is nearer the truth: not only does the octave not arise, but neither does any other thing arise through [additions of] the same magnitude of difference throughout,[99] whether it is tuned on all [the strings] or always on the same ones. Yet if in the same way we take in succession the fourth and the fifth, the extremes will form the octave, since these are more easily determined by ear.

If, however, we construct six tones in succession by ratio [or 'reason', *logos*], the extreme notes will make a magnitude slightly greater than the octave; and
26 it will always be by the same degree of difference, that is, double the difference between the *leimma* and the half-tone, which, in accordance with the first of our postulates, comes very close to being in the ratio 65:64.[100]



AK: HP = 531,441:262,144

AK:FR = 2:1

[99] No significant harmonic structure is constituted by a series of equal intervals.

[100] This is an approximation of double the difference between the *leimma* and the half-tone, i.e., of the difference between a true octave and six 9:8 tones. The extremes of an interval spanning six tones are in the ratio $9^6:8^6$ = 531,441:262,144 (see 8 Eucl. *Sect. Can.* proposition 9). The estimate of 65:64 as the difference between this and 2:1 has an error of only about one part in 530.

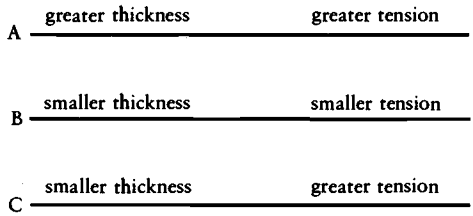
This sort of result will be easily understood if we fasten seven more strings on the *kanōn*, in association with the one string, on the basis of the same kind of selection and placing. For if we accurately attune the eight notes at equal pitch in equal lengths of the strings, the notes ABCDEZHF, and then, by application of the measuring-rod [*kanonion*], divided into six epogdoic [9:8] ratios in succession, we place an identical bridge at the proper division corresponding to each note, to make distance AK the epogdoic of BL, BL that of CM, CM that of DN, DN that of EX, EX that of ZO, ZO that of HP, while AK makes with FR the duple ratio, the latter notes will sound accurately the homophone of the octave, but PH will be slightly higher than FR, and always to the same degree.

That the strings do not differ, even when there is more than one of them, if they are made to be of equal pitch in equal lengths, will be clear from the following.[101] Since in strings there are three causes of difference in respect of high and low, of which one lies in the density of the strings, one in their
27 thickness, and one in their length [*diastasis*], and the sound made by the denser string, the thinner string, and that with smaller length is higher; and since in strings tension is substituted for increased density – for it tenses and stiffens, and for that reason is more uniform in strings with lesser lengths – it is clear that if the other factors are the same, then as the greater tension is to the smaller, so is the sound based on the greater to that based on the smaller; and as the greater thickness is to the smaller, so is the sound based on the smaller to that based on the greater. I say, then, since these things are so, that in dissimilar strings, when they are made to be of equal pitch in equal lengths, the deficiency of the sound arising from the greater thickness is made up for in exchange by the excess of sound arising from the greater tension. And the ratio of the greater thickness to the lesser is always the same as that of the greater tension to the lesser.[102]

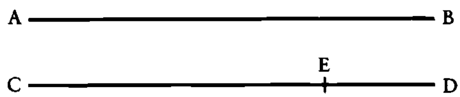
Thus let there be two notes, A and B, of equal pitch in equal lengths, and let the thickness of A – and therefore its tension too, of course – be greater than that of B. Let another, C, be taken in an equal length, having a thickness equal to B and a tension equal to A. Since then C differs from B only in tension, then as is the tension of C to the tension of B, so will be the sound of C to the sound of B. Again, since C differs from A only in thickness, then as is the thickness of A to the thickness of C, so will be the sound of C to the sound of A, while the sound of C has the same ratio to each [of the others], both to the sound of A and to that of B, for the sounds of A and B are equal. Therefore as is the

[101] Compare the tests for a true string in chapter 8 above. Here Ptolemy is concerned to show that even if the strings differ from one another in thickness and tension, still there will be no relevant difference between them so long as equal lengths of each give equal pitches.

[102] Related points are marshalled to a different conclusion at 6 Theophrastus ap. Porph. *Comm.* 63.1–15; cf. 9.3 Adrastus ap. Theon Smyrn. 65–6. Ptolemy presumably lacked accurate means of measuring thickness and tension. Observation showed that greater tension and greater thickness could compensate for one another, but the thesis that this compensation is achieved when both are increased in the same ratio is plainly speculative; in the sense probably intended it is mistaken.



tension of C to that of B, so is the thickness of A to that of C: and as is the tension of C to that of B, so is the tension of A to that of B, for the tensions of A and C are equal: and as is the thickness of A to that of C, so is the thickness of A to that of B, for the thicknesses of B and C are equal. Hence as is the tension of A to the tension of B, so is the thickness of A to the thickness of B. This would be true of them even if they were in all respects unvarying and indistinguishable from a single string. But again, if in strings that are like that, AB and CD, we make the lengths unequal by diminishing the latter to CE, then as is the distance AB to the distance CE, so will be the sound of CE to the sound



of AB. For since as the length CD is to the length CE, so is the sound of CE to the sound of CD, and since the length AB is equal to CD and the sound of AB is equal to the sound of CD, then as is the length AB to the length CE, so is the sound of CE to the sound of AB.

12 *Concerning the division of the genera and of the tetrachords in each, according to Aristoxenus*[103]

Let these points complete our account of the greater differences between the notes. We must now turn to the smaller ones that measure the first of the concords, which are found when the fourth is divided into three ratios in the way corresponding to what has already been determined, so that the first homophone, which is one, may be put together from the two first concords, and the first concord from three melodics, up to the number that bounds this proportion. Now the fact is that the division of the fourth is not in all cases the same, but is constituted differently on different occasions, the two extreme notes remaining stationary, to maintain the concord in question (for which reason people call them 'standing' notes), while the two in between move, to make the differences between the notes in it [i.e., in the fourth] unequal.[104] This

[103] For Aristoxenus' analyses of the genera and their subordinate 'shades' or 'nuances' (*chroai*) see 7 *El. Harm.* 21.31–27.14, 46.19–52.32.

[104] Cf., for example, 7 Aristox. *El. Harm.* 46.19–23. 'Standing' is a literal rendering of the word I have usually represented in this context by the term 'fixed'.

sort of movement is called modulation [*metabolē*] in respect of genus,[105] and a genus in *harmonia* is a relation, of some [determinate] quality, that the notes composing the concord of the fourth have to one another.[106] The first distinction of genus is into two sorts, corresponding to its being softer or more
29 tense:[107] the softer is that which is more inclined to draw the character together, the more tense that which is more inclined to divide it.[108] The second distinction is into three, the third being placed somehow between the two mentioned, and this is called 'chromatic'. Of the others, the one softer than it is enharmonic, the tenser one diatonic. Peculiar to the enharmonic and the chromatic is what is called the '*pyknon*', when the two ratios at the lower end are together smaller than the remaining one, and peculiar to the diatonic is what is called the '*apyknon*', when no one of the three ratios is greater than the remaining two together.[109] The more recent writers make several more distinctions within these, but for the present at least let us write out the Aristoxenian ones, which are as follows. He divides the tone sometimes into two equal parts, sometimes into three, sometimes into four and sometimes into eight, and he calls the fourth part of it an enharmonic diesis, the third part a diesis of the soft chromatic, the fourth part together with the eighth part a diesis of the hemiolic chromatic, and the semitone ['half-tone'] common to the tonic chromatic and the diatonic genera; and from these he posits distinctions between six unmixed genera, one in the enharmonic, three in the chromatic – soft, hemiolic and tonic – and the remaining two in the diatonic – one of the soft diatonic, one of the tense.[110] Then in the enharmonic genus he makes the

[105] The word *metabolē* is not used in this sense in *El. Harm.* But later 'Aristoxenian' classifications of forms of modulation often include modulation of genus (e.g., Anon. Bell. 27, 65, Cleonides *Eisagoge* 205.1–4, Bacchius *Eisagoge* 50, 52). Ptolemy's own discussion of *metabolē* in Book II chs. 5–11 is not primarily concerned with changes of genus, but a reference to *metabolai* of genera occurs at 53.12.

[106] No general definition of genus appears in *El. Harm.* The commonest in later Aristoxenian sources is exemplified at 12 Arist. Quint. *De Mus.* 15.21, where a genus is defined as a specifically qualified division of a tetrachord; see also n. 103 on that passage.

[107] 'Softer' refers to relaxed or lowered pitch, 'more tense' to raised pitch. A 'softer' genus is one whose moveable notes (in particular the higher of them, e.g., *lichanos*, 35.1–3, 49.11–13, etc.) are lower in pitch relative to the fixed notes. *El. Harm.* does not use these terms of genera, but does use them in a similar way to distinguish *chroai* within a genus.

[108] By 'character' (*ēthos*), here and in similar contexts elsewhere (e.g., 32.15, 38.5), Ptolemy means that of a form of melody, not that of a person affected by it. The expressions 'more inclined to draw...together' and 'more inclined to divide' translate *synaktikōteron*, *diastatikōteron*. The terms evidently belong to the same tradition of classification as do the distinctions of *ēthos* or *tropos* at 12 Arist. Quint. *De Mus.* 30.12ff., 40.15; see the notes on those passages.

[109] Aristoxenus defines the *pyknon* similarly (of course without reference to ratios) at 7 *El. Harm.* 24.11–14 and elsewhere. Its restriction to the enharmonic and chromatic is made clear at 25.21ff., 51.19ff. The word *apyknon* appears in *El. Harm.* only at 29.2 (a different use of the word is at 12 Arist. Quint. *De Mus.* 9.15). Ptolemy's definition of *apyknon* is inexact: the term refers to the lower two intervals of a tetrachord where the highest interval is greater than those two together. A tetrachord whose central interval is greater than the outer two together is admitted by Aristoxenus at *El. Harm.* 52.31–3. Neither Aristoxenus nor Ptolemy would conceive it as containing a *pyknon*, but it fails to fit the present description of a tetrachord containing an *apyknon*.

[110] For Aristoxenus' quantification of these divisions see the passages cited in n. 103 above.

interval next to the lowest [note], which is last [lit. 'following'], and the middle interval, each consist of an enharmonic diesis, and the remaining, 'leading' interval of two tones: for instance, if the number assigned to the tone is 24, each of the intervals of the *pyknon* is 6 of these, and the remaining one 48.[111] In the soft chromatic he makes each of the intervals of the *pyknon* consist of a third part of a tone, and the remaining interval of one and a half and a third: for instance, if each of the former is 8, and the latter is 44. In the hemiolic chromatic he makes each of the two intervals of the *pyknon* consist of a quarter and an eighth of a tone,[112] and the remaining interval of one and a half and a quarter: for instance if each of the former is 9 and the latter is 42. In the tonic chromatic he makes each of the two intervals of the *pyknon* consist of a half-tone, and the remaining interval of one tone and a half: for instance if each of the former is 12 and the latter is 36. In the cases of the two remaining genera, which are *apykna*, he again keeps the 'following' interval as consisting of a half-tone in both, and of those that succeed it in the soft diatonic he makes the middle one consist of a half and a quarter of a tone, the 'leading' one of one and a quarter, for instance 12 and 18 and 30; and in the tense diatonic he makes the 'following' interval consist of a half-tone, and makes each of the others, the middle and the 'leading' interval, a tone, for instance 12 and 24 and 24. The numbers are set out below.

enharmonic	soft chromatic	hemiolic chromatic	tonic chromatic	soft diatonic	tense diatonic
48	44	42	36	30	24
6	8	9	12	18	24
6	8	9	12	12	12
60	60	60	60	60	60

13 *Concerning the division of the genera and the tetrachords according to Archytas*

From these facts too, therefore, it seems that Aristoxenus gave no thought to ratio [*logos*], but defined the genera only by what lies between the notes, and not by their differences considered in relation to one another, passing over the causes of the differences as being no causes, as nothings, as mere limits, while

[111] Here and usually elsewhere when commenting on Aristoxenus, Ptolemy follows his practice of treating intervals as 'distances', not as ratios. The *El. Harm.* describes small intervals as fractions of a tone, the smallest mentioned being twelfths: it does not assign numbers to the tone and its parts, as Ptolemy does on Aristoxenus' behalf, following later Aristoxenians (see 12 Arist. Quint. *De Mus.* 17.21 with n. 111). Elsewhere Ptolemy usually represents the Aristoxenian tone by the number 12; the variant 24 is used here to make possible a division by 8 (for the hemiolic chromatic, below).

[112] Aristoxenus (7 *El. Harm.* 51.1–4) puts this by saying that each of its intervals is the 'hemiolic' of the equivalent enharmonic interval, which is a quarter-tone. That is, it is one and a half times as great – hence its name (lit. 'half-and-whole').

attaching the distinctions to things that are bodiless and empty.[113] Hence it is of no concern to him that in almost all cases he is dividing melodic distances in half, though those that are epimoric by no means admit such treatment.[114]

But Archytas of Tarentum,[115] of all the Pythagoreans the most dedicated to the study of music, tried to preserve what follows the principles of reason [*logos*] not only in the concords but also in the divisions of the tetrachords, believing that a commensurable relation between the differences is a characteristic of the nature of melodic intervals. But though he sets off from this presupposition, at several points he seems to fall hopelessly short of it; and though in most cases he is well in control of this sort of thing, he is patently out of tune with what has already been straightforwardly accepted by the senses, as will be seen at once from the division of the tetrachords that he proposes. He posits three genera, the enharmonic, the chromatic and the diatonic, and he makes his division of each of them in the following way. He makes the 'following' ratio the same in all three genera, 28:27; the middle one
31 in the enharmonic 36:35 and in the diatonic 8:7, so that the 'leading' interval in the enharmonic turns out to be 5:4, in the diatonic 9:8. In the chromatic genus he locates the note second from the highest by reference to that which has the same position in the diatonic. For he says that the second note from the highest in the chromatic stands to the equivalent note in diatonic in the ratio of 256 to 243. Such tetrachords, on the basis of the ratios set out, are constituted in their lowest terms by the following numbers. If we postulate that the highest note of each tetrachord is 1,512, and the lowest, in epitritic [4:3] ratio with this, is 2,016, this latter term will make the ratio 28:27 with 1,944, and that will be the quantity of the second note from the lowest in all three genera. As to the second note from the highest, that in the enharmonic genus will be 1,890, since that makes with 1,944 the ratio 36:35, and with 1,512 the ratio 5:4. The equivalent note in the diatonic genus will be 1,701, since that makes with 1,944 the ratio 8:7, and with 1,512 the ratio 9:8. The equivalent note in the chromatic genus will be 1,792, since that has a ratio to 1,701 as is 256 to 243. The table of these numbers is set out below.

enharmonic		chromatic		diatonic	
1512		1512		1512	
	5:4		32:27		9:8
1890		1792		1701	
	36:35		243:224		8:7
1944		1944		1944	
	28:27		28:27		28:27
2016		2016		2016	

113 Compare chapter 9 above.

114 See 24.10–11 above.

115 The remainder of this chapter is quoted above as 1.21, where notes will be found.

32 14 *A demonstration that neither of the divisions [i.e., those proposed by Aristoxenus and by Archytas] preserves what is truly melodic*

Now the chromatic tetrachord, as we said, was put together by him [Archytas] in a way contrary to his own premises (for the number 1,792 makes an epimoric ratio neither with 1,512 nor with 1,944);[116] while both the chromatic and the enharmonic were put together in a way contrary to the plain evidence of the senses. For we grasp the 'following' [i.e., the lowest] ratio of the familiar chromatic as greater than 28:27, while the 'following' ratio in enharmonic, once again, which appears much smaller than its equivalents in the other genera, he supposes to be equal to them;[117] and further, he makes the middle ratio smaller than it [i.e., than the 'following' ratio], setting it in the ratio 36:35, though wherever it occurs such a thing is always unmelodic, that is, where the magnitude next to the lowest note is made greater than the middle one.[118]

These things, then, seem to set up a slanderous accusation against the rational [*logikos*] criterion, since when the division of the *kanōn* is made according to the ratios set out by his proposals, that which is melodic is not preserved. For the majority of those set out above, and of those that have been worked out by virtually everyone else, are not attuned to the characters [*ēthē*] generally agreed on.[119] Again, the number of genera admitted by Archytas falls short of the norm [*to metrion*], since he supposes that not only the enharmonic, but the chromatic and diatonic too, are each single in form, while those admitted by Aristoxenus are too many in the chromatic (where the dieses of the soft and of the hemiolic differ by a twenty-fourth part of a tone, which imprints no noticeable variation on the hearing), but fall short in the diatonic, where it is obvious that those that are sung are more, as we shall be able to see from things that will shortly be demonstrated.[120] Again, he too is wrong both in

[116] Ptolemy's use of the term 'commensurable relation' at 30.13 is to be construed along the lines of 16.13 etc., and as implying that the ratios must be epimoric. Archytas does indeed breach this principle if it means (what Ptolemy intends) that every scalar step should be of epimoric ratio. But the principle may be taken differently, in such a way that Archytas' divisions do not offend against it: see the Appendix to chapter 1.

[117] The equality of the lowest intervals in all the Archytan genera is a notable peculiarity of his system, briefly discussed in the Appendix to chapter 1.

[118] The principle Ptolemy draws on here, and uses in his own constructions in chapter 15, is originally Aristoxenian: see 7 *El. Harm.* 52.8ff. It is breached also by Didymus in the chromatic attributed to him by Ptolemy in Book II ch. 14.

[119] Here *ēthē* are apparently 'perceived distinguishing features'. For a similar idea see 7 Aristox. *El. Harm.* 48.15–49.22, where the term *ēthos* is used in a comparable way (48.32–3).

[120] Ptolemy's own account of the genera occupies chapters 15–16 below. In the case of the diatonic he may have misconstrued Aristoxenus' intention: the latter's divisions are exemplary, not exhaustive (see 7 *El. Harm.* 50.19ff., cf. 48.9ff.). As to the minute difference between soft and hemiolic chromatic, Aristoxenus' treatment suggests that the focus of the distinction is the interval by which each *pyknon* falls short of a tone – a chromatic diesis, one third of a tone, in soft chromatic, and an enharmonic diesis, one quarter of a tone, in hemiolic (see 7 *El. Harm.* 51.4–7). Given that there is a clearly

respect of the *pykna*, where he makes the two 'following' magnitudes equal to one another, whereas the middle one is always grasped as being greater,[121] and in making equal the intervals next to the lowest note in the tense diatonic and the tonic chromatic, so making the chromatic too big.[122]

33 ### 15 *Concerning the division of the tetrachords by genus, according to what is rational* [eulogon] *and evident to perception* [phainomenon]

Since, then, not even these people have divided the primary genera of the tetrachords in a way that agrees with perception, let us ourselves try, here as well, to preserve what is consistent both with our postulates concerning melodic relations and with the appearances [*ta phainomena*], in accordance with those conceptions of the divisions that are primary and natural.[123]

To find the positions and orders of the quantities, we adopt as our primary postulate and rational principle the thesis that all the genera have the following feature in common: that in the tetrachords too, the successive notes always make those epimoric ratios in relation to one another which amount to divisions into two or three that are nearly equal.[124] By these divisions the differences [lit. 'excesses'] in the first concords were also found to be bounded, and they go up only to the number 3 there too, since that completes all the intervals. For beginning from the octave homophone and the duple ratio, in which the difference between [lit. 'excess of'] the extremes is equal to the one that is exceeded, we took for its reduction by equals the hemiolic ratio of the concord of a fifth, in which the difference between the extremes contains a half of that which is exceeded, and the epitritic ratio of the concord of a fourth, in which the difference between the extremes contains a third part of that which

perceived aesthetic difference between chromatic and enharmonic intervals, he can insist not unreasonably on the reality of the distinction between the two *chroai*, despite the near-identity of the sizes of their constitutent intervals. See also n. 122 below.

121 Ptolemy maintains this principle consistently, restating it at 33.22–4. Of the predecessors whose divisions he records in Book II ch. 14, only Eratosthenes preserves the principle throughout. Both he and Didymus, unlike Ptolemy, seem to have aimed to make the intervals of the enharmonic *pyknon* as nearly equal as possible: the same is true of Eratosthenes' chromatic.

122 The lowest interval of Ptolemy's soft diatonic (21:20) is indeed greater than that of his higher chromatic (22:21): see the tables on pp. 308 and 310 below. But when expressed in relation to a common term the former is 441:420, the latter 440:420. It is hard to believe that this difference 'imprints a noticeable variation on the hearing' if that of Aristoxenus, criticised above, does not.

123 Ptolemy continues to insist on the importance of both criteria, reason and perception, in his constructions as well as in his criticisms. The present chapter constitutes 'rational' construction, for the most part, though certain principles are adopted 'on the basis of agreed perception'; the authority of the ear is invoked to confirm and qualify its results in ch. 16 and Book II ch. 1; cf. also Book II ch. 16, Book III chs. 1-2.

124 'In the tetrachords too', because this is a further application of the principle that was at work in chapter 7 (15.22ff.), as Ptolemy explains below. The rule that divisions are to be made into 'nearly equal' epimoric ratios reflects also the thesis of 16.17ff. that a 'more melodic' division is one in which the difference between the terms constitutes a larger simple part of each term: see notes 66 and 68.

is exceeded; and for its augmentation by equals we took the triple ratio of the concord of an octave and a fifth, in which the difference between the extremes makes two of that which is exceeded, antithetically to the half, and the quadruple ratio of the double octave homophone, in which the difference between the extremes makes three of that which is exceeded, antithetically, once again, to the third part.[125] Secondly, on the basis of agreed perception, we adopt similarly, as common to all the genera, the thesis that the 'following' [lowest] magnitudes of the three are smaller than each of the remaining ones; as peculiar to the genera that have *pykna* the thesis that the two magnitudes next to the lowest note are together less than the one next to the highest note; and as peculiar to the *apykna* the thesis that none of the magnitudes is greater than the remaining two together.[126]

With these principles laid down, then, we first divide the epitritic ratio of the concord of the fourth, as many times as is possible, into two epimoric ratios: such a thing, once again, occurs only three times, when we adopt in addition the three epimoric ratios in succession below it, the ratios 5:4, 6:5 and 7:6. For the ratio 16:15 added to the ratio 5:4 fills out the epitritic, as does the ratio 10:9 added to the ratio 6:5, and the ratio 8:7 with the ratio 7:6; and after these we cannot find the ratio 4:3 put together from just two other epimorics.[127]

Now in the genera that contain the *pyknon*, since in them the 'leading' [highest] ratios are greater than the remaining ones together, they fitted the greater ratios in the pairings [*syzygiai*] set out – that is, the ratios 5:4, 6:5 and 7:6 – to their leading ratios, and the remaining and smaller ones – that is, the ratios 16:15, 10:9 and 8:7 – to the remaining ones taken together.[128] The division of each of these in respect of the two 'following' ratios is achieved when it is divided into three sections (because by these means the three ratios of the tetrachord are at once produced), the differences being kept equal, and the ratios nearly equal, since it is not possible for them to be equal.[129] For if

125 Like the Pythagoreans, whose embarrassment on the subject was noted in chapter 6, Ptolemy avoids giving structural importance to the octave plus fourth, 8:3, though he claims to have circumvented the Pythagoreans' special difficulty (16.10–12). The interval had apparent structural significance only in the so-called LPS: see Ptolemy's discussion of this system in Book II ch. 6.

126 This group of principles is based on what is acceptable to perception, whereas the 'primary postulate' discussed above is an independent principle of reason. With the first of the 'perceptual' principles compare 32.23–5, and on the others see 29.5–9, where they are implicitly and correctly attributed to Aristoxenus. They are stated here almost as a definition: they are 'perceptual' in that it is perception, not mathematics, that finds a significant distinction between *pykna* and *apykna*. Though the difference can be described quantitatively, it is not a mathematical intuition that makes it important. See 7 Aristox. *El. Harm.* 48.20–31.

127 Ptolemy's procedure for dividing the tetrachord involves two steps, of which this is the first. It divides the fourth into two intervals, each of epimoric ratio. As Ptolemy says, there are only three ways of doing this.

128 This move makes use of the second 'perceptual' principle stated above. The apparent reference to other theorists ('they fitted') seems out of place. Since in Greek 'they fitted' differs from 'I fitted' by only one letter, it is quite likely that the text is mistaken.

129 The smaller ratio in each pair is to be divided to form the two intervals of the *pyknon*. The obvious way to do this is to divide it into two 'near-equals'. Thus if we double the terms, for example, of 16:15, giving 32:30, we can divide it as 32:31 × 31:30. This is

we take the first numbers making the ratio 15:16, I mean 15 and 16, and triple them, we shall get 45 and 48, and their mean numbers in equal excesses are 46 and 47. Now since 47 does not make an epimoric ratio with both the extremes, but only 46 does so, making with 48 the ratio 24:23, and with 45 the ratio 46:45, the greater, the ratio 24:23, because of our initial postulates, will be conjoined with the ratio 5:4, and the remaining ratio, 46:45, will fill up the 'following' ratio.[130] Again, if we take the first numbers that make the ratio 10:9, that is, 9 and 10, and triple them, we shall get 27 and 30, and their means in equal excesses are 28 and 29. But 29 does not make an epimoric ratio with both the extremes, whereas 28 makes with 30 the ratio 15:14, and with 27 the ratio 28:27, so that here too the ratio 15:14 is conjoined with the ratio 6:5, and the ratio 28:27 is left in the 'following' position. Similarly, if we take the first numbers that make the ratio 8:7, which are 7 and 8, and triple them, we shall get 21 and 24, and their means in equal excesses are 22 and 23. Since the latter does not make an epimoric ratio with both the extremes, but only 22 does so, making with 24 the ratio 12:11, and with 21 the ratio 22:21, here too the ratio 12:11 will be conjoined with the ratio 7:6, and the ratio 22:21 will possess the 'following' position.

Now since of all the genera the enharmonic is softest, there is as it were a road from it towards the more tense, by a process of increase through first the softer chromatic, then the tenser, towards the succeeding genera that are *apykna* and diatonic: in general those appear softer that have the larger leading ratio, and those appear tenser that have the smaller one.[131] We have thus attached the tetrachord put together from the ratios 5:4, 24:23 and 46:45 to the enharmonic genus; that put together from the ratios 6:5, 15:14 and 28:27 to the softer of the chromatics; and that put together from the ratios 7:6, 12:11 and 22:21 to the more tense of the chromatics. The first numbers that contain these three tetrachords are these: common [to all three genera], those of their extremes, 106,260 and 141,680; peculiar [to each genus individually], those of the ones second from the leaders [the highest], 132,825 and 127,512 and 123,970; and those of the ones that come third, 138,600 and 136,620 and 135,240. These are shown in the table below.

As to the *apykna* genera, it follows from our previous definitions that the smaller ratios of those arising from the first division of the epitritic ratios, the

evidently the procedure adopted by Didymus in generating his enharmonic, and an application of it will give Eratosthenes both his chromatic and his enharmonic (though these were in fact probably derived in a different way): see Book II ch. 14, and compare 12 Arist. Quint. *De Mus.* Book III ch. 1. Ptolemy varies this procedure, multiplying the terms of the ratio by 3 instead of 2, so that, for example, 16:15 becomes 48:45. The difference between the terms can now be divided into three equal steps. He next chooses the intermediate term which forms epimoric ratios with each extreme: here it is 46, since 48:46 (= 24:23) and 46:45 are epimoric, while 47:45 is not. The two epimoric ratios are then assigned to the intervals of the relevant *pyknon*. Ptolemy's reasons for taking this indirect route are not clear. Unless it was the mere fascination of the number 3, it was probably his wish to make the difference in size between the two intervals of the *pyknon* substantial and noticeable.

130 The 'initial postulate' operating here is that the ratio of the lowest ('following') interval be smaller than each of the ones above it (33.22–4).

131 See n. 107 above.

enharmonic	soft chromatic	tense chromatic
106,260	106,260	106,260
5:4	6:5	7:6
132,825	127,512	123,970
24:23	15:14	12:11
138,600	136,620	135,240
46:45	28:27	22:21
141,680	141,680	141,680

division into two, should here, by contrast, be placed in the leading positions, and that the greater ratios coupled with them should be divided in the same way into the two 'following' ratios.[132] Now the ratio 16:15 is found to be incapable of occupying the leading position. For if we take the numbers making the remaining ratio, 5:4, which are 4 and 5, and once again triple them, to make 12 and 15, there will fall as means with equal excesses 13 and 14. Now 13 will not make an epimoric ratio with both the extremes, while 14 will make with 12 the ratio 7:6 and with 15 the ratio 15:14, neither of which is it permissible to place in the 'following' position, since each will be greater than that in the 'leading' position, that is, than the ratio 16:15, contrary both to what is perceptually evident and to our initial thesis [*logos*].[133]

However, when the ratio 8:7 is arranged in the leading position, the first numbers bounding the remaining ratio, 7:6, which are 6 and 7, when tripled in the same way will make 18 and 21, whose means, taken in equal excesses, are 19 and 20. Then 19, once again, will not make an epimoric ratio with both the extremes, but 20 will make with 18 the ratio 10:9, and with 21 the ratio 21:20, of which the greater, 10:9, will be conjoined as before with the ratio 8:7, while the lesser, 21:20, will fill out the 'following' ratio. In the same way, when the ratio 10:9 is arranged in the leading position, if the numbers bounding the remaining ratio, 6:5, which are 5 and 6, are tripled, they will make 15 and 18, whose means falling in equal excesses are 16 and 17. Now 17 will not make an epimoric ratio with both the extremes, but 16 will make with 18 the ratio 9:8, and with 15 the ratio 16:15, so that the greater, 9:8, is conjoined with the ratio 10:9, while the remaining one, the ratio 16:15, is attuned to the 'following' position.

But prior to all these ratios, the ratio 9:8 was found in its own right to contain the tone arising from the difference between the first two concords;[134] and this, according to what is both rational and necessary, ought also to occupy the leading position, those closest to it being conjoined with it, since none of the epimoric ratios fills out with it the epitritic ratio.[135] The ratio 10:9 has

[132] The relevant definition is that of *apykna* genera at 33.26–7; in their tetrachordal divisions the two lower intervals are jointly greater than the highest.

[133] This is the thesis stated at 33.22–4.

[134] See 16.25–8.

[135] That is, the first step of the process of division cannot be made in the same way as before (33.27ff.), since there is no epimoric ratio n:m such that 9:8 × n:m = 4:3. It is notable

already been conjoined with it in the division set out above, but the ratio 8:7 has not yet. Hence we shall conjoin this with it, in the middle position, and allocate the remainder making up the epitritic ratio, which is the ratio 28:27, to the 'following' position.

Here, once again, in correspondence with the magnitude of the leading ratios, we shall attach the tetrachord put together from the ratios 8:7, 10:9 and 21:20 to the soft diatonic, that put together from the ratios 10:9, 9:8 and 16:15 to the tense diatonic, and that put together from the ratios 9:8, 8:7 and 28:27 to the one lying somehow between the soft and the tense, which is called 'tonic' [*toniaios*], reasonably enough, because that is the size of its leading position.[136]
37 The first numbers that contain these three tetrachords are these: common [to all three genera], those of the extremes, 504 and 672; peculiar [to each genus individually], those of the ones second from the leaders, 576 and 567 and 560; and those of the ones that come third, 640 and 648 and 630. These are shown in the table below.

soft diatonic	tonic diatonic	tense diatonic
504	504	504
8:7	9:8	10:9
576	567	560
10:9	8:7	9:8
640	648	630
21:20	28:27	16:15
672	672	672

The fact that the divisions of the genera set out above do not contain only what is rational [*eulogos*] but also what is concordant with the senses can be grasped, once again, from the eight-stringed *kanōn* that spans an octave, once the notes are made accurate, as we have said, in respect of the evenness of the strings and their equality of pitch.[137] For when the bridges set under [the strings] are aligned with the divisions marked on the measuring-rods [*kanonia*] placed beside them – the divisions that correspond to the ratios in each genus – the octave will be so tuned that the most musical of men would not alter it any more, even a little. We would be astonished at the nature of the ordering of attunement [*to hērmosmenon*], if on the one hand the reasoning [*logos*] that deals with it moulded, as it were, and shaped the differences that preserve melody, and if hearing followed the lead of reason to the greatest degree

that Ptolemy is prepared to modify his methods of construction in the light of additional musical and 'rational' considerations; he does not abandon his principles, but finds new ways of deploying them to suit the case. Here the principle of 'near equality' will be preserved by placing next to the 9:8 tone an interval as close as possible to it in size. Compare also his attitude to the 'familiar genera' in the next chapter.

136 That is, its upper interval is the *toniaion diastēma*, the 9:8 tone.

137 The procedure has been described in chapter 11.

possible (being thus arranged alongside the ordering arising from reason, and recognising the appropriateness of each of its propositions), while on the other hand the outstanding experts in the subject condemned it, though they are unable, by themselves, to initiate an investigation of the rational divisions, and neither do they think fit to try to discover those that are displayed by perception.[138]

38 16 *The genera more familiar to the hearing – how many there are, and which ones they are*

Now of the genera that have been set out, we would find all the diatonic ones familiar to our ears, but not to the same extent the enharmonic, nor the soft one of the chromatics, because people do not altogether enjoy those of the characters [*ēthē*] that are exceedingly slackened, but it is enough for them in the movement towards the soft to stop at the tense chromatic.[139] For the *pyknon*, by which, in a way, the nature of the soft is distinguished from that of the tense, finds its limit in this genus, beginning from here in the progression towards the softer, and ceasing here again in that towards the more tense. Again, in the segmentation of the whole tetrachord into two ratios, it is defined by the ratios that are nearest to equality and are consecutive, that is, by the ratios 7:6 and 8:7, which divide in half the whole difference between the extremes.[140] For the reasons given, then, this genus seems most agreeable to the ears.

It also suggests to us another genus, when we set out from the melodic-ness [*emmeleia*] that is constituted in accordance with equalities, and investigate the question whether there is any appropriate ordering of the tetrachord when it is initially divided into the three nearly equal ratios, again in equal excesses.[141] The ratios comprising this sort of genus are 10:9, 11:10 and 12:11, when we have again tripled in the same way the first numbers that display the ratio 4:3,

[138] This echoes the criticisms levelled at Aristoxenus and his followers at the beginning of chapter 9.

[139] Such comments on the enharmonic are common (e.g., 12 Arist. Quint. *De Mus.* 16.13ff.); I know no parallel in this period to Ptolemy's claim about the soft chromatic. Aristoxenus traces the unpopularity of the 'true' enharmonic to the aesthetic impact of the large upper interval of its tetrachord (7 *El. Harm.* 22.31ff.), not, like Arist. Quint., to the difficulty of singing quarter-tones. Ptolemy extends the Aristoxenian diagnosis further. His evidence suggests that contemporary taste would not tolerate an interval greater than 7:6 in this position – perhaps because larger ones could no longer be grasped as incomposite scalar steps – whereas the testimony of Aristoxenus and Archytas indicates that in the fourth century B.C. practical musicians still remembered (if they did not much use) the ditone as an incomposite interval, and that audiences found the 5:4 major third perfectly acceptable in that role. But Ptolemy's remark at 40.6ff. seems to imply not only that the enharmonic survived in his day, but that it did so in its Aristoxenian form, with a ditone of 81:64.

[140] The ratio 7:6 makes the upper interval of this tetrachord, 8:7 the lower two together. Here Ptolemy finds another application for his mathematical principle of 'near equality'.

[141] Another application of the same principle, the tetrachord being divided from the start into three 'near equals' (rather than into two, one of which is later subdivided). This is a somewhat more simple-minded application, a quality that Ptolemy finds reflected in the melodic character of this division.

making the successive numbers 9, 10, 11 and 12, and the successive ratios that have been set out. When here too the greater ratios are put first in order, there arises a tetrachord close to the tense diatonic,[142] and more even than it, both in itself and still more in association with the filling-out of the fifth. For when the [interval of] disjunction, which makes an epogdoic ratio [9:8], is conjoined with the 'leading' note, the characteristic of equality is no longer produced only in the three excesses, but in the four that are contained by the successive ratios from the epogdoic to the ratio 12:11. The first numbers that make this kind of octave, when the [interval of] disjunction is placed in the middle, are [from the top] 18 and 20 and 22 and 24 and 27 and 30 and 33 and 36. When a division is taken in [strings] of equal pitch on the basis of these numbers, the character that becomes apparent is rather foreign and rustic, but exceptionally gentle, and the more so as our hearing becomes trained to it, so that it would not be proper to overlook it, both because of the special character of its melody, and because of the orderliness of the division. Another reason is that when a melody is played in this genus by itself, it gives no offensive shock to the hearing, which is true, pretty well, only of the intermediate one of the other diatonics,[143] the others being attuned by forcible constraint [*bia*] when taken by themselves, but capable of being successful in a mixture with the diatonic just mentioned, when those softer than it are taken in the tetrachords lower than the disjunctions, the tenser in those that are higher.[144] So let us call this genus the 'even diatonic', from the characteristic it has.

When an investigation of the other familiar genera is undertaken, the intermediate or 'tonic' one of the diatonics, when it is examined in itself and unmixed, will fit the *sterea* in the *lyra*, and in the *kithara* those corresponding to the attunements of the *tritai* and the *hypertropa*; the mixture of the tense chromatic with it, which has been mentioned, will fit the *malaka* in the *lyra* and the *tropika* in the *kithara*; the mixture of soft diatonic with tonic will fit the *parhypatai* in the *kithara*; the mixture of tense diatonic with tonic will fit the *metabolika* characters, which the *kitharōdoi* call Lydian and Iastian[145] – except

[142] The ratios of the tense diatonic are 10:9, 9:8, 16:15, so that the 'leading' interval is the same in each of these divisions.

[143] This is the 'tonic' diatonic, whose ratios are 9:8, 8:7, 28:27.

[144] Ptolemy is the only source to report these important facts about musical practice, and to describe the 'mixed' genera in common use (see the next paragraph, with Book II ch. 1 and especially ch. 16). The expression 'lower than the disjunctions' does not mean 'in the lower half of the two-octave system'. It refers to any tetrachord that is placed immediately below a tone of disjunction. One of these is the tetrachord *mesōn*, the other that which stands highest of all in the GPS, the tetrachord *hyperbolaiōn*. This tetrachord counts as being below a disjunction, because in the progressive rotation of the series of intervals in the GPS by which different *tonoi* are produced (Book II chs. 5–11), the tetrachords come to occupy different positions in the tonal space, and the tetrachord *hyperbolaiōn* is carried round from the top of the system to the bottom, reappearing below the disjunctive tone that separates *proslambanomenos* and *hypatē hypatōn*. 'Mixed' genera will inevitably break Aristoxenus' primary law of melodic succession (e.g., 7 *El. Harm.* 53.33ff.). Though versions of the rule are repeated by later authors too (e.g., 9.2 Adrastus ap. Theon Smyrn. 51.4ff.), it was apparently in Ptolemy's time a 'truth' only of theory.

[145] These 'attunements' (*harmogai*) represent ways of tuning stringed instruments found by Ptolemy in current practice, named according to the usage of the musicians themselves.

that while they sing in accordance with the tense diatonic that has been set out, as can be seen from a comparison with the ratios proper to that genus, they actually tune to another genus, close to that one, but plainly different: for they make the two leading intervals tones [9:8] and the remainder, as they think, a half-tone, but as reason implies, what is called the *leimma* [256:243].[146] This works for them well enough, since there is no noticeable difference either in the leading positions between the ratios 9:8 and 10:9, nor in the 'following' positions between the ratio 16:15 and the *leimma*. For if we take numbers standing to 72 in the ratios 10:9 and 9:8, the latter will make 81, the former 80; and the ratio 9:8 will be in the ratio 81:80 to the ratio 10:9. This also is the ratio between the ditone – that is, the ratio 9:8 taken twice – and the ratio 5:4, which was the leading ratio of the enharmonic genus. For the ratio 5:4 applied to the number 64 makes 80 once again, while the ratio 9:8 taken twice makes 81. Similarly, since the ratio of the *leimma* is 256 to 243, while the number standing to 243 in the ratio 16:15 is 259, the ratio in which the ratio 16:15 stands to the *leimma* will be that of 259 to 256. This is again the same as the ratio 81:80, and that is because the ratio 5:4 is equal to the ratios 9:8 and 10:9 taken together. For this reason, in neither of the genera set out does any noticeable offence arise, when in the tense diatonic they wrongly use the ratio 9:8 instead of 10:9 in the leading position and the *leimma* instead of the ratio 16:15 in the 'following' position, and when in the enharmonic they use the ratio 9:8 taken twice instead of the ratio 5:4 in the leading position, and the *leimma* once again instead of the ratio 16:15 for the two 'following' ratios taken together.[147]

They are further discussed in Book II ch. 1 and crucially in ch. 16, where other distinguishing features are mentioned. For discussion of the *kithara* tunings see the Appendix to Book II. The *lyra* tunings are more straightforward. *Stereos* means 'hard', *malakos* 'soft'; they correspond approximately to a diatonic and a chromatic respectively. Porphyry (*Comm.* 154.15–16) says that *sterea tetrachorda* are ones which have as a component a tone of the size used in disjunctions, 9:8, and that the expression is equivalent to 'diatonic': cf. 12 Arist. Quint. *De Mus.* 110.22–3 and, for example, 77.19–20. Compare also the common musical use of *malakos*, for example, by Ptolemy at 28.28ff. (But the scholium to 43.11 makes the term *sterea* refer to the fact that the system 'stands firm' and does not modulate.) In the forms given by Ptolemy, most of the names of the attunements are neuter plural adjectives. Porphyry indicates that they are to be understood as governing the word 'tetrachords'; an alternative would be *systēmata*. The exceptions are *tritai* and *parhypatai*, which are the plural forms of note-names and are discussed in the Appendix to Book II.

146 This is the familiar 'Pythagorean diatonic' implied by 1.12 Philolaus frag. 6, 2.3 Plato *Tim.* 35b–36b, 8 Eucl. *Sect. Can.* propositions 19–20 and many later sources. Ptolemy's account suggests that in some sense the tense diatonic (10:9 × 9:8 × 16:15) is the correct form of a tetrachord in the system intended, and is indeed what is sung, the other being adopted for convenience in tuning an instrument (since that can be done purely by the 'method of concordance': see 40.14ff. with n. 149). This is confirmed at 40.2ff. and reflected again in Book II ch. 1 (43.19–44.12). The 'Pythagorean' system is called 'ditonic diatonic' at the end of the present chapter. In Book II ch. 16 it alone is mentioned in this connection, the tense diatonic being ignored (presumably because it is not reflected in instrumental attunements, which are the subjects under discussion there).

147 It is surprising to find the suggestion that the enharmonic survived in practice at this date, especially in the 'Aristoxenian' form indicated here: see the opening lines of this chapter, with n. 139.

Let us then accept this genus too, both because of the ease of the modulations to it from the tonic diatonic genus,[148] in the case of its mixture with this one, and also because the ratio of the *leimma* has a certain affinity with the fourth and the tone, marking it out from the other ratios that are not epimoric, since it follows inevitably when two epogdoics [9:8 tones] have been inserted into the epitritic [the 4:3 fourth]. For the *leimma*, too, can be constructed by itself by means of concords, just as can the tone, the latter from the difference between the first two concords, the former from the difference between the ditone and the concord of a fourth.[149] The first numbers that make this genus are 192, 216, 243 and 256. It may reasonably be called 'ditonic', since it has tones as its two leading ratios.

even diatonic	ditonic diatonic
18	192
10:9	9:8
20	216
11:10	9:8
22	243
12:11	*leimma*
24	256

41 **Book II**

Contents

[148] This 'ease of modulation' (*metabolē*) may arise from the fact that both genera (and the tense diatonic) include the 9:8 tone in their tetrachords. I do not think that the point made here is relevant to Ptolemy's designation (39.13) of these systems as *metabolika*, since the other 'mixed' systems modulate in comparable ways (for a guess at the word's significance see the Appendix to Book II).

[149] These constructions by means of concords are already implicit in 2.3 Plato *Tim.* 35b (and probably earlier in 1.12 Philolaus frag. 6, 1.21 Archytas ap. Porph. *Comm.* 30.9ff.): cf. also 7 Aristox. *El. Harm.* 55.11ff. For an articulation of the method in the context of the enharmonic genus see 8 Eucl. *Sect. Can.* proposition 17.

7 Concerning modulations with respect to what are called the *tonoi*

8 That the outermost *tonoi* must be bounded by the octave

9 That we must posit just seven *tonoi*, equal in number to the species [*eidē*] of the octave

10 How the differences between the *tonoi* may be soundly established

11 That one should not increase the *tonoi* by semitones

12 On the difficulty of using the single-stringed *kanōn*

13 Concerning the alterations to the *kanōn* proposed by the music-theorist Didymus

14 An exposition of the numbers making up the division of the octave in the changeless *tonos* and for each of the genera individually

15 Exposition of the numbers that make up the divisions of the familiar genera in the seven *tonoi*

16 Concerning the things played on the *lyra* and the *kithara*

Book II

42 1 *How the ratios of the familiar genera can also be found through perception*

There is also another way in which we can find the same sets of proportions, those of the genera that are familiar and readily accepted by the ear,[1] not generating their differences from what is rational [*eulogon*] alone, as we did just now, and then submitting them by means of the *kanōn* to evidence drawn from what is perceived [*ta phainomena*], but reversing the procedure, first setting out the attunements put together through perception alone, and then showing from them the ratios that go with the equalities and differences between notes that are adopted in each genus. We shall assume here too only those things that are straightforwardly agreed by everyone,[2] that the concord of the fourth bounds an epitritic ratio [4:3], and that the tone bounds one that is epogdoic [9:8].

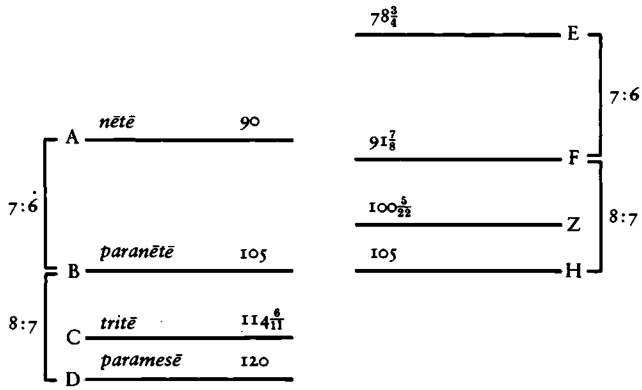
Of the tetrachords played by the *kitharōdoi* let there be constructed, first, the fourth from *nētē* to *paramesē* belonging to what is called the *tropoi*.[3] Let this

[1] As will become clear, these are the forms of attunement described in the previous chapter and in Book II ch. 16. There is no discontinuity between Books I and II.

[2] That is, these are the only theoretical principles adopted. The argument that follows relies also on the reader's capacity to follow Ptolemy's constructions in practice, and to recognise the 'familiar' tunings by ear: written argument has to be supplemented by real experiments with instruments, attuned by the reader to fit what he himself hears as the appropriate systems.

[3] In chapter 16 below it is explained that the tunings are made in specific *tonoi*: the *tonoi* are discussed at length in chapters 7–11. Ptolemy's account depends on his distinction between two ways of naming notes (chapter 5), by *thesis* ('position') and by *dynamis* ('function'). A note's *thesis* is its position (first, second, third, etc.) in the two-octave system of which it is a part, and these positions are identified by the names of the corresponding notes of the GPS. (Thus the first or lowest note is 'thetic'

be ABCD, with A assigned to *nētē* [see next figure]. I say that what this contains is the genus of the tense chromatic that has been set out, and first that the ratio of AB is 7:6, while that of BD is 8:7. Those of BC and CD will be shown



subsequently. Now each of AB and BD will be found to make a magnitude greater than a tone,[4] that is, greater than the ratio 9:8, and the ratio of AD is
43 4:3; and no two ratios[5] greater than 9:8 fill out the ratio 4:3 except 7:6 and 8:7, so that of the ratios AB and BD, one will be 7:6, the other 8:7. Next let there be taken the note H, equal in pitch to B, and let there be constructed upwards from it the tetrachord EFZH, similar to ABCD.[6] Now A will be found to be higher than F[7] – B and H being of equal pitch – and hence the ratio of AB is greater than that of FH, while it was laid down that the ratio of FH is the same as that of BD. Hence the ratio of AB is greater than that of BD, and hence that of AB will be 7:6, while that of BD will be 8:7.[8]

proslambanomenos, the second *hypatē hypatōn*, and so on, regardless of the intervals between them.) But in different *tonoi*, a given position is occupied by a note with a different *dynamis*, this being defined by the relation in which the note stands to the locations of the disjunctive tones in its particular *tonos*. Thus from a 'dynamic' point of view, the note immediately below the higher disjunctive tone is always *mesē*, no matter where it falls in the double-octave space. The nomenclature is confusing, since the same names are used for both *theseis* and *dynameis*, and Ptolemy does not always specify which is intended. In the present chapter the notes are named throughout by their *theseis*: this can be shown as follows. The tuning called *tropoi* belongs to the Hypodorian *tonos* (ch. 16). The chromatic tetrachord described here always falls just below a disjunction (see 39.2–5), and there is no disjunction above the tetrachord between dynamic *paramesē* and *nētē* (*diezeugmenōn*). But in Hypodorian, the tetrachord between dynamic *nētē diezeugmenōn* and *nētē hyperbolaiōn*, above which there is a disjunction (see n. 144 to Book I ch. 16), is positioned within the two-octave space on the degrees from thetic *paramesē* to *nētē diezeugmenōn*. Ptolemy uses the latter names to designate the tetrachord in question, and hence is using the thetic nomenclature. Similar considerations apply to the other examples discussed in this chapter.

[4] By perception. The construction must be done on real strings, not just in diagrams: cf. n. 2 above.

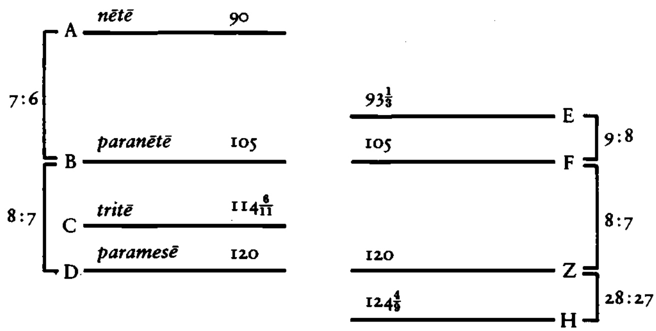
[5] That is, no two epimoric ratios. See Book I ch. 15.

[6] 'Similar' here means 'in the same intervals or ratios', but at a different pitch. A new set of four strings is required.

[7] Again, by ear, not by calculation.

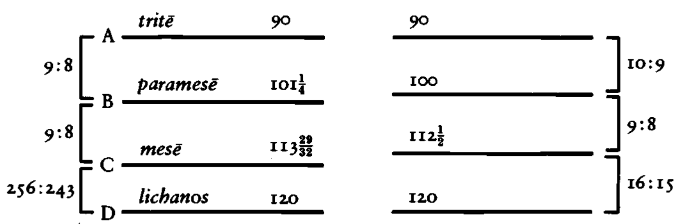
[8] Ptolemy postpones consideration of the intervals BC and CD until the last paragraph of the chapter (45.11ff.).

Again, letting the tetrachord ABCD stand unchanged, let the note F be taken, of equal pitch to B, and when it is established let there be constructed the tetrachord from *paramesē* to *chromatikos* belonging to the *sterea*.[9] Let this be EFZH, with E assigned to *paramesē* (see next figure). I say that what is



contained by it is the genus of the tonic diatonic, and that the ratio of EF is 9:8, that of FZ is 8:7, and that of ZH is 28:27. For EF will make exactly a tone, that is, the ratio 9:8, and Z will be found to be of equal pitch to D, so that the ratio FZ is the same as that of BD, that is the ratio 8:7; and the ratio of ZH is left as 28:27, which together with 9:8 and 8:7 makes up the ratio 4:3.

Next let there be constructed the tetrachord from *tritē* to *diatonos* belonging to what is called the *lastiaiolia*.[10] Let this be ABCD, with A assigned to *tritē*. I say that what is contained by it is the genus of the ditonic diatonic, according to which each of the leading ratios is 9:8, and the remaining one is that of the
44 *leimma*. This is immediately obvious. For the *kitharōdoi* make their attunement in such a way that a tone – that is, the ratio 9:8 – is produced both by AB and by BC, and there is left for CD the ratio of 256 to 243, which together with the



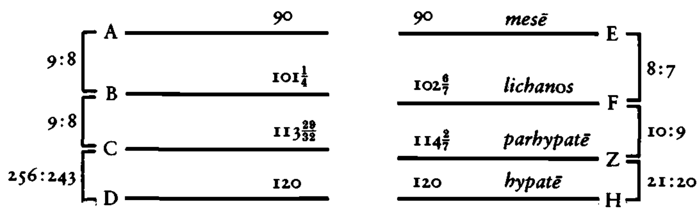
[9] The notes are again identified by *thesis*, the note *chromatikos* (i.e., chromatic *lichanos mesōn*) being treated as lying a 9:8 tone above *hypatē mesōn*. The *lyra* tuning called *sterea* can apparently occur in any *tonos*, but only in Phrygian and Hypodorian will there be a tetrachord of the form required bounded by these *theseis*. Probably Ptolemy intends the Hypodorian, where the tetrachord will be that bounded by dynamic *paramesē* and *nētē diezeugmenōn*, since this is the *tonos* proper to the *kithara* tuning that corresponds to *sterea*, the one called *tritai* (see ch. 16).

[10] The tuning *lastiaiolia* (called *lastia* in Book I ch. 16) is in the Hypophrygian *tonos* (Book II ch. 16). Its dynamic *nētē diezeugmenōn* and *paramesē* will fall on thetic *tritē diezeugmenōn* and *diatonos* (i.e., diatonic *lichanos mesōn*) respectively. The strings ABCD have now been retuned without reference to any previous arrangement.

two 9:8 ratios fills out the ratio 4:3: it is smaller than the ratio 19:18, greater than 20:19.

If, however, we hold fast to the precise character [of this tuning-system] and not to what is easy, and construct the tetrachord that was set out as belonging to the *metabolē*,[11] BC will once again produce a tone and the ratio 9:8, but AB will make something a little less than a tone, so that its ratio amounts to the greater of those that are smaller than 9:8, which is 10:9, and the ratio of CD amounts to 16:15, which along with 10:9 and 9:8 fills out the ratio 4:3, and the genus of the tense diatonic is put together.

Again, letting the tetrachord ABCD stand unchanged – I mean, according to the attunement of the ditonic diatonic – let there be constructed the note H, equal in pitch to D, and let there be attuned upwards from it the tetrachord from *mesē* to *hypatē* in the *parhypatai*. Let this be EFZH, with Z making *parhypatē*[12] [see next figure]. I say that what this contains is the genus of the



soft diatonic, according to which we would find the leading ratio to be 8:7, the middle one 10:9, and the remaining one 21:20. That the ratio of EF is 8:7 has been shown in the case of the *sterea*, for not one of them has been altered here.[13] But it must be shown that the ratio of FZ comes to 10:9, and that of ZH to 19:20. Now C will be found to be a little higher than Z, so that the ratio of ZH is smaller than that of CD, that is, than 19:18. But FZ will make something smaller than a tone, so that the ratio of FZ is also smaller than 9:8; and the ratio of FH is 7:6, since that of EF is 8:7. And there are no two ratios that fill out the ratio 7:6, of which one is smaller than 9:8 and the other smaller than 19:18, except 10:9 and 21:20. But it is the ratio of ZH that is smaller than 19:18. Hence it will be 21:20, while the ratio of FZ will be 10:9.

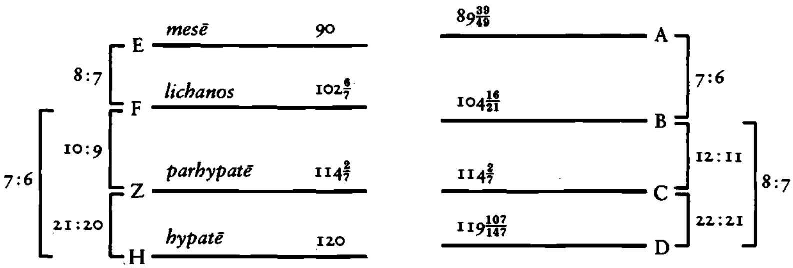
Finally, letting the tetrachord EFZH stand unchanged, let the note C be

[11] See Book I ch. 16, 39.12ff. The tense diatonic about to be described is the 'correct' tuning for this Iastiaiolian tetrachord, the ditonic diatonic an 'easy' alternative. On the sense of 'belonging to the *metabolē*' see the Appendix to Book II.

[12] The tuning *parhypatai* requires the Dorian *tonos*, where thetic and dynamic designations coincide (chapter 16 below). The natural expectation that the name of this tuning makes some special allusion to the note *parhypatē* (see the Appendix to Book II) is reinforced by Ptolemy's unnecessary reference to the note here.

[13] The present tuning has not been constructed by steps involving the *sterea*. Ptolemy must mean that if EF were thought of as the central interval of a tetrachord in *sterea*, we would find ourselves able to use it unaltered as the highest interval of this tetrachord in *parhypatai*. Further, if *sterea* is constructed in the Hypodorian *tonos* (see n. 9 above), an instance of its 8:7 interval will fall immediately below thetic *mesē*, in the place here occupied by EF.

constructed, of equal pitch to Z, and when it is established let there be attuned the tetrachord ABCD belonging to the original chromatic,[14] A being once again assigned to the highest note, so that the ratio of BD is 8:7. It is to be shown that the ratio of BC is 12:11, while that of CD is 22:21 [see next figure]. Now



D will be found to be a little higher than H, so that the ratio of CD is smaller than that of ZH, that is, than the ratio 21:20; while B is perceptibly lower than F, so that the ratio of BC is smaller than that of FZ, that is, than the ratio 10:9. Once again, no ratios fill out the ratio 8:7, one of which is smaller than 10:9, the other smaller than 21:20, except 12:11 and 22:21; and it is the ratio of CD that is smaller than 21:20, so that it will be 22:21, and that of BC is left as being 12:11. And that is what was to be shown.[15]

46 ## 2 *Concerning the use of the* kanōn *in connection with the instrument called the* helikōn[16]

The differences between tetrachords corresponding to the genera have thus been established for us by these methods, through the assessment and comparison of notes of unequal pitch.

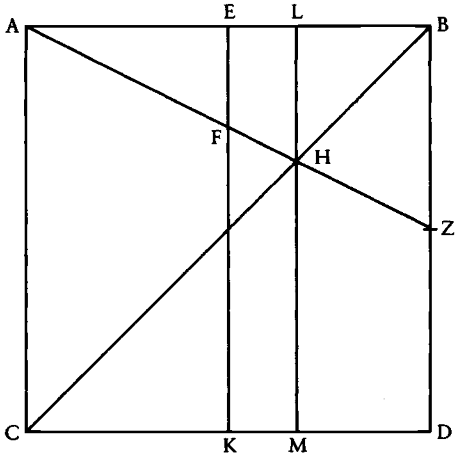
It is possible to use the eight-stringed *kanōn* of the octave in a different way too, in conjunction with the instrument called the *helikōn*, which has been made by students of mathematics to display the ratios in the concords, in the following sort of way.

They construct a square, ABCD, and after dividing AB and BD in half at E and Z, they join up AZ and BHC, and draw parallel to AC the line EFK through E, and the line LHM through H. It follows from this that AC is double each of BZ and ZD, and each of these is double EF, since AB is also double

[14] That is, the tense chromatic which Ptolemy has set off to analyse earlier in the chapter (42.10ff.). He has established the ratios of AB and BD, but not yet those of BC and CD.

[15] Ptolemy has offered ways of confirming his analyses of five kinds of tetrachord: the tense chromatic tetrachord in *tropoi*, the tonic diatonic in *sterea*, the ditonic diatonic in *lastiaiolia*, its more 'proper' variant the tense diatonic in the same system, and the soft diatonic in *parhypatai*. No other genera of tetrachord are involved in any of the 'familiar' tunings discussed in Book I ch. 16, Book II ch. 16.

[16] The name, as Porphyry notes (*Comm.* 57.15–16), is taken from Mount Helikon, home of the Muses. The device is mentioned also at 12 Arist. Quint. *De Mus.* 99.1ff.



AE,[17] so that it is also the case that AC is four times EF, and is the epitritic of the remainder FK.[18] It is also shown that MH is double HL, since as is DC to CM, so is DB to HM, and as is BA to AL, so is BZ to LH. And for this reason, as is BD to HM, so is BZ to LH, and conversely as is BD to BZ, so is MH to
47 LH. Then AC is the hemiolic [3:2] of HM and triple HL; so that when four strings of equal pitch are arranged in the same positions as those of the straight lines AC, EK, LM and BD, and when a *kanonion*[19] is placed under them in the position of AFHZ, and when the following numbers are assigned to them, 12 to AC, 9 to FK, 8 to HM, and 6 to each of BZ and ZD, then there are produced all the concords and the tone, that of the fourth, according to the epitritic ratio, being constituted by AC and FK and by HM and ZD and by LH and FE, that of the fifth, in hemiolic ratio, by AC and HM and by FK and ZD and by BZ and LH, that of the octave, according to the duple ratio, by AC and ZD and by HM and LH and by BZ and FE, that of the octave and a fourth, in the ratio of 8 to 3, by HM and FE, that of the octave and a fifth, according to the triple ratio, by AC and LH, that of the double octave, according to the quadruple ratio, by AC and EF, and finally the tone, according to the epogdoic ratio, by FK and HM.

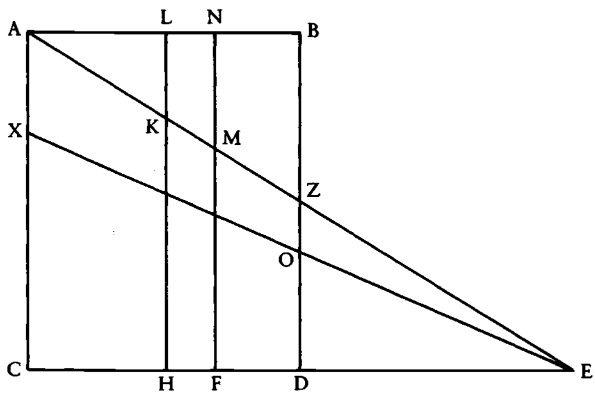
Next to this instrument, suppose that we draw up a rectangle ABCD, and
48 think of AB and CD as determining the vibrating lengths of the strings,[20] and

[17] This follows because ABZ and AEF are similar triangles; the same principle is repeatedly used in the sequel.

[18] That is, AC:FK = 4:3.

[19] Here the word designates a rod serving as a bridge. The segments on both sides of the bridge are used as sounding-lengths of the strings.

[20] Ptolemy writes '...think of AC and BD in accordance with the *apopsalmata* of the strings'. The general sense, clearly, is that AC and BD are the ends at which the strings are fixed, but the word *apopsalma* itself refers to the section of the string that gives out a note (sometimes elsewhere in the treatise to the point on a string under which a moveable bridge is placed to divide off such a section). These points are confirmed by Porphyry at *Comm.* 158.30.



AC and BD as the extreme notes of the octave. Then we add DE, equal to and extending CD, and cut the side CD, by the application of rulers [*kanonia*], in the ratios proper to the genera,[21] making E the limit of high pitch.[22] Through the resulting points of division on it we stretch strings parallel to AC and equal to one another in pitch, and when this is done we place under them what will be the bridge common to the strings in the position, AZE, that joins the points A and E. In this way we shall make all the lengths of the strings in the same ratios [as those marked on CD], so that it makes possible the assessment of the ratios that have been assigned to the genera. For as the lines taken from E along CD stand to one another, so will those drawn from the limits of these parallel to AC and as far as AZ stand to one another: for instance, as is EC to ED, so is CA to DZ. Hence these lines will make the octave, since their ratio is the duple.

Suppose that we cut off from CD, once again, the line CH, as a fourth part of EC, and CF as a third of the same line, and locate strings through H and F, HKL and FMN, equal in pitch to the first ones, so that AC becomes the epitritic of HK and the hemiolic of FM, and again FM becomes the epitritic of DZ and HK its hemiolic, and again HK is to FM in the ratio 9:8. Then these will make in relation to each other the concords that go with these ratios, and the equivalent result will follow for the divisions in the interior of the tetrachords, taken in the ratios appropriate to those that are being assessed.

The first method[23] is easier to apply than this in that it is not necessary to alter the distances between the strings, but this one is easier than the other in that it has a common bridge, which is single and has a single position, and further in that it is possible to move the bridge down, pivoting on E, to the position of XOE, and so to make the whole *tonos* ['pitch' or 'key'] higher,

[21] In this construction, any set of ratios may be tested for its accuracy of fit with the expectations of the ear. Ptolemy intends us to check all the ratios assigned to the generic divisions in Book I chs. 15–16.

[22] That is, the sounding-lengths of strings become progressively shorter, and hence higher in pitch, as they are placed closer to E, until at E no sounding length remains.

[23] That is, that using the square *helikōn*.

while the special character of the genus remains unchanged. For as CA, for instance, is to ZOD, so is XC to OD, and similarly for the others.[24] Again, let us repeat, the former method is at a disadvantage by comparison with this one, in that there one has to move several small bridges [*hypagōgidia*] to
49 accommodate each attunement, while this one is at a disadvantage by comparison with the other in that here one has to move the strings to completely new positions, and the changes in the points of contact are brought about no longer with equal distances between strings, but often at distances that differ by a large amount.[25]

3 *Concerning the forms* [eidē, '*species*'] *of the first concords*

Let that be our outline of the theoretical issues concerning the concordant and melodic relations between notes that are established in conformity with the length of string plucked, the homophones being included along with the concords. The next topic for discussion after these is that dealing with the *systēmata*, but first we must define the differences between the first concords in respect of what is called 'form' [*eidos*]. They are as follows. A form is a particular kind of positioning of the ratios which, when they lie between the appropriate boundaries, are peculiar to each genus.[26] In the fifth and the octave these will be the tones of disjunction, while in the fourth they will be the ratios between the two 'leading' notes, those which create variations in the direction of greater softness or tenseness.[27] Thus we call it the 'first form' in all cases when the peculiar ratio holds the leading place, since what leads is first; the second when it holds the second place, next after the leading one; the third when it holds the third place, and so on in order. Hence in each case there are as many forms as there are places belonging to ratios, three in the fourth, four in the fifth and seven in the octave. Now it turns out that there is only one form of the fourth that is bounded by fixed notes (it is the first form), only two of

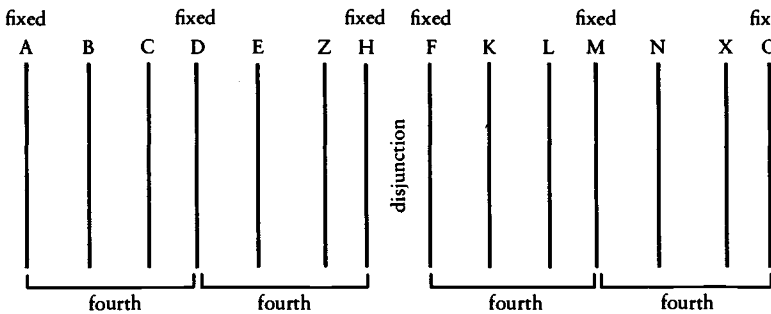
[24] The principle is no longer that involving similar triangles, but the claim plainly holds, since the construction is independent of the proportions of the original rectangle ABCD.

[25] With these comments compare Ptolemy's discussion of Didymus' modifications of the *kanōn* in chapter 13 below. Ptolemy does not explain how the additional bridges would be placed on the first instrument, but his general point is evidently correct: if the strings are kept stationary, no one straight bridge, however placed, will divide the strings so as to yield all the ratios of any generic tetrachord. It is not hard to invent simple examples of how new bridges might be placed for these purposes, but it would be a clumsy method to pursue very far. For further discussion of complex experimental instruments see ch. 16 below and Book III chs. 1–2.

[26] Compare the definition given at 7 Aristox. *El. Harm.* 74.9ff., where he too sets out to enumerate the *eidē* or *schēmata* of the concords, and, for example, 12 Arist. Quint. *De Mus.* 14.12–14, 14.27–15.2, 15.6–8.

[27] Ptolemy is explaining how he will identify the various forms of sequence spanning an octave, a fifth or a fourth. Structurally significant fifths and octaves always include the tone of disjunction, and we can therefore identify them by the place held by this tone in their sequence of intervals, highest, second-highest, etc. The fourths considered contain no disjunctions: their forms are identified by the position of the 'leading' interval (e.g., that between dynamic *lichanos* and *mesē*), the one that is most important in creating difference of genus (cf. 35.1–3).

the fifth (the first and fourth forms), and only three of the octave (the first, fourth and seventh forms).[28] For if we set out a fourth ABCD, treating A as the highest note, and conjoin with it another similar fourth below, DEZH, then conjoin with this in the same way a tone HF, conjoin with this, once again, a fourth FKLM, and with that another fourth MNXO, then of the notes the fixed ones will be A, D, H, F, M and O.[29] The first form of the fourth will be MO, the second LX, the third KN, and it is clear that it is only MO, the first, that is bounded by fixed notes. Of the fifth, the first form will be HM, the second ZL, the third EK, the fourth DF, and it is clear that of these only HM, the first, and DF, the fourth, are bounded by fixed notes. Of the octave, the first form will be HO, the second ZX, the third EN, the fourth DM, the fifth CL, the sixth BK and the seventh AF, and of these, once again, only HO, the first, and DM, the fourth, and AF, the seventh, are bounded by fixed notes.



4 *Concerning the complete* systēma, *and that only the double octave is of this kind*

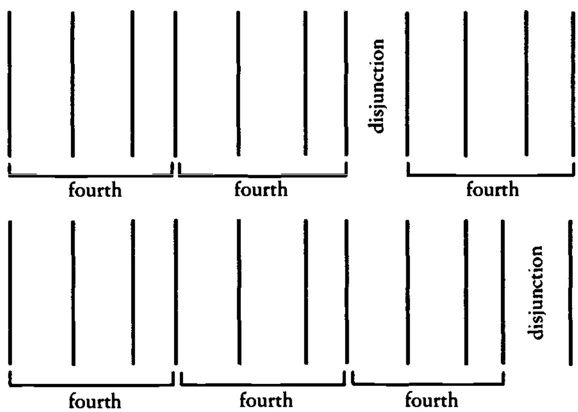
With these points set out in advance we may say that the name '*systēma*', unqualified, is given to a magnitude put together out of concords, just as a concord is a magnitude put together out of melodics, and a *systēma* is, as it were, a concord of concords.[30] The name 'complete *systēma*' is given to that

[28] This is argued below, but examples may be helpful. A fourth bounded by fixed notes, e.g., by *mesē* and *hypatē mesōn*, always has the first form, with the 'peculiar' interval at the top. There is a fifth of the first form (disjunction at the top) between, for example, *paramesē* and *hypatē mesōn*, and one of the fourth form (disjunction at the bottom) between, for example, *nētē diezeugmenōn* and *mesē*. Examples of the three octave-forms mentioned appear, for example, between *paramesē* and *hypatē hypatōn*, between *nētē diezeugmenōn* and *hypatē mesōn*, and between *nētē hyperbolaiōn* and *mesē*.

[29] The letters may be construed as representing the notes of the GPS from *nētē hyperbolaiōn* down to *hypatē hypatōn*. The remaining note, *proslambanomenos*, is not given a letter, because in Ptolemy's theory of the *tonoi* (to which all this is a preliminary), the highest and lowest notes come to be treated as equivalent in *dynamis*: see in the first instance 52.19–21. The tetrachord *synēmmenōn* is not used by Ptolemy as a genuine element of the basic system: see chapter 6 below.

[30] Ptolemy's usage is highly specialised, and tailored to the needs of his exposition. Other writers commonly use *systēma* of any scalar sequence of intervals, as few as two in the case of Aristoxenus (e.g., 7 *El. Harm.* 29.1–6), three or more in some other authors (e.g.,

which contains all the concords, together with the forms proper to each of them, since in general something is complete if it contains all its own parts.[31] By the first definition the octave is a *systēma* – and indeed this one was thought by the ancients to be sufficient in itself[32] – as are the octave and a fourth,[33] the octave and a fifth, and the double octave. For each of them is bounded by two or more concords. By the second definition only the double octave can be a complete *systēma*, for only in it are there all the concords, together with the forms that have been set out. Those greater than it will not contain any additional form beyond those capable of being found in this one, while those
51 smaller than it will lack some of those that are in this one: hence it is not correct to call 'complete' the *systēma* put together out of the octave and a fourth. For it will never contain the seven forms of the octave, and will not always contain the four forms of the fifth: when it is so positioned that the tone disjoins two conjunct tetrachords from one tetrachord, it will contain the four forms of the fifth, but only four of the seven forms of the octave – those starting from either of the extremes. But when it is so positioned that the tone is at the limit, and the three tetrachords are conjoined, it will contain only one form of the fifth and one of the octave, either the first or the last of both of them. This can be seen from the diagram set out below, if one adds to it one similar tetrachord at one end or the other. But in the double octave, when the two octaves are



12 Arist. Quint. *De Mus.* 13.4–5). Ptolemy is building towards an account of the structure of melodic 'space' as a whole. Having discussed melodic intervals and concords, he now needs a designation for the larger structures constituted by sequences of several concords.

[31] Thus Ptolemy is not content to adopt the GPS as something 'given' by tradition and practice, as most authors are. Its perfection or completeness lies in the fact that it is the smallest system that incorporates every form of the fourth, the fifth and the octave.

[32] That is, they thought that an analysis of forms of sequence existing within the octave would be a sufficient analysis of all melodic relations. Compare particularly 7 Aristox. *El. Harm.* 2.15–25, 6.21–31, 36.30–2. The expression *teleion systēma* ('complete' or 'perfect' system) is quite commonly used to mean 'octave', as frequently in 12 Arist. Quint. *De Mus.*, e.g., 9.2, 14.16 (where the obvious reason is given).

[33] Often known as the Lesser Perfect System (LPS). For further discussion of it see chapter 6 below.

similar and are put together in the same direction, in every case, corresponding to every position in which the first of the disjunctions is placed,[34] we shall find that all the forms of the octave, of the fifth and of the fourth are contained; and we shall find no further form in the concords that exceed the double octave.

5 *How the names of the notes are understood in relation to* thesis [*'position'*] *and to* dynamis [*'function', 'capacity'*]

The reason why the *systēma* of an octave and a fourth was teamed up [*parezeuktai*] with the double octave is an issue we shall look at later.[35] But to the notes of the genuinely complete *systēma*, the double octave – which are 52 fifteen in all, because one is common to the lower and the upper octave, and is the middle note of them all – we give the following names, sometimes with respect to their actual position [*thesis*], that is, to their being higher or lower absolutely.[36] The one common to the two octaves, mentioned just now, is called *mesē*; the lowest note is *proslambanomenos* and the highest *nētē hyperbolaiōn*; then those after *proslambanomenos*, in the sequence upwards as far as *mesē*, are called *hypatē hypatōn*, *parhypatē hypatōn*, *lichanos hypatōn*, *hypatē mesōn*, *parhypatē mesōn* and *lichanos mesōn*; those after *mesē*, similarly, as far as *nētē hyperbolaiōn*, are called *paramesē*, *tritē diezeugmenōn*, *paranētē diezeugmenōn*, *nētē diezeugmenōn*, *tritē hyperbolaiōn* and *paranētē hyperbolaiōn*. Sometimes we name them with respect to function [*dynamis*],[37] that is, to the way in which they are related to something else. Here we first adjust to their positions [*theseis*] the functions [*dynameis*] that they have in what is called the changeless [*ametabolon*, 'unmodulated'] *systēma* of the double octave, so that we may use, in this case, the same names for both the positions and the functions, and then alter them in the other cases.[38] That is, we take one of the two [disjunctive] tones in the double octave, from an origin

[34] That is, every form of the double octave contains every form of the fourth, the fifth and the octave.

[35] In the next chapter.

[36] Not, of course, by reference to 'absolute pitch', but simply to the order in which they stand, from lowest to highest. In this designation by *thesis*, a note's name indicates only its ordinal position in the two-octave sequence of which it is treated as a part. See n. 3 above.

[37] The conception of *dynamis* that gives its title to this method of naming is related to that of Aristoxenus in 7 *El. Harm.* Books II and III. A note's *dynamis* is defined by the complex of relations in which it stands to other notes of the system. The dual nomenclature may have been formalised by Ptolemy himself, in an attempt to reduce the ambiguities of ordinary musical talk to clarity and order.

[38] The *ametabolon systēma*, in this usage, is the GPS set out with the tetrachords and disjunctive tones distributed in the most familiar way. That is, it has a tone at the bottom, then two conjoined tetrachords, then a further tone, and finally two more tetrachords in conjunction. It is 'changeless' or 'unmodulated' because modulation of *tonos* will involve the projection of a different order of intervals onto the same tonal space, and this *systēma* is the starting point from which such modulations are conceived as beginning (see 54.9–11). It is therefore identified with the most fundamental of the *tonoi* between which modulation takes place, that is, the Dorian (see especially chapter 10 below).

at what is *mesē* by position,[39] and place next to it on either side two tetrachords in conjunction, of the tetrachords that there are in the whole, and then assign the other tone to the remaining interval, the lowest. We then give the name '*mesē* by function', from its positioning here, to the lower note of the higher disjunction, and the name '*paramesē*' to the higher note, '*proslambanomenos*' and '*nētē hyperbolaiōn*' to the lower note of the lower disjunction,[40] and '*hypatē hypatōn*' to the higher note. Then we give the name '*hypatē mesōn*' to the note common to the two lower conjoined tetrachords after the lower
53 disjunction, and '*nētē diezeugmenōn*' to the note common to the two higher conjoined tetrachords after the higher disjunction; and again, '*parhypatē hypatōn*' to the second from lowest note of the tetrachord after the lower disjunction, '*lichanos hypatōn*' to the third; '*parhypatē mesōn*' to the second from lowest note of the tetrachord before the higher disjunction, and '*lichanos mesōn*' to the third; then '*tritē diezeugmenōn*' to the second from lowest note of the tetrachord after the higher disjunction, and '*paranētē diezeugmenōn*' to the third; '*tritē hyperbolaiōn*' to the second from lowest note of the tetrachord before the lower disjunction, and '*paranētē hyperbolaiōn*' to the third. Then it is only on the basis of these names, that is, the names of the functions, that we can properly describe the following notes as 'fixed' in the modulations of the genera – *proslambanomenos*, *hypatē hypatōn*, *hypatē mesōn*, *mesē*, *paramesē*, *nētē diezeugmenōn* and *nētē hyperbolaiōn*, which last is one and the same as *proslambanomenos* – and the rest as 'moveable'. For when the functions are altered in position, the points belonging to the fixed or moveable notes no longer correspond to the same places.[41] And it is clear that the first form of the octave[42] in the *systēma* set out above, the one called 'changeless', is bounded for the reason given by *paramesē* and *hypatē hypatōn*, the second by *tritē diezeugmenōn* and *parhypatē hypatōn*, the third by *paranētē diezeugmenōn* and *lichanos hypatōn*, the fourth by *nētē diezeugmenōn* and *hypatē mesōn*, the fifth by *tritē hyperbolaiōn* and *parhypatē mesōn*, the sixth by *paranētē hyperbolaiōn* and *lichanos mesōn*, and the seventh by *nētē hyperbolaiōn* or *proslambanomenos* and *mesē*. For ease of reference, the designations [*parasēmeiōseis*] forming the basis of the changeless *systēma* are set out below.

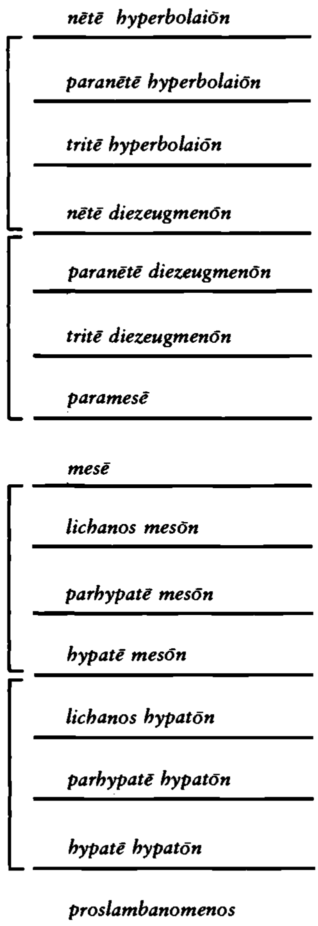
[39] The next sentence clarifies this: the disjunction is placed, for these purposes, immediately above the note that is *mesē* 'by position'. It thereby becomes also *mesē* 'by function'.

[40] When the system is taken in a *tonos* other than Dorian, dynamic *proslambanomenos* will no longer be the lowest note of the double octave, and some note prior to *nētē hyperbolaiōn* will be the highest. Then the intervals lost from the top of the system reappear at the bottom, together with the *dynameis* associated with their bounding notes. *Proslambanomenos* will have below it the interval that stands below dynamic *nētē hyperbolaiōn*, and the two will thus become the same note.

[41] Thus for example if dynamic *proslambanomenos*, a fixed note, falls on the third degree of the two-octave system, thetic *parhypatē hypatōn* will have become a fixed note, and thetic *proslambanomenos* will have ceased to be one, since its place is now occupied by dynamic *tritē hyperbolaiōn*.

[42] The first form is that with the disjunctive tone at the top, the second that with it second from the top, and so on (chapter 3 above).

The complete, changeless
systēma diezeugmenon



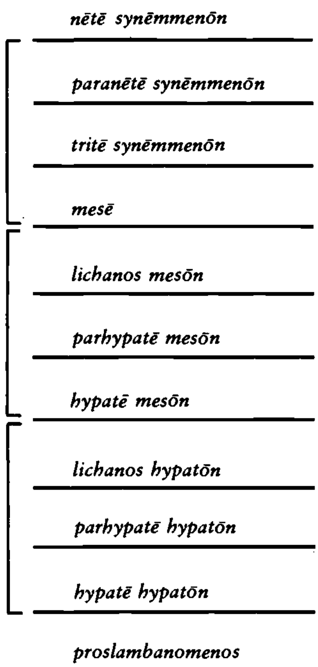
6 How the magnitude of the octave conjoined with a fourth acquired the reputation of being a complete systēma

30 The *systēma* considered above is also called 'disjunct' [*diezeugmenon*] to
54 distinguish it from that which is constructed on the basis of the magnitude put together from the octave and a fourth, which is called 'conjunct' [*synēmmenon*] because, instead of the disjunction, it has another tetrachord in conjunction with *mesē* and above it.[43] This tetrachord is called 'conjunct' too, on account of the property it has (the 'disjunct' tetrachord gets its name in the same way),

[43] These designations of the *systēmata* are quite common, e.g., 12 Arist. Quint. *De Mus.* 116.20ff.

and in it the note after *mesē* is called *tritē synēmmenōn*, the one next in succession *paranētē synēmmenōn*, and the 'leading' note of the tetrachord, a fixed note, is called *nētē synēmmenōn*. It appears that this sort of *systēma* was invented by the ancients to accommodate a different form of modulation, being treated as 'modulating' [*metabolikon*] by contrast with the other one which is 'changeless' [*ametabolon*]. For the latter is not given this name from its not modulating with respect to genus, since it is common to all the genera, but from its not modulating with respect to the function of the *tonos*.[44]

The *systēma synēmmenon*



In relation to what is in this sense called '*tonos*' there are two primary varieties of modulation, one in which we go through a whole melody at a higher pitch or a lower one, keeping the sequence the same throughout, and a second in which not the whole of the melody is altered in pitch, but a part is altered in contrast to the original sequence. Hence the latter should be called modulation of melody, rather than of *tonos*. For in the former it is not the melody, but the *tonos* throughout the whole that is altered, while in the latter

[44] See n. 38 above. The 'changeless' system is indeterminate or 'common' in genus, in that it remains the same system, for present purposes, no matter what generic divisions the tetrachords display, since change of genus does not affect the order in which *dynameis* appear within the double octave. Bacchius *Eisagoge* 308.3ff. gives the title 'changeless' to both the 'disjunct' and the 'conjunct' systems (i.e., the GPS and the LPS).

the melody is turned aside from its proper ordering, while the pitch [*tasis*] is not altered as such, but as having an effect on the melody. Hence the former kind does not implant in the perception the impression of a difference in respect of function, through which the character [of the melody] is changed, but only of a difference in respect of height or depth of pitch.[45] But the latter as it were expels the perception from the melody that is familiar and expected, when it has first strung together a coherent sequence at some length, and then changes in some way to a different form, either in respect of genus or in that of pitch – for instance, when it modifies the genus from continuous diatonic to chromatic, or when, beginning from a melody that has habitually made its transitions to notes concordant at the fifth, there occurs a change of course to notes concordant at the fourth, as in the case of the *systēma* set out above.[46] For when the melody has risen to *mesē*, and then does not go on, as its habit has been, to the tetrachord *diezeugmenōn*, which is related in the concord of the fifth to the tetrachord *mesōn*, but is pulled round, as it were, and linked to the tetrachord conjunct [*synēmmenon*] with *mesē*, so that it makes a fourth instead of a fifth with the notes prior to [i.e., below] *mesē*, when this happens, contrary to expectation, then the senses are presented with a complete change and a deviation – an agreeable one when the linkage is well proportioned and melodic, disagreeable when it is the opposite. Hence just about the finest modulation, single in its function, is that which is like the one just mentioned in taking, as the addition that makes the change, the interval of a tone, that by which the fifth exceeds the fourth. For in that it is common to the genera,[47] the tone can make this modulation in all of them: in that it is different from the ratios inside the tetrachords it can thoroughly alter the melody:[48] and in that it is well proportioned, in accordance with its constitution as the first of the melodics, it makes the transitions of the melody neither very large nor altogether insignificant. For either of these is difficult for the ear to distinguish.

56 Now three successive conjunct tetrachords are produced, to create the special feature of this sort of modulation, by a kind of partial mixture of two disjunct *systēmata*, when the latter, taken as wholes, differ from one another in respect of *tonos* by a fourth.[49] But since a way of increasing the *tonoi* this far had not been invented by the ancients – for they knew only the Dorian and the Phrygian and the Lydian, which differ from one another by one tone, so that

[45] Such a 'modulation' is merely a transposition from one pitch-range to another. With the admirably clear and careful discussion of modulation that follows, here and in chapter 7, compare the relatively confused account in 12 Arist. Quint. Book 1 chs. 10–11.

[46] These kinds of modulation are plainly different, and are classified elsewhere (e.g., Cleonides *Eisagoge* 205.2–6) as modulation 'by genus' and modulation 'by *systēma*'. The shift between conjunct and disjunct sequences evidently played a significant part in elaborate forms of Greek music, as the separate classification in Cleonides suggests: cf. 12 Arist. Quint. *De Mus.* 16.24–17.2.

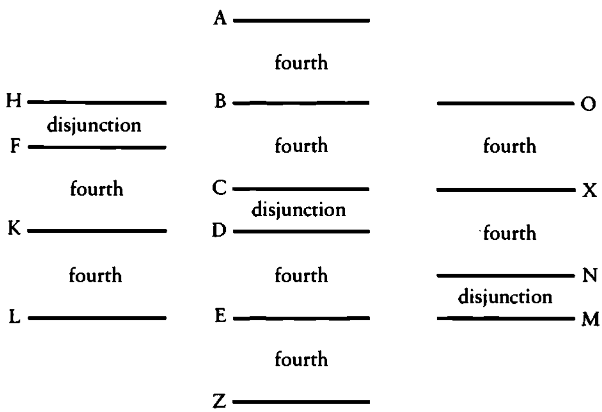
[47] Because the interval disjoining tetrachords is always the same, regardless of genus.

[48] In certain genera there is a 9:8 tone inside the tetrachord. But since it never appears at the bottom of the tetrachord, a melody that moves through the tone from, for example, *mesē* to *paramesē* is always clearly distinguishable from one that moves from *mesē* to *tritē synēmmenōn*.

[49] That is, when the dynamic *mesē* of one *tonos* is a fourth away from that of the other.

they could not reach a *tonos* that was higher or lower by a fourth[50] – and since they had no way of making three successive conjunct tetrachords from the disjunct *systēmata*, they adopted the conjunct structure, giving it the name '*systēma*' too, so that the modulation set out above might be available to them.[51] For in general, where there are *tonoi* exceeding one another by a fourth, if in each of the tetrachords before their equivalent disjunctions that of the higher is conjoined with that of the lower, at its upper end, it will make three conjunct tetrachords in the lower, of which the highest is the one that has been brought across: and if in the tetrachords after their equivalent disjunctions that of the lower is conjoined with that of the higher, at its lower end, this too makes three conjunct tetrachords in the lower, of which the one brought across is the lowest.

For let there be a tetrachord AB, downwards from A, the highest note, then another conjunct with it, BC, next a tone of disjunction, CD, and again below it two other tetrachords in conjunction, DE and EZ. Of the *tonos* that is higher by a fourth let there be taken HF, the disjunction equivalent to CD,[52] and in conjunction with it, below, two other tetrachords, FK and KL. Of the *tonos* that is lower by a fourth in relation to the first, let there be taken MN, the disjunction equivalent to CD, and in conjunction with it, above, two tetrachords NX and XO.



57 Since then the note F is the equivalent of D,[53] it will be higher than it by a fourth: and it is higher than K by the same amount. Hence D and K are of equal

[50] The thesis that only these *tonoi* were known in early times appears elsewhere, for example, at ps.-Plut. *De Mus.* 1134a–b, Athenaeus *Deipn.* 635d, cf. Bacchius *Eisagoge* 303.3–4. The reports are linked to names belonging to the sixth century B.C. Compare also chapter 10 below.

[51] This hypothesis is historically quite improbable: the ancient seven-note system (or systems) can scarcely have been devised with an eye to the theoretical problems of modulation. On its antiquity see particularly 4.27 ps.-Ar. *Probs* XIX.47, 10 Nicomachus *Ench.* chs. 3 and 5.

[52] That is, if CD in its *tonos* is the tone between dynamic *mesē* and *paramesē*, so is HF in its *tonos*.

[53] Each is, for example, dynamic *mesē*, in its own *tonos*.

pitch, so that it will be possible to conjoin the tetrachord KF with D on its upper side, and to make three tetrachords successive in the *tonos* AZ, of which this one will be the highest: they are ZE, ED and DF. Again, since note N is equivalent to C, it will be lower than it by a fourth, and it is lower than X by the same amount. Hence C and X are of equal pitch, so that it will be possible to conjoin the tetrachord XN with C, on its lower side, and to make again three successive tetrachords in the *tonos* AZ, of which this one is the lowest: they are AB, BC and CN.[54]

7 *Concerning modulations with respect to what are called the* tonoi

Let this be enough to show that when, alongside disjunct complete *systēmata*, there is placed their equivalent at the interval of a fourth, this makes the conjunct *systēma* redundant,[55] in addition to the fact that it does not even have the nature of something complete, as we were saying. It must next be made clear that in the case of modulations that involve shifts in complete structures [*systaseis*], which we give the special name *tonoi* because it is from pitch that they take their differences, the quantity of them is potentially infinite, as is that of the notes – for a *tonos*, in this usage, differs from a note only in that it is composite while the other is incomposite, like the difference between a line and a point, where once again nothing prevents us from moving either the point alone or the whole line to the adjacent position – but in the actuality available to the senses it is determinate, since that of the notes is so too.[56] Hence among what is understood concerning the *tonoi* there must be three determinate limits [*horoi*], as there are for each of the concords, the first being that according to which the ratio of the outermost *tonoi* is constructed, the second that according to which the quantity of those between the extremes is determined, and the third that according to which the differences between successive *tonoi* are established – just as in the case of the fourth, for instance, where the extreme notes make an epitritic ratio, where there are only three ratios constituting the whole, and where the differences between the ratios are such-and-such.[57] In that case, however, each of these limits [*horoi*] has its own special basis,

[54] These manoeuvres amount to transitory modulations of *tonos*, the original one (represented by AZ) remaining as the basic framework of the sequence; but having reached a note above which it would normally proceed through a tone of disjunction, it proceeds instead to a tetrachord belonging to a different *tonos*, a tetrachord whose lowest note stands at the same place in the system as does the note below the disjunction in AZ.

[55] That is, it need no longer be conceived as an independent structure, since it arises out of modulations of the one fundamental 'disjunct' series.

[56] Compare 7 Aristox. *El. Harm.* 68.18–69.28. The context is quite different, but the underlying conception is the same. Compare also *El. Harm.* 47.8–50.14, 53.21–32.

[57] The three questions to be answered are: (i) What is the interval (ratio) between dynamically equivalent notes in the *tonoi* that lie furthest apart? (Ptolemy will in fact use the note *mesē* as his point of reference.) (ii) How many *tonoi* lie between these extreme ones? (iii) What are the intervals between dynamically equivalent notes in each *tonos* and its neighbours?

whereas in the *tonoi* the other two follow in a way upon the first, constrained by one and the same restriction. Most people, not grasping the consequence of
58 this restriction, set out each of the limits in ways that disagree with one another, some settling on a ratio smaller than the octave, some just on the octave, some on one greater than that. More recent writers have continually hunted out some advance of this kind over their predecessors, a procedure inappropriate to the nature of attunement [*to hērmosmenon*] and to its periodicity [*apokatastasis*]. It is by this alone that one must determine the interval between what are to be the outermost *tonoi*: for neither change of position in the voice, nor that in anything else that makes sounds, can have one and the same limit [*horos*].[58] For we shall not find that the construction of modulation with respect to *tonos* exists for the sake of higher and lower voices – as when whole instruments are raised or lowered in pitch, to accommodate that sort of difference, and no alteration in the melody results, or when the whole melody is completed in just the same way by lower-voiced or higher-voiced performers [lit. 'competitors'] – but it exists in order that the same melody, in the same voice, starting sometimes from a higher position and sometimes from a lower, may produce a change [*tropē*] in character [*ēthos*].[59] This is achieved, in shifts between *tonoi*, by the voice's limits no longer being attached to each of those of the melody, but one always ceases before the other, in one direction the limit of the voice occurring before that of the melody, and in the opposite direction that of the melody preceding that of the voice, so that the melody that was originally fitted to the compass of the voice, by falling short of it in one place in the modulations and exceeding it in another, provides for the ear the impression of a different character.[60]

[58] The 'periodicity' is that of the octave, as becomes clear in the following chapter. Ptolemy's more general point is that the question must be settled by reference to harmonically structural considerations, not ones to do with practical convenience, for the latter would give no clear cut answer. There is no unique compass common to all producers of melody, and hence the limits in question cannot be set by them. As it stands, the argument is unpersuasive; the real point is in the next two sentences.

[59] The negative thesis is clear: modulation of *tonos* is not a simple matter of moving a tune to the level of pitch most convenient for the performers. Positively, it occurs during the course of a single melody, and it moves the 'starting point' to different pitch-levels, so generating a difference of *ēthos*. So far it would be possible to construe the phenomenon Ptolemy is describing as a transitory change of 'key', and from one point of view that is what it is. But it gradually becomes clear that his conception is primarily that of the projection, on to the same tonal range, of a different order of intervals, creating a shift in that range's dynamic centre of gravity. It is only secondarily the projection of the same order of intervals on to a different range.

[60] The 'limits of the voice' here are simply those of the two-octave range being used. Those of the melody are defined by the *dynameis* of the extreme notes, where the 'extremes' are those that appear as such in the ordinary GPS. If the melody stays in the basic Dorian *tonos*, the limits of voice and of melody, so conceived, will coincide. But when the *tonos* changes, notes with *dynameis* that lie high in the Dorian *tonos* will be removed from the top and reappear at the bottom (or conversely). The 'limits of melody' will now lie elsewhere than at the 'limits of voice', and the melodic relations presented will be differently ordered. Ptolemy's notion of *tonos* seems closer to that of 'mode' than to that of 'key' (cf. 12 Arist. Quint. *De Mus.* Book 1 chs. 10–11 with notes). But even the term 'mode' may be misleading. Two melodies may be played on the same note-series

8 *That the outermost* tonoi *must be bounded by the octave*

Let us agree that the first and most important cyclic recurrence [*apokatastasis*] of similarity in attunement [*to hērmosmenon*] is, once again, in the first of the homophones, that is, in the octave, whose bounding notes are no different from a single note, as we have shown.[61] And just as those of the concords that are put together with it produce the same result as they would produce if they were alone, so each melody that spans just the interval of the first homophone, or an interval formed from the first homophone by combination,[62] can run through its course in exactly the same way taking either of the outermost notes as its starting point. Hence in modulations of *tonos* too, when we want to move to one an octave higher or lower, we shall not alter any of the notes[63] (whereas
59 in all the remaining modulations we always do alter some), and the *tonos* itself remains the same as the original one. Correspondingly, the *tonos* differing from the original one by a fourth is the same as that differing from the same one by an octave and a fourth; that differing from the original one by a fifth is the same as that differing from the same one by an octave and a fifth, and so on for the others. Thus those who limit the outermost of the *tonoi* to a range smaller than the octave cannot have completed the cycle of attunement [*to hērmosmenon*] – for there will be, over and above them, some *tonos* different from all the first ones[64] – while those who exceed the octave posit those outside the octave superfluously, since they are always the same as those taken previously, the one at an octave being the same as the original one, and those at given distances from the octave being the same as those at equal distances in the same direction from the original one. Hence even those who go just as far as the octave are wrong in numbering as one of the *tonoi* the one at an octave from the original. For it is clear that they fall into the same error as those who go beyond the limit laid down, except in so far as the former do so in respect of one *tonos*, the latter

(e.g., the white notes of a piano), and differ in mode by having a different tonal focus (as a white-note tune focussing on C is in the major mode, and one focussing on A in the minor; other quite familiar kinds of mode may focus on D or on E). Ptolemy's *tonoi* are not described like that: the *dynameis* remain the same in each of them. The crucial difference is that the tonal focus is higher or lower in the tonal space employed. Take a simple instrument with, for example, eight open strings, spanning an octave. Then two melodies, each covering an octave or less, and both in, for example, the 'A' mode, may require quite different tuning systems, if in one the melodic focus is at the bottom of its range and in the other it is, for instance, on the fourth note from the bottom. For Ptolemy it is apparently the movement of the same modal dynamic focus to a different position in the range that is fundamental to difference of *tonos*.

61 See the opening paragraph of Book I ch. 6.

62 That is, it spans some exact whole number of octaves.

63 When we modulate through an octave, for example, by placing on thetic *mesē* the 'dynamic' note previously located on thetic *proslambanomenos*, the notes on given thetic degrees could be renamed by *dynamis* accordingly. But since their actual relations to one another would not be altered, the renaming would correspond to no real change, and is best avoided.

64 The cycle has been completed only when every *dynamis* in the octave series has appeared on each one of the *theseis* of the central octave. If it is not complete, some form of the octave will not have been exemplified.

in respect of several; so that it would be right for them [i.e., the former] to accept responsibility for the offences complained of, on the grounds that they provided the original grounds for the transgression [*hyperbolē*, 'excess']. For if a *tonos* that is the same is adopted just once, in addition to those preceding it, in the way that the one at the octave is the same as the original one, what, they might well ask, is to prevent those that follow on from the remaining ones from being added in the same way? Further, the functions [*dynameis*] in the octave should not be measured by the quantity of its terms, but by the quantity of the ratios that jointly constitute it;[65] and we have here the most apt exemplification of this, in the species [*eidē*] that are contained by it. For we all assume quite unambiguously that these are just seven, while the notes that constitute them are eight, and no one would say that the one taken downwards from the lowest note, for example, makes a different species from the first – the one in the same direction from the highest note – because it is true in general that any species taken in the same direction, beginning from each of the extremes of the octave, produces the same function [*dynamis*].[66]

60 ### 9 *That we must posit just seven* tonoi, *equal in number to the species* [eidē] *of the octave*

Our discussion has brought us, then, to the point where we can discover the quantity of the *tonoi*. We would do well to make them equal in number to the species [*eidē*] of the octave, since that is also the number of the species of the two first concords together, these being taken in accordance with the ratios in each case, whose nature will not allow either more or fewer ratios to be posited.[67] For if one wished to make their divisions into a greater number of parts – more than three, for instance, in the case of the fourth – or, if it comes to that, into that same number of divisions with random differences, or with ones that were determinate, but different from those taken according to the harmonic ratio, both reason [*to eulogon*] and perception [*to phainomenon*]
61 would immediately object.[68] In the same way, since the *tonoi* contained in the octave correspond to the nature of the concords and take their origin from them, so that the *systēmata*, taken as wholes, may have differences that are concordant,[69] if people seek either to make them more in number than the

[65] That is, the number of functions corresponds to the number of intervals, not of notes.

[66] The *dynameis* mentioned in these sentences are not those of individual notes, but those of each octave-system taken as a whole, octaves of different species or forms being treated as having melodically different functions or characters. (For another parallel between Ptolemy's treatments of notes and *tonoi* see 57.16–19.)

[67] On the species of the concords see chapter 3 above.

[68] Ptolemy does not explain here just what would be wrong with such divisions, for the excellent reason that the bulk of Book 1 has been devoted to showing which divisions are rationally and melodically acceptable, and why. The present sentence is a reminder rather than an argument.

[69] There are two points here. First, the *tonoi* 'take their origin' from the concords. This means that the rearrangements that constitute different *tonoi* have as their elements the intervals reached by 'natural' divisions of the concords, and are constrained by the principles of organisation that apply to them. Secondly, the *tonoi* are to have

seven species and ratios of the octave, or to make the differences between all of them equal, we must not agree with them: for they have no persuasive reason to offer, either for the equality of the augmentations between one whole *tonos* and another – such a thing being condemned as altogether inappropriate in *harmonia*[70] – or for the claim that all the differences are tones, for example, or again semitones or dieses, from the adoption of which they determine the number of the *tonoi* in accordance with the quantity of them that makes up an octave.[71] For why should they make them of that size, in particular, given that according to these people a concordant interval can accommodate both these differences and those and several others, both in the arrangements of the genera and in those of the intervals? It is not open to them to say that this magnitude divides the octave exactly while that one does not do so exactly,[72] or that this one divides it into an even number of parts while the other divides it into an odd number of parts.[73] However, if the tone divides the octave into six and the semitone divides it into twelve, the third of a tone into eighteen and the quarter into twenty-four, and if none of these differs only imperceptibly from another,[74] what, one might ask, are we to define as the differences between the
62 seven *tonoi*, since the octave is not divided into seven equal ratios; and given that they are unequal, it is not obvious what kinds of ratio it would be appropriate to apply.[75] We should answer that it is the ratios found on the basis of a succession of the first concords, that is, the ratios after the addition of fourths in one direction within the octave, this addition being the same as that of the fifth constructed in the opposite direction. For since a note lower by a fourth than a given note is higher by a fifth than the note that is the lower homophone of the given note, it is also the case that the note higher by a fourth than a given note is lower by a fifth than the note that is the higher homophone of the given note. And it is essential not only here, but everywhere, that the homophones should take precedence over the concords and be adopted as

concordant 'differences'. This is explained at the end of the chapter. Notes equivalent in *dynamis* in two different *tonoi* must be so related that the pitch of one can be reached from that of the other by movements through concordant intervals. See n. 76 below.

70 Compare particularly 25.13–16, 32.23–5.

71 Ptolemy's criticisms are directed principally against Aristoxenian writers, who developed a system of thirteen (later fifteen) *tonoi* set out at intervals of a semitone. Their procedure is pointless from Ptolemy's point of view, and in its final result (whatever its original motivation may have been) must have tended towards a conception of *tonos* as 'key'. See 12 Arist. Quint. *De Mus.* 20.5ff.

72 Because for the Aristoxenians the octave can be divided into equal intervals in many different ways.

73 Such distinctions are merely irrelevant for Ptolemy, but a classification of intervals into 'odd' and 'even' seems to have had a place in the Aristoxenian tradition. See, for example, 12 Arist. Quint. *De Mus.* 11.14–17, 22.21, and compare ps.-Plut. *De Mus.* 1145b–c.

74 That is, if there is a genuinely perceptible difference between, for example, a semitone and a third of a tone, or between a third of a tone and a quarter. Cf. 32.18–21.

75 The principle of equal division not only fails to determine which of such divisions should separate the *tonoi*, but must in fact lack application here at all. It has nothing to say about the way in which unequal divisions are to be made. Aristoxenus is as scornful as is Ptolemy of attempts to explicate melodic relations in terms of the equality of intervals: see, for example, 7 *El. Harm.* 28.17ff., 53.11ff., cf. 48.15ff.

principles prior to them, and that the concords should have the same priority over the melodics, so that in the case of the *tonoi* too those that are concordant should be taken first, and then those that are found by means of the difference between these, of whatever sort they may be, since transition to the next *tonoi* in succession does not make so acceptable a modulation as does transition to *tonoi* that stand at the distance of the first concords.[76]

10 *How the differences between the* tonoi *may be soundly established*

The people who go on up to eight *tonoi*, by way of the one included superfluously in their number along with the seven, seem to have stumbled, somehow or other, on the differences that are appropriate to them, but not on the basis of the proper approach. They straightforwardly adopt the three oldest *tonoi*,[77] called Dorian, Phrygian and Lydian by derivation from the names of the races from which they originated – or for whatever reasons anyone else wants to think up – and assume that these differ from one another by a tone (perhaps this is why they called them '*tonoi*'). From these they make the first concordant modulation the one to a fourth upwards from the lowest of the three,[78] the Dorian, entitling this *tonos* Mixolydian from its proximity to the Lydian, since it no longer made the difference between itself and the Lydian a
63 whole tone, but made it correspond to the part of the fourth that is left after the ditone between the Dorian and the Lydian. Next, since the Dorian lay at a fourth below this one, they decided to put in also the *tonoi* at a fourth below the remaining ones: the one that was to be below [i.e., a fourth below] the Lydian they called Hypolydian, the one below the Phrygian Hypophrygian, and the one below the Dorian Hypodorian; and the one that was to be at an octave above this last *tonos*, and so was the same as it, they entitled Hypermixolydian on the basis of what happened to be true of it, on the grounds, that is, that it was placed above the Mixolydian – using '*hypo*' to indicate lower pitch and

[76] The most acceptable modulations are those in which dynamically equivalent notes in the two *tonoi* involved are separated by a concordant interval. Of the concords only the fourth and the fifth are available (since equivalent notes in all the *tonoi* all fall within an octave), and these amount to the same thing, an upwards fifth being relevantly the same as a downwards fourth, and conversely. For similar views about the 'best' modulations see, for example, 12 Arist. Quint. *De Mus.* 22.15–18, Cleonides *Eisagoge* 205.10ff. If every *tonos* is to be accessible by a series of 'acceptable' modulations from every other, the intervals between them must be constructible by moves through fourths and fifths, and they will remain the same independently of genus.

[77] On these *tonoi* as historically primary see 56.4ff.

[78] Ptolemy thinks of *tonoi* as 'higher' or 'lower' according to the positions of their dynamic *mesai* in the central octave of the thetic series (from thetic *hypatē mesōn* to *nētē diezeugmenōn*). Thus in Dorian, dynamic *mesē* coincides with thetic *mesē*. In Phrygian, where the *thesis* of *hypatē mesōn* is occupied by dynamic *lichanos hypatōn*, dynamic *mesē* falls one degree higher than in Dorian, on thetic *paramesē*; the difference is a tone. The dynamic *mesē* of Lydian is a degree higher still in the thetic series (after another tone), and that of Mixolydian occupies the next degree, a *leimma* above the Lydian, as the present sentence shows.

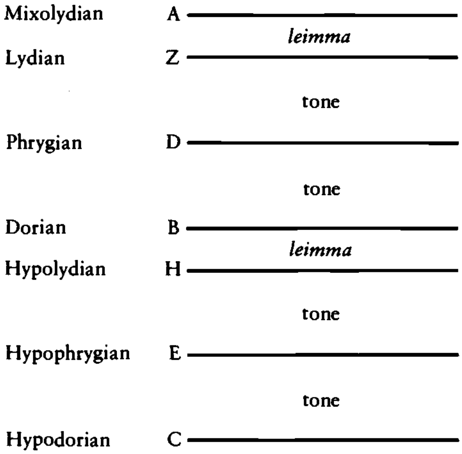
'*hyper*' to indicate higher.[79] In correspondence with the sequence of the first *tonoi*, the difference between Hypodorian and Hypophrygian is then a tone, and so, similarly, is that between Hypophrygian and Hypolydian: that between Hypolydian and Dorian is a *leimma*, which they wish to make a half-tone. But as we have said, one should not construct the concords out of the melodics, but conversely the melodics from the concords,[80] since the concords are both easier to construct and more important both for other purposes and for producing modulations.

This can be done according to the proper method, if we begin by setting down a higher *tonos*, which we will call A, then take first the one lower than it by a fourth, B, and next the one lower than B by a fourth, C, which will still be within the compass of an octave. Next, since the one lower than C by a fourth falls outside the octave, we take the one functionally equivalent [*isodynamounta*] to it, that is, the one higher than C by a fifth, D. Then, once again, we set down the one lower by a fourth than this one, E, and next, instead of the one lower than E by a fourth, since that too falls outside the octave, we make Z the one higher than E by a fifth; and we set down once again the one lower than Z by a fourth, H. When these are constructed in this way, strictly on the basis of successive reductions to lower pitch through the first concord, the fourth – which is, as we have said, the same as increase to higher pitch by a fifth – it will unquestionably follow that the differences between C and E, between H and E, between B and D, and between D and Z are constituted as tones, while those between H and B and between Z and A contain what is called the *leimma*. For since *tonos* D is placed a fourth higher than E and a fifth higher than C, the difference between C and E will be a tone. Similarly, since
64 Z is a fourth higher than H and a fifth higher than E, the difference between E and H will be a tone too. Again, since C is lower than H by a ditone and lower than B by a fourth, the difference between B and H will contain the *leimma*. For the rest, since BC, DE, ZH, and BA are all fourths, so that the difference between E and C is made equal to that between D and B, the difference between E and H to that between Z and D, and the difference between B and H to that between A and Z, it follows that the difference between D and B and that between Z and D will each be a tone, while the difference between A and Z will amount to a *leimma*. And if we take one at an octave from C or from A [i.e., above C or below A], it will plainly differ by a tone from the one next to it, because AC, which makes up a double fourth, is less than an octave by a tone.

Now A corresponds to Mixolydian, Z to Lydian, D to Phrygian, B to Dorian,

[79] The terminology of 'hypo-' and 'hyper-' versions of the *tonoi* was further developed in the Aristoxenian tradition: see 12 Arist. Quint. *De Mus.* 21.1ff. with notes 119, 120. This 'Hypermixolydian' is of course not required according to Ptolemy's conception of the subject.

[80] This is not a criticism of the equation of *leimma* and half-tone, but a reflection on the first steps of the construction, in which Dorian, Phrygian and Lydian were separated, without explanation, by 'melodic' intervals of a whole tone each.



H to Hypolydian, E to Hypophrygian, and C to Hypodorian, so that the differences between them, which have been somehow or other handed down, have now been discovered by reason [*logos*].[81]

11 *That one should not increase the* tonoi *by semitones*

It is clear that in these *tonoi* that we have set out there will be, peculiar to each of them, a specific note of the octave that belongs to dynamic *mesē*, since the *tonoi* are equal in number to the species.[82] For if we set out an octave in the intermediate range of the complete *systēma*, that is, the range from thetic *hypatē mesōn* to *nētē diezeugmenōn* (to allow the voice to move about and exercise itself comfortably upon melodies of middling compass, for the most part, going out only infrequently to the extremes because of the hard work and force involved in slackening or tension that goes beyond the norm),[83] the dynamic *mesē* of the Mixolydian will be attuned to the position of *paranētē diezeugmenōn*, so that the *tonos* may make the first species of the octave in the range set out;[84] that of the Lydian will be attuned to the position of *tritē diezeugmenōn*, corresponding to the second species; that of Phrygian to the position of *paramesē*, corresponding to the third species; that of Dorian to the position of *mesē*, making the fourth and central species of the octave; that of

[81] Ptolemy assigns the names on the basis of the tradition, and finds the intervals that the tradition incorporates to have a sound rational basis. This squares with his general methodological principle that reason, properly applied, will confirm the findings of unprejudiced (but accurate) perception. The 'tradition' here will then be that of the musicians, and contrasts sharply with that of theorists who both ignore musical practice and develop their abstract schemata from the wrong intellectual starting point.

[82] That is, each *tonos* has as its *mesē* a different thetic degree of the central octave.

[83] Hence the thetic system is not associated with any absolute pitch. It is located in relation to the voice or instrument used, in such a way that its central octave fits the range of pitch most convenient to the performer.

[84] For the species of the octave and their numbering from first to seventh see chapter 3 above.

Hypolydian to the position of *lichanos mesōn*, corresponding to the fifth species; that of Hypophrygian to the position of *parhypatē mesōn*, corresponding to the sixth species; and that of Hypodorian to the position of *hypatē mesōn*, corresponding to the seventh species. Thus it is possible to maintain some notes in the *systēma* unchanged in alterations between *tonoi*, keeping the range of the voice constant because in different *tonoi* the same functions [*dynameis*] never fall on the positions of the same notes.[85] But if we adopt more *tonoi* than these, as do those who make the differences between them increase by semitones, it will be inevitable that the *mesai* of two *tonoi* are always attuned to the position of a single note, so that the *systēmata* will be moved intact in a shift from one of these two *tonoi* to the other, while no longer preserving the original pitch as something shared, against which the special quality of the voice should be aligned.[86] Thus the dynamic *mesē* of Hypodorian, for example, is conjoined with thetic *hypatē mesōn*, and that of Hypophrygian with *parhypatē mesōn*: the *tonos* constructed between these (which they call 'lower Hypophrygian' in contrast with the other, higher one)[87] must have its *mesē* either corresponding to *hypatē*, like Hypodorian, or to *parhypatē*, like the higher Hypophrygian. Since that is so, when we shift from one of the *tonoi* that have this common note to the other, the note too will be moved, tensed or slackened by a semitone, because it has the same function, that of *mesē*, in each of the *tonoi*; and the tensings or relaxations of all the remaining notes will follow suit, to keep their ratios with respect to *mesē* the same as those taken before the modulation in accordance with the genus
66 common to both the *tonoi*.[88] Thus the *tonos* will not even seem to be different in species from the previous one, but will be the same again, Hypodorian or

[85] It is a criterion of 'different *tonos*' that a given *dynamis* moves to a different position. It is not enough that it moves to a different pitch, as in mere change of key; it must move to a different thetic degree while the *theseis* occupy the same overall range of pitch. In fact, though changes of *tonos* will always leave some individual thetic degrees at the same pitches, others will move, and in certain cases the overall pitch range is forced to move up or down by a little. Thus, for instance, dynamic *mesē* in Lydian falls on the sixth thetic degree (thetic *tritē diezeugmenōn*), two tones above the *mesē* of Dorian, and a tone plus a *leimma* below Dorian *nētē diezeugmenōn*, the highest Dorian note of the central octave. But the Lydian *dynamis* that falls on the eighth thetic degree of this octave is its *tritē diezeugmenōn*, a moveable note, which in some genera will lie more or less than a tone and a *leimma* above its *mesē*, and hence the locus of the central octave will shift upwards or downwards in pitch by a small amount. See the tables appended to chapter 15.

[86] Since there are only seven available positions (*theseis*) for the *mesai*, additional ones, though differing in pitch, must coincide in *thesis* with ones already present. Then if they are to belong to different *tonoi* in any sense, it can only be by a transposition of the same octave species to a new pitch, rather than by the generation of a new species in the same pitch-range. That, on Ptolemy's view, does not amount to change of *tonos* (see 58.7ff.).

[87] These additional *tonoi* belong to the Aristoxenian systems described in 12 Arist. Quint. *De Mus*. ch. 10.

[88] Ptolemy assumes (for simplicity's sake at least) that modulation of *tonos* is taking place without any alteration of genus. But the two kinds of modulation may sometimes have gone hand in hand, especially since some genera were in practice associated with specific *tonoi*. On the account given in chapter 16 below, a modulation from the attunement called *tritai* to that called *parhypatai*, for instance, would have involved alterations of both *tonos* and generic division.

Hypophrygian, merely sung at a higher or lower pitch. Let that, then, complete our exposition of a rational and adequate account of the seven *tonoi.*

12 *On the difficulty of using the single-stringed* kanōn

Since our remaining task, in the enterprise of displaying with complete clarity the agreement of reason with perception, is that of dividing up the harmonic *kanōn* – not just in one *tonos*, such as the changeless [*ametabolon*] *systēma*, nor in one genus, or two, following the practice of our predecessors, but in absolutely all the *tonoi* and each of the melodic genera, so that we may also include, set out jointly, all together, the positions that notes have in common – we shall first discuss briefly the imperfection of this single-stringed *kanōn*.[89] No substitute for it seems so far to have been devised, to provide a readily assessable form of comparison with the senses for the attunements worked out by reason for melodic sequences running through wholes. For this kind of instrument seems to have passed into disuse, both for practical performance and for the theoretical study of the things that produce the ratio of what is attuned, since the *kanonikoi* had as their domain only the study of theorems,[90] and practical usage was involved with *lyrai* and *kitharai*, and similar instruments, while neither of the activities mentioned seemed to be the proper concern of anyone else. In *lyrai* and *kitharai* the melodic intervals were constituted according to the proper ratio, but this is not demonstrated on them, given that such a thing is not even achieved with accuracy on *auloi* or *syringes*, instruments that would have a completer competence at both sorts of presentation, since they construct the differences between notes in conformity with lengths.[91]

[89] Ptolemy indicates that we should proceed here as he did earlier in relation to the concords (Book I chs. 7–8) and the generic divisions (Book I chs. 14–16, Book II chs. 1–2). That is, having given a reasoned analysis of the subject in hand, he proceeds to show the agreement of his conclusions with perception, by the use of such devices as the monochord. Here his ambitious project is to set out the divisions corresponding to all the different *tonoi* in every genus, so pulling the threads of his discussion together. He intends also to indicate the 'common positions', that is, the pitches, relative to a given tonal 'space', on which there fall several different *dynameis* in different *tonoi* and their generic forms. Since the task is complex, and the pitch-differences often minute, a discussion of the imperfections of the *kanōn* on which they will be assessed is apposite enough. But the present chapter concentrates, unexpectedly, on the monochord's deficiencies as a performing instrument (the problems associated with scientific demonstration are hardly touched on until the latter part of chapter 13), and the instruments on which the constructions are in fact to be performed are not monochords, but ones related to those described in chapter 2 above. They are fully discussed in the first two chapters of Book III (cf. also Book II ch. 16).

[90] The word *kanonikos* is used by Ptolemaïs of Pythagorean theorists: she implies that they used the *kanōn* only to display the attunements that mathematical reasoning declared authoritative, not to test the accuracy of this reasoning by recourse to perception (see 9.10 Porph. *Comm.* 22.22ff.). Ptolemy's point here seems to be similar. No doubt the *kanonikoi* used the *kanōn*, but only to give perceptible form to the results of their theorems.

[91] In these wind instruments, unlike *lyrai* and *kitharai*, differences of pitch are (roughly) correlated with measurable differences of length, but too many other relatively uncontrollable variables are involved for accuracy to be attainable. See the opening of Book I ch. 8.

The degree to which it [the single-stringed *kanōn*] is inferior to the other instruments can be seen most plainly in the fact that they can do at any rate one of the two things accurately, while it can do neither.[92] This is because, in the first place, neither the evenness of the string nor the positions of its limits is thoroughly checked, and hence the ratios of the segments that are derived from it are not sound.[93] Further, people in general do not make their divisions by reason [*logos*, 'ratio'], but after stretching out the string, and then altering the
30 position of the bridge until each of the notes they are seeking presents itself to the ear, they mark that position as the appropriate division, so abandoning the purpose for which the instrument exists[94] – proceeding in the same way as do people who manufacture wind instruments. Next, even if the length has been properly divided, it is possible, when ample time is taken in shifting the bridge,
67 for the notes to be fairly well attuned to each other, but if its position is altered more quickly on account of the rhythmic continuity of the melody, this is no longer possible in the same way, since the appropriate marks themselves are not accurately located or precisely touched, because of the speed with which the shift in position is made.[95] Indeed, so far as practical usage is concerned, this instrument would be the last and feeblest. This is partly because one attunes it with one hand and plucks it with the other, working separately and at the same time, so that it is deprived of the finest products of manual technique – I mean such things as added accompaniment, the plucking of notes together, transilient motion in ascent or descent, legato playing, and in general melodic interweaving that proceeds through notes some distance apart[96] – since, there

[92] The two tasks are the accurate production of musically acceptable melodic intervals, in the context of performance, and the demonstration of their correlations with the ratios excogitated by reasoning.

[93] For Ptolemy's methods of checking the adequacy of his apparatus see Book I chs. 8 and 11.

[94] That is, that of testing the fit between what reason 'hypothesizes' and what perception accepts. In its present form the criticism cannot apply to the *kanonikoi* (n. 90 above). It is levelled at people who tune *kanones* merely by ear. Nothing is said to suggest that they go on to offer mathematical representations of their results; the sequel suggests that Ptolemy is thinking of the single-stringed *kanōn*'s use as a performing instrument. As such it was rare. Nicomachus at 10 *Ench.* ch. 4, 243.14–15, implies that in this usage it was identical with the *pandoura* or *phandoura*, a necked instrument played like a lute or guitar, but this usually had three strings. Pollux IV.60 distinguishes the monochord, 'an invention of the Arabs', from the *trichordon* 'which the Assyrians called the *pandoura*'. On the latter see particularly Athenaeus *Deipn.* 176b, 182e, 183b, 183f–184a (with *GMW* vol. 1, p. 265 n. 23, p. 292 n. 164). But the performer's *kanōn* as Ptolemy conceives it is radically different from the *pandoura*: see the next note.

[95] That Ptolemy is focussing on its role in practical music is now evident (see also n. 99 below). The present sentence implies that the positions of the divisions were only indicated by marks, to which a bridge was moved: the instrument was not fretted and stopped at the frets by the fingers. Representations of the *pandoura* show performers plucking with the right hand and stopping strings with the fingers of the left, but I know of none that shows clearly whether frets were present or not (see, for example, Paquette (1984), p. 201 plates H15, H16). Ptolemy's *kanōn*, with its moveable bridges, was evidently distinct in structure and mode of performance from the *pandoura* that Paquette illustrates. It is, at least in general conception, the same as the theorist's instrument described by Ptolemy in Book I ch. 8.

[96] The list of musical techniques impossible on the monochord is hard to interpret: see Düring (1934), pp. 245–7. The etymology of the first, *epipsalmos*, suggests 'plucking in

being only one hand that plucks, it cannot easily traverse the larger distances, nor can it touch two different places at once.[97] It is also because it is inevitable here that continuity between sounds, which embraces the most unmelodic species, since it makes no sound stationary and determinate, follows for most of the time upon the shifting of the bridges, which drag after them sounds of that sort in conjunction with the friction along the string;[98] for it is impossible for them to leap, as it were, and jump to the marked-out positions, and hence it is also impossible to use the swifter rhythms with any facility. This, it seems to me, is why practitioners of this instrument, conscious of its deviations from the notes belonging to proper attunement [*to hērmosmenon*], never exhibit it alone to the assessments of the senses, but accompanied either by the *aulos* or by the *syrinx*, so that its errors may be undetected, disguised by the accompanying sounds of these instruments.[99]

13 *Concerning the alterations to the* kanōn *proposed by the music-theorist Didymus*[100]

The music-theorist Didymus was the first person to try to introduce some kind of improvement [*diorthōsis*, 'correction'] in the *kanōn*. He failed to achieve what was necessary, however, in that he concentrated solely on making the bridge easier to manipulate, being unable to find any cure for the other more numerous and more serious defects which we have described. His procedure is to take the distances belonging to the notes not from one of the limits only, but from the opposite one too, corresponding to positions such that the lengths from each of the limits are unequal, and each has, in relation to the whole length, a ratio appropriate to some note – for instance, when the two parts are

addition', perhaps indicating an accompaniment adding decorative figures around a melody. *Synkrousis*, 'striking together', might be the simultaneous playing of two notes, perfectly possible on *kitharai* etc., but severely limited on the monochord (Düring's alternative interpretation, 'trill', is also plausible). 'Transilient motion...' translates *anaplokē* and *kataplokē*: the former involves movements such as (in modern notation) C-E-D-F upwards, the latter F-D-E-C downwards. See 12 Arist. Quint. *De Mus.* 16.18–21 on the general conception of *plokē* (lit. 'weaving'). 'Legato playing' translates *syrma*, used literally of something dragged or trailed along. Gevaert suggested a connection with *syrigma* or *syrigmos*, used of the production of flageolet tones (see Athenaeus *Deipn.* 638a, cf. Xenophon *Anabasis* VII.3.32–3). As Düring says, this sense would probably demand an emendation of the text. The key word of the last phrase is *symplokē*, lit. 'weaving together', cognate with *anaplokē* and *kataplokē*. It might, as Düring supposes, mean a combination of those two, but its position in the list and its air of summing things up suggests something rather more general – perhaps merely the smooth melodic connection of notes widely spaced in pitch. ('Some distance apart' represents *hyperbatōn*, 'transilient', used also of notes at 12 Arist. Quint. *De Mus.* 14.27.)

97 The latter consideration might explain the difficulty of the first two techniques on the list, the former the remaining ones.

98 Every shift of the bridge's position would be inclined to generate an 'unmelodic' glissando. Compare 6 Theophrastus ap. Porph. *Comm.* 64.32–65.3.

99 This is unambiguous evidence that the instrument, for all its defects, was indeed used sometimes in performance.

100 On Didymus see the introduction to part 2 of chapter 9.

in duple ratio to one another, and in relation to the whole the greater part
68 evidently stands in hemiolic ratio [2:3], corresponding to the fifth, while the smaller stands to it in triple ratio corresponding to the octave and a fifth. For if the whole length is assigned to *proslambanomenos*, the larger of its sections, which is two parts of it [i.e., two thirds], will make *hypatē mesōn*, while the smaller, a third of it, will make *nētē diezeugmenōn*; and similarly for the others which admit that sort of treatment. We can agree that this sort of modification gives some help with the defect associated with continuous shifts of position [of the bridge], since the bridges can often remain in place during several pluckings, in the location common to two notes, the impact corresponding to the notes being made in each of the two sections. But it makes the method more difficult, when the melody does not conjoin common notes,[101] in that the different positions of the same notes raise the question which of them is to be used, since the continuous activity of plucking does not allow any time for thought; and by comparison with a choice between several possibilities, an approach through a succession moving always in one and the same direction would be more ready to hand.[102]

As to the ratios of the division, he takes no account of the consequences of what is perceived [*ta phainomena*], but while he himself posits three genera, diatonic, chromatic and enharmonic, he makes his divisions for only two genera, the chromatic and the diatonic, and only in the changeless [*ametabolon*] *systēma*;[103] and even in these the ratios are not soundly constituted. For he places the leading [highest] notes of the tetrachords in the ratio 5:4 to the notes third in succession from them [inclusive] in both of the genera, and to the second notes in succession in the ratio 6:5 in chromatic and 9:8 in diatonic. Thus the 'following' [lowest] differences in both genera constitute the ratio 16:15, while the middle ones in chromatic constitute the ratio 25:24, and in diatonic 10:9, contrary to what appears to the senses. For in the chromatic genus, of the ratios containing the *pyknon* he has made the 'following' ratio greater than the middle one, whereas such an occurrence is in no way melodic;[104] and in diatonic he has made the leading ratio greater than the middle one, when the opposite should be the case, to conform to what diatonic essentially is;[105] and again, he has made the 'following' ratios of the two

[101] That is, notes both of which are made available by a single position of the bridge.

[102] Ptolemy is no doubt right in principle, but the objection seems captious: the complications are scarcely beyond the powers of a practised executant. With the structure of his comments ('it gives some help...but it makes the method more difficult') compare the last paragraph of chapter 2 above.

[103] That is, in the 'unmodulated' system corresponding to Ptolemy's Dorian *tonos*. Didymus' divisions are set out in tables, with those of other authors, in the next chapter. Ptolemy's claim that he gave no enharmonic division (cf. Thrasyllus at 9.5 Theon Smyrn. 93.1) is odd, since one appears under his name in the tables, and in the text at 71.4–5.

[104] Compare Ptolemy's criticism of Archytas' division of the enharmonic, 32.6–10. For further comment see n. 123 below.

[105] Ptolemy's own 'soft' and 'even' diatonics themselves breach this alleged principle (see Book I chs. 15 and 16, and the tables in Book II ch. 14). But they are for him somewhat odd forms of the diatonic, not representative.

genera equal, when that of the diatonic ought to be smaller.[106] The reason for all these things was his failure to embark on the imposition of the ratios with sufficient circumspection, having failed to consider in advance the way in which they are used in practice: only this makes it possible for them to be
69 brought into conformity with the impressions of the senses. This is why they seem to have constructed the ratios of the concords, which are capable of being tested on a single string on the basis of its division into two, while those of the melodics can be understood only through the composition of the whole *systēma*, which could not be seen accurately on a single string, and their treatment of these is utterly mistaken.[107] For they would be plainly refuted if one were to construct the divisions they propose on the eight strings of equal pitch that we have discussed, these being adequate to display to the hearing the sequence belonging to a melody, so that the genuine and the spurious can be distinguished.[108] In order to make readily available the contrast between our divisions of the genera and those that have previously been handed down – those, at any rate, that we have come across – we shall set out a partial comparison of them, in the middle *tonos*, the Dorian, to display in just that case the difference that there is.

In general, we have not undertaken our approach to the divisions in the same way as the older writers, dividing the whole length into the ratios indicated for each note, because of the laboriousness and difficulty of this sort of measurement. Instead, on the *kanonion* that is placed up against the strings, we have begun by dividing the length cut off, from the highest limit of the sounding length [*apopsalma*] to the mark there will be to indicate the lowest note, into divisions that are equal and proportionate in size. We have placed numbers against these, beginning from the highest limit, through however many parts may be involved, so that now that we have got set out the numbers related in the ratios appropriate to each of the notes, starting from the common limit mentioned, we may always find it easy to bring the dividing points on the moveable bridges up against the positions indicated by the *kanonion*.[109] And since it turns out that the numbers containing the differences shared by the genera run into tens of thousands, we have used the nearest sixtieth parts of

[106] This is the obvious meaning of the text as it stands, but it gives the wrong sense: all Ptolemy's diatonics except the tonic diatonic have a larger 'following' ratio than his chromatics (see the tables in chapter 14 below). There are several ways in which the text could suitably be emended. On the relevant feature of Didymus' division see n. 123 below.

[107] Ptolemy now shifts his attack from Didymus to previous theorists in general. The two forms of 'testing' that he considers are those exemplified in Book I ch. 8, Book II chs. 1–2 (cf. Book I ch. 11).

[108] Compare Book I ch. 1, 4.4–7.

[109] Ptolemy thus marks his *kanonion* into a number of equal segments (120 of them, as appears below). Each boundary is labelled with a numeral. We now consider the octave bounded (in the Dorian divisions, at least) by the notes given by the 60-unit length and the 120-unit length. To each set of ratios into which this octave is divided, there corresponds a table of numerals indicating segment-boundaries. By locating the bridge successively at each boundary designated by the numerals in a table, we shall produce divisions of the octave in the appropriate sequence of ratios.

complete whole units, down to the first sixtieths of a single unit, so that our comparisons are never in error by more than one sixtieth of one of the parts into which the *kanonion* is divided.[110]

In order that the distance covered by the fourth below the disjunction may span thirty parts, the number proposed by Aristoxenus,[111] and in order that when we take his divisions in the larger context we may still understand the segment consisting of a tetrachord through the same numbers, we have posited that the length from the common limit to the lowest note of the octave set out consists of one hundred and twenty segments, and the note higher than this by a fourth is ninety, in epitritic ratio [4:3], so that the note a fifth higher than the lowest is eighty, on the basis of hemiolic ratio [3:2], and the highest note of the octave is sixty, in duple ratio [2:1].[112] The intermediate, moveable notes take their numbers in accordance with the ratios of each genus.

[110] The number that would run into tens of thousands is the denominator of a fraction of the whole length that could serve as the common unit-measure for all the generic divisions. Ptolemy's choice of sixtieths of a unit for his approximations may be influenced by a feeling for symmetry (since the octave span is itself divided into sixty). It is also related to his use in astronomy of the Babylonian sexagesimal system (see Toomer (1984), pp. 6–7). Ptolemy's tables in chapters 14–15 do indeed represent *all* fractions sexagesimally, approximated to the nearest sixtieth. For convenience (but with some hesitation) I have followed Düring in setting them out in a modern fractional form that removes the approximations, except in the last table of chapter 15.

[111] For Aristoxenus the fourth contains two and a half tones, or thirty twelfths of a tone: see Book I ch. 12 with n. 111.

[112] Ptolemy's attempt to accommodate his measurements to Aristoxenian conceptions is misleading. To divide a sounding string into equal parts is not equivalent to a division of tonal 'space' into equal intervals. Thus while there are 30 divisions of the string in the fourth by which 90 differs from 120, there are only 20 in that by which 60 differs from 80; this is why Ptolemy speaks specifically of the 'fourth below the disjunction'. Again, within that fourth, equal numbers of parts do not represent equal intervals (since, for example, the interval between notes sounded by lengths of 90 and 100 units is different from that between notes sounded by lengths of 100 and 110 units: the interval whose ratio is 10:9 is different from that of ratio 11:10). Aristoxenus represented equal intervals as equal tonal 'distances', expressed in terms of tones and their fractions. But he was certainly not committed to the view that if two strings differ in length by a certain amount, measured in unit-lengths, then the interval between their pitches is equal to that between the pitches of any other two strings whose lengths differ by the same amount. The proposition is quite false, and Aristoxenus would certainly have recognised the fact. But in the tables of chapter 14 below, Ptolemy represents Aristoxenus' divisions in terms that require equal differences of number (within any one tetrachord) to represent equal intervals. It follows that the numbers he assigns to the boundaries of the Aristoxenian intervals either fail to capture Aristoxenus' intentions, or cannot be mapped directly on to the *kanonion* in the manner required. Ptolemy can hardly have missed the point – or so at least one would have supposed. But an inspection of the tables suggests that another theorist, Eratosthenes, had indeed construed the Aristoxenian twelfths of a tone as though they could be represented, in each tetrachord, by equal distances on a length of string, and had worked out his ratios accordingly (at least for the enharmonic and the chromatic). See notes 116, 117 below, and cf. n. 85 to Book I; see also 12 Arist. Quint. *De Mus.* Book III ch. 2 with n. 16.

14 *An exposition of the numbers making up the division of the octave in the changeless* tonos *and for each of the genera individually*[113]

We have written out three *kanonia*, each of eight lines, of which the first has five columns, the second eight, and the third ten. In the column placed at the beginning of each of them the order of the notes is set out beside them.[114]

71 The first *kanonion*, then, contains the enharmonic genera. In the first column is that of Archytas, in the ratios 5:4, 36:35 and 28:27;[115] in the second that of Aristoxenus, with distances of 24, 3 and 3 parts;[116] in the third that of Eratosthenes, in the ratios 19:15, 39:38 and 40:39;[117] in the fourth that of Didymus,[118] in the ratios 5:4, 31:30 and 32:31; in the fifth our own, in the ratios 5:4, 24:23 and 46:45.[119]

[113] Conceived as a *tonos*, the changeless (*ametabolon*) system is the Dorian, that in which the notes named by *dynamis* coincide with their namesakes in the series of *theseis*.

[114] This seems to mean that the note-names are listed in an initial column, beside the numbers assigned to each note in the other columns. If so, the list is missing from the MSS tables. The notes are in fact those of the tetrachords *mesōn* and *diezeugmenōn*, the central octave of the GPS, the smallest number being assigned to the highest note (*nētē diezeugmenōn*), since numbers correspond here directly to lengths of string.

[115] On Archytas' divisions see Book 1 chs. 13–14.

[116] This applies only to the lower tetrachord; in the higher the differences are 16, 2 and 2. See note 112 above, and note that Ptolemy represents the interval of disjunction, here as in the other columns, as that between the numbers 90 and 80. As a ratio this gives the correct value to the tone, 9:8, but it makes no sense at all in terms of Aristoxenian 'distances'. Ptolemy is trying to combine Aristoxenian and 'Pythagorean' constructions in a way that is hardly possible in the absence of a logarithmic system of 'cents'. On Aristoxenus' divisions see also Book 1 chs. 12 and 14.

[117] The numbers assigned to notes in Eratosthenes' divisions of enharmonic and chromatic (but not diatonic) are identical with those given to Aristoxenus. It seems clear that Eratosthenes (an immensely learned Alexandrian scholar of the third century B.C.) represented his divisions in terms of ratios, not of tonal 'distances', otherwise Ptolemy would certainly have mentioned the fact (cf. also the reference to Eratosthenes' divisions at the end of 10 Nicomachus *Ench.* ch. 11). The fact that Ptolemy could express the terms of his ratios by the same numbers as those assigned to Aristoxenus can hardly be coincidental. Probably Eratosthenes misconceived the relation between Aristoxenian and Pythagorean representations in the way outlined in n. 112 above. Alternatively, his ratios were not those of string lengths at all, or those of some comparable variable such as speed of transmission, but simply those between Aristoxenian tonal distances, but there is no place in any known form of Greek harmonic theory for ratios of that sort. Certainly he was trying somehow to represent Aristoxenian intervals in the terminology of ratio theory, a fact that helps to explain the ungainly and un-Pythagorean character of his highest enharmonic interval, 19:15. Since the arithmetical differences between terms in Pythagorean ratios are quite different forms of quantity from the 'distances' between Aristoxenian pitches, the attempt is quite incoherent.

[118] On Didymus see chapter 13 above and the excerpts at 9.13–9.14. The presence of an enharmonic division under his name conflicts with Ptolemy's indication at 68.18–19 that he gave none; I do not know how the anomaly is to be resolved. The division is evidently based on the acceptance (as in most of the other enharmonic tables) of an upper interval of a major third, 5:4, instead of the Aristoxenian ditone (81:64, to which Eratosthenes' 19:15 is, by accident or design, a very close approximation), and an attempt to make the two remaining intervals as nearly equal epimorics as possible. See also n. 123 below.

[119] See Book 1 ch. 15. The passage from here to the end of the chapter (though not the tables) was missing from the texts on which the scholars and copyists who transmitted

The enharmonic genera

According to Archytas	According to Aristoxenus	According to Eratosthenes	According to Didymus	According to us
60	60	60	60	60
75	76	76	75	75
$77\frac{1}{7}$	78	78	$77\frac{1}{2}$	$78\frac{6}{23}$
80	80	80	80	80
90	90	90	90	90
$112\frac{1}{2}$	114	114	$112\frac{1}{2}$	$112\frac{1}{2}$
$115\frac{5}{7}$	117	117	$116\frac{1}{4}$	$117\frac{9}{23}$
120	120	120	120	120
5:4 × 36:35 × 28:27 = 4:3	24+3+3 = 30	19:15 × 39:38 × 40:39 = 4:3	5:4 × 31:30 × 32:31 = 4:3	5:4 × 24:23 × 46:45 = 4:3

The second *kanonion* contains the chromatic genera. In the first column is that of Archytas,[120] in the ratios 32:27, 243:224 and 28:27; in the second the soft [*malakon*] chromatic of Aristoxenus, with distances of 22, 4 and 4 parts; in the third the hemiolic chromatic of Aristoxenus, with distances of 21, $4\frac{1}{2}$ and $4\frac{1}{2}$ parts; in the fourth the tonic chromatic of Aristoxenus, with distances of 18,
72 6 and 6 parts;[121] in the fifth that of Eratosthenes,[122] in the ratios 6:5, 19:18 and 20:19; in the sixth that of Didymus,[123] in the ratios 6:5, 25:24 and 16:15; in the seventh our own soft [*malakon*] chromatic, in the ratios 6:5, 15:14 and 28:27; and in the eighth our own tense [*syntonon*] chromatic, in the ratios 7:6, 12:11 and 22:21.[124]

all existing MSS relied. It was reconstructed in the fourteenth century by the Byzantine scholar Isaac Argyrus, whose work was revised by John Wallis in the seventeenth century. The text of the Argyrus-Wallis supplement is printed by Düring and translated here. It has no claim to reproduce Ptolemy's words exactly, but since it does little more than to describe the tables, following the manner of the earlier part of the chapter, it is unlikely to misrepresent them seriously.

120 See Book 1 chs. 13–14 for Ptolemy's account of this odd division, and his criticisms of it.

121 Aristoxenus' divisions are set out in Book 1 ch. 12. For references to 7 *El. Harm.* see the notes to that passage.

122 The numbers representing terms in Eratosthenes' chromatic ratios again coincide with a set ascribed to Aristoxenus, those of his tonic chromatic, the one most commonly adopted by later theorists. See n. 117 above.

123 For Ptolemy's comments on this division see 68.27–32. It seems clear that Didymus' constructions are designed precisely to ensure one of the results of which Ptolemy complains, in effect that of locating the diatonic and the chromatic *parhypatē* at the same pitch. He also makes this pitch identical with the position of enharmonic *lichanos* (since the two upper chromatic or diatonic ratios are together equal to the highest interval of the enharmonic, 5:4). The same equivalences hold between the relevant notes in Aristoxenus' enharmonic, his tonic chromatic and his diatonic (in both the diatonic shades listed by Ptolemy), and in 'Pythagorean' divisions that are derived directly from Plato, e.g., those of Thrasyllus at 9.5 Theon Smyrn. 90.22ff.

124 For the derivation of these divisions see Book 1 ch. 15.

The chromatic genera

According to Archytas	According to Aristoxenus: soft chromatic	According to Aristoxenus: hemiolic chromatic	According to Aristoxenus: tonic chromatic	According to Eratosthenes	According to Didymus	According to us: soft chromatic	According to us: tense chromatic
60	60	60	60	60	60	60	60
$71\frac{1}{9}$	$74\frac{2}{3}$	74	72	72	72	72	70
$77\frac{1}{7}$	$77\frac{1}{3}$	77	76	76	75	$77\frac{1}{7}$	$76\frac{4}{11}$
80	80	80	80	80	80	80	80
90	90	90	90	90	90	90	90
$106\frac{2}{3}$	112	111	108	108	108	108	105
$115\frac{5}{7}$	116	$115\frac{1}{2}$	114	114	$112\frac{1}{9}$	$115\frac{5}{7}$	$114\frac{6}{11}$
120	120	120	120	120	120	120	120
$32:27 \times 243:224 \times 28:27 = 4:3$	$22+4+4 = 30$	$21+4\frac{1}{2}+4\frac{1}{2} = 30$	$18+6+6 = 30$	$6:5 \times 19:18 \times 20:19 = 4:3$	$6:5 \times 25:24 \times 16:15 = 4:3$	$6:5 \times 15:14 \times 28:27 = 4:3$	$7:6 \times 12:11 \times 22:21 = 4:3$

The third *kanonion* contains the diatonic genera. In the first column is that of Archytas, in the ratios 9:8, 8:7 and 28:27; in the second the soft [*malakon*] diatonic of Aristoxenus, with distances of 15, 9 and 6 parts; in the third the tense [*syntonon*] diatonic of Aristoxenus, with distances of 12, 12 and 6 parts; in the fourth that of Eratosthenes, in the ratios 9:8, 9:8 and that of the *leimma* [256:243]; in the fifth that of Didymus, in the ratios 9:8, 10:9 and 16:15; in the sixth is our own soft [*malakon*] diatonic, in the ratios 8:7, 10:9 and 21:20; in the seventh is our own tonic diatonic, in the ratios 9:8, 8:7 and 28:27; in
74 the eighth is our own ditonic diatonic, in the ratios 9:8, 9:8 and that of the *leimma* [256:243]; in the ninth is our own tense [*syntonon*] diatonic, in the ratios 10:9, 9:8 and 16:15; and in the tenth is our own even [*homalon*] diatonic, in the ratios 10:9, 11:10 and 12:11.[125]

73 The diatonic genera

According to Archytas	According to Aristoxenus: soft diatonic	According to Aristoxenus: tense diatonic	According to Eratosthenes	According to Didymus
60	60	60	60	60
67$\frac{1}{2}$	70	68	67$\frac{1}{2}$	67$\frac{1}{2}$
77$\frac{1}{7}$	76	76	75$\frac{15}{16}$	75
80	80	80	80	80
90	90	90	90	90
101$\frac{1}{4}$	105	102	101$\frac{1}{4}$	101$\frac{1}{4}$
115$\frac{5}{7}$	114	114	113$\frac{29}{32}$	112$\frac{1}{2}$
120	120	120	120	120
9:8 × 8:7 × 28:27 = 4:3	15+9+6 = 30 7:6 × 38:35 × 20:19 = 4:3	12+12+6 = 30 17:15 × 19:17 × 20:19 = 4:3	9:8 × 9:8 × 256:243 = 4:3	9:8 × 10:9 × 16:15 = 4:3

125 For references to the divisions of Archytas, Aristoxenus and Ptolemy see notes to the previous paragraph. On Didymus no comment is needed beyond that in n. 123. On Eratosthenes see notes 112, 117. But here the terms of his divisions do not coincide exactly with those of Aristoxenus. The ratios are those of what Ptolemy calls the 'ditonic diatonic' (see also Book 1 ch. 16, 39.16ff.), the system familiar from Plato's *Timaeus*, the *Sect. Can.* and many other sources. No doubt the simplicity of its construction recommended it to Eratosthenes: the equality of its two upper intervals echoes an obvious feature of Aristoxenus' higher diatonic (and in both systems will be described as 'tones'), and the terms are in fact closer to those assigned to the latter than are those of any other system that Ptolemy lists. Eratosthenes may have construed it as a rationally corrected version of the Aristoxenian division.

According to us				
soft diatonic	tonic diatonic	ditonic diatonic	tense diatonic	even diatonic
60	60	60	60	60
$68\frac{4}{7}$	$67\frac{1}{2}$	$67\frac{1}{2}$	$66\frac{2}{3}$	$66\frac{2}{3}$
$76\frac{4}{21}$	$77\frac{1}{7}$	$75\frac{15}{16}$	75	$73\frac{1}{3}$
80	80	80	80	80
90	90	90	90	90
$102\frac{6}{7}$	$101\frac{1}{4}$	$101\frac{1}{4}$	100	100
$114\frac{2}{7}$	$115\frac{5}{7}$	$113\frac{29}{32}$	$112\frac{1}{2}$	110
120	120	120	120	120
8:7 × 10:9 × 21:20 = 4:3	9:8 × 8:7 × 28:27 = 4:3	9:8 × 9:8 × 256:243 = 4:3	10:9 × 9:8 × 16:15 = 4:3	10:9 × 11:10 × 12:11 = 4:3

74.6 15 *Exposition of the numbers that make up the divisions of the familiar genera in the seven* tonoi

It is for one purpose only, as we said, that we have started by setting out the shape of these divisions, that of assessing the differences between genera. To accomplish our remaining task, that of expounding the practice of modulations of the octave,[126] we took in the same way the constitutive numbers for each of the seven *tonoi*, those that accommodate the familiar genera of melody. We did this, further, in the way that each of them is naturally linked together throughout its whole series: we took, that is, for the one that can be sung just by itself, the numbers divided up for ratios within the same genus, but for those that are sung in part, in a special combination with the one mentioned (unless one is prepared to use force), we took from the combination of ratios the numbers attuned to the positions appropriate to the mixture, in order to disguise the fact that we too have gone beyond the limits of what we ought to do, since we have already busied ourselves too much with the divisions of unfamiliar genera.[127]

[126] This corresponds to the task announced at the opening of chapter 12 above.

[127] Two points are being made here. First, the tetrachords in the systems set out in the tables will all be divided according to the ratios of the 'familiar' genera (see the beginning of Book 1 ch. 16), these are: the tense chromatic, and the soft, tonic, ditonic and tense diatonics. The other genera recognised by Ptolemy are less familiar in practice. Consideration of them is 'beyond the limits of what we ought to do' because, though they are rationally acceptable and could be used in performance, the contemporary ear is unaccustomed to them and is therefore, perhaps, less competent to check their credentials. Secondly, as he has also said in Book 1 ch. 16 (38.33ff.), tunings spanning more than a single tetrachord do not in practice use tetrachords of the same genus throughout, except in the case of the tonic diatonic. Tetrachords of other genera normally appear only in association with tonic diatonic tetrachords. Ptolemy's tables

Here we have set out fourteen *kanones*, twice the number of the seven *tonoi*, each having eight lines, as before, equal in number to the notes in the octave, and five columns, corresponding to the quantity of the familiar genera.[128] The seven *kanones* set out at the top contain the numbers making up the octave from thetic *nētē diezeugmenōn* downwards, and those set out below them contain the numbers making up the octave from thetic *mesē* or *nētē hyperbolaiōn* downwards, so that we may be able to make the attunements from whichever of the starting-points we choose.[129] The first two *kanones* contain the Mixolydian *tonos*,[130] the second two the Lydian, the third the Phrygian, the fourth, those in the middle, the Dorian, the fifth the Hypolydian, the sixth the Hypophrygian, and the last the Hypodorian.[131] Of the columns, the first ones for each *tonos* make up the mixture of tense [*syntonon*] chromatic and tonic diatonic, the second the mixture of soft [*malakon*] diatonic and tonic diatonic, the third the tonic diatonic by itself and unmixed, the fourth the mixture of tonic and ditonic diatonic, and the fifth that of tonic diatonic and tense diatonic.[132] Once again, the number belonging to the arrangement of the notes is placed beside the first columns,[133] and the titles belonging to each *tonos* and genus are placed above, in the appropriate positions. We have also added here a *kanonion* with ten lines and eight columns, containing, combined together, all the different numbers belonging to each note, so that there may be clearly displayed for us the quantity of the positions and the magnitude of the interval occupied by each of the notes in all the modulations that are set out.[134]

span an octave each, and each scalar form represented is of a sort commonly used in practice (tonic diatonic, or one of the familiar combinations), not an artificially purified theoretical construction in a single genus.

128 These are listed later in the paragraph.

129 That is, the first seven tables represent forms of the central octave of the thetic system, while the second seven represent forms of the lower octave (from *mesē* downwards) or of the higher (from *nētē hyperbolaiōn* downwards). In any given system the latter two are identical in structure and do not need separate representations.

130 The sense is that the first table in each of the two sets of seven represents Mixolydian, and so on for the rest.

131 For the distinctions between the seven *tonoi* see chapter 10 above. In Ptolemy's scheme they are most conveniently identified by reference to the thetic degree on which the dynamic *mesē* of each falls. Thus the dynamic *mesē* of Mixolydian falls on the seventh degree of the central octave, thetic *paranētē diezeugmenōn*, that of Lydian on the sixth degree, and so on for the rest, in the order given. The position of dynamic *mesē* (and of *proslambanomenos* or *nētē hyperbolaiōn*) in any table can easily be recognised by the invariable disjunctive tone, 9:8, lying immediately above it.

132 These four 'mixtures' and the unmixed tonic diatonic constitute the five 'familiar genera' mentioned at the beginning of the paragraph. We have been told in Book I ch. 16 (39.2–5) that in the mixtures the 'softer' tetrachords are always placed below the disjunctions, the 'tenser' above (the softer being those with a larger interval between the highest note of the tetrachord and the second). Thus the tetrachords immediately below dynamic *mesē* and dynamic *nētē hyperbolaiōn* will contain the softer divisions in a given pair; over the whole double octave the two kinds of tetrachord alternate. The tables consistently locate the tetrachords according to this principle.

133 See note 114 above; here too the implied lists are missing.

134 This final table collects, in each column, all the different numerical values assigned to a given note in the preceding tables. The greatest number of different values assigned to any one note is ten, hence Ptolemy's 'ten lines'. Here 'note' means 'degree of the

[note continues on p. 356.

11.1 *Mixolydian from* nētē

1	2	3	4	5
60	60	60	60	60
$67\frac{1}{2}$	$67\frac{1}{2}$	$67\frac{1}{2}$	$67\frac{1}{2}$	$67\frac{1}{2}$
$78\frac{3}{4}$	$77\frac{1}{7}$	$75\frac{15}{16}$	$75\frac{15}{16}$	$75\frac{15}{16}$
$85\frac{10}{11}$	$85\frac{5}{7}$	$86\frac{11}{14}$	$86\frac{11}{14}$	$86\frac{11}{14}$
90	90	90	90	90
$101\frac{1}{4}$	$101\frac{1}{4}$	$101\frac{1}{4}$	$101\frac{1}{4}$	100
$115\frac{5}{7}$	$115\frac{5}{7}$	$115\frac{5}{7}$	$113\frac{29}{32}$	$112\frac{1}{2}$
120	120	120	120	120
9:8 × 7:6 × 12:11 × 22:21 × 9:8 × 8:7 × 28:27	9:8 × 8:7 × 10:9 × 21:20 × 9:8 × 8:7 × 28:27	9:8 × 9:8 × 8:7 × 28:27 × 9:8 × 8:7 × 28:27	9:8 × 9:8 × 8:7 × 28:27 × 9:8 × 9:8 × 256:243	9:8 × 9:8 × 8:7 × 28:27 × 10:9 × 9:8 × 16:15

11.2 *Lydian from* nētē

1	2	3	4	5
$60\frac{20}{21}$	$60\frac{20}{21}$	$60\frac{20}{21}$	60	$59\frac{7}{27}$
$63\frac{17}{81}$	$63\frac{17}{81}$	$63\frac{17}{81}$	$63\frac{51}{243}$	$63\frac{17}{81}$
$71\frac{1}{9}$	$71\frac{1}{9}$	$71\frac{1}{9}$	$71\frac{1}{9}$	$71\frac{1}{9}$
$82\frac{26}{27}$	$81\frac{17}{63}$	80	80	80
$90\frac{1}{2}$	$90\frac{170}{567}$	$91\frac{3}{7}$	$91\frac{3}{7}$	$91\frac{3}{7}$
$94\frac{17}{21}$	$94\frac{22}{27}$	$94\frac{22}{27}$	$94\frac{22}{27}$	$94\frac{22}{27}$
$106\frac{2}{3}$	$106\frac{2}{3}$	$106\frac{2}{3}$	$106\frac{2}{3}$	$105\frac{85}{243}$
$121\frac{19}{21}$	$121\frac{19}{21}$	$121\frac{19}{21}$	120	$118\frac{14}{27}$
28:27 × 9:8 × 7:6 × 12:11 × 22:21 × 9:8 × 8:7	28:27 × 9:8 × 8:7 × 10:9 × 21:20 × 9:8 × 8:7	28:27 × 9:8 × 9:8 × 8:7 × 28:7 × 9:8 × 8:7	256:243 × 9:8 × 9:8 × 8:7 × 28:27 × 9:8 × 9:8	16:15 × 9:8 × 9:8 × 8:7 × 28:27 × 10:9 × 9:8

11.3 *Phrygian from* nētē

1	2	3	4	5
$60\frac{1}{3}$	$60\frac{1}{5}$	$60\frac{20}{21}$	$60\frac{20}{21}$	$60\frac{20}{21}$
$63\frac{13}{63}$	$63\frac{21}{100}$	$63\frac{17}{81}$	$63\frac{17}{81}$	$63\frac{17}{81}$
$71\frac{3}{28}$	$71\frac{89}{800}$	$71\frac{1}{9}$	$71\frac{1}{9}$	$70\frac{170}{729}$
$81\frac{13}{49}$	$81\frac{27}{100}$	$81\frac{17}{63}$	80	$79\frac{1}{81}$
$84\frac{52}{189}$	$84\frac{7}{25}$	$84\frac{68}{243}$	$84\frac{68}{243}$	$84\frac{68}{243}$
$94\frac{17}{21}$	$94\frac{163}{200}$	$94\frac{22}{27}$	$94\frac{22}{27}$	$94\frac{22}{27}$
$110\frac{11}{18}$	$108\frac{9}{25}$	$106\frac{2}{3}$	$106\frac{2}{3}$	$106\frac{2}{3}$
$120\frac{2}{3}$	$120\frac{2}{5}$	$121\frac{19}{21}$	$121\frac{19}{21}$	$121\frac{19}{21}$
22:21 × 9:8 × 8:7 × 28:27 × 9:8 × 7:6 × 12:11	21:20 × 9:8 × 8:7 × 28:27 × 9:8 × 8:7 × 10:9	28:27 × 9:8 × 8:7 × 28:27 × 9:8 × 9:8 × 8:7	28:27 × 9:8 × 9:8 × 256:243 × 9:8 × 9:8 × 8:7	28:27 × 10:9 × 9:8 × 16:15 × 9:8 × 9:8 × 8:7

11.4 *Dorian from* nētē

1	2	3	4	5
60	60	60	60	60
$67\frac{1}{2}$	$67\frac{1}{2}$	$67\frac{1}{2}$	$67\frac{1}{2}$	$66\frac{2}{3}$
$77\frac{1}{7}$	$77\frac{1}{7}$	$77\frac{1}{7}$	$75\frac{15}{16}$	75
80	80	80	80	80
90	90	90	90	90
105	$102\frac{6}{7}$	$101\frac{1}{4}$	$101\frac{1}{4}$	$101\frac{1}{4}$
$114\frac{6}{11}$	$114\frac{2}{7}$	$115\frac{5}{7}$	$115\frac{5}{7}$	$115\frac{5}{7}$
120	120	120	120	120
9:8 × 8:7	9:8 × 8:7	9:8 × 8:7	9:8 × 9:8	10:9 × 9:8
× 28:27 × 9:8	× 28:27 × 9:8	× 28:27 × 9:8	× 256:243	× 16:15 × 9:8
× 7:6 × 12:11	× 8:7 × 10:9	× 9:8 × 8:7	× 9:8 × 9:8	× 9:8 × 8:7
× 22:21	× 21:20	× 28:27	× 8:7 × 28:27	× 28:27

11.5 *Hypolydian from* nētē

1	2	3	4	5
$60\frac{1}{3}$	$60\frac{1}{5}$	$60\frac{20}{21}$	$60\frac{20}{21}$	$60\frac{20}{21}$
$63\frac{13}{63}$	$63\frac{21}{100}$	$63\frac{17}{81}$	$63\frac{17}{81}$	$63\frac{17}{81}$
$71\frac{3}{28}$	$71\frac{89}{800}$	$71\frac{1}{9}$	$71\frac{1}{9}$	$70\frac{170}{729}$
$81\frac{13}{49}$	$81\frac{27}{100}$	$81\frac{17}{63}$	80	$79\frac{1}{81}$
$84\frac{52}{189}$	$84\frac{7}{25}$	$84\frac{68}{243}$	$84\frac{68}{243}$	$84\frac{68}{243}$
$94\frac{17}{21}$	$94\frac{163}{200}$	$94\frac{22}{27}$	$94\frac{22}{27}$	$94\frac{22}{27}$
$110\frac{11}{18}$	$108\frac{9}{25}$	$106\frac{2}{3}$	$106\frac{2}{3}$	$106\frac{2}{3}$
$120\frac{2}{3}$	$120\frac{2}{5}$	$121\frac{19}{21}$	$121\frac{19}{21}$	$121\frac{19}{21}$
22:21 × 9:8 × 8:7	21:20 × 9:8	28:27 × 9:8	28:27 × 9:8	28:27 × 10:9
× 28:27 × 9:8	× 8:7	× 8:7 × 28:27	× 9:8 × 256:243	× 9:8 × 16:15
× 7:6 × 12:11	× 28:27 × 9:8	× 9:8 × 9:8	× 9:8 × 9:8	× 9:8 × 9:8
	× 8:7 × 10:9	× 8:7	× 8:7	× 8:7

11.6 *Hypophrygian from* nētē

1	2	3	4	5
$62\frac{2}{9}$	$60\frac{20}{21}$	60	60	60
$67\frac{29}{33}$	$67\frac{137}{189}$	$68\frac{4}{7}$	$68\frac{4}{7}$	$68\frac{4}{7}$
$71\frac{1}{9}$	$71\frac{1}{9}$	$71\frac{1}{9}$	$71\frac{1}{9}$	$71\frac{1}{9}$
80	80	80	80	$79\frac{1}{81}$
$91\frac{3}{7}$	$91\frac{3}{7}$	$91\frac{3}{7}$	90	$88\frac{8}{9}$
$94\frac{22}{27}$	$94\frac{22}{27}$	$94\frac{22}{27}$	$94\frac{22}{27}$	$94\frac{22}{27}$
$106\frac{2}{3}$	$106\frac{2}{3}$	$106\frac{2}{3}$	$106\frac{2}{3}$	$106\frac{2}{3}$
$124\frac{4}{9}$	$121\frac{19}{21}$	120	120	120
12:11 × 22:21	10:9 × 21:20	8:7 × 28:27	8:7 × 28:27 × 9:8	8:7 × 28:27
× 9:8 × 8:7	× 9:8 × 8:7	× 9:8 × 8:7	× 9:8 × 256:243	× 10:9 × 9:8
× 28:27 × 9:8	× 28:27 × 9:8	× 28:27 × 9:8	× 9:8 × 9:8	× 16:15 × 9:8
× 7:6	× 8:7	× 9:8		× 9:8

11.7 *Hypodorian from* nētē

1	2	3	4	5
60	60	60	60	60
70	$68\frac{4}{7}$	$67\frac{1}{2}$	$67\frac{1}{2}$	$67\frac{1}{2}$
$76\frac{4}{11}$	$76\frac{4}{21}$	$77\frac{1}{7}$	$77\frac{1}{7}$	$77\frac{1}{7}$
80	80	80	80	80
90	90	90	90	$88\frac{8}{9}$
$102\frac{6}{7}$	$102\frac{6}{7}$	$102\frac{6}{7}$	$101\frac{1}{4}$	100
$106\frac{2}{3}$	$106\frac{2}{3}$	$106\frac{2}{3}$	$106\frac{2}{3}$	$106\frac{2}{3}$
120	120	120	120	120
7:6 × 12:11	8:7 × 10:9	9:8 × 8:7	9:8 × 8:7	9:8 × 8:7
× 22:21 × 9:8	× 21:20 × 9:8	× 28:27 × 9:8	× 28:27 × 9:8	× 28:27 × 10:9
× 8:7 × 28:27	× 8:7 × 28:27	× 8:7 × 28:27	× 9:8 × 256:243	× 9:8 × 16:15
× 9:8	× 9:8	× 9:8	× 9:8	× 9:8

11.8 Mixolydian from *mesē* = 11.4
11.9 Lydian from *mesē* = 11.5
11.10 Phrygian from *mesē* = 11.6
11.11 Dorian from *mesē* = 11.7

11.12 *Hypolydian from* mesē

1	2	3	4	5
$56\frac{136}{729}$	$56\frac{136}{729}$	$56\frac{136}{729}$	$56\frac{136}{729}$	$56\frac{136}{729}$
$63\frac{17}{81}$	$63\frac{17}{81}$	$63\frac{17}{81}$	$63\frac{17}{81}$	$63\frac{17}{81}$
$73\frac{181}{243}$	$72\frac{136}{567}$	$71\frac{1}{9}$	$71\frac{1}{9}$	$71\frac{1}{9}$
$80\frac{400}{891}$	$80\frac{1360}{5103}$	$81\frac{17}{63}$	$81\frac{17}{63}$	$81\frac{17}{63}$
$84\frac{68}{243}$	$84\frac{68}{243}$	$84\frac{68}{243}$	$84\frac{68}{243}$	$84\frac{68}{243}$
$94\frac{22}{27}$	$94\frac{22}{27}$	$94\frac{22}{27}$	$94\frac{22}{27}$	$93\frac{1409}{2187}$
$108\frac{68}{189}$	$108\frac{68}{189}$	$108\frac{68}{189}$	$106\frac{2}{3}$	$105\frac{85}{243}$
$112\frac{272}{729}$	$112\frac{272}{729}$	$112\frac{272}{729}$	$112\frac{272}{729}$	$112\frac{272}{729}$
9:8 × 7:6	9:8 × 8:7	9:8 × 9:8	9:8 × 9:8	9:8 × 9:8
× 12:11 × 22:21	× 10:9 × 21:20	× 8:7 × 28:27	× 8:7 × 28:27	× 8:7 × 28:27
× 9:8 × 8:7	× 9:8 × 8:7	× 9:8 × 8:7	× 9:8 × 9:8	× 10:9 × 9:8
× 28:27	× 28:27	× 28:27	× 256:243	× 16:15

11.13 Hypophrygian from *mesē* = 11.2
11.14 Hypodorian from *mēsē* = 11.3

11.15 *A table putting together all the different numbers assigned to each note*

In this table I have followed Ptolemy's procedure in expressing non-integral numbers by seximagesimals, instead of exact fractions, as in the preceding tables. Thus, for instance, '60.12' is to be read as '$60\frac{12}{60}$'. In many cases a single sexagesimal approximation corresponds to several different exact fractions in tables 11.1–11.14.

56·11	63·13	70·14	79·1	84·17	93·39	105·21	112·22
59·16	66·40	71·7	80	88·53	94·49	106·40	118·31
60	67·30	72·14	80·16	90	100	108·22	120
60·12	67·43	73·45	80·27	90·18	101·15	110·37	120·24
60·20	67·53	74	81·16	90·30	101·35	112·30	120·40
60·57	68·34	74·56	82·58	91·26	101·49	113·54	121·54
62·13	70	76·11	85·43	93·20	102·51	114·17	124·27
—	—	76·22	85·55	—	105	114·33	—
—	—	77·9	86·47	—	—	115·43	—
—	—	78·45	—	—	—	—	—

80.6 ## 16 *Concerning the things played on the* lyra *and the* kithara

Let that be enough to give us an understanding of the straightforward method of division, and of the exposition of ratios and their interrelations, as far as this can be grasped from the *kanonia* by themselves. Now what in the *lyra* are called the *sterea* in a given *tonos* are contained by the numbers of the tonic diatonic in that same *tonos*, and the *malaka* by the numbers in the mixture [of tonic diatonic] with the tense [*syntonon*] chromatic in that same *tonos*.[135] Of those played on the *kithara*,[136] the *tritai* are contained by the numbers starting from *nētē* of the tonic diatonic in the Hypodorian *tonos*, and the *hypertropa*, similarly, by the numbers of the tonic diatonic in the Phrygian; the *parhypatai* are contained by those in the mixture [of tonic diatonic] with soft diatonic in the Dorian; the *tropoi* by those in the mixture with tense chromatic in the Hypodorian; the ones that they call '*lastiaiolia*' by those in the mixture with ditonic diatonic in the Hypophrygian; and the *lydia* by those in the mixture with ditonic diatonic in the Dorian.[137]

It turned out that the highest of all the notes is at a distance of about 55 parts from the common limit, and the lowest at a distance of about 125.[138] Hence after this segment we must leave a distance before the limit at the other end, which will accommodate half the thickness of both the fixed and the moving
81 bridge, by taking away from the whole length [of the string] a distance equal to the combination of these two thicknesses or even more. We divided the remainder into twenty-five segments of five parts each, though this further division – that into parts – is sufficient if it is done only in the fourteen five-part segments that contain the seventy parts between the outermost notes, that is, the parts from number 55 to number 125. It will also be useful to attach additional *kollaboi*, equal in number to the others, at the opposite limits of the *kanōn*, to make it easy to shift the lengths of the strings along, when they are

octave', regardless of which octave in the (thetic) system is being represented. Ptolemy assumes that divisions will be made, for the purpose of testing their accuracy, over eight strings at a time, not the full fifteen (but see Book III ch. 1). The same eight strings can be used for testing any octave-structure. Hence the final table tells us, for each string, the various positions at which it will be stopped by the bridge in the procession through the genera and *tonoi*. I should repeat that the numbers in the principal tables are expressed here as exact fractions; I have retained Ptolemy's sexagesimal approximations (see n. 110 above) only in table II.15.

135 Ptolemy now seeks to match structures mapped out in his tables with the common attunements of *lyra* and *kithara* already discussed in Book I ch. 16. On those of the *lyra*, mentioned in this sentence, see 39.6–11 with n. 145. It becomes clear below that the *kithara* attunements are restricted to certain *tonoi*; the present sentence indicates that those of the *lyra* are not. The *sterea* are the tunings represented in the third column of each table, the *malaka* those represented in the first. In the latter case the chromatic tetrachord, being 'softer', always lies immediately below a disjunctive tone (see n. 132 above).

136 On the *kithara* attunements described below see the Appendix to Book II.

137 These 'mixtures' are all with the tonic diatonic. For details see the Appendix below.

138 These numbers refer to the terms in the tables of chapter 15, conveniently put together in the last table. The smallest number is in fact a little over 56 (thetic *mesē* in Hypolydian, table 12), the largest a little under 125 (thetic *hypatē mesōn* in the first generic division of Hypophrygian, table 6, col. 1). On the variation in the numbers assigned to terms bounding the octave see n. 85 above.

being tested, one of the *kollaboi* that hold them being relaxed, the other tightened.[139] It will also be useful to make them moveable on their *pelekēseis*[140] over the breadth of the *kanōn*, for the sake of a second form of usage, in which a single flat bridge is placed under the strings, and the sideways movements of the strings make the appropriate attunements. For when two *kanones*, equal to the length of the fixed bridges, are divided once again into the parts lying between the outermost notes, and when one of the two *kanones* is placed against each of the two bridges, in such a way that equal numbers are placed opposite one another against the same points, the sideways movements of the strings will be displayed by these numbers, for people who are capable of making an attunement.[141] If the *kollaboi* themselves go along with them too, the notes will maintain the same pitches, but if the *kollaboi* stay still, the result will be that the strings, being sometimes slackened and sometimes tightened as a consequence of the sideways movement, will once again need to be restored to their original equality of pitch [*isotonia*].

Appendix to Ptolemy, *Harmonics* Book II

In the sixteenth chapters of Book I and Book II (and in Book II ch. 1) Ptolemy mentions two forms of attunement used on the *lyra*, and six proper to the *kithara*. Of the *lyra* tunings enough has been said in the notes, but those of the *kithara* are more problematic. In Book I they are named as *Tritai*, *Hypertropa*, *Tropika*, *Parhypatai*, *Lydia* and *Iastia*, the last two being described jointly as *metabolika*, 'modulating'. In

[139] The *kollabos* or *kollops* was a device onto which the string was wound, and which could be turned to adjust its tension. In this case the *kollopes* were attached to bars running close to the edges of the instrument, at a right angle to the strings. Ptolemy is describing an expedient by which a string longer than the distance between its points of attachment may be tested for evenness along various parts of its length (by the methods described in Book I chs. 8 and 11).

[140] The word is related to *pelekys*, 'axe'. The sense is that the *kollaboi* might be fixed not directly onto the bar, but onto wooden blocks shaped like axe-heads, drilled and fitted to the bar so that they could slide along its length. No doubt they were shaped in this way so that the sharp edge would be pressed against the sound-board, to prevent them turning on the bar under the strings' tension. (The term may possibly mean 'wood-chip' rather than 'axe-head', but the shape intended will be much the same.)

[141] A method of this sort is described in Book II ch. 2, and discussed further in Book III chs. 1–2. The idea is clearly articulated in Ptolemy's account of the second instrument mentioned in the former passage (47.18ff.). The instrument is a rectangular device with a bridge placed diagonally. The 'fixed bridges' of the present passage are those over which all the strings pass at each end of their total span. As a string is moved sideways, its *pelekēseis* sliding along the bars parallel to the adjacent fixed bridges, its sounding-length, formed by the distance along it between one fixed bridge and the diagonal bridge, is correspondingly altered. The ratios between the sounding-lengths of two strings is equal to the ratio between the distances along the fixed bridge from its intersection with each string to the point at which the fixed bridge meets the diagonal bridge. Two *kanonia* are marked at points bounding such distances, according to the ratios of a given system, and each is aligned with one of the fixed bridges. Then strings of equal tension are slid along the bars on their *pelekēseis*, until each crosses the first fixed bridge at one of the points marked, and reaches the other, on the opposite side of the instrument, at the equivalent point. (The point of having a *kanonion* aligned with each of the fixed bridges is mainly to ensure that the strings run parallel to one another and at a right angle to the fixed bridges.) Their lengths will be cut by the diagonal bridge in the same ratios as those in which the distances on the *kanonia* were marked, and will thus deliver notes sounding at the intervals appropriate to the system.

Book II *Tropika* becomes *Tropoi* and *Iastia* becomes *Iastiaiolia*, but the same systems are plainly intended.

In Book I the attunements are distinguished only by reference to the genera of the tetrachords they contain. The relevant generic divisions of the tetrachord are as follows.

Tense chromatic 7:6 × 12:11 × 22:21
Soft diatonic 8:7 × 10:9 × 21:20
Tonic diatonic 9:8 × 8:7 × 28:27
Ditonic diatonic 9:8 × 9:8 × 256:243
Tense diatonic 10:9 × 9:8 × 16:15

Book I tells us, and Book II agrees, that the generic constitutions of the six *kithara* attunements are these.

Tritai tonic diatonic throughout
Hypertropa tonic diatonic throughout
Tropika or *Tropoi* a mixture of tonic diatonic and tense chromatic
Parhypatai a mixture of tonic diatonic and soft diatonic
Lydia a mixture of tonic diatonic and ditonic diatonic
Iastia or *Iastiaiolia* a mixture of tonic diatonic and ditonic diatonic

In every case of a mixture, the 'softer' tetrachord (that with the greater highest interval) falls below the disjunctive tone, the 'tenser' above it. In *Lydia* and *Iastia*, so Book I informs us, musicians actually sing in an attunement that mixes tonic diatonic with tense diatonic, and this is theoretically correct, but they tune their instruments to the ditonic version in place of the tense. This complication is ignored in Book II, where only the instrumental tunings are considered.

Book I gives us no way of distinguishing between tunings (e.g., *Tritai* and *Hypertropa*) that employ the same genus. The reason becomes clear in Book II: the differences are to do with *tonos*, and of course the subject of the *tonoi* had not been opened in Book I. Book II tells us that each of the *kithara* tunings (but not those of the *lyra*) is specific to a particular *tonos*, and that the interval structures projected on to the set of strings by, for example, *Tritai* and *Hypertropa*, or by *Lydia* and *Iastiaiolia*, are therefore different, even though their genera are the same. The structures are identified as they appear in the central octave of the thetic series, on the eight strings of which thetic *nētē diezeugmenōn* is the highest (this fact is stated explicitly only for the first case, *Tritai*, but must apply to the rest too). Hence each is represented in one column of one of the first seven tables in chapter 15, the table being determined by *tonos*, the column by genus. The ratios proper to each tuning, and their order, can thus be read off directly from the tables. The specifications are these.

Tritai tonic diatonic in Hypodorian: table 11.7 col. 3
Hypertropa tonic diatonic in Phrygian: table 11.3 col. 3
Parhypatai tonic diatonic with soft diatonic in Dorian: table 11.4 col. 2
Tropoi tonic diatonic with tense chromatic in Hypodorian: table 11.7 col. 1
Iastiaiolia tonic diatonic with ditonic diatonic in Hypophrygian: table 11.6 col. 4
Lydia tonic diatonic with ditonic diatonic in Dorian: table 11.4 col. 4

Consideration must be given to the names of these attunements and to their interrelations. The following table identifies the notes, specified 'dynamically', which each attunement locates at the thetic degrees between *hypatē mesōn* and *nētē diezeugmenōn* (the degrees of the central octave) and it specifies the ratios between them. Comparisons between the columns in the table may go some way towards resolving the difficulties.

Thetic degrees	Dynameis and ratios in each attunement					
	Tritai (Hypodorian)	*Hypertropa* (Phrygian)	*Parhypatai* (Dorian)	*Tropoi* (Hypodor.)	*Iastiaiolia* (Hypophryg.)	*Lydia* (Dorian)
nētē diezeug.	*nētē hyperb.* 9:8	*paranētē diezeug.* 8:7	*nētē diezeug* 9:8	*nētē hyperb.* 7:6	*paranētē hyperb.* 8:7	*nētē diezeug.* 9:8
paranētē diezeug.	*paranētē hyperb.* 8:7	*tritē diezeug.* 28:27	*paranētē diezeug.* 8:7	*paranētē hyperb.* 12:11	*tritē hyperb.* 28:27	*paranētē diezeug.* 9:8
tritē diezeug.	*tritē hyperb.* 28:27	*paramesē* 9:8	*tritē diezeug.* 28:27	*tritē hyperb.* 22:21	*nētē diezeug.* 9:8	*tritē diezeug.* 256:243
paramesē	*nētē diezeug.* 9:8	*mesē* 9:8	*paramesē* 9:8	*nētē diezeug.* 9:8	*paranētē diezeug.* 9:8	*paramesē* 9:8
mesē	*paranētē diezeug.* 8:7	*lichanos mesōn* 8:7	*mesē* 8:7	*paranētē diezeug.* 8:7	*tritē diezeug.* 256:243	*mesē* 9:8
lichanos mesōn	*tritē diezeug.* 28:27	*parhypatē mesōn* 28:27	*lichanos mesōn* 10:9	*tritē diezeug.* 28:27	*paramesē* 9:8	*lichanos mesōn* 8:7
parhypatē mesōn	*paramesē* 9:8	*hypatē mesōn* 9:8	*parhypatē mesōn* 21:20	*paramesē* 9:8	*mesē* 9:8	*parhypatē mesōn* 28:27
hypatē mesōn	*mesē*	*lichanos hypatōn*	*hypatē mesōn*	*mesē*	*lichanos mesōn*	*hypatē mesōn*

Note that in *Tropoi* and in *Iastiaiolia* the 'softer' tetrachord lies higher in the central octave than the 'tenser', due to the position of the disjunctive tone in the *tonoi* to which they belong. The tenser tetrachord is always immediately about the disjunction, not necessarily higher absolutely than the softer.

Let us treat *Tritai* as in some sense basic, since it always appears first in Ptolemy's lists. Its name must be connected with that of the notes called *tritē* in the upper tetrachords, and we would expect it to contrast in some corresponding respect with *Parhypatai*, the only other attunement named in this manner. In justification of its name, commentators point out the coincidence of its *tritē hyperbolaiōn* with thetic *tritē diezeugmenōn*. It may also be noted that any such Hypodorian system brings both the upper tetrachords of the series of *dynameis*, and hence both the *tritai*, into the range of the central octave. Neither consideration seems very strong, but it is not clear that we can do better. So far as *Parhypatai* is concerned, a comparison between it and *Tritai* allows us to make a rather more cogent point. Conceived simply as projections of interval structures onto eight strings, *Tritai* and *Parhypatai* are identical except in their two lowest intervals. The way in which those of *Parhypatai* differ from those of *Tritai* is determined by the locus of the former's *parhypatē mesōn*. This explains the name better than does the mere fact that this *parhypatē* coincides with thetic *parhypatē mesōn*, since (because it is Dorian) all its *dynameis* coincide with *theseis* of the same name.

The assumption that *Tritai* is basic gives some clue to the character of *Hypertropa*. The '*tropos*' stem should plainly have reference to modulation, '*hyper*' indicating either 'excess' or upwards direction. The latter gives an appropriate sense, for *Hypertropa* is indeed a straightforward modulation of *tonos* upwards from the *Tritai* structure. It has another point of interest, in that the two structures imposed on the octave are the same, except that *Hypertropa* replaces *Tritai's* tetrachordal conjunction at the fifth degree (thetic *paramesē*) with a disjunction between the fifth degree and the sixth. Ptolemy has explained (Book II ch. 6) that such a change is a form of modulation by *tonos*: in a shift from *Tritai* to *Hypertropa*, as the relative positions of their dynamic *mesai* show, what we have is modulation upwards by a fifth. (Compare the scholium to 42.12, though this implies, misleadingly, that such an account holds of *Tropoi* rather than *Hypertropa*.)

Tropoi presents no obvious problem. It has the same root structure as *Tritai*, both being Hypodorian, and half of its tetrachords are the same. The others have undergone a change (*tropos*, 'turning') or modulation (*metabolē*) of genus from diatonic to chromatic. From a similar point of view the system taken by itself 'turns' as it progresses from one tetrachord to the next.

Iastiaiolia and *Lydia* are best considered together, and separately from the others. Some sense must be given not only to their individual names, but to their joint designation in Book I as *metabolika*. Now 'Iastian' (as the former is called in Book I) is an old name that the theorists reckoned equivalent to 'Hypophrygian', and *Iastiaiolia* is in Ptolemy's Hypophrygian *tonos*. But *Lydia* is in Dorian. There are serious puzzles here, and the following is only a suggestion. If we begin from the 'changeless' system, onto which the Dorian *tonos* maps directly, we can move to the Lydian *tonos* by shifting each *dynamis* upwards by two thetic degrees. (Thus, for example, Lydian *mesē* falls on thetic *tritē diezeugmenōn*.) But suppose we treat the Iastiaiolian attunement, rather than the Dorian *tonos*, as the basic starting point. What we get if we move each of its *dynameis* upwards by two degrees is precisely the attunement called *Lydia*. Conversely, if *Lydia* is taken as basic, *Iastiaiolia* is generated when its *dynameis* are moved downwards by two degrees: and if such a movement begins instead from the fundamental Dorian series, it will produce the Hypophrygian or 'Iastian' *tonos*. Thus *Lydia* and *Iastiaiolia* are related to one another as the Lydian and 'Iastian' *tonoi* are to Dorian. They are transformations or 'modulations' of one another, and take their names from the form of these intermodulations: hence the description *metabolika*.

Even if this explanation were accepted, the suffix '*-aiolia*' would still need to be investigated. 'Aeolian' is another old name, treated by the theorists as equivalent to

'Hypodorian'. It is of course easy to generate an 'Aeolian' central octave from an 'Iastian', simply by removing a tone from the bottom and adding it at the top, or by construing as the focal octave a series beginning one degree higher (on thetic *parhypatē mesōn*). In a tuning through nine strings, both Iastian and Aeolian could be captured together, and the whole complex might then receive the combined designation. But this suggestion (like many of the others in this appendix) is only speculative. For further hypotheses and associated discussion see Düring (1934), pp. 201–15.

The attunements of which Ptolemy writes are plainly said to be those of familiar practice, and we have no reason to doubt that his tabulations approximate closely to what was commonly heard in his day. It is a signal merit of his approach that he insists on bringing his theoretical results for judgement to the court of educated perception, and that in so doing he makes clear that many of the constructions developed out of his own mathematical operations (and those of other theorists) are not actually employed by musicians at all. Even when the theory has been properly worked out, practical systems instantiate only a small selection of those that it reveals as 'rational'. In that case only this selection can be thoroughly assessed for conformity to usage (compare Book II ch. 1, where just these 'familiar' attunements are applied to the *kanōn* and offered to the judgement of the ear). The others are confirmed only indirectly, by the fact that they form the 'rational' whole that coordinates what is otherwise a disconnected jumble of practices. The musicians whose attunements Ptolemy considers no doubt inherited and developed their systems without benefit of Ptolemaic analysis. If Ptolemy's evaluation of them were indeed aesthetically acceptable, this would say a good deal both for the theoretical credentials of his overall approach to genera and *tonoi*, and for his ingenuity in locating the data of practice within the scope of the whole. For one additional point of considerable interest see Book III ch. 1, 85.28ff., with n. 10.

82 ## Book III

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Book III

83 1 *How the ratios throughout the whole* [*system*] *can be employed and assessed by means of the fifteen-stringed* kanōn

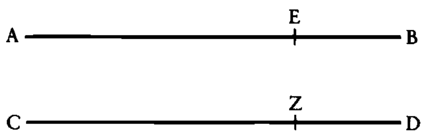
It would appear that at least for the purposes of the exposition we have set out above, the use of something only as large as the octave is sufficient, this being the first that can contain in itself the whole pattern [*idea*, 'appearance', 'form'] of melody; and this, so it seems, is why it was called '*dia pasōn*', and not '*di' oktō*' in the same way as the *dia pente* and the *dia tessarōn*, which are named by derivation from the number of the notes that comprise them.[1] But suppose that someone's enthusiasm should make him want to fill out on the *kanōn* the double-octave *systēma*, to achieve a complete repertoire of complexity, so as to add to the eight notes the remaining seven, adding up to the fifteen of the magnitude of the double octave on the *lyra*. A method can be found of achieving this objective too, in such a way that the short lengths left for the highest notes do not make them lack sonority, and the *kanonia* that are to be introduced do not have to be marked with divisions up to the double octave.[2] [This will be done] if we distinguish the two outermost octaves by the

[1] *Dia pasōn* means 'through all (the strings)', and is the usual expression for 'octave'. *Dia pente*, 'through five', is the fifth, and *dia tessarōn*, 'through four', the fourth; *di' oktō* would mean 'through eight'. Greek commentators often remark on this feature of their terminology: cf., for example, ps.-Ar. *Probs* XIX.32.

[2] These problems are superficially only matters of practical convenience, but the first at least has genuine repercussions within the science, given the emphasis Ptolemy lays on empirical testing. If the strings lack sonority, their pitches cannot be accurately discriminated. Ptolemy may also be thinking of the conditions required for public demonstrations.

tensions and finenesses of the strings, and if we keep at equal pitch with one another the eight finer strings,[3] going upwards from the middling ones, at the pitch appropriately attuned to *mesē*, and the remaining seven thicker ones, again at equal pitch with one another, at the pitch corresponding to *proslambanomenos*, making an octave with the ones at the opposite end, the interval by which, as we saw, *proslambanomenos* is lower than *mesē*. Thus a division of only one octave will be fitted to the two orderings, making the ratio of an octave between each of those that ought to be homophones.[4]

If we think of two notes contained in equal distances of the length, AB and CD, and if AB is an octave higher than CD, and if we then cut off the equal distances AE and CZ, AE will also be an octave higher than CZ. For in general,



since as distance AB is to AE, so is the sound of AE to that of AB, and as
84 distance CD is to CZ, so is the sound of CZ to that of CD; and since as distance AB is to AE, so is CD to CZ, it follows that as the sound of AE is to that of AB, so is the sound of CZ to that of CD. And conversely, as the sound of AE is to that of CZ, so is the sound of AB to that of CD. Thus since ex hypothesi the sound of AB is an octave higher than that of CD, the sound of AE will also be an octave higher than that of CZ, and this will be true of all the notes bounding the seven intervals throughout the instrument, when the same section of the *kanōn* is placed against both.[5]

In this way an attunement can be made even by someone who is capable of assessing only strings of equal pitch; and on the other hand, it will be possible for a person who can accurately discriminate the correct differences between the notes in respect of each species to perform the reverse process – that is, no matter what pitch the notes may have, to set up the bridges to correspond to the division belonging to any one genus and *tonos*, and then to attune the strings to the hearing in a way that conforms to what is laid down. For when this has been done once, then when the bridges are shifted to the positions belonging to another genus or *tonos*, this too will be attuned, and so will all the

[3] That is, their pitches are to be equal before the insertion of the moveable bridges: cf. Book I ch. 11.

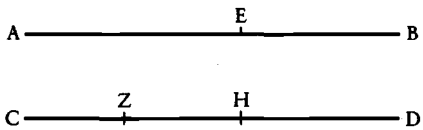
[4] Octaves and double octaves are classified as 'homophones' in Book I ch. 7. Since the interval structure of the octave above *mesē* is always identical with that below it, the same measurements can be used to find the proper lengths of strings in each octave, so long as the two sets of strings are tuned, initially, an octave apart. The argument of the next paragraph is designed to prove the underlying assumption that if two strings of equal length sound an octave apart, and if each is then stopped by a bridge at the same distance from the origin, the resulting equal sounding-lengths will still sound an octave apart.

[5] That is, when each is stopped by a bridge at the same point marked on a measuring-rod (*kanōn*).

others in the same way, because the initial attuning has made the notes once again equal-pitched in equal lengths.[6]

Thus, let there be two notes, as before, AB and CD, and let there be cut off from each the unequal sections AE and CZ, and let them be so attuned that the sound of part CZ has the same ratio to that of AE as length AE has to CZ. I say that equal divisions of the notes [i.e., of the lengths of string] will also be of equal pitch.[7]

Let distance CH be cut off, equal to AE. Since as distance AE (that is, CH) is to CZ, so is the sound of CZ to the sound of AE (because that is how they were attuned) and also to the sound of CH (because of what we initially laid down), the sound of CZ will have the same ratio to both that of AE and that of CH. Hence parts AE and CH of the notes, which have been constructed in equal lengths, will be of equal pitch.



What has been shown will also be evident, with immediate clarity, when after the notes have been attuned in the way we described,[8] the bridges are shifted to the positions which cut off distances that are all equal. For we shall find that each of the arrangements is equal in pitch to itself [i.e., in all its strings], and that the two are related to one another in the octave, just as we laid down this relation according to our previous method.

Let no one be disturbed by the plurality of notes [strings], in that potentially, and in accordance with the condition assigned to them in common, they are no different from a single string: if this is not completely true of all of them, the whole would be in error. For the task we assigned to the *kanōn* was not that

[6] A person who can recognise equality of pitch can set the strings' tensions appropriately. Then when he places bridges at the points demanded by the relevant ratios, he can be confident that they will sound correctly (even if he cannot rely on his ear to recognise this correctness). Conversely, someone who can recognise at least one correct attunement by ear can begin with the strings tuned at random. He can then place the bridges as the ratios of the attunement dictate, and adjust the strings' tension until the requisite interval pattern is heard. He can then be sure that if the bridges were removed, the strings' pitches would be equal, and hence that their division by bridges placed in accordance with the ratios derived mathematically for other attunements would also give aesthetically correct results. The point is further argued in the next paragraphs.

[7] 'Notes', here as quite often in Ptolemy, means 'strings'. Strings AB and CD are initially tuned at random. When unequal parts of them have been cut off by bridges, the strings are so attuned that the interval between the pitches sounded by the parts is the one whose ratio is the same as that between their respective lengths. (For example, if the lengths of the parts are in the ratio 3:2, they are to be tuned so that the note from the longer is a fifth below that from the shorter.) Ptolemy will argue, correctly, that in this case the pitch from any given length of AB is the same as that from an equal length of CD (so long, of course, as the strings are of even constitution).

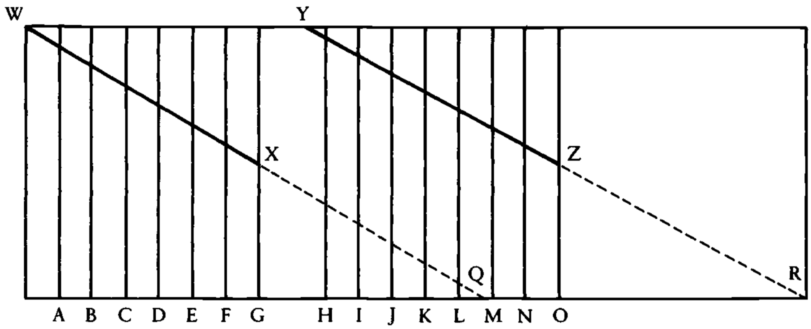
[8] Ptolemy now returns to the lay-out of strings he is recommending, with a set of seven tuned to equal pitches, an octave below a set of eight.

of displaying the ratios of the melodies through a string that is numerically one, or through a plurality of strings whose number is determined, but that of using any number whatever of strings of equal pitch, such that they present themselves as no different from a single string, to make by reason alone that same attunement which the most musical people would make by ear. For the sake, most importantly, of exhibiting the quite incomparable skill with which the works of nature are crafted, and for the sake, secondly and in consequence, of promoting the practice that makes use of it, it is essential that this sort of method be adopted as a foundation, for the discovery and the exhibition of the ratios that make attunement accurate.[9]

Now with one of the two methods of using the *kanōn*, I mean that according to which one bridge is placed under each of the strings, we have no fault to find, so long as the whole *systēma* is divided up into two sets of similar divisions, in order that the differences that have been expounded may all be attuned. But in the other method, in which it will only be necessary to use two bridges, placing one under each of the two orderings, it will often happen that the strings located by the ends of the bridges, in the middle of the span of the *kanōn*, come up against the ends of the bridges lying opposite to them, in the sideways movements involved in shifts of tuning, and so can no longer maintain their proper lengths. Hence it is possible by this latter method to determine only those *systēmata* in which one or other of the notes mentioned keeps the same position in the shifts of tuning: this happens particularly in those played on the *kithara*. Only in these *systēmata* is it sufficient to use these continuous bridges in the way described; and as a result the *kollaboi* of the fixed, common notes in these *systēmata* can stay still, without any sideways displacement.[10]

[9] This is a reminiscence of the theme of Book I ch. 2.

[10] For the method that uses one or two continuous bridges see Book II ch. 2, 47.18ff., Book II ch. 16, 81.9ff. Here the strings are to be arranged as suggested earlier in the present chapter, 83.12ff. The eight strings for the notes from thetic *mesē* upwards are tuned to equal pitches, an octave above the seven for those from *proslambanomenos* to *lichanos mesōn*. In the diagram below (which is my construction, not Ptolemy's), strings A–G are the lower set, H–O the upper. The two diagonal bridges, WX, YZ, are placed at the same angle to the frame, and the sounding-lengths of the strings (those in the lower part of the diagram) are determined by sideways displacements, as explained in Book II ch. 2. (The dotted lines XQ, ZR, represent the equivalents of ZE in the diagram on p. 321; they need not be physically present in their entirety as extensions of the bridges.) The problem Ptolemy has in mind is a practical one: in modulating from one attunement to another, strings G and H (thetic *lichanos mesōn* and *mesē*) will sometimes need to be shifted sideways, and their displacements may bring them up against the bridges of the other set of strings. One would have supposed that the difficulty was not too hard to resolve in practice. It is an interesting fact about the *kithara* tunings that Ptolemy mentions (see Book I ch. 16, Book II ch. 16) that these strings need shifting in only two cases. Thetic *mesē* is the same in all of them, and thetic *lichanos mesōn* is always the same except in *lastiaiolia* and *Lydia*. This can be seen from the specifications in Book II ch. 16 and the tables in ch. 15: see also the Appendix to Book II. The numbers attached to the *lichanoi* are those of the second note from the top in the relevant parts of tables II.8–II.14. Those proper to *mesē* have to be found indirectly. On the instrument, *mesē* is treated as the lowest note of the higher octave. It does not appear in the tables as the lowest note of any octave, but the upper octave is identical in structure to the lower,



86 *2 Methods for the division of the double octave by means of the eight notes only*

The division of the double octave can also be worked out through no more than the eight notes originally set out, in the following way.[11] Take AB as the *kanonion* fitted to the whole length,[12] and divide it at point C, so as to make segment AC double segment CB. Then take, in either direction from C, first CD, in the direction towards B, and then CE, in the direction towards A, so that the whole of DE divides off the width of one of the moveable bridges, or a little more, while EC is double CD, so that the remainder AE is still double the remainder DB.



If we then divide each of segments BD and AE into parts extending down to the lowest note, making A and B the starting points of the numbers,[13] and then make two bridges move under [the strings] to make a comparison between the octaves at either end, when the positions of the bridges are in agreement, being placed, for each note, against the same numbers, they will maintain the segment next to A at double the length of that next to B, so that the whole of the octave next to B is constituted an octave higher than that next to A.

Let the *kanonion* be divided in the following way. Since when the eight notes [i.e., strings] are originally of equal pitch, the highest notes of the two octaves – taken at the halves of AE and DB – have poor resonance, particularly that

which the tables represent, once again, in tables 11.8–11.14. Then the *mesai* have lengths corresponding to those of the lowest notes in these tables, the *proslambanomenoi*. It will be found that thetic *mesē* in the *kithara* tunings (but by no means always elsewhere) is invariably 120: *lichanos* (initially tuned, of course, an octave below the pitch of strings on the set to which *mesē* belongs) is usually $68\frac{4}{7}$, but a little over 63 in *Iastiaiolia* (table 11.13, col. 4) and $67\frac{1}{2}$ in *Lydia* (table 11.11, col. 4). *Iastiaiolia*, then, might well pose the problem to which Ptolemy refers.

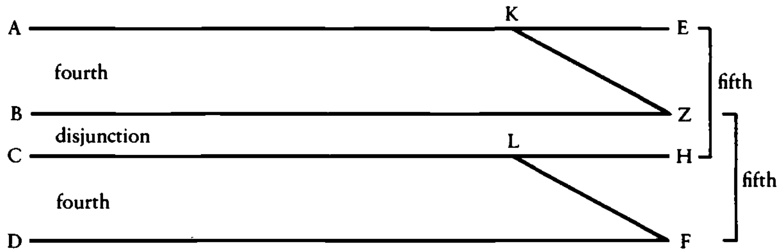
[11] 'Notes', here again, is to be read as 'strings'.

[12] In this case the *kanonion* is placed along the length of each string, between its points of contact with the fixed bridges.

[13] The method according to which BD and AE are to be divided is that described at 69.13ff., cf. 80.18ff., but modified in the way explained in the next paragraph.

next to B, because the segments that produce them are constrained by their shortness,[14] we shall once again take precautions, making the four upper strings finer, equal in pitch to one another, but higher in pitch by a fifth than the four below them, these also being kept equal in pitch to one another. Thus a division up to only a fourth in each of the tetrachords from the lowest notes upwards will make up the octave, this being put together from increase in length over a fourth, and increase in tension over a fifth.

Suppose that the common boundaries at one end of the segments belonging to the tetrachords[15] are on the line through A, B, C and D, and that of the notes
87 [strings] in equal lengths the highest is AE, the fourth from it [inclusive] is BZ, the fifth CH and the eighth DF. AE and BZ are higher by a fifth than CH and DF, in correspondence with their tension. Let equal sections AK and CL be marked off, so that BZ and DF are their epitritics: then when the *apopsalmata* on the bridges[16] are placed under points F, L, Z and K, it is clear that AK will be a fourth higher than BZ, and CL than DF. And since it was laid down that



BZ is higher by a fifth than DF, and that AK is higher by a fifth than CL (since the whole of AE is related in this way to CH, and so is BZ to DF), it is plain that BZ is also higher than CL by a tone, and that AK is higher than DF by an octave. The equivalent is true of the segments that fall in between, since when the division of the higher four notes is transposed down, as a whole, by a set of hemiolic ratios, those of the fifth, all the notes are established at unequal pitches such that, when they are reduced to a lower pitch to the same degree that they were raised by the tension to a higher pitch, they are restored, by reduction in length, to the quantities belonging to their original ratios.

When we construct the positions of the higher tetrachords, in hemiolic ratio to those indicated by the table of numbers, we must take care to ensure that we introduce them to the divisions taken at both ends of the *kanonion*. We shall extend these to $130\frac{11}{60}$ parts, so that we may be able to construct the hemiolic of the number belonging to the lowest of the four notes that start from the highest, which comprises $86\frac{47}{60}$ parts.[17]

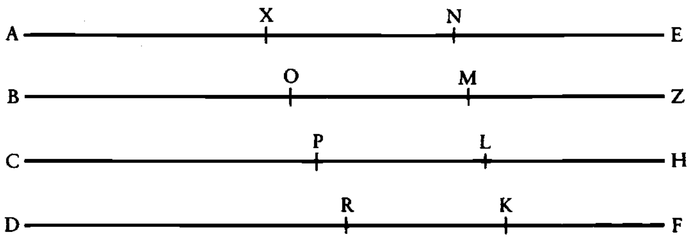
[14] Compare ch. 1 above, 83.9ff.

[15] These boundaries are formed by the fixed bridge, e.g., that which would run through point A in the first diagram in this chapter.

[16] *Apopsalma* here seems to mean the point of contact between bridge and string.

[17] The largest number assigned to the fourth-highest string is $86\frac{47}{60}$ to the nearest sixtieth, $86\frac{11}{14}$ in exact fractions (see Book II ch. 15, tables 11.1 and 11.15). The hemiolic of the

88 The lengths of the higher notes [strings] will be increased still further if we make the four notes in question a whole octave higher than the ones below them. In that case each of the two octaves will no longer be constituted, as before, by both tetrachords together, but instead one will be constituted by one, the other by the other – that is, the whole higher octave by the higher tetrachord and the lower by the lower, the same division being aligned with each.[18]



Thus suppose that the proposed arrangement comprises the whole length of one of the tetrachords. Let us divide off the four lower notes of the octave starting from the end-points A, B, C and D, and the higher ones from the end-points E, Z, H and F. String DF is divided into the lowest and the highest notes of the octave, the next one, CH, into the two next to those mentioned, BZ into the two that are third from the same ones, and AE into the two that are fourth from the extremes, so that the arrangement is contained in a circle from the highest to the lowest, through F, H, Z, E and A, B, C, D. Then if we take the *kanonion* previously described,[19] and for each of the two sets of divisions into lengths in duple ratio that were previously constructed, we align just the larger segment with each of the notes in turn, so as to fit the origin of the parts to F, H, Z and E for the first four numbers, taking the smallest from F, and so as to associate it with points A, B, C and D for the remaining four, in their case
89 taking the smallest from A, and if we place the bridges under the divisions indicated by the numbers, it is clear that note FK will make the highest note of the octave, HL the second [inclusive] from it, ZM the third, EN the fourth, and again AX the fifth, BO the sixth, CP the seventh and DR the eighth. And if we associate with it the other tetrachord, constructing upon it too the division

approximated number (the number standing to it in the ratio 3:2) is $130\frac{11}{60}$. The point is introduced because this method is in fact less straightforward than Ptolemy's treatment may have suggested. Not every note in the upper 'tetrachord' (here meaning 'group of four strings') stands, in each attunement, a fifth above its counterpart in the lower. The sense is rather that when the strings are initially tuned in the way described, then if the numbers proper to the lower strings are read off from the tables, those required for the upper four must be raised in the ratio 3:2. These new numbers will be needed 'at both ends of the *kanonion*', that is, in the segments of the upper strings on both sides of the bridge labelled DE in the opening paragraph of this chapter.

[18] Each octave will now be contained in four strings, with four of its notes played on each side of the bridge DE.

[19] This refers to the measurements required in the first and simplest construction in this chapter, 86.8ff. Only the division of what is there called AE is needed (the 'larger segment' mentioned below).

made up from the same numbers, we shall make two octaves, in which if both the tetrachords are of equal pitch the octaves will also be equal to one another in pitch and as it were duplicated, while if the two tetrachords differ from one another in pitch by an octave, the octaves will differ by the same magnitude, and when conjoined will add up to the double octave.

It is immediately clear that here the division in the higher positions no longer cuts off sections smaller even than the length FK, as it did in the previous series, where we used another set of notes as the highest ones. It is also clear that with this method, only the first procedure can succeed, and that the one that works by means of common bridges is no longer possible.[20] For since the lateral distances between the strings are necessarily kept the same along their whole length, the previous arrangement kept the ratios bounded by given notes the same at both ends, in accordance with the sameness of the lateral distance between them, since they were all set up so as to make an octave between the parts lying at opposite ends. But this one, on the whole, requires that dissimilar ratios at either end be bounded by the same notes [strings] and the same lateral distances: and hence it can no longer encompass the sequence of intervals by means of the similarities pervading the whole length.[21] Such, then, are the more reliable methods by which we can divide up the *systēmata* of the doubled notes, by means of notes that are contained in the halves of the numbers.

In general, to those forms of practical usage that cover the span of an octave, we should attach the numbers set out as comprising the division from *nētē diezeugmenōn* [downwards], in order that the melody may be taken at middling pitches; and to those covering the double octave we should attach the numbers set out from *nētē hyperbolaiōn* [downwards], or from *mesē*, so that the melody may be able to be attuned at both the extremes, which are alike.[22] We must also ensure that even if the width of the moveable bridges is less than that of those fixed at the end-points (which is a consequence of preventing them from taking away a large part of the length), at least their convexities all make the circumferences of equal circles, and that no erroneous variation occurs in the lengths between the *apopsalmata*, since the moveable bridges ought not to have a position higher than that of those at the extremes.[23]

Suppose that the base of the *kanōn* is on the straight line AB, and let ACD and BEZ be erected on it at right angles, and let there be drawn segments HD and FZ of circles with centres C and E, representing the convex curves of the

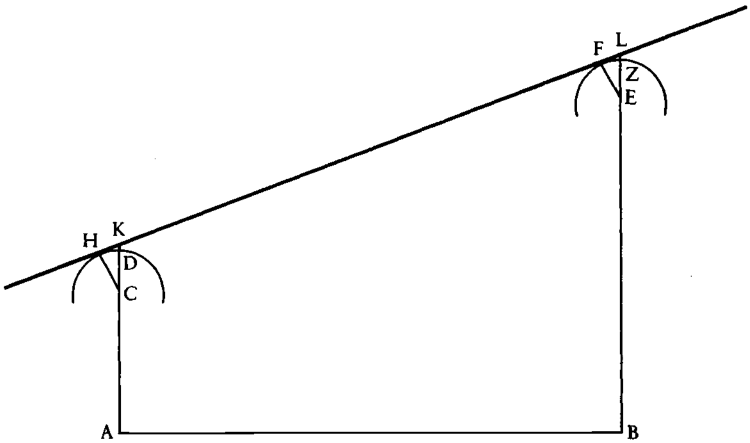
[20] The first method uses a separate bridge for each string. On the second see n. 10 above.

[21] The point of these remarks will be clear from a study of the original description of the 'common bridge' procedure, Book II ch. 2, 47.18ff. The method determines ratios between string-lengths by ratios of lateral distances. Hence if the ratio between, for example, HL and ZM, is different from that between CP and BO, the method would require that the lateral distance between the longer segments of strings BZ and CH is different from that between their smaller segments, and this is not possible.

[22] The first group are set out in Book II ch. 15, tables 11.1–11.7, the second in tables 8–14. The designations of the notes are of course by *thesis*.

[23] The bridges are designed in the way described in Book I ch. 8; see also the next paragraph of the present chapter. Here and below the word *apopsalma* must again refer to the point of contact between bridge and string.

bridges, in such a way that BZ is greater than AD. Let the straight line FH be drawn, touching the curves at H and F, and let H be joined to C and F to E. Let HF be cut at K by the extension of CD, and at L, similarly, by the extension of EZ. Now since the straight lines drawn through the middle of the thickness



of the bridges fall on points D and Z, through which lines AD and BZ were extended, the [theoretical] points of contact with the strings, and the *apopsalmata*, are established at points D and Z. It is also clear that an application of the *kanonion*, laid under the end-points K and L, will display the length KL, whereas its application between the true points of contact and
91 *apopsalmata* will make HF. Now the triangle CHK has angles equal to those of EFL, since CK is parallel to EL and CH to EF. For this reason, as is EF to CH, so is FL to HK: and when CH and EF are equal – that is, when the curves ZF and HD constitute segments of equal circles – FL will also be equal to HK, and KL to the whole of HF, so that there is no difference between the distance measured by the *kanonion* and the true one. But if the circles are unequal, this result will no longer hold, but the division [marked on the *kanonion*] will display a different section from the one that has actually been constructed. And if it were possible for this sort of error to be in the same ratio over absolutely all the notes [strings] – which would happen if all the bridges were at the same distance from the ends[24] – no error would be involved in the use of these parts when the ratios were augmented or diminished on each of the strings. But since it is entirely necessary that the locations of the bridges make the lengths unequal, from which it follows that the differences in the greater distances are smaller and those in the smaller ones, conversely, are greater, there would be no merely negligible error in the lengths of the sections in an attunement of this sort, if we do not construct the positions and movements of the fixed and moveable bridges according to the method we have prescribed.[25]

[24] Because only then would the angle of each string to the horizontal be the same.

[25] In practice it will be essential for the moveable bridges to be slightly higher than the fixed ones, in order to make firm contact with the strings. Ptolemy's argument shows

3 *In what class the power* [dynamis] *of* harmonia, *and the science of it, are to be located*

It seems to me, then, that we have demonstrated adequately and in several ways that the nature of attunement [*to hērmosmenon*] possesses its own proper ratios right down to the melodics, and that we have shown which ratio belongs to each of them, in such a way that those who strive eagerly to master both the rational grounds of the principles laid down and their assessment in practice – that is, the methods of using the *kanōn* that we have expounded – can be in no doubt that they conform, throughout all the species [of *systēma*], to what we accept on the basis of the senses.[26]

92 Since it is natural for a person who reflects on these matters to be immediately filled with wonder – if he wonders also at other things of beauty – at the extreme rationality of the power of *harmonia*, and at the way it finds and creates with perfect accuracy the differences between the forms [*eidē*, 'species'] that belong to it, and since it is also natural for him to desire, through some divine passion, to behold, as it were, the class to which it belongs, and to know with what other things it is linked [*synēptai*, 'is conjoined'] among those included in this world-order, we shall try, in a summary way, so far as it is possible, to investigate also this remaining part of the study we have undertaken, to display the greatness of this kind of power.

Since all things, then, have as their first principles [*archai*] matter and movement and form, matter corresponding to what underlies a thing and what it comes from, movement to the cause and agency, and form to the end and purpose, we should not accept that *harmonia* is that which underlies (for it is something active, not something passive), nor that it is the end, since on the contrary it is what produces some end, such as good melody, good rhythm, good order and beauty, but that it is the cause, which imposes the appropriate form on the underlying matter.[27] Now causes fall into three kinds, at the highest level, one corresponding to nature and concerned only with being, one corresponding to reason and concerned only with being good, and one corresponding to God, concerned with good and eternal being.[28] The cause

that this is permissible just so long as the parts of the surfaces of the fixed and moveable bridges over which the strings pass all form parts of the circumferences of equal circles.

[26] Thus the task indicated in Book I ch. 2 has been completed, and this ends the part of Ptolemy's treatise that deals with harmonic science in its application to strictly musical phenomena. The rest of Book III considers *harmonia* in a wider context, identifying analogues of musical structures and relations in the human soul and in the heavens. It argues that wherever beauty and appropriateness are found, they are expressions of 'harmonic' ratios and proportions: music itself is just one domain governed by such principles. The motivation is comparable to that of 12 Arist. Quint. *De Mus.* Book III, though Ptolemy's approach is more systematic, and his conclusions are often entirely different.

[27] On these distinctions see also the opening of Book I ch. 1. Ptolemy draws his classification of principles, and his conceptions of God, reason and nature in the sequel, principally from Aristotle *Metaph.* Book XII, *Physics* Books II and VIII, though with modifications due probably to Stoic influence.

[28] Things caused by nature are thereby brought into being: the natural imposition of form on appropriate material produces a thing of some determinate natural kind. Reason (for

involved in *harmonia* is not to be identified as corresponding to nature, since it does not implant being in the underlying matter, nor to God, since it is not the primary cause of eternal being, but, clearly, to reason, which falls between the other causes mentioned and joins with them in producing the good: the gods have it with them always, since they are always the same, whereas not all natural things do so, nor do they do so in all ways, for the opposite reason.[29] Of the cause that is in accordance with reason, one aspect is intelligence [*nous*], corresponding to the diviner form, one is skill [*technē*], corresponding to reason itself, and one is habit [*ethos*], corresponding to nature.[30] Hence we can find *harmonia* fulfilling its proper purpose in connection with all of them. For reason, considered in general and without qualification, is productive of order and proportion, while harmonic reason, in particular, is productive of them in the class of what is heard, just as is imagistic [*phantastikos*] reason in the class of what is seen, and critical reason in that of what is thought. It makes correct
93 the ordering that exists among things heard, which we give the special name *emmeleia* ['melodiousness'], through the theoretical discovery of proportions by means of intelligence, through their practical exhibition by means of skill, and through experience in following them by means of habit. When we consider this – that reason in general also discovers what is good, establishes in practice what it has understood, and brings the underlying material into conformity with this by habituation – it is to be expected that the science that embraces all the species of science that rely on reason, which has the special name 'mathematics', is not limited solely by a theoretical grasp of beautiful things, as some people would suppose, but includes at the same time their exhibition and practice, which arise out of the very act of understanding.[31]

This sort of power [*dynamis*] employs as its instruments and servants the highest and most marvellous of the senses, sight and hearing, which, of all the senses, are most closely tied to the ruling principle, and which are the only senses that assess their objects not only by the standard of pleasure but also, much more importantly, by that of beauty. For on the basis of each of the senses one

instance, that of the skilled craftsman) further imposes on naturally existing objects excellences of various sorts, but is powerless to bring into being anything new that is not merely a reorganisation of what already exists by nature. In God the principles of existence and excellence are fused. God is that which, because of its perfection, must exist eternally and changelessly. See especially Aristotle *Metaph.* Book XII chs. 6–7.

29 That is, because they change, and cannot therefore be in all respects perfect at all times.

30 Intelligence in its perfect form is identical with God; through it the contents of its awareness are perfectly constituted. Skill is the capacity to impose excellence on material under the guidance of reason. Through habit or natural disposition things may be brought towards excellence independently of any deliberate rational guidance. Cf. Aristotle *Phys.* Book II chs. 1 and 8, *Metaph.* Book XII ch. 9, *Eth. Nic.* Book X ch. 7.

31 That the findings of reason must be exhibited to the senses through the skilful devising of suitable instruments, and that the senses must be trained in the practical grasp of what reason discovers, have been guidelines of Ptolemy's procedure from the start. Those who restrict 'mathematics' to pure theory are perhaps Platonists (also, no doubt, any mathematicians who pursue their studies merely in the abstract): see especially Plato *Rep.* Book VII. But Plato would agree that – in the ideal case – excellence of disposition and behaviour arises 'out of the very act of understanding', though he denies that theoretical truth can be exhibited and confirmed directly in perceptible phenomena.

can discover, in perceptibles, distinctions proper to that sense alone, such as white and black, for example, in the field of things seen, high and low pitch in that of things heard, good and bad odour in that of things smelled, sweet and bitter in that of things tasted, and soft, for instance, and hard in that of things touched; and one can also discover, of course, the agreeable or disagreeable nature of each of these different qualities. But no one would classify the beautiful or the ugly as belonging to things touched or tasted or smelled, but only to things seen and things heard, such as shape and melody, or the movements of the heavenly bodies, or human actions;[32] and hence these, alone among the senses, give assistance with one another's impressions in many ways
94 through the agency of the rational part of the soul, just as if they were really sisters. It is only hearing that displays things seen, by means of [spoken] explanations, and only sight that reports things heard, by means of writings, and the result is often clearer than if either of the two had announced the same things by itself – as when things transmitted by speech are made easier for us to learn and remember when accompanied by diagrams or letters, and things recognisable by sight are displayed more vividly through a poetic representation, the appearance of waves, for instance, or scenery or battles or the external circumstances of suffering [or 'emotion'], so that our souls are brought into sympathy with the forms of the things reported, as if we were seeing them. It is therefore not just by each one's grasping what is proper to it, but also by their working together in some way to learn and understand the things that are completed according to the appropriate ratio,[33] that these senses themselves, and the most rational of the sciences that depend on them, penetrate progressively into what is beautiful and what is useful. Related to sight, and to the movements in place of the things that are only seen – that is, the heavenly bodies – is astronomy: related to hearing and to the movements in place, once again, of the things that are only heard – that is, sounds – is harmonics.[34] They employ both arithmetic and geometry, as instruments of indisputable authority, to discover the quantity and quality of the primary movements; and they are as it were cousins, born of the sisters, sight and hearing, and brought up by arithmetic and geometry as children most closely related in their stock.

[32] On the special status of sight and hearing see Book I ch. 2, cf. 2.1 Plato *Rep.* 530d. The expression 'the ruling principle' (*to hēgemonikon*) at the beginning of the paragraph refers to the mind; the terminology is Stoic. The distinction between the pleasant and the beautiful is familiar from Plato. A related distinction is that between what is pleasant and what expresses moral value: in Peripatetic sources the audible is sometimes privileged in this respect above even the visible, e.g., Arist. *Pol.* 1340a–b, ps.-Ar. *Probs* XIX.27, 29.

[33] Or perhaps more generally 'rational formula': the Greek is *logos*.

[34] Compare 2.1 Plato *Rep.* 530c–d with 1.19 Archytas frag. 1, which also provide the sources of the metaphor of 'sisterhood' engagingly developed below.

4 *That the power of attunement* [to hērmosmenon] *is present in all things that are more perfect in their natures, but is revealed most fully through human souls and through the movements in the heavens*

Let this be enough to show that the power of *harmonia* is a form of the cause corresponding to reason, the form that concerns itself with the proportions of movements, and that the theoretical science of *harmonia* is a form of mathematics, the form concerned with the ratios of differences between things heard, this form itself contributing to the good order that comes from theoretical study and understanding to people habituated in it. We must also insist that this sort of power must necessarily be present to some extent in all things that have in themselves a source of movement,[35] just as must the other powers, but especially and to the greatest extent in those that share in a more complete and rational nature, because of the suitability of the way in which they were generated. In these alone can it be revealed as preserving fully and clearly, to the highest degree possible, the likeness of the ratios that create appropriateness and attunement in the various different species. For in general, each of the things put in order by nature is characterised by some ratio both in its movements and in its underlying materials. For those in which this ratio can be maintained in due proportion, there is birth and nourishment and preservation and everything which belongs to the class of the better; but when it is deprived of its proper power, in the things in which that is possible, there arise all the things that are opposite to the ones we mentioned and tend towards the worse.[36] It is not found, however, in movements that alter the matter itself, since because of its inconstancy neither the quality of the matter nor its quantity is capable of being defined; but it is found in those movements that are involved most closely with forms.[37] These, as we said, are those of things that are most perfect and rational in their natures, as among divine things are the movements of the heavenly bodies, and among mortal things those of human souls, most particularly, since it is only to each of these that there belong not only the primary and complete sort of movement (that in respect of place), but also the characteristic of being rational. It reveals and displays, so far as it is possible for a human being to grasp it, the pattern of organisation that is based on the harmonic ratios of the notes, as we can see if we divide up each kind in turn, beginning with that to which human souls belong.

[35] This phrase is a reminiscence of Aristotle, e.g., *Phys.* Book II ch. 1, where the possession of such a source is a mark of something's existing 'by nature' (not as an artefact or a merely accidental accretion).

[36] The triple division 'birth, nourishment and preservation' and their opposites are found at Aristotle *De Mundo* 397b, but the notion of a mathematical order underlying them is more Platonist than Aristotelian, and is found also in the Hippocratic medical writings. See 12 Arist. Quint. *De Mus.* Book III ch. 18 with notes, and cf. his ch. 24.

[37] This idea is again derived from Plato: cf. particularly *Timaeus* 27c–29d, 47e–56c.

5 *How the concords are attuned to the primary distinctions in the soul, along with the species proper to them*

There are three primary parts of the soul, the intellectual, the perceptive and the animating,[38] and there are also three primary forms of homophone and concord, the homophone of the octave, and the concords of the fifth and the
96 fourth. Hence the octave is attuned to the intellectual part, since in each of these there is the greatest degree of simplicity, equality and stability; the fifth to the perceptive part; and the fourth to the animating part. For the fifth is closer to the octave than is the fourth, since it is more concordant, due to the fact that the difference between its notes is closer to equality;[39] and the perceptive part is closer to the intellectual than is the animating part, because it too partakes in a kind of apprehension [*katalēpsis*].[40] Now things that have animation do not always have perception, and neither do things that have perception always have intellect: things that have perception, conversely, always do have animation, and things that have intellect always have both animation and perception.[41] In just the same way, where there is a fourth there is not always a fifth, and neither is there always an octave where there is a fifth: where there is a fifth, conversely, there is always a fourth too, and where there is an octave, there are always both a fifth and a fourth. The reason is that the former are made up of the less perfect melodic intervals and combinations, the latter of the more perfect.

One can say that the animating part of the soul has three forms [*eidē*], equal in number to the forms [or 'species'] of the fourth, related respectively to growth, maturity and decline[42] – for these are its primary powers. The perceptive part has four, equal in number to those of the concord of the fifth, related respectively to sight, hearing, smell and taste (if we treat the sense of touch as being common to them all, since it is by touching the perceptibles in one way or another that they produce our perceptions [*antilēpseis*] of them).[43] The intellectual part, finally, has at most seven different species, equal in number to the species of the octave: these are imagination [*phantasia*, 'imaging'], concerned with [the reception of] communications from perceptibles, intellect [*nous*], concerned with the first stamping-in of an impression, reflection [*ennoia*], concerned with the retention and memory of the stamped

[38] The classification is related to Aristotle's (e.g., *De Anima* Book II ch. 2), but the third term, *hektikon* (lit. 'that which preserves a condition or state') is Stoic. It is likely that in this chapter Ptolemy is making substantial use of Stoic sources, even where the content of his remarks has an Aristotelian pedigree.

[39] In the sense explained in Book I ch. 7.

[40] Another Stoic term, as is the doctrine. The associations suggested here have some affinities with the opening of 12 Arist. Quint. *De Mus.* Book III ch. 11.

[41] This hierarchy of capacities, in which each higher power includes and presupposes the lower ones, is again both Aristotelian and Stoic.

[42] Compare Aristotle *De Anima* 411a, *De Mundo* 397b.

[43] The idea that every sense is a form of touch was rejected by Aristotle, who had found it in Democritus. Versions of it were revived by both Stoics and Epicureans. For a different treatment of the relation between the senses and the elements of melody see 12 Arist. Quint. *De Mus.* Book III chs. 14–15.

impressions, thought [*dianoia*], concerned with recollection and enquiry, opinion [*doxa*], concerned with superficial conjecture, reason [*logos*], concerned with correct judgement, and knowledge [*epistēmē*, 'scientific knowledge'], concerned with truth and understanding [*katalēpsis*].[44]

Our soul is also divided in another way, into the rational [*logistikos*, 'calculative'], the spirited [*thymikos*] and the appetitive [*epithymētikos*].[45] For reasons that explain their equality, similar to those just mentioned, we may reasonably link the rational part to the octave, the spirited, which is closely related to the rational, to the fifth, and the appetitive, which is lowest in order of importance, to the fourth. All other facts about the relative value of these
97 parts and about the ways in which they encompass one another can be derived in a similar manner from these points; and we would find, once again, that the more notable distinctions between the virtues proper to each of them are equal in number to the distinctions between the species of the first concords, since melodiousness [*to emmeles*] among notes is a virtue of them, while unmelodiousness [*to ekmeles*] is a vice, and conversely virtue among souls is a melodiousness belonging to them, while vice is an unmelodiousness. A feature common to both classes is the attunement [*hērmosmenon*] of their parts, when they are in a condition conforming to nature, and lack of attunement [*to anharmoston*] when they are in a condition contrary to nature.

Thus there will be three species of virtue belonging to the appetitive part, as in the concord of the fourth:[46] moderation [*sōphrosynē*] in contempt for pleasures, self-control [*enkrateia*] in the endurance of deprivations, and shame [*aidōs*] in the avoidance of what is disgraceful. There will be four species of virtue belonging to the spirited part, as in the concord of the fifth: gentleness [*praotēs*] in refusal to be stirred up by anger, fearlessness [*aphobia*] in refusal to be perturbed by the anticipation of what is dreadful, courage [*andreia*] in contempt for dangers, and steadfastness [*karteria*] in the endurance of hardships. The seven species of virtue belonging to the rational part will be acuteness [*oxytēs*], the virtue to do with agility; cleverness [*euphuia*], the one to do with perspicacity; shrewdness [*anchinoia*], the one to do with clarity of vision; judgement [*euboulia*], the one to do with assessing; wisdom [*sophia*], the one to do with the theoretical; prudence [*phronēsis*], the one to do with the practical; and experience [*empeiria*], the one to do with the dispositions. Again, just as in attunement the accurate construction of the homophones must

[44] Six of these seven 'species' of intellection are regularly (though usually informally and not always consistently) distinguished by both Plato and Aristotle. Only *ennoia* plays no substantial role in their work. All seven occur frequently in Stoic writings. Their interrelations are examined by Ptolemy in his *On the Criterion*.

[45] This classification is derived from Plato *Rep*. Book IV. (Aristotle makes little or no use of the distinction between the 'spirited' and the 'appetitive' in his ethical writings.) It is crucial to Arist. Quint. (e.g., 12 *De Mus*. Book II chs. 5–6, Book III chs. 8 and 16), and common elsewhere.

[46] In the complex classification of virtues that follows, Ptolemy goes well beyond Plato. Düring (1934), p. 271 compares it with one attributed to Speusippus: all the distinctions used, including the seven species of intellectual excellence, are deployed in Stoic writings. Compare also 12 Arist. Quint. *De Mus*. Book III ch. 16.

take the lead, and those of the concords and melodics follow on after it – since a small error in the lesser ratios does not hamper the melody as much as one in the larger and more important ones – so also in souls it is natural for the intellectual and rational parts to govern the others,[47] which are subordinate, and they [i.e., the former] need greater accuracy in the imposition of correct ratio, since they are themselves responsible for the whole or the greater part of any error among the others.

The best condition of the soul as a whole, justice, is as it were a concord between the parts themselves in their relations to one another,[48] in correspondence with the ratio governing the principal parts, the parts concerned with intelligence and rationality being like the homophones, those concerned with good perception and skill, or with courage and moderation, being like the concords, while those concerned with the things that can produce and the things that participate in the *harmoniai* are like the species of the melodics.[49] The whole condition of a philosopher[50] is like the whole *harmonia*
98 of the complete *systēma*, comparisons between them, part by part, being made by reference to the concords and the virtues, while the most complete comparison is made by reference to what is, as it were, a concord of melodic concords and a virtue of the soul's virtues, constituted out of all the concords and all the virtues.[51]

[47] A common theme in Plato, notably in *Rep.* Book IV, and (*mutatis mutandis*) in both Aristotle and the Stoics.

[48] Cf. Plato *Rep.* 443e, 554e.

[49] The three 'parts' here considered are conceived as: (a) the faculty whereby we understand, in the abstract, what excellence consists in, (b) the faculty whereby we identify it in particular instances, together with the disposition to pursue it, and (c) the practical capacity to create it, once it is understood, identified and pursued. The reference to the *harmoniai* is not here purely musical: the capacity is that of making in things external to us, and building in our own character, structures conforming to the 'harmonious' patterns discerned by reason. The idea seems to be that though (as Ptolemy insists) the true nature of the melodic intervals must be derived from that of the concords, and that of the concords from that of the homophones, still the species of concords and homophones which have been drawn on in the preceding analyses cannot be instantiated in practice except through appropriate concatenations of melodic ratios.

[50] This conception of the 'philosopher' originates with Plato, but carries traces of the Stoic 'sage', whose characteristics include 'good perception', 'skill', and the capacity for good practical action as well as the strictly moral and intellectual features emphasised by Plato.

[51] Ptolemy's treatments of the 'parts of the soul' in this chapter have involved three major classifications: (a) into intellectual, perceptive and animating, (b) into rational, spirited and appetitive, (c) into intelligence, perceptiveness along with virtue, and the capacity to produce and become. His procedures in each case are closely analogous to one another, and it is possible that he intends the third classification, which is complex and difficult, to combine the two others, as a preparation for his comparison of the best human condition with the complete *systēma*. But the way in which the three analyses are to be coordinated remains less than clear. For the expression 'concord of concords' see 50.14–15.

6 *A comparison between the genera of attunement* [to hērmosmenon] *and the genera to which the primary virtues belong*

For each of the two kinds of principle [*archē*], that is, the theoretical and the practical, there are three genera, natural, mathematical and theological in the case of the theoretical, and ethical, domestic and political in that of the practical.[52] These do not differ from one another in character, since the virtues of the three genera are shared, and dependent on one another; but they do differ in magnitude and value and in the compass of their structure. Hence one might appropriately compare with each of these three genera the three items called by the same name, 'genera', in the field of *harmonia* – I mean the enharmonic, the chromatic and the diatonic – since these also differ from one another in magnitude, and in the bulk that corresponds to their expansion and contraction: for in these genera both the *pyknon* and the *apyknon* undergo that sort of modification,[53] both in position and in function.

Thus the enharmonic is to be compared to the natural and the ethical, because of its decrease in magnitude by comparison with the others; the diatonic to the theological and the political, because of the similarity of its order and its majesty to theirs; and the chromatic to the mathematical and the domestic, because of the shared nature [*koinotēs*, 'commonness'] of what is intermediate in relation to the extremes. For the mathematical genus is involved to a high degree both in the natural and in the theological, and the domestic shares with the ethical in being private and subordinate, and with the political in being corporate and controlling; while the chromatic is linked in a way both with the relaxation and softness found in the enharmonic, and with the gravity and tension found in the diatonic, having in relation to each of them the opposite character to theirs, just as *mesē* is the note an octave higher in relation to *proslambanomenos*, and the note an octave lower in relation to *nētē hyperbolaiōn*.[54]

99 7 *How the modulations of attunement resemble those of souls in crises of life*

In the same way, we can relate modulations [*metabolai*] of *tonos* in *systēmata* to changes [*metabolai*] in souls brought about by crises in the circumstances of life.[55] For just as in the former, when the genera are kept the same, there can be an alteration in the melody, depending on whether or not its sequence of familiar steps adopts different positions for the expression of its activity, so also

[52] The distinctions are Aristotelian, see, for example, *Metaph.* Book VI ch. 1 (on theoretical enquiries), *Eth. Nic.* Book VI ch. 8, cf. Book X ch. 9 and *Pol.* Book I chs. 1–2 (on practical ones). Cf. also 12 Arist. Quint. *De Mus.* Book I ch. 5.

[53] That is, the 'bulk' corresponding to a genus is to be judged by the size of the interval between the lowest note of a tetrachord and the higher of its two moveable notes.

[54] Ptolemy's treatment of the genera in this chapter has no direct counterpart in 12 Arist. Quint., but contrast the implications of the latter's discussion in Book III ch. 11.

[55] On modulation of *tonos* see Book II chs. 7ff. The subject discussed here is considered also in 12 Arist. Quint. *De Mus.* Book III ch. 26.

in changes [*metabolai*] in human life the same species of psychological disposition are sometimes turned to different courses of behaviour, being drawn along with the customs of the political systems [*politeiai*] they happen to encounter into conditions more suitable to these systems: this sort of thing happens in the laying down of laws itself, the laws often being altered [*metharmozomenōn*] to fit the forms of judicial administration that are appropriate to the prevailing circumstances. Thus just as peaceful conditions turn the souls of the citizens towards greater stability and reasonableness [or 'fairness', *to epieikesteron*], while conditions of war, by contrast, turn them towards greater boldness and disdainfulness [*to kataphronētikōteron*], and again conditions of poverty and shortage of necessities turn them towards greater moderation and thriftiness, while those of plenty and abundance turn them towards greater liberality and lack of restraint, and similarly with all other conditions, so in the same way, in modulations in *harmonia*, the same magnitude is turned in the higher *tonoi* towards a greater capacity to excite [*to diegertikōteron*], and in the lower ones towards a greater capacity to calm [*to katastaltikōteron*], because among notes, too, the higher is the more intensifying, the lower the more relaxing. Hence it is reasonable to compare the intermediate *tonoi*, those around the Dorian, with moderate and stable ways of life, the higher ones, those like the Mixolydian, to ones that are disturbed and more vigorously active, and the lower ones, those like the Hypodorian, to ones that are relaxed and more lethargic.[56] Indeed, our souls are quite plainly affected in sympathy with the actual activities of a melody, recognising the kinship [*syngeneia*, 'common genus'], as it were, of the ratios belonging to its particular kind of constitution, and being moulded by the movements specific to the idiosyncracies of the melodies, so that they are led sometimes into pleasures and relaxations, sometimes into griefs and contractions; they are sometimes stupefied and lulled to sleep, sometimes invigorated and aroused; and they are sometimes turned towards peacefulness and restraint [*katastolē*], sometimes towards frenzy and ecstasy [*enthousiasmos*], as the melody itself modulates in different ways at different times, and draws ours souls towards the conditions constituted from the likenesses of the ratios. I suppose it was because he understood this fact that Pythagoras advised people that when they arose at dawn, before setting off on any activity, they should apply themselves to music [*mousē*, 'the Muse'] and to soothing melody, so that the confusion of their souls resulting from arousal out of sleep should first be transformed into a pure and settled condition and an orderly gentleness, and so make their souls well-attuned and concordant for the actions of the day. It also seems to me that the fact that the gods are invoked with music and melody of some sort – with hymns and *auloi*, for instance, or with Egyptian *trigōnoi* – shows that we desire them to listen to our prayers with kindly gentleness.[57]

[56] Cf. 12 Arist. Quint. *De Mus.* Book II ch. 14, particularly 80.7ff.

[57] *Trigōnoi* are triangular harps. The designation 'Egyptian' is unusual, as is the mention of harps rather than *lyrai* or *kitharai* in this connection, but Ptolemy was working in Alexandria, and may have had local customs in mind.

8 *Concerning the similarity between the complete* systēma *and the circle running through the middle of the signs of the zodiac*

Let that be enough to have given us a view of the affinities between human souls and musical attunement [*to hērmosmenon*]. We have found, to sum it up, that the homophones and concords correspond to the primary parts of the soul, the forms of the melodic sequences to the forms of the virtues, the differences between tetrachords in respect of genus to the genera of the virtues, in their relative worth and magnitude, and modulation of *tonos* to the alterations of character that occur in changing circumstances of life. Our next task is to display the fundamental postulates [*hypotheseis*] about the heavenly bodies as being completely determined in accordance with the harmonic ratios.[58] One of our ways of investigating this will deal with all or most of them together, while the other will deal individually with each of them, taken one by one. Of these the first, comprehensive approach comes first, and it is in it that we make our starting point.

In the first place, then, the truth of our proposition is plainly indicated by the very fact that both the notes [of music] and the courses of the heavenly bodies are determined by intervallic movement alone, upon which there attends none of the changes [*metabolai*] that alter a thing's being.[59] It is indicated also by the fact that all the circuits of the aetherial things are circular and orderly, and that
101 the cyclic recurrences of the harmonic *systēmata* have the same features; for the order and pitch of the notes apparently advances, as it were, along a straight line, but their function [*dynamis*] and their relation to one another, which constitutes their special character, is determined and enclosed within one and the same circuit, since in their nature there is no starting point among them, and their starting point in respect of position [*thesis*] is shifted in different ways at different times to the various successive places in the series.[60] Then if one takes the circle through the middle of the signs of the zodiac and cuts it, theoretically, at one of the two equinoctial points, and after opening it up, as it were, one fits it by equal lengths to the complete *systēma* of the double

[58] This brings together the two fields treated as analogous in Book I ch. 2, where the language is similar. In both places the term *hypothesis* probably refers to the rational principles at work in the world, underlying the behaviour of perceptible things, rather than to the scientist's theories or assumptions.

[59] Here Ptolemy treats change of pitch as analogous to a form of local movement. Incautiously handled, this sort of description would suggest a conception of the 'movement of sound' close to that of Aristoxenus, for example, 7 *El. Harm.* 3.5ff., which Ptolemy rejects. The thesis that the heavenly bodies are subject to movement in place but to change of no other sort is Aristotelian: see, for example, *Metaph.* Book XII ch. 2 (1069b 25–6), cf. Book VIII ch. 1 (1042a 5–6).

[60] This refers to the treatment of the two-octave series of *dynameis* in Book II ch. 5, and in the succeeding chapters on the *tonoi*. As successive *tonoi* are mapped onto the system of *theseis*, note-functions that were located in the upper part of the system move round to reappear at the bottom: the 'lowest' note-function, *proslambanomenos*, becomes identical with the 'highest', *nētē hyperbolaiōn*. (See 52.18–19, and the next paragraph of the present chapter.) Hence the series of *dynameis* has no 'natural starting point', and the 'starting point in respect of *thesis*' stands at a different place in the dynamic cycle of each *tonos*.

octave, the equinoctial point at which no cut was made will correspond to *mesē*, while of the one that was cut, one end corresponds to *proslambanomenos*, the other to *nētē hyerbolaiōn*.

Next, if one bends the double octave round into a circle, in conformity with its function [*dynamis*], and joins *hyperbolaia* [i.e., *nētē hyperbolaiōn*] to *proslambanomenos*, so uniting the two notes, this conjunction will obviously lie diametrically opposite *mesē*, and will stand to it in the homophone of the octave. The reasonableness of the comparison described is established through the fact that the locus of the diameter in the circle has equivalent properties to those that have been shown to belong to the octave. For in the diameter is contained the duple ratio of the whole circle to the semicircle, and also, by comparison with the others, the highest degree of equality, since it is only the diameter that must pass through the centre of the circle – the centre being the source [*archē*] of the figure's equality – and because lines drawn in other ways, even if they divide the whole circumference into some number of equal parts, do not so divide the whole area, while the diameter so divides both area and circumference. Hence the configurations of stars that are diametrically opposite one another in the zodiac are the most invigorating [or 'active', *energētikōtatoi*] of all of them,[61] as are those among the notes that make an octave with one another.

9 *How the concords and discords of attunement* [to hērmosmenon] *are similar to those in the zodiac*

Again, just as the concords of melodies involve division into four parts and no more, since in the greatest, the double octave, the larger term is quadruple the
102 smaller, while in the smallest, the fourth, the larger exceeds the smaller by a fourth part of itself, so in the same way divisions of the circle into four parts and no more produce the complete set of configurations in the zodiac that are understood as being concordant and active.

Let us draw a circle, AB, and divide it, starting from some one point, A, into two equal parts by means of line AB, into three equal parts by means of line AC, into four equal parts by line AD, and into six equal parts by line CB.[62] Then arc AB will make the configuration of diametrical opposition, AD that of a square, AC that of a triangle, and CB that of a hexagon. And the ratios of the arcs, starting from the same point, that is, from A, will include those of the homophones and the concords, and that of the tone besides, as we can see if we suppose the circle to consist of twelve segments, since this is the first number

[61] These expressions indicate what will be seen more fully in the sequel, and what is to be expected in the light of his other writings, that Ptolemy is prepared to treat the movements and configurations of the heavens both from the point of view of scientific astronomy and astrologically.

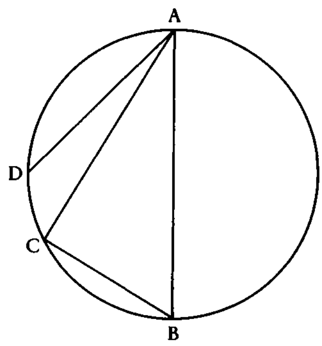
[62] See the diagram on p. 382. Line AC is said to divide the circle into three equal parts in the sense that it cuts off an arc that is one third of the whole circle. Later this is called the 'arc belonging to the triangle', since an equilateral triangle inscribed in a circle divides its circumference into three such arcs. Ptolemy's descriptions of the other lines follow the same pattern.

to have a half, a third and a fourth part. For arc ABD will be 9 of these segments; arc ABC will be 8; semicircle AB will be 6; arc ADC will be 4; and arc AD will be 3. The segments will make the duple ratio belonging to the first of the homophones, that is, the octave, in three ways: the 12 segments of the whole circle will make it in relation to the 6 of the semicircle, the 8 of arc ABC in relation to the 4 of AC, and the 6 of arc ACB in relation to the 3 of AD. They will make the hemiolic [3:2] ratio of the greater of the first concords, that is, the fifth, again in three ways: the 12 segments of the whole circle will make it in relation to the 8 of arc ABC, the 9 of arc ABD in relation to the 6 of AB, and the 6 of arc AB in relation to the 4 of AC. They will make the epitritic [4:3] ratio of the smaller of the first concords, that is, the fourth, in three ways too, on the same basis: the 12 segments of the whole circle will make it in relation to the 9 of arc ABD, the 8 of arc ABC in relation to the 6 of AB, and the 4 of arc AC in relation to the 3 of AD. Again, they will make the triple ratio of the concord of an octave and a fifth in two ways: the 12 segments of the whole circle will make it in relation to the 4 of arc AC, and the 9 of arc ABD in relation to the 3 of AD. They will make the quadruple ratio of the homophone of the double octave in one way: the 12 segments of the whole circle will make
103 it in relation to the 3 of arc AD. They will make the 8 to 3 ratio of the concord of the octave and a fourth in one way: the 8 segments of arc ABC will make it in relation to the 3 of arc AD. They will make the epogdoic [9:8] ratio of the tone, again in one way: the 9 segments of arc ABD will make it in relation to the 8 of ABC. The different relations between the numbers compared are set out in the diagram below.[63]

The octave
ABCD to AB
ABC to AC
AB to AD

The fifth
ABCD to ABC
ABD to AB
AB to AC

The fourth
ABCD to ABD
ABC to AB
AC to AD



The octave and a fifth
ABCD to AC
ABD to AD

The double octave
ABCD to AD

The octave and a fourth
ABC to AD

The epogdoic [tone]
ABD to ABC

The circle	12	ADC	4
ABD	9	AD	3
ABC	8	CB	2
AB	6	DC	1

[63] Thus the geometric configuration formed by an arc of the zodiac, XY, is treated as a magnitude (a distance round the circumference, defined by its relation to the whole circumference) capable of being harmonically related to another that starts from the same origin, XZ. In the key to the diagram, 'ABCD' designates the whole circle, while 'ABD' designates the arc of 9 segments.

Of the first concords, the fifth, by these same principles, can be treated as corresponding to the configuration of the triangle,[64] and the fourth to that of the square; the tone can be treated as corresponding to the twelfth part. For the circle makes the duple ratio with semicircle AB, while AB makes the hemiolic ratio with arc AC, the arc belonging to the triangle, and arc AC makes the epitritic ratio with arc AD, the arc belonging to the square; and the differences between them, corresponding to the tone, is arc CD, which contains a twelfth part of the circle.[65]

The fact, then, that nature gave the circle of the signs of the zodiac a twelve-part constitution, is what we should rationally expect, since the complete
104 *systēma* of a double octave is very close to twelve tones, and the interval of a tone was fitted to the twelfth part of the circle.[66] In a marvellous way, too, those of the points on the zodiac that are separated by one twelfth part are not concordant, but are only in the class of the melodics, while those separated by five twelfth parts are in that of the unmelodics [*ekmelē*]; they are called 'uncoordinated', and are so too in function [*dynamis*]. For in relation to the two arcs based on the straight line subtending a twelfth part, the whole circle makes the ratios of 12 to 1, or 12 to 11, which are foreign to the concords but not to the melodics.[67] But in relation to the two arcs arising from the straight line subtending five twelfth parts, the whole circle makes the ratios of 12 to 5, or 12 to 7, which are foreign to both the concords and the melodics, since neither is either epimoric or multiple, nor is it compounded from any ratios that are appropriate to concords.[68] Further, in all the arrangements based on the

[64] See n. 62 above.

[65] To say that the fifth corresponds to the configuration of the triangle, etc., as Ptolemy does at the outset of the paragraph, is inexact: neither the triangle on its own, nor the arc cut off by one of its sides, corresponds to or represents the fifth. The arc does so in relation to that of the diameter (the semicircle), similarly for the octave and the fourth. The case of the tone is different again. The tone is not represented by a relation between arc CD and some other arc, but by that between two arcs (ABC, ABD) whose difference is CD. There is a further awkwardness of expression here. Before the semicolon in the last sentence, AC and AD are the arcs from A to C and to D by the shortest routes. But the difference between AD and AC is a tone (i.e., the distances are in the ratio 9:8) only if they are taken by the longer route, through B. Yet a thesis of the sort required could have been presented consistently. The ratio of circle to semicircle is 2:1, the octave, and their difference is arc AB. The ratio of the circle to arc ABC is 12:8 = 3:2, the fifth, and their difference is arc AC. The ratio of the circle to arc ABD is 12:9 = 4:3, the fourth, and their difference is arc AD. The ratio of arc ABD to arc ABC is 9:8, the tone, and their difference is arc CD.

[66] Here again Ptolemy's expression is misleading. He appears to be saying that one twelfth of the way round the circle constitutes, by itself, the 'distance' of a tone. The fact is rather that the tone corresponds to the relation between certain arcs (two thirds and three quarters of the circle respectively) that differ by one twelfth of the circle.

[67] Ptolemy is mistaken about the ratio 12:1, since it is the product of 8:1 and 3:2, and is thus the ratio of three octaves and a fifth. It exceeds the range of the usual system, but by the criterion stated in Book 1 ch. 6 (13.3ff.) it must count as a concord. The ratio 12:11 is melodic, occurring as an interval in Ptolemy's tense chromatic tetrachord (see, for example, 35.6–7).

[68] The facts stated by Ptolemy as his reasons for treating these ratios as unmelodic are correct, but his conclusion seems misleading. His last clause ('nor is it compounded...') is presumably meant to take account of the 'concordance' of such ratios as 8:3, that of the octave plus fourth, which is neither multiple nor epimoric but is compounded from 2:1 and 4:3 (see Book 1 chs. 6 and 7). Now the ratio 12:5 can be treated as 2:1 × 6:5,

points making twelfth parts of the circle, there are only three species of squares, equal in number to the species of the concord of the fourth, and only four species of triangles,[69] equal in number to the species of the concord of the fifth, since it is only these among the concords that have the property of being incomposite.[70]

10 *That continuous sequence* [to ephexēs] *among notes resembles the longitudinal movement of the stars*

Let that be a sufficient account of the facts concerning the circular movement itself, considered in respect of both kinds of *harmonia*, and of the arrangements [*schēmata*] that share the general titles 'concordant' and 'discordant'. We must next investigate the facts about the principal kinds of difference between movements in the heavens. There are three kinds, the longitudinal [*kata mēkos*], in the direction of forwards or backwards, in accordance with which the differences between their risings and their settings are brought about, and the converse;[71] the vertical [*kata bathos*], in the direction of lower and higher, in accordance with which they make their movements further from the earth or closer around it; and finally the lateral [*kata platos*], in the direction of the sides, in accordance with which their passage comes to be more to the north of us or more to the south. It would be reasonable for us to attach the first, the longitudinal, to the progress of the notes towards notes that are higher or lower absolutely – for continuous sequence [*to ephexēs*] is the same in each of the two kinds of movement – and to relate the parts near the risings and settings to the lowest pitches, those in the middle of the heavens to the highest. For the risings

and 12:7 as 8:7 × 3:2. These are not all ratios 'appropriate to concords', of course, but those that are not, 6:5 and 8:7, are both legitimate melodic ratios, constituting scalar steps in Ptolemy's soft chromatic and in his soft and tonic diatonics respectively. Hence each of the original ratios is compounded from a concordant and a melodic ratio, and each represents an interval between notes in a legitimate system. They are not 'melodic' ratios in the sense used at 15.14 and elsewhere (simple scalar steps), but it seems odd to call them flatly 'unmelodic'. It is clearly false to call them 'uncoordinated in function', if this means that they do not represent relations between notes with determinate and legitimate melodic *dynameis*. With the use of 'unmelodic' involved here compare Book I ch. 10, 24.27–8.

[69] That is, there are only three sets of points on which squares inscribed in the circle can be based, and four admitting (equilateral) triangles.

[70] In this context an 'incomposite' concord is one not compounded out of lesser concords. As the notes above have indicated, this chapter is dotted with uncharacteristic errors, misleading claims and confusions of expression. Short of denying Ptolemy's authorship of the passage, which we have no positive grounds for doing, I have no explanation to offer. There are affinities between this chapter and 12 Arist. Quint. *De Mus.* Book III ch. 23.

[71] Here Ptolemy is referring to movement along the Zodiac, either through the signs in the order in which they rise over the horizon, or the reverse ('forwards or backwards'). A body may move some distance along the Zodiac in the time between its rising and its setting, or in the time between its setting and its rising ('the converse'). The 'differences between their risings and settings' are thus the differences between their positions, along the Zodiac, at the times of their risings and of their settings. See further ch. 13. 'Longitudinal' here, and 'lateral' below and in ch. 12, are to be understood by reference to the line of the ecliptic, not to the more familiar equatorial coordinates. I am grateful to Dr J. V. Field for her attempts to enlighten my ignorance of astronomy, and especially for her help in formulating this note.

and settings contain both the beginning and the end of their appearances, the beginning, as they come out of the unperceived, the end, as they go into the unperceived; and the lowest pitches contain both the beginning and the end of sound [*phōnē*], the beginning, as they come from silence, the end, as they go towards silence, since the lowest is nearest to the vanishing-point of sound, and the highest is furthest from it. That is why people training their voices begin by singing from the lowest notes, and as they come to an end cease upon the same ones. The positions in mid-heaven, since they are at the greatest distance from the vanishing-points, should be made to correspond with the highest notes, these being also at the greatest distance from silence. Further, it is the places that are low down that make the lowest of the vocal sounds, and the upper ones that make the highest, which is why we say that the lowest pitches travel from the midriff, the highest from the forehead.[72] Again, the risings and settings are lowest down, the positions in mid-heaven the highest up. The latter are reasonably compared to the highest [*oxytatoi*] notes, the former to the lowest [*barytatoi*];[73] and hence the movements of the stars towards the positions in mid-heaven correspond to the progress of the notes from lower pitches to higher, while the movements away from the positions in mid-heaven correspond, conversely, to their progress from higher pitches to lower.

11 *How the vertical movement of the stars is comparable to the genera in* harmonia

We shall find that the second of these differences, the vertical, is similar to the difference between what in *harmonia* are called the genera. For these latter, again, include three forms, the enharmonic, the chromatic and the diatonic, distinguished by the magnitude of the ratios in the tetrachords; while the
106 former includes three forms of distance, those corresponding to the least, the intermediate and the greatest, these too being measured in comparison with one another by the magnitude of their velocities. Then their passages at intermediate distances, which always comprise intermediate velocities, can fairly be compared most closely to the chromatic genera, since in these too the *lichanoi* make intermediate divisions in the tetrachords; those corresponding to the least movements, whether these go with the distances further from the earth or with those closer around it, can be compared to the enharmonic genera, since the two intervals taken together make a smaller interval than the remaining one, in accordance with the form that is called *pyknon*; and those corresponding to the greatest movements, whether these go, once again, with the distances further from the earth or with those closer around it, can be compared to the diatonic genera, since in these the two intervals are never less

[72] This remark probably reflects singing-teachers' jargon, not unlike that of today. But there is a serious physiological theory behind Ptolemy's use of it: see Book I ch. 3 (8.25–9.15).

[73] The reasoning here, if there is any, is unclear. Probably Ptolemy is making a linguistic point, based on the fact that Greek writers sometimes (but rarely) adopt the practice of calling treble notes 'high' and bass ones 'low'. He is pointing out the implied correlation between the notions of 'sharpness' (*oxytēs*) and height, 'heaviness' (*barytēs*) and lowness.

than the remaining one, in accordance with the form that is called *apyknon*.[74] Another reason is that the enharmonic genus and the least of the velocities make for contraction, of the melody in the one case and of the speed in the other; the diatonic and the greatest of the velocities make for expansion; and the chromatic genus and the intermediate velocity occupy a position somehow intermediate between the extremes.[75]

12 *That the modulations between* tonoi *correspond to the lateral movements of the stars*

The third and last distinction between heavenly movements, I mean the lateral, should be related to modulations between *tonoi*. For in the one case no change of genus is consequent upon a shift between *tonoi*, and in the other no perceptible variation in velocity is found to result from lateral movements. Among them we should compare the Dorian *tonos*, which is right in the middle of the others, with the middle positions of their lateral movements, those set along the celestial equator, as it were, in the case of each of the spheres; the
107 Mixolydian and the Hypodorian, as being the extremes, with the most northerly and southerly positions, conceived in the guise of tropics; and the remaining four *tonoi*, which are between the ones mentioned, with those falling on the parallels between the tropics and the celestial equator, these being themselves four in number, because of the division of the slantwise circle [i.e., the ecliptic] into twelve, corresponding to the twelve parts of the zodiac. For each of the points at the tropics will make one parallel, and the two points at equal distances in each direction from these will again make one and the same parallel, so that five pairings are produced, corresponding to the lengths of the divisions into twelve parts, and five parallels between them, making seven

[74] The paths of the planets, in Ptolemy's view, are not pure circles. As each planet moves in a large circle round the earth, it also describes smaller circles (epicycles) around this main line of travel, but in the same plane. When the epicycle is carrying a planet 'backwards', against the direction in which it moves round the large circle's circumference, the effective speed of its overall forward movement is slower than at other times; when it is carrying it in the same direction as the main circle, it is swifter. Since an epicycle may rotate either 'backwards' at the top and 'forwards' at the bottom or the converse, the planet's slower overall movement may occur either when it is further from the earth or when it is nearer; hence Ptolemy's insistence on leaving both possibilities open. The issue is discussed by Ptolemy in *Almagest* Book III ch. 3 (Toomer (1984), pp. 144–5). He later argues that while the moon's epicycle turns in the opposite direction to that of its main circle (Book IV ch. 5, Toomer (1984), pp. 181–9), those of the other planets turn in the same direction as their main circles (Book IX ch. 5, Toomer (1984), p. 442). Hence while the moon's overall movement is swiftest when it is closest to the earth (at perigee), those of the planets are swiftest when they are furthest away (at apogee).

[75] The words 'make for contraction', 'make for expansion', represent the adjectives *systaltikos* and *diastatikos*, used at 12 Arist. Quint. *De Mus.* 30.12–15 (cf. 40.15) of the moral and emotional characters of different forms of melody. The 'systaltic' creates gloom, the 'diastatic' arouses the spirit, while an intermediate type generates emotional calm. Ptolemy may mean to evoke these associations here, and in that case the emotional influence of more and less rapid movements is presumably astrological. Otherwise it is unclear what these remarks add to the argument of the rest of the chapter.

along with the tropics, equal in number to the modulations of the *tonoi*.[76] Those higher than the Dorian *tonos* will be arranged, because of the greater elevation of the melody, to correspond to the movements near the higher pole, those of summer, as it were;[77] that is, where the northern pole is higher they correspond to those nearest the northerly, and where the southern pole is higher, to the opposite ones. Those lower than the Dorian, because of the greater lowering of the melody, will be arranged to correspond to the movements near the invisible pole, those of winter, as it were; that is, where the southern pole is higher they correspond to those nearest the northerly, and where the northern pole is higher, to the opposite ones.[78]

13 *Concerning the correspondence* [analogia] *between the tetrachords and the configurations in relation to the sun*

The remaining arrangement, that of the tetrachords and tones in the complete *systēma*, will next be shown to be involved in the arrangement of configurations in relation to the sun. The disjunctive tones are associated with
108 the distances from heliacal settings [*krypseis*] to heliacal risings [*phaseis*] and with the sunset-rising or full-moon positions.[79] The notes that conjoin the two pairings of tetrachords, *hypatē mesōn* and *nētē diezeugmenōn*, are associated with the positions on either side of these that form with them a square, that is, in the positions of the half-moon.[80] Thus the configuration starting from the [heliacal] rising of each, that of the first moon-shape [crescent], is comparable to the tetrachord *hypatōn*, since, once again, the beginning of rising and the beginning of the lowest notes are the same.[81] The one in succession [*ephexēs*] to this, that of the first half-circle [half-moon], is comparable to the tetrachord *mesōn*;[82] the one starting either from a contrary rising, as in the case of Mercury and Venus, or from the sunset-rising of the other three planets, or from the beginning of waning in the case of the moon, that of the second half-

[76] The circle of the zodiac is at an angle to the celestial equator. From the zodiacal points furthest from the equator, circles parallel to the equator are imagined: these are the tropics. Four additional parallels are constructed, two between each tropic and the equator. We thus have seven parallel circles, with six spaces between them. Then the angled circle of the zodiac, touching the outer parallels and intersecting with the others, will be divided into twelve. (The figure is hard to construct on paper. I could see it properly only after drawing the lines on a tennis ball; the reader may find it helpful to follow suit.)

[77] That is, those visited by the sun in summer.

[78] Hence, which parallels will count as higher and which as lower will depend on the terrestrial position (northern or southern hemisphere) in relation to which they are considered.

[79] The position of a planet on a circle round the earth varies in relation to that of the sun. The 'distance' referred to here is the amount of arc on that circle by which the relation has changed between the time when a planet sets with the sun and the time it rises with it (heliacal setting and rising), or between the time when it rises opposite the sun at sunset and the time when it sets opposite to it at dawn (like the moon when it is full).

[80] These are the positions of the planets on the circle, relative to the sun, when they rise half-way through the sun's diurnal or nocturnal course.

[81] A planet traverses the lowest tetrachord between the occasion when it rises with the sun and that on which it reaches the next corner of the square, the first 'half-moon' position. In the case of the moon, this corresponds to the 'crescent' phase.

[82] The expression 'half-circle' refers to the phase between half-moon and full moon positions, not to the half-moon position itself.

circle,[83] is comparable to the tetrachord *diezeugmenōn*, and makes in relation to the first moon-shape and the tetrachord *hypatōn* the figure of diametrical opposition and the homophone of the octave. The one in succession to these, going as far as the heliacal settings, that of the second moon-shape [decrescent], is comparable to the tetrachord *hyperbolaiōn*; and it too makes, in relation to the first half-circle and the tetrachord *mesōn* the figure of diametrical opposition and the homophone of the octave. The distances from heliacal settings to risings, and at the sunset-risings from evening-risings to dawn-settings, or those during the apparent full moons,[84] are close to a twelfth part,
109 like the tone-intervals at the disjunctions: consequently the distances during each of the four configurations are close to two and a half twelfth parts, just as each of the four tetrachords is close to two and a half tones. Again, in the case of the moon, the configurations diametrically opposite one another, taken together, make a complete one, corresponding to the appearance of the whole,[85] just as notes related in the octave produce a unity, in accordance with the sameness of our perception of them.

14 *Which are the first numbers in which the fixed notes of the complete* systēma *may be compared with the first spheres that there are in the universe*

From such demonstrations of similarity as these we can grasp most clearly the general affinities between the features distinguishing melodic intervals and those distinguishing the heavenly movements. It remains to investigate the affinities in each particular case, which can be reliably detected through the ...[86] numbers that are present, and through the ratios that are contained by them. If the whole circle is divided into 360 parts, when the moon, or any one of the planets, comes to a position diametrically opposite the sun, the intervening distance will be of 180 parts, these being conceived as lying on the

[83] This is the phase marked, in the case of the moon, by the shift from full moon to the second half-moon. The 'other three planets' (Saturn, Jupiter, Mars) rise on these occasions, as the scholiast says (in a note to 108.1), diametrically opposite the sun, 'in opposition' in the strict sense. Mercury and Venus, by contrast, are never diametrically opposite the sun: when they rise at sunset they do so above a sector of the western horizon. This is why their sunset appearances are described separately here. The expression 'contrary rising' renders the Greek more or less literally, and is designed to avoid the misleading use of the term 'opposition'. [84] See n. 79 above.

[85] That is, two half-moons, or a full and a new moon, add up to a complete whole.

[86] Like the latter part of Book II ch. 14, the passage from here to the end of chapter 15 was missing from all texts on which the compilers of existing MSS relied. The version we have is due to the fourteenth-century Byzantine scholar Nikephoros Gregoras (an extant letter shows that his work on the *Harmonics* was finished by 1335). He had laboured to produce a sound text of the existing parts of Ptolemy's book, on the basis of MSS which he describes as badly corrupted by poor copyists, with many omissions and confusions. When he reached the present point, he himself wrote a supplement to do the work of the missing sections. Thus, the text printed by Düring and translated here is not Ptolemy's at all, but Gregoras' own attempt at a reconstruction, based on his own admirable scholarship and his sense of what was demanded by the chapter titles recorded in the MSS on which he drew. I have included the passage for completeness' sake, but in view of its origins, and the relative simplicity of the argument, I have not thought it necessary to provide commentary.

circular circumference. For when these are doubled they yield the number of the whole circle, 360. When they are related in triangular configuration with one another, then, we say, they stand from one another at an interval of 120 parts; for when these are tripled they yield the number of the whole circle, 360. When they are separated from one another in square configuration, then, we say, the interval between them is of 90 parts; for four times 90, similarly, makes 360, once again. When they are in hexagonal configuration, then, we say, the distance is of 60 parts; for six times 60 is again 360. Then when the complete *systēma* of music is related to these, the notes that are fixed will be related to the positions of these numerical intervals in the following way. *Proslambanomenos* will be related to the position at 180 parts, *hypatē mesōn* to that at 120, *nētē diezeugmenōn* to that at 90, *nētē hyperbolaiōn* to that at 60; and the two fixed notes bounding the tone of disjunction to the position which is the starting point of the distances mentioned, or in other words the place in which we locate the position of the sun or of any other planets, the one from which the measures of the distances are disjoined in each direction.

110 15 *How the ratios of the movements proper to each can be found in terms of numbers*

Given these points, the number of the distance belonging to the square, 90, taken as a mean between the 120 parts of the distance belonging to the triangle and the 60 of that belonging to the hexagon, will make two intervals, in hemiolic [3:2] and epitritic [4:3] ratios, like those of the two first concords of *harmonia*, the fifth and the fourth. Just as in music these two first concords, the fifth and the fourth, when put together make the homophone of the octave, so here too the intervals of the two ratios mentioned, that is, the hemiolic and the epitritic, when put together will make the duple ratio, the one corresponding [*analogon*] to the homophone of the octave. When the number of 360 parts of the whole circle is brought into relation with these, it will make with 90 the quadruple ratio, that corresponding to the complete *systēma* of the double octave in music. One could also find the same correspondence [*analogia*] by investigating in a different way, starting from the twelfth parts of the zodiac themselves. For the 120 parts contain an interval of four twelfth parts, the 90 contain one of three, and the 60 one of two. Of these, 3 is a mean: in subordination to 4 it makes the epitritic ratio, and in relation to 2 it makes the hemiolic ratio. From the two latter the duple ratio is constituted (I mean that of 4 to 2); and when the number 12, that of the zodiacal signs in the whole circuit, is brought into relation with these, it will make with 3 the quadruple ratio, corresponding to what properly goes with the complete *systēma* of the double octave in music. Since we have been mentioning polygons, that is, the configuration of the triangle, the square and the hexagon, there would by all means follow yet another way of showing similarly, from their angles, the ratios proper to *harmonia*: but we have decided that the method set out above is sufficient for present purposes, and have postponed the fuller account to a time of greater leisure.

16 How the interrelations of the planets are to be compared with those of the notes[87]

Let no one think[88] that the note of Jupiter is in concord with each of the lights, whereas that of Venus is in concord only with that of the moon, since the tone is not in a ratio of concord. For the latter [the note of Venus] was made part
111 of the dominion of the moon, while that of Jupiter has been included in that of the sun, for the same reasons. Since each note of the ones that bring destruction makes the concord of a fourth with one of the two that bring good (that of Saturn, *nētē hyperbolaiōn*, with that of Jupiter, *nētē diezeugmenōn*, and that of Mars, *nētē synēmmenōn*, with that of Venus, *mesē*),[89] it followed that the note of Saturn belonged rather to the dominion of the sun, and that of Mars to that of the moon.[90] Hence among the configurations, those of

[87] The text of this chapter was found by Gregoras (see note 86 above) embedded in one MS text of Book III ch. 9, where, as he saw, it certainly does not belong. He relocated it here, as chapter 16. Whether he was right to do so is disputed: it has been denied both that it belongs here, and indeed that it is Ptolemy's own work. Sceptics on these points have included Barlaam as early as the fourteenth century, Wallis in the seventeenth and Jan in the nineteenth. (Barlaam's discussion of the last three chapters of Ptolemy's treatise is printed in Düring (1930), pp. 112–21.) Düring (1930, Introduction, pp. LXXXIV–LXXXVIII) presents their objections, and offers a detailed case for its authenticity as a fragment (lacking at least the opening sentences) of Ptolemy's original chapter 16. The sceptics' most telling point is that this passage assumes a scheme in which the LPS, including *nētē synēmmenōn* as a fixed note, has equal status with the two-octave system, which omits the tetrachord *synēmmenōn* and is Ptolemy's main focus. Ptolemy has indeed argued (Book II ch. 6) that the LPS is not an independent 'perfect system', but arises out of the other through a form of modulation. But as Düring points out, though Ptolemy refuses to treat the LPS as 'perfect' or as independent of the two-octave system, he nevertheless accords it the title of a *systēma* (the *systēma synēmmenōn*), and at 54.7 he unequivocally describes *nētē synēmmenōn* as a fixed note. None of the arguments, I think, is decisive. The question has a bearing on the guess of Byzantine scholars later than Gregoras that Ptolemy never finished the work, but died without writing Book III chapters 14–16. If chapter 16 is genuine and in the right place, we may fairly suppose that chapters 14 and 15 had also been written. If it is not, that by itself gives no grounds for accepting these Byzantine conjectures.

[88] I follow Düring, with some misgivings, in treating these opening words as a misleading scribal addition. The proposition that they introduce seems to be genuinely Ptolemaic doctrine, not a theory to be rejected.

[89] To the correspondences of notes to planets made here we can add the implications of the initial sentence (rejecting the first four words). Jupiter is in concord with each of the lights (Sun and Moon); Venus stands at a tone from the Sun. In that case the Sun must be *paramesē*, and the Moon is most probably *hypatē mesōn*, a fourth below Venus, an octave below Jupiter. This gives a scheme identical with that in the inscription, probably by Ptolemy, that was found at Canopus (included in the Teubner edition of Ptolemy's works, ed. J. L. Heiberg, Leipzig, 1898–1907), which adds the information that Mercury, not mentioned here, was assigned the same note as Venus. The inscription also gives correspondences between the remaining fixed notes and other cosmic entities. To *proslambanomenos* is assigned the pair of heavy elements, water and earth; to *hypatē hypatōn* is assigned the pair of light elements, air and fire; and to an additional note a tone above *nētē hyperbolaiōn*, probably entitled *hyperhyperbolaia*, is assigned the sphere of the fixed stars. The systems sketched here may be compared with those of 10 Nicomachus *Ench.* ch. 3, and 12 Arist. Quint. *De Mus.* Book III ch. 22.

[90] The reasoning is that if Jupiter belongs to the Sun, so does Saturn, and if Venus belongs to the Moon, so does Mars. Ptolemy's astrological treatise, the *Tetrabiblos*, confirms that the Moon's dominion (or 'party', *hairesis*) includes Moon, Venus, Mars, and the

Saturn in relation to Jupiter are all bringers of good, while of those between Saturn and the sun only the triangular ones are so, since they are more concordant than the others. Similarly, of those between Mars and Venus, and between Mars and the moon, not all, but only the triangular ones are bringers of good:[91] conversely, those between Saturn and the moon and between Saturn and Venus are all evil, while those between Mars and the sun and between Mars and Jupiter are all treacherous.[92]

tetrachords whose boundaries they are (*mesōn, synēmmenōn*). The Sun's dominion includes Sun, Jupiter, Saturn, and the tetrachords bounded by them (*diezeugmenōn, hyperbolaiōn*) (see *Tetrabiblos* I. 9). *Tetrabiblos* I. 5 also agrees with the present text in making Venus and Jupiter 'bringers of good', Mars and Saturn 'bringers of destruction'. A scholiast to the present chapter explains that Jupiter and the Sun belong to day, with which is linked heat, activity and maleness, Venus and the Moon to night, with its dampness, idleness and femaleness. Each of the 'bringers of destruction' must be assigned to one of these parties, and this is done in such a way that their mingling with corresponding goods mitigates their specific sort of evil. Hence Saturn, whose characteristic evil is cold, is assigned to the 'hot' party of the Sun and Jupiter, while Mars, whose special evil is dryness, is placed in the 'damp' faction of the Moon and Venus. (He adds that Mercury belongs equally to both camps.) Compare also 12 Arist. Quint. *De Mus.* Book III ch. 21. The system as a whole may be set out as follows.

fourth	*nētē hyperbolaiōn*	Saturn (destruction, cold)	*Hairesis* of Sun (hot, active, male)
	nētē diezeugmenōn	Jupiter (good, hot)	
fourth	*nētē synēmmenōn*	Mars (destruction, dry)	*Hairesis* of moon (damp, idle, female)
fourth	*paramesē*	Sun (hot)	
	mesē	Venus (good, wet) with Mercury?	
fourth	*hypatē mesōn*	Moon (wet)	

[91] The triangular configurations are associated with the concord of a fifth (see 103.5–6). The considerations at work here are evidently concerned with the relations, within each *hairesis*, between the bringer of good, the bringer of evil, and the lord of the dominion. The details of the reasoning are unclear. In particular, Ptolemy does not explain the asymmetry of his treatment: why are all configurations between Saturn and Jupiter favourable, but only triangular ones between Mars and Venus?

[92] Here the focus is on relations in which the two bringers of destruction, Saturn and Mars, stand to other bodies. But now (by contrast with the preceding sentence) Ptolemy is considering their relations with members of the rival *hairesis*. If we draw on the scholiast's comment (n. 90 above), it appears that he is condemning as evil all combinations of cold and damp, and as treacherous all combinations of dryness and heat. The assignment of notes to planets in this chapter belongs to a system based on the relative distances of bodies from the earth: cf. 10 Nicomachus *Ench.* ch. 3, 12 Arist. Quint. *De Mus.* Book III ch. 22. The doctrine of 'configurations' is concerned, by contrast, with their relative positions in the circle of the zodiac. Ptolemy seems to have intended to bring together facets of both schemes, but how the programme was to be pursued in detail is not clear. The astrological notions of 'configuration' and 'aspect' are treated more fully in *Tetrabiblos* I. 5.

CHAPTER 12

Aristides Quintilianus

Of the life of Aristides Quintilianus nothing is known, and though he refers in the *De Musica* to a treatise on poetics, we have nothing from his pen except the present work. He cannot be earlier than the first century A.D., since he mentions Cicero as a figure from the past, and the use made of his book by Martianus Capella guarantees that he is no later than the fourth. Beyond this the matter is uncertain. The most recent commentator (Mathiesen 1983) inclines to the view that his brand of Neoplatonism aligns him with Plotinus and Porphyry, which would place him in or after the late third century, and Mathiesen identifies a number of passages that may have been drawn from Porphyry's *Commentary on Ptolemy's Harmonics*. (Aristides does not seem to have been directly acquainted with Ptolemy's work.) This suggestion has a good deal in its favour, though a few considerations might point to an earlier date; Mathiesen's introduction should be consulted for details.

Aristides' aim, as he explains in Book I ch. 2, is to put everything relevant to the study of music together in a single treatise. This, he says, no previous writer had done, and in view of the vast scope of his conception of the subject, he may well be right. Most of his themes had been touched on by others, but no author we know of (not even Ptolemy) had attempted to articulate them all in such detail in one work, and within a single unifying framework. Aristides has often been called an eclectic, and so he is, but his treatise is not just an assemblage of disparate ideas. It is designed under an overarching vision of the divine order of things, and of the unitary, divine source of musical structures in their three major instantiations: in the audible music of human practice, in the soul and in the natural universe at large.

The programme of the *De Musica* is simple in outline, though the interweaving of its themes becomes increasingly complicated as the work proceeds. It is best approached through the classification of the parts of music and of musical understanding set out in Book I ch. 5. The subject is divided first into a theoretical and a practical side. Theoretical studies encompass two main fields, the 'technical' and the 'physical'. Of these the technical investigates the structures to be found in audible music, and it is subdivided into harmonics, rhythmics and metrics. The physical branch pursues music's 'highest principles, its natural causes, and its concordant relations to the things that there are', that is, to elements and structures in the cosmos as a whole. It in turn has two parts, of which one is concerned with numbers, the other with the parts of nature themselves, natural or 'physical' things, their material, formal and animating principles and their interrelations. The practical or educational side of musical study investigates the ways in which 'technical' knowledge of music can be used, and specifically how it should be used in practice for the soul's good. It is also subdivided. One branch directs the use in melodic, rhythmic and poetic composition of structures that have already been technically analysed; the second concerns their proper modes of expression in instrumental performance, singing and acting. Aristides' first book deals with the technical side of the theoretical branch, his second with the practical or educational branch, and his third with the principles and metaphysical sources of the truths displayed in both the others, embraced under the two aspects of the physical side of the theoretical branch. Since the treatise is long and densely packed with detail, the most helpful form of introduction may be a schematic summary.

Book I begins with a chapter explaining in outline Aristides' guiding conception of his subject and its value. Chapter 2 discusses the reasons for its previous neglect and shows their inadequacy. After a prayer for divine assistance (chapter 3), he proceeds to a preliminary definition of the science and its branches, and to a description of the 'material' of music, distinguishing three kinds of sound, continuous, intervallic and intermediate (chapter 4). Chapter 5 presents the classification of studies sketched above: after a brief discussion of pitch and its qualifications it goes on to list the seven parts of harmonics, which is the first division of the 'technical' branch.

Each of the next seven chapters deals with one of these parts: notes (chapter 6), intervals (chapter 7), *systēmata* (chapter 8), genera (chapter 9), *tonoi* (chapter 10), modulation (chapter 11), and melodic composition (chapter 12). Aristides' accounts of each are drawn almost entirely from the Aristoxenian tradition, with few substantial contributions or modifications of his own. Many parallels can be found either in the *El. Harm.* itself, or in the later 'Aristoxenian handbooks'. But his treatment is fuller than any of the latter, and includes a host of minor details not found elsewhere. Together with the *Eisagoge* of Cleonides it is the best surviving source for those aspects of Aristoxenian harmonics that are missing from our texts of the *El. Harm.* It must be recognised, however, that Aristides was evidently working from several different compilations of Aristoxenian doctrines, not always, if ever, from Aristoxenus' writings themselves. He does not always excerpt skilfully: his discussions are riddled with inconsistencies, and in several passages it is pretty clear that he did not understand what he was writing. These confusions are irritating, but have their own value: we can find out more from them about the different versions of 'Aristoxenian' harmonics that had developed than we could from a more carefully processed and homogenised product. Of special interest are Aristides' descriptions of harmonic systems and modes of analysis which belonged, so he says, to a time even earlier than Aristoxenus (see especially 12.5ff., 15.8ff., 18.5ff.). Though their authenticity is uncertain, it is at least possible that they are derived (perhaps at several removes) from the lost treatise on his predecessors to which Aristoxenus refers at 7 *El. Harm.* 2.30–1.

Chapters 13–19 deal with rhythmics, the second division of the technical branch of musical studies. The principal basis of analysis is the division of the rhythmical foot into thesis (strong beat, down-beat) and arsis (weak beat, up-beat), and the classification of feet into genera in accordance with the ratios between the durations of these elements. This approach involves the development of the notion of an elementary or 'primary' duration, whose definition is the task of the first of the five parts of rhythmics listed in chapter 13. It is discussed in chapter 14, along with various other concepts fundamental to the subject. Chapters 15–16 consider the structures of rhythms in each of three genera, the genera being the second subject listed in chapter 13. Rhythmic forms involving mixtures of genera are described in chapter 17. Chapter 18 is a digression, reviewing the basis of a different mode of rhythmical analysis. Finally, chapter 19 looks briefly at the three remaining parts of rhythmics, which deal with rhythmical tempo, modulation and composition. Aristides' main source throughout the passage is again Aristoxenus, and the notes draw attention to some of the links between Aristides' treatment and that of *El. Rhythm.* Book II, which is translated in the Appendix to chapter 7 above.

The last ten chapters of Book I (chapters 20–9) deal with the third subject to fall under the technical branch of musicology, which is metrics. Unlike rhythm, which exists in dance and in instrumental music as well as in poetry and song, metre as conceived here belongs exclusively to patterns of words. The basis of metrical analysis is not in the relation of a strong beat to a weak one, but purely in the relative durations of syllables, without reference to the rhythmic categories of arsis and thesis. Syllables themselves acquire their durations from characteristics of their constituent letters and the ways in

which these are sounded in combination. Hence the first three parts of metrics, listed in chapter 20, deal with the elements (letters or simple sounds, chapter 20), syllables (chapter 21) and the classes of metrical foot formed by combinations of syllables (chapter 22). Feet are in turn combined into metres (the fourth part of the subject, chapter 23). Particular classes of metre are discussed individually in chapters 24–5, and their compounds and combinations in chapters 26–8. The fifth part of the subject deals from a metrical point of view with the structure of the whole that is formed by a sequence of 'metres' or metrical 'lines', that is, the poem (chapter 29). Aristides' account of these matters follows a well established tradition of metrical scholarship that had been developed in the context of literary, rather than musical analysis, and is not directly connected with Aristoxenian writings, but it stands, on the whole, consistently enough beside his doctrines about rhythm. Two important figures in this rather dry and dusty field of scholarship were Hephaestion and Dionysius of Halicarnassus, and they are probably the main sources for this part of Aristides' work.

Questions about the value of music in education and in psychiatric therapy, the subject of Book II, cannot be answered, Aristides says (chapter 1), without some understanding of the nature of the soul. His account of the soul is not set out all at once, but in three phases (chapters 2, 8, 17, with further elaborations in Book III), each well accommodated to the current level of his educational analysis. Chapter 2 concentrates on a largely Platonic distinction of the soul's parts into the rational and the irrational, the latter related to its involvement with things of this world and with the body, the former dependent on memory of the perfect beauties of the pure realm from which it came. Two corresponding forms of education are identified in chapter 3, philosophy to cultivate the rational part, music the irrational. Thus the existence of a proper educational role for music is given a preliminary explanation. Chapter 4 explores the basis of music's effectiveness, which comes through its powers of 'imitation', *mimēsis*: the forms of 'conception' (*ennoia*), diction, melodic structure (*harmonia*) and rhythm incorporated into music have uniquely direct and vivid similarities to actions, characters and emotions. Chapter 5 identifies the role of music more closely as the cure or redirection of emotions, each class of which belongs to its own part of the soul, and is characteristic of people of different sexes and ages. Music works by inducing in the sufferer a moderate and well-proportioned emotional condition of the same sort as that by which it has been excessively affected. In the long chapter 6 Aristides seeks to confirm his claims for the educational value of music through a discussion of the practices of earlier times. He comments on the doctrines of Plato's *Republic*, and reflects critically on views expressed in the *De Re Publica* of Cicero. He adds observations drawn from the musical culture of various ethnic groups, designed to demonstrate further the ways in which different musical usages create different forms of character and behaviour, both in individuals and in political communities, and to show how music, properly cultivated, creates social concord and human friendship.

These chapters have established the possibility and sketched the foundations of the educational uses of music. Aristides now turns to details. What kinds of melody and rhythm, he asks in chapter 7, will suitably discipline the emotions? 'Suitability' must in fact be sought in four areas, already mentioned in chapter 4: conception (*ennoia*), diction, *harmonia* and rhythm. But first a fuller account of the soul is needed, to explain the sources of its passions and the ways they vary from person to person. Here (chapter 8) a crucial distinction is introduced which will govern much of the subsequent analysis, the distinction between male and female. This is not just a matter of biological gender. All objects of enjoyment, all qualities of perceptible things and the skills that deal with them, and all traits of character and disposition are classified as either male or female, or as some combination of the two. When the soul desires to be attached to

a body, its desire is specifically for a body of male, female or combined form, and it is from its affinities with one gender or the other that all its passions arise.

These affinities appear at the most basic level in the soul's 'conceptions' of things. By a 'conception' (*ennoia*) Aristides means the way in which the soul represents an object to itself in thought, where this representation includes an affective element of approval or disapproval, desire or rejection. Our conceptions are continually remodelled 'in the light of ideas aroused through words', and it is the moulding of conceptions by words that is the subject of chapter 9. Here, after distinguishing ethical education into two kinds, 'therapy' and 'cultivation', Aristides proceeds to examine in detail a series of rhetorical and poetic devices for presenting and arousing appropriate conceptions. The discussion continues in chapter 10, with a classification of different forms or styles of discourse built up from combinations of conceptions, and of the uses to which each is suited.

Chapter 11 turns to the study of diction, conceived as a quest for appropriateness of articulate sound. The sounds of syllables, words and the rhythmic feet of poetry get their character from those of their constituent letters. These are now carefully described and classified, along with their combinations, and given preliminary characterisations of an ethical and aesthetic sort.

Three chapters (12–14) are devoted to the investigation of the ethical and emotional character of harmonic structures and melodies. The male–female distinction is again fundamental. Aristides begins by explaining that though all melody is 'female' by comparison with rhythm, still melodies and their elements are themselves graded between the more female and the more male. The source of the character of any melodic complex, as of any stretch of speech, lies in the character of its elements, notes in the one case, letters in the other. The differences in character between melodic forms will also give a clue to the nature of those between kinds of instrument, since different styles of melody are appropriate to each. An account of instruments is promised (see chapters 16–19). In chapter 13 Aristides announces his intention of attaching to each note of the system a vowel 'suitable for its vocalisation', suitable in the sense that each note is given a vowel of corresponding 'gender'. The genders of the vowels are scrutinised, and then, in chapter 14, four of them are selected to generate a system of solmisation, in which each is prefaced by the letter 't'. This form of solmisation is met in other sources too, and may be very old, but its connection with a characterisation of notes by their genders is unparalleled, and is quite probably due to Aristides himself. The rest of the chapter reflects on the *systēmata*, *harmoniai* and forms of melody that arise from combinations of notes according to the rules studied in Book 1, discussing their different emotional qualities and the ways in which they can appropriately be used. Chapter 15 discusses rhythms, again associating the categories identified in Book 1, and their combinations, with corresponding emotional effects.

So far, Aristides has concentrated on just one part of the 'practical' branch of musical studies, that of the 'use', in composition, of the structures which the 'technical' branch has analysed. He now turns to the other part, which deals with the nature and value of different modes and means of musical 'expression'. In chapter 16, after a brief discussion of manners of 'delivery' (the bodily actions involved in musical performance), he introduces the topic of instruments, as promised in chapter 12, each type being classified under the categories of male and female. There follows an important paragraph describing the different ways in which musical therapy can suitably be applied. Chapter 17 returns, once again, to the soul, offering an elaborate account of its constitution and of the manner in which it builds for itself a body out of elements from each region of the universe. The purpose is to explain how it is that the soul is sympathetically affected by the activity of instruments. The theme is developed further

in chapter 18, which reflects on the phenomenon of sympathetic vibration, and on resemblances in kind between properties of instruments and those of elements drawn by the soul from each cosmic region. Chapter 19 confirms these views about instruments by reference to the theories, legends and practices of antiquity, and to the customs and characters of various races (the plan of this chapter parallels that of chapter 6). This completes Book II. (The MSS add a paragraph, certainly from a different source, on the characters of the three melodic genera.)

It is evidently to his second book that Aristides gave the most careful and imaginative attention. Many of its central ideas, admittedly, can be found in earlier writers, but he has developed them in his own way, coherently and elegantly, and the book deserves to be treated as a serious, ingenious and substantially original contribution to its field. By contrast, though Aristides makes some effort to organise the borrowings that constitute the bulk of Books I and III within his overall plan, this 'framing' is loose, and on the whole merely collects and orders the ideas that are placed inside it, rather than transforming them.

Book III contains Aristides' account of the 'physical' domain of musicology, divided, as explained in Book I ch. 5, into number theory and the field specifically called 'physics'. The former study occupies chapters 1–6; chapters 7 and 8 are transitional, though still focussing principally on numbers; the remaining nineteen chapters (9–27) fall under the heading of 'physics'.

In chapter 1 we are told of the discovery and analysis of the musical ratios; much of the story is familiar from other documents in the 'Pythagorean' tradition. Chapter 2 describes the monochord, and gives a method of constructing a 'division of the *kanōn*'. Chapter 3 gives a brief account of the *helikōn* (more fully described in 11 Ptol. *Harm.* Book II ch. 2). In chapter 4 Aristides offers a geometrical explanation of the restriction of concordant relations within the octave to three kinds of ratio. Chapter 5 approaches the issue from a numerical point of view, analysing concordant relations in terms of arithmetic, geometric and harmonic proportion. Up to this point the majority of Aristides' ideas have come from the mainstream of mathematical harmonics, relying heavily (but not exclusively) on doctrines derived from Plato. Chapter 6 discusses number from a different point of view, attaching to each number from 1 to 12 a symbolic significance of its own. This practice has very ancient roots, but was especially characteristic of early Pythagoreanism. The 'physical' part of the book draws indiscriminately both on the theorems of mathematics proper, and on the mystical numerology of chapter 6.

Chapter 7 seeks to explain how it is that musical relations as we find them in this world reflect the perfect patterns of numbers, but yet reflect them imperfectly. The explanation is cosmological. Things here are 'constituted in imitation of higher things', and whereas the latter belong to a pure and incorruptible region, and represent perfectly the activity of the divine source, the former exist in a region that is 'turbid and murky', where the effects of this activity become confused and faint. Nevertheless, what happens here arises through 'sympathy' with the activity of things in the higher region. Our music has its source in the divine, but the grossness of bodily processes detracts from its precision and accuracy. Chapter 8 reinforces the thesis that numerical relations are essential to music by displaying their involvement, in analogous ways, in all significant kinds of art and skill, and by arguing that 'concordant' numerical proportions underlie good order and successful functioning in every domain.

Chapter 9 proposes an explanation of the correspondence of each kind of musical entity and structure with some element or mode of order crucial to the workings of the universe as a whole. The thesis is not that the organisation of the universe is based on music. It is the converse, that music's patterns of construction are drawn from those of

the cosmos, and that this fact is the basis of music's power. Thus the structures described in Book I, and the powers discussed in Book II, are to be drawn together in an account that shows their dependence on the order of all things and its divine source. The importance of the new step towards this final undertaking is marked by a renewed invocation of the god.

Chapters 10–23 present detailed correlations between musical items on the one hand and elements, structures and entities in the physical realm on the other. In four of these chapters the musical side of the relation is analysed in terms of number. I shall give only a very cursory summary of their contents.

Chapter 10 relates the characteristics of musical 'matter', the movement of the voice, to matter in the universe at large. It then turns to the kinds and qualifications of notes, linking them to the forms and properties of matter. Chapter 11 discusses briefly the three basic concordant *systēmata* (those of the fourth, fifth and octave), corresponding respectively to corporeal, intermediate and incorporeal being, and it goes on to make correlations between the three melodic genera and various triadic divisions of being in general, and of the soul. The opening paragraph of chapter 12 calls on numerological ideas to explain an apparent inconsistency in the preceding discussion. The rest of chapters 12 and 13 are also numerological, linking numbers relevant to music with cosmic entities, relations and activities, and especially with the behaviour of the moon. Chapter 14 discusses the five tetrachords of the system, relating them to the five sense organs and the five primary elements. Chapter 15 undertakes a parallel analysis for three structures spanning a fifth, and for the two octaves. Chapters 16 and 17 are concerned with the human soul. The former relates the fourths, fifths and octaves discussed in chapters 14–15 to the psychic virtues. The latter associates the progression from the lower octave of the system to the conjunct tetrachord *synēmmenōn* with the effortless development from early life to an easy path of vice, and the progression into the disjunct upper octave with conversion to the hard road of virtue.

Chapters 18 and 19 reintroduce numerical conceptions, first linking harmonic and rhythmic proportions and ratios with human periods of gestation, then applying the analysis of concordant ratios to the relations between the material elements, and between the seasons.

Chapter 20 returns to the fourth, fifth and octave, correlating the first two with the elements, the last with the planets and their melodious movement. This leads to a general account of the doctrine of the harmony of the heavens, treated as involving real sound (whose imperceptibility, so far as we are concerned, is explained by Aristides in an unusual and characteristic way). We can reach some understanding of it by considering the affinities of heavenly bodies with the notes of the musical system, and these affinities will be explained not in terms of quantitative relations between distances or velocities, as was usual, but by reference to Aristides' pervasive duality of male and female.

The basis is laid down in chapter 21, where the Sun, Moon and planets are first assigned their principal male or female characters, and are linked to seven notes of corresponding quality. Four further notes belong to the four main regions of the Zodiac, while the final seven are attached to the same bodies as are the first, under a complementary analysis of their powers. The whole scheme is saturated with notions derived from astrology. Chapter 22 links each harmonic *tropos* (or *tonos*) with the planet or region of the Zodiac whose note is its origin, and each class of rhythm and of instrument with the planet or region governing the *systēma* most appropriate to it. Chapter 23 is a meditation on the number 12, which is the number of tones in the octave, the sum of the sides of a paradigmatic right-angled triangle, the sum of three terms related in the basic rhythmic ratios, and the number of segments in the Zodiac. The

terms are manipulated arithmetically to generate further calculations about periods of human gestation, and are applied to the analysis of the Zodiac's geometry. It is not surprising, Aristides concludes at the beginning of chapter 24, that a basis can be found for astrological predictions concerning human births.

By the end of the first paragraph of chapter 24, Aristides has effectively completed his account of musical relations among the elements, among the major perceptible phenomena of the cosmos and in the human soul. He turns next to the thesis that the soul of the universe is also structured by relations of 'concordant numbers' (the discussion that follows is closely based on 2.3 Plato *Timaeus* 35–6). This investigation is clearly necessary for his programme. Matter does not organise itself: the systematic musical coherence of perceptible things must ultimately be drawn from an intelligence whose nature is the perfection of order and unity. This intelligence (the 'Craftsman, Form and Unitary Ratio' of Book I ch. 3) is not identical with the world-soul. That soul is its creation or offspring, developed as an instantiation of form that is specifically appropriate to the peculiar conditions of spatiality in the material cosmos.

The derivation of the world-soul from a unitary, intelligible principle is symbolised from one point of view by its representation in terms of numbers, themselves intelligible and incorporeal, and from another by the development of both major aspects of its structure out of the same seminal, indivisible unit. The structure itself is based on two modes of 'progression towards depth', that is, of complexity developed to the point at which it can describe and govern relations in three dimensions. The two progressions are the series of doubles, 1, 2, 4, 8, and the series of triples, 1, 3, 9, 27, on which Plato's construction is founded.

These progressions represent different aspects of the structure informing and animating the universe, analogous to the two primary aspects of the human soul. In the latter, there is a set of psychic impulses that arises directly from the soul's involvement with body. They are irrational and shifting. They can be ordered for better or for worse, but they are indefinite and unstable, inseparable from that which is not in its own nature orderly or perfect, and even the best ordering of them is not something determinate, but shifts with circumstance. The other aspect of the human soul is reason, and though it can be applied to the control and organisation of bodies, it belongs in essence to a different domain. Its perfection lies in pure understanding, and in a form that is changeless and eternal. In the soul of the universe, the series of doubles represents that part of its structure which has to do directly with its organisation of the bodily, changeable, perishable and divisible. Its progression 'towards depth' is to be understood literally: it develops the mode of complexity required to organise bodies in three-dimensional space. The series of triples represents the incorporeal, indivisible and active. Its progression 'towards depth' is an abstract, non-spatial analogue of development into three dimensions, the same order of complexity being detached from the special conditions of bodily existence. Its configurations are those of forms embraced by changeless, eternal and intelligible being, simply as such. These distinctions are the basis of Aristides' detailed reflections on the Platonic construction, to which the whole of chapter 24 is devoted.

Chapter 25 draws a general moral. In view of the divine basis of musical order in the universe as a whole, it is clear that the most fundamental and compelling source of our impulse to make music is divine 'possession' or inspiration, filling us with the beauties of the perfect upper realm, and dispelling the ignorance and madness that arise from the soul's conjunction with body. There is some reason, then, behind the ecstasies of Bacchic rites. The chapter goes on to present summary correlations between the male and female characters of the four vowels used in solmisation, and the natures and activities of the four lower material elements: the letter tau, with which each is prefixed

in the solmisation, symbolises the fifth element, aether, whose activity transmits life. It ends by linking the inseparable thesis and arsis of rhythm with the equally inseparable phases of generation and destruction.

Two chapters remain. Their musical theme is modulation: it is related to the capacity of small changes, 'new starts', in human, political or natural life, to generate major shifts in their course, turning them away from the destiny that would naturally have flowed from their origins. Chapter 26 reflects at some length on the relation between destiny and the possibility of new directions in the course of events, and on various kinds of distinction between what is necessary and what is contingent. Chapter 27 grants that just as striking musical modulations can arise from a shift in a single note, so a small initial effort can radically change a life. But such efforts are unreliable means for escaping the vicissitudes of life in this world. The only secure escape is through philosophy, through conversion to a composed acceptance of all that life can bring. Music, ultimately, is only a preliminary form of instruction, the servant of philosophy, everywhere providing the foundations of understanding whose pinnacles philosophy builds (compare Book II ch. 3). Here, with a brief valediction, Aristides ends his work.

Aristides' treatise was influential in later times, especially the technical analyses of Book I. Its sections on harmonics and rhythmics were used extensively by Martianus Capella in the early fifth century, and in his Latin version became well known in Western Europe during the Middle Ages. Book I was also known in Byzantium, where parts of it were drawn on by George Pachymeres, and much more by Manuel Bryennius. The work was familiar to Renaissance theorists, though no printed edition appeared until Meibom's in 1652. Despite its importance, modern studies have mostly confined themselves to detailed considerations of limited parts of the text, and the only published translation in English is Mathiesen (1983). His introduction and notes are useful especially for the light they shed on the treatise's date, sources and structure; they are less helpful over the interpretation of detail. The translation itself, unfortunately, is quite unreliable (see my review in *Ancient Philosophy*, 4 (1984)). For a bibliographical survey of previous scholarship on Aristides see pages 4–10 of Mathiesen's introduction.

12 The *De Musica*

1 **Book I**

Chapter 1 I am constantly astonished, my most admirable friends Eusebius and Florentius, at the enthusiasm with which the ancient philosophers devoted themselves to every branch of learning, and at the way in which, after discovering some things for themselves and inheriting some which other people had found out, they carried them on to their proper completion, and ungrudgingly explained and bequeathed to their successors the benefits which flow from them. But I marvel at their greatness of mind most especially on the occasions when we discuss music with one another, as we often do. This pursuit was not for them among those that are of merely casual interest, as many who are ignorant about the matter have supposed, especially nowadays: rather, it was held in honour for its own sake, and was also exceedingly admired for its value in relation to the other sciences, to which it offers an account both of a first principle and, one might almost say, of a final objective.[1]

[1] First principle (*archē*) and final objective (*telos*) come together because a description of a thing's essence is also a description of its perfection. As argued in Book III, it is musical

Another merit peculiar to the art, and one which seems to me especially significant, is this. Unlike the others, its usefulness is not thought to be restricted to one subject matter or to a short period of time: every stage of life, life as a whole, and every action can be perfectly ordered only through music. Painting and all similar arts, in their pursuit of visual beauty, yield only a tiny fragment of benefit, and since they are easy for everyone to grasp, they display no increase, as time goes on, in the intricacy of the understanding. Medicine
2 and gymnastics benefit the body, but cannot begin in childhood to confer on those who pursue them the rewards which flow from learning. Dialectic and its counterpart advance the soul towards wisdom, so long as the soul they lay hold on is purified by music, but without music they not only fail to advance it: they sometimes even bring it to ruin.[2] The art we have mentioned is the only one that extends through virtually every subject matter, and lasts through the whole of time, putting the soul in order with the beauties of *harmonia* and setting up the body with decorous rhythms.[3] It provides children with the benefits that melody brings, and to those who are getting older it gives the beauties of metrical diction, and indeed of discourse in general: when they are older still, it explains to them the nature of numbers and the intricacies of proportions, and reveals the *harmoniai* that arise from them in all bodies. Most important and most perfect of all is its capacity to yield the ratios of that which men find hardest to understand, the soul, and not only the individual soul, but the soul of the universe as well.[4] The inspired utterance of Panaceus the Pythagorean bears witness to what I say: he asserts that the task of music is not merely to relate to one another the parts of musical sound, but to bring together in a harmonious relation all natural things.[5] These points will be demonstrated later, as the argument progresses.[6]

Chapter 2 What has principally stimulated me to attempt this treatise is the low opinion which most people have of the subject, and I am determined to show how excellent a branch of learning it is that they unjustifiably despise.

analysis that reveals the complex pattern of relations constituting the harmonious order of the universe, and of its major sub-systems. Music also provides an objective for the sciences in their practical role, by describing the forms of excellence that the sciences seek to promote. This idea is applied especially to excellences of the soul in Book II, and another example is adduced in the medical analogies of Books II and III.

2 'Dialectic' is philosophical reasoning, as, for example, at Plato *Rep.* 531d, *Phaedrus* 266c, etc. Its counterpart is rhetoric, often contrasted with it by Plato (e.g., *Phaedrus* 266c), referred to in this way by Aristotle *Rhet.* 1354a 1, etc. The dangers of expertise in argument, in the absence of musical culture, is another Platonic theme: e.g., *Rep.* 401d–402a, cf. 535a–539e.

3 Here *harmonia* is music's melodic aspect, broadly contrasted with rhythm, but implying the possession of proper 'order'. The thesis that *harmonia* affects the soul, rhythm the body (echoed at 55.4–6) is not meant to imply that rhythms do not influence the soul. See 30.17–24, 40.14–20 with 30.12–15, and Book II throughout, especially chapter 15: cf. Plato's remarks on the relations between music and gymnastics, *Rep.* 410b–412b.

4 The idea of the 'soul of the universe' and its ratios is rooted in Plato (2.3 *Tim.* 35a ff. with context). It is mentioned at 92.3ff., and discussed with direct reference to the *Timaeus* in Book III ch. 24.

5 Nothing substantial is known of Panaceus.

6 See Book III chs. 6ff.

Other sciences are neglected either, like medicine, because they are difficult, or,
like geometry, because most people get no pleasure from them. Music is not to
be rejected for either of these reasons. It presents no great difficulty, and
through its presence it cannot fail to give well-proportioned pleasure to those
who pursue it: it can quickly give benefits to those who are prepared to work,
3 and yield honourable and outstanding pleasure as its fruits. Besides, anyone
who labours in the other arts needs some different activity by way of diversion,
whereas those who study music find their recreation in the work itself, which
brings no less joy to the spirit than profit to the mind. Most people give this
fact no weight, preferring the pleasure which comes from idleness and
ignorance to that which accompanies reason and brings benefit.

It is also, however, because there are those who have brought to the art no little love, but who, because they have not attended to every part of it, passed on nothing of significance to their associates, and earned from them no praise on account of music. Again, it is also because virtually none of the ancient writers put together their accounts of music as a complete whole in a single study: instead, they discussed particular topics piecemeal in disconnected writings.[7] They said nothing about most of its principles and natural causes, and more or less confined their attention to technical matters and to the subject of the use of melodies.[8] But that is enough of preliminaries.

Chapter 3 Now we must set out upon our course, after calling in due form on the god who leads the Muses. For the poets, even though what they are working on is not music – of which they use only a small fraction in their descriptions of deeds of old – nevertheless call upon the Muses, and Apollo their leader. But we shall not be narrating ancient myths with the help of a part of music. We are attempting to present music itself, in its entirety, and to describe its nature and character. We are seeking to reveal all the forms which it takes in sound and all its instantiations in bodies; to reveal whether certain relations to numbers are also ratios of resemblance between it and our most valuable element, the soul; and to explain everything about this whole universe

[7] Aristides speaks again of omissions by earlier authors in Book II ch. 7; he is referring there to the alleged Pythagorean concealment of esoteric knowledge. Here his point is more general: he is probably thinking partly of Aristoxenus (cf. next note), who wrote independent essays on various aspects of music rather than a single compendious treatise, and who did not link them directly with psychological and cosmic themes. Those who did (e.g., Theon Smyrn. and his sources, Nicomachus, etc.) dealt relatively superficially with musical matters. We know of no earlier writer who put together so many elements of the subject in a work of this scale (not even Ptolemy).

[8] 'Technical matters', *technologia*, any expert and systematic discussion of a subject. Here the reference is to the sciences of harmonics, rhythmics and metrics, the contents of Aristides' first book. The 'principles and natural causes' are no part of this *technologia* (see 6.8ff.): they concern mathematics, physics and cosmology, and are represented by the speculations of Book III. All such discussions are treated as irrelevant to harmonics by Aristoxenus (e.g., 7 *El. Harm.* 32.18–28). 'The use of melodies' (that is, the ways in which different kinds of melody may be used for different psychological and moral purposes) occupies much of Book II. Among earlier writers, Aristides might be thinking of Damon and Plato, as well as Aristoxenus, or of more recent authors such as Diogenes of Babylon or Plutarch, or any of a host of others.

4 which a man who advances higher may reveal without violating the principles of music.[9] Upon whom should we first call to aid us in so great an undertaking? Should it not be him who has fitted together harmoniously the whole of this visible world with invisible devices, and devised every soul most perfectly with the ratios of *harmonia*? – whether it is right to call him Craftsman,[10] giving him a title suitably derived from the things he has made, or reverent to call him Form, thus indicating his powers to men through that which he imparts to his subordinates,[11] or proper to call him Ratio or Unit, or, as men of divine wisdom have done, Unitary Ratio, revealing by the first title how he harmonises and orders everything, and by the second how he has arrested the multiplicity and diversity of things, and holds them together in one with unbreakable bonds.[12] Upon him let us call, and pray him to supply us with every secure perception, and to grant us complete leisure to speak adequately of the subject we have undertaken. Let that be a sufficient supplication; and let us at once begin to present our account of the whole of music, as we have promised.

Chapter 4 The science of music concerns melody and everything associated with melody. People also define it as follows: 'the theoretical and practical skill concerned with melody, both complete and instrumental'. Others define it like this: 'the skill that deals with what is appropriate in sounds and in movements'. We, however, will give a more complete definition, in accordance with the aims we have expressed: 'knowledge of what is appropriate in sounds and in the movements of bodies'.[13] It is thus a science, in which there is secure

[9] Lit. 'not unmusically': cf. 107.25–30. Aristides occasionally speaks of music as a 'mystery', some of whose secrets it may be improper to reveal (e.g., 23.22–5). Here it is the inner secrets of the divine order at large which may need to be treated with caution: cf., for example, 65.16–17, 104.3ff.

[10] *Dēmiourgos*, the title of the constructor of the universe in Plato's *Timaeus*.

[11] For God as giver of form compare 104.4–26. The 'subordinates' are probably the lesser divinities (i.e., primarily the stars and planets: cf. 123.6 and Plato *Tim.* 41a), but might include human souls and the contents of the material universe as a whole.

[12] This theme also goes back to the *Timaeus*, e.g., 29e–32c, 47e–52c. The language of the titles was probably developed in the Platonist-Pythagorean tradition of the early centuries A.D.; it reappears in Plotinus and Proclus.

[13] Aristides mentions four definitions of musical science, preferring the last. (i) The first is echoed at Bacchius *Eisagoge* 293.1, where the sequel suggests that 'melody' (*melos*) is restricted to the narrow sense, a succession of notes abstracted from rhythm etc. (compare the opening of 7 Aristox. *El. Harm.* Book 1, where the interpretation of *melos* is uncertain). Aristides wants a definition of wider scope. (ii) The second appears, with 'science' (*epistēmē*) for 'skill' (*technē*), at *Anon. Bell.* 29. The theoretical–practical distinction is Peripatetic, and in this context may go back to Aristoxenus (cf. Cleonides *Eisagoge* 179.3). It is elaborated in chapter 5 below. 'Complete melody' (*teleion melos*) includes melody, rhythm and words: see 5.4, 28.8–10, 30.17–24, *Anon. Bell.* 29 and 46. The addition of 'instrumental' belongs to the same tradition: *Anon. Bell.* 29, cf. 7 Aristox. *El. Harm.* 33.1–4. (iii) A definition in terms of what is 'appropriate' or 'proper' is also given at *Anon. Bell.* 29. The word *prepon*, 'appropriate', is common in aesthetic contexts from Plato onwards (e.g., *Rep.* 397b, 399a, 400b). The conjunction 'sound (or 'voice') and movement' is also found in Plato, *Laws* 665a. (iv) This translates Winnington-Ingram's emendation of the text: the MSS have '...appropriate in bodies and movements'. The emendation is persuasive. The definition (which may be Aristides' own) is then 'more complete', in that it states which movements are intended, and it is 'in accordance with the aims we have expressed', in that it reflects the requirement that

and infallible knowledge: for it will admit no change or alteration in the statements which it contains either in the role of propositions or in that of conclusions.[14] At the same time we can reasonably call it a skill, since it is an organisation [*systēma*] of perceptions, and of perceptions trained to accuracy:[15] nor, as the ancient writers understood and our discussion will show, is it useless for the conduct of life. That it concerns 'complete' melody is to be expected, since melody, rhythm and diction must all be considered, if it is to express fully the completeness of song.[16] In respect of melody in the strict sense, this involves sound qualified in certain ways; in respect of rhythm, the movement of sound;[17] in respect of diction, the metre:[18] while the things involved in 'complete' melody are the movement of sound and of body, and durations of time and the rhythms which arise from them.[19] That it is a skill concerned with what is appropriate is not surprising, since nothing that lacks propriety is of any account: for the proper consists in the communication, by things that are noble and praiseworthy, either of order or of mutual concordance, to things that are of some significance.[20] Music is described as practical and theoretical for the following reasons. When it considers its own elements and concerns

the science should reveal 'all the forms which music takes in sound, and all its instantiations in bodies' (3.26ff.): cf. 5.19.

14 'Propositions': *problēmata*, often 'problems', but 'propositions' at, for example, Aristotle *Topics* 101b 28, and consistently in Aristoxenus. Its etymology, and its uses in 7 *El. Harm.* (44.4, 11, 60.17, 62.19, 68.13, 22), suggest the sense 'propositions used as starting points in discussion', perhaps also at Plato *Rep.* 530b and *Theaet.* 180c. *Apotelesmata*, 'conclusions', is not a technical term, but means 'results' rather generally. Presumably Aristides has in mind remarks such as 7 *El. Harm.* 43.25–44.20.

15 The general idea is Aristoxenian (cf. 7 *El. Harm.* 33.1–26), but has passed through other hands. The word translated 'perception' is *katalēpsis*, a Stoic coinage for the direct grasp of an object by the mind.

16 On 'complete melody' (*teleion melos*) see n. 13 above. The threefold distinction goes back at least to Plato (*Rep.* 398d: it forms the background to *Laws* 653–73, where numerous complications are introduced: cf. also Aristotle *Poetics* 1447a 22).

17 This expression is not often used to refer to rhythm: in the Aristoxenian tradition it normally applies to movement in pitch (the *locus classicus* is 7 *El. Harm.* 8.13ff., cf. 3.5ff.). Rhythm is not here said necessarily to involve movement of body, but this is reintroduced below (similar apparent vacillations exist in Plato *Laws* 653ff.). Perhaps Aristides means that musical science studies rhythm in the guise of patterns of movement in sound, but that in actual performance, *teleion melos* as the complete musical product, this same rhythm is always instantiated also in bodily movement: cf. *Laws* 672e. This may also be why 'bodily movement' appears again as part of the 'matter' of music in association with references to music's practical aspect, contrasted with the theoretical: see 5.19.

18 Metre has to do with relative lengths of syllables and their ordering, rhythm with patterns of stress and relaxation; hence the latter is applicable also to instrumental music and dancing. See Aristides' rather confused distinctions at 45.20–9. He gives two accounts of rhythmics, the first (32.8–38.14) said to be that of theorists who combine it with metrics (see 38.15–16), the second (38.17–39.25) that of those who distinguish them. The passage at 39.26–40.25 may be reckoned common to both schools. Metrics is treated separately at 40.28–52.23.

19 'Durations of time' represents *chronoi*, sometimes rendered simply 'durations' in the sequel. *Chronos* is the ordinary word for 'time', but is also a technical term of rhythmics: see especially 31.8–9, 32.11–33.11.

20 The text of this last sentence is in some doubt. I suggest that it begins *to gar prepon esti*, and ends, as Jan suggests, *ē hē pros allēla symphōnia*. Winnington-Ingram's emendation to *symphōnias* is, I think, unnecessary.

itself with definitions and technical propositions it is said to engage in theory: when it translates these elements into action in the useful and proper composition of melodies, it is described as engaging in practice.[21]

The material of music is sound and bodily movement.[22] Sound has been described by some as air which has received an impact, by others as an impact which air has received, the former defining as a noise the body itself which has been qualified, the latter, more accurately, the qualification which the body has received.[23] Movement is subject to different durations of time, time being the measure of movement and rest. Some movement is by nature simple, some not simple:[24] and of the latter some is continuous, some intervallic and some
6 intermediate. Continuous sound is that which makes its relaxations and tensions imperceptibly, because of their rapidity. Intervallic sound is that in which the pitches are clearly apparent, while what lies between them is imperceptible. Intermediate sound is a combination of both the others. The continuous type is that by which we converse, the intermediate type that by which we read poetry, while the intervallic is that which makes intervals of definite size between simple sounds, and comes to points of rest. It is this type that is also called 'melodic'.[25]

[21] The distinction is elaborated in chapter 5 below.

[22] See n. 17 above, and contrast 108.18, which has no reference to bodily movement. This technical usage of 'matter' (*hylē*) is Peripatetic.

[23] Aristides prefers Aristotle's account (3.15 *De Anima* 419b–420b) to the looser definitions suggested, for example, at 4.2 ps.-Ar. *Probs* XI.6, cf. 23, 51, 58.

[24] Despite the preceding reference to time, this concerns movement in pitch, not rhythmic movement. Though there is much that is Aristoxenian in the rest of this paragraph and in the complementary passage at 6.25ff., the treatment of rest on a pitch as 'simple movement' is entirely contrary to Aristoxenus' principles (see 7 *El. Harm.* 9.1ff., 12.1ff.). It reappears at 6.27 and at 7.6–7, and belongs rather among Pythagorean conceptions (cf. especially 10 Nicomachus *Ench.* ch. 4, 11 Ptol. *Harm.* Book I chs. 3–4). This may be an instance of Aristides' uncritical conflation of different sources, or perhaps of his (or an intermediary's) confused reading of Aristoxenus, of which there are further signs below (see 6.29ff.).

[25] The ancestor of this passage is plainly 7 Aristox. *El. Harm.* 8.13ff., but there are several differences. The description of 'continuous sound' as making its relaxations and tensions imperceptibly is perfectly comprehensible, but it seems to be an echo of 7 *El. Harm.* 10.11–18, which is part of a description of intervallic movement. This odd reversal occurs also at Cleonides *Eisagoge* 180.13, but in none of the other compilers. At 6.28ff. Aristides has in mind a non-Aristoxenian sense of 'relaxation and tension' (*anesis* and *epitasis*), and this might underlie his unusual treatment, but Cleonides has the Aristoxenian account (*Eisagoge* 180.20–181.6). The description of intervallic sound originates with 7 *El. Harm.* 10.14–15, though in view of what has been said, Aristides can hardly have had the passage in front of him. The association of the continuous with speech, the intervallic with melody, reflects 7 *El. Harm.* 9.21–9: cf. 10 Nicomachus *Ench.* 238.22, Cleonides *Eisagoge* 180.12ff., Porph. *Comm.* 10.1ff. Aristides' reference to an intermediate variety has few parallels: compare 10 Nicomachus *Ench.* 239.8ff., and perhaps 7 Aristox. *El. Harm.* 9.30–3. It would be incautious to base much on Aristides' claims about poetry-reading: he seems to find triadic groupings irresistable (two extremes and an intermediate), even when the intermediate item has little real significance. Here he may be searching for a parallel to the notion of an 'intermediate' between rhythmic and non-rhythmic movement, suggested by references to *parakatalogē* ('recitative') at ps.-Plut. *De Mus.* 1141a, ps.-Ar. *Probs* XIX.6, and to 'melted songs', *kechymena asmata*, at 31.27 below (cf. 51.27), *Anon. Bell.* 3, 85, 95.

Chapter 5 Of music as a whole one part is called theoretical, the other practical. The theoretical is that which carefully examines its technical propositions, both the major theses and those that fall under them, and which studies its highest principles, its natural causes, and its concordant relations to the things that there are.[26] The practical is that which acts in accordance with the technical propositions, and pursues the ultimate objective: it is also called the 'educational'.[27] The theoretical is divided into the physical and the technical. Of these the physical has two parts, one of which is arithmetical, while the other bears the same name as the genus,[28] and also discusses the things that there are: the technical has three parts, harmonics, rhythmics and metrics. The practical is divided into one part which employs the elements just mentioned, and another which expresses them: the parts of the one which employs them are melodic composition, rhythmic composition and poetry, while the one which expresses them has as its parts instrumental performance, vocal performance and acting, in which are included the bodily movements corresponding to the melodies that underlie them.[29]

Movement is a change of qualifications into things of the same genus.[30] The movement of sound has two forms, the multiple and the indivisible.[31] Of the multiple we have spoken already: the indivisible or simple form is called 'pitch'. Pitch is rest or immobility of the voice, and this has two forms,
7 relaxation and tension.[32] Relaxation is when the voice moves from a higher-pitched region to a lower, tension when it shifts from a lower-pitched region to

[26] On the theoretical–practical distinction see n. 13 above. For the distinction between major and subordinate propositions see 7 Aristox. *El. Harm.* 43.25–44.20. The study of 'natural causes' etc. takes the scope of Aristides' project beyond that of Aristoxenus (cf. n. 8 above). 'The things that there are' (*ta onta*) is Aristides' usual phrase for 'the contents of the universe'. He seldom makes explicit the Platonic distinction between things that 'are' (eternally) and things that 'become' (but, see, for example, 133.16), but he is thinking mainly of the former, and specifically of the gods, the soul and the heavenly bodies.

[27] The identification of the 'practical' with the 'educational' reveals what the objective of music is, and why at 5.18 its practical side was confined to the 'useful' composition of melodies. Education is a major preoccupation of Book II. Music's capacity to give pleasure (cf., for example, 60.10ff.) is always subordinate to its role in moral education.

[28] That is, 'physics', the study of *physis*, nature.

[29] Of the two branches of the theoretical, the 'technical' is expounded in Book I, the 'physical' in Book III, while Book II discusses the 'practical' side, the psychological and educational uses of all six of the musical skills listed here. But there is inevitably some overlapping in Aristides' treatment of these topics.

[30] The definition is based on Aristotle: see, for example, *Physics* 224a–225b. *Kinēsis*, translated 'movement', has a wider application than the English expression; it may refer to change of any kind.

[31] These are the forms referred to as 'not simple' and 'simple' in chapter 4. The present terminology is not found in this context elsewhere.

[32] By describing pitch (*tasis*) first as 'simple movement' and then as 'immobility', Aristides is uttering a non-Aristoxenian and an Aristoxenian sentiment in one breath (see n. 24 above). From Aristoxenus' point of view, the treatment of relaxation and tension as forms or species of pitch is also wrong. It is precisely the error he warns us against at 7 *El. Harm.* 10.11–13.30 (compare the clear summary of Aristoxenus' position at *Anon. Bell.* 36–38). But Aristides' next sentence represents Aristoxenus' views correctly.

a higher. The results of these movements we call depth and height respectively.[33] Depth occurs when the breath is carried up from low down, height when it is projected from the surface.[34] Every simple movement of sound is a pitch, but that of melodic sound is given the special title, a note.[35] The note is what we must next discuss, after remarking as a preliminary that harmonics as a whole has seven parts. It deals with (i) notes, (ii) intervals, (iii) *systēmata*, (iv) genera, (v) *tonoi*, (vi) modulations, (vii) melodic composition.[36] Let us then speak first of notes. Forgive me if I use some words that are hard to understand: the special terminology is demanded by the technicality of the subject.

Chapter 6 A note, then, is the smallest part of melodic sound.[37] The functions (*dynameis*) of notes are by nature unlimited in number, but those handed down to us, in each of the genera taken together, are twenty-eight.[38] These are their names: *proslambanomenos, hypatē hypatōn, parhypatē hypatōn, enharmonios hypatōn, chrōmatikē hypatōn, diatonos hypatōn, hypatē mesōn, parhypatē mesōn, enharmonios mesōn, chrōmatikē mesōn,*

[33] *Oxytēs* (lit. 'sharpness') and *barytēs* (lit. 'heaviness'), as usual in Greek. That they are products of tension and relaxation, not identical with them, is the doctrine of 7 Aristox. *El. Harm.* 10.27ff., cf. Cleonides *Eisagoge* 180.20ff.

[34] Aristides discusses the physiological causes of different kinds of sound at 81.7ff.

[35] *Phthongos*: compare 7 Aristox. *El. Harm.* 15.15–21, and, for example, Cleonides *Eisagoge* 179.9–10. Melodic sound has been identified with intervallic at the end of chapter 4. Aristides means that the term 'pitch' applies to any 'stationary sound' or 'simple movement of sound', while 'note' is reserved for a sound of this sort occurring in the context of a series of others, separated by intervals. For the distinction see also 9.1, 9.2 Thrasyllus and Adrastus ap. Theon Smyrn. 47.18ff., 49.6ff.

[36] The list corresponds to that of Aristoxenus' second book, 7 *El. Harm.* 35.25–38.26, though the order is different. Most later compilers have this list in one order or another (e.g., *Anon. Bell.* 20, Cleonides *Eisagoge* 179.6–8), rather than that of *El. Harm.* Book I (cf. ps.-Plut. *De Mus.* 1142f), which excludes composition. Its inclusion is at first sight inconsistent with Aristides' location of melodic composition on the 'practical' side at 6.21, but the subject in fact divides into the theoretical study of melodic forms, considered in chapter 12 of the present book, and the analysis of their use for different practical purposes, considered in Book II. (Cleonides circumvents the problem in a different way, by defining harmonics itself as both theoretical and practical, *Eisagoge* 179.3–4.) Aristides now proceeds to devote one chapter to each of the seven departments of harmonics.

[37] The definition is not the commonest one: most sources give a version of 7 Aristox. *El. Harm.* 15.15–16, '...the incidence of the voice on one pitch' (e.g., 10 Nicomachus *Ench.* 261.5, Cleonides *Eisagoge* 179.9, and many others). But there are parallels with the present definition at Porph. *Comm.* 86.26, Bacchius *Eisagoge* 292.22, and compare, for example, the (possibly interpolated) passage at 10 Nicomachus *Ench.* 261.4, *Anon. Bell.* 21 and 49, Porph. *Comm.* 81.20. It is not out of place in an Aristoxenian context, and may have originated with his 'harmonicist' precursors.

[38] The Aristoxenian doctrine to which this is related is summarised at Cleonides *Eisagoge* 181.10: 'notes are unlimited in respect of pitch, but in respect of *dynamis* there are eighteen in each genus'. On the contrast between an indefinite number of pitches (and of magnitudes of interval between them) and a determinate number of *dynameis*, compare 7 Aristox. *El. Harm.* 47.8ff., 68.13ff. Aristides has perhaps misunderstood a legitimate occurrence of the claim that the number of notes is *dynamei apeiron*, as at 11 Ptol. *Harm.* 57.15, where it means 'potentially unlimited', taking it to mean 'unlimited in respect of function' (the use of *dynamei* at Cleonides). But the notion that the number of *dynameis* is not unalterably fixed by nature may be hinted at in the cautious phrasing of 7 *El. Harm.* 53.25–32.

diatonos mesōn, mesē, tritē synēmmenōn, enharmonios synēmmenōn, chrōmatikē synēmmenōn, paranētē synēmmenōn, nētē synēmmenōn, paramesos, tritē diezeugmenōn, enharmonios diezeugmenōn, chrōmatikē diezeugmenōn, paranētē diezeugmenōn, nētē diezeugmenōn, tritē hyperbolaiōn,
8 *enharmonios hyperbolaiōn, chrōmatikē hyperbolaiōn, paranētē hyperbolaiōn, nētē hyperbolaiōn.*[39]

Proslambanomenos is so called because it does not belong to any of the named tetrachords, but is adopted in addition outside them because of the concord it forms with *mesē*.[40] It stands to *hypatē hypatōn* in the ratio of a tone, the ratio in which *mesē* stands to *paramesos*.[41] *Hypatē hypatōn* is so called because it is placed first in the first tetrachord, since the word '*hypatos*' was used by the ancients to refer to what is first.[42] *Parhypatē* is the note which lies beside *hypatē*. *Enharmonios, chrōmatikē* and *diatonos hypatōn* are the notes which reveal the genera of melody, for there are various different arrangements of the tetrachords.[43] Taken as a class they are called *hyperhypatai*.[44] *Hypatē mesōn*, once again, is the first note of the tetrachord *mesōn*, [which is so called] because it is the only tetrachord recognised as lying between the tetrachords *hypatōn* and *synēmmenōn*.[45] *Parhypatē mesōn* is the note which follows it, and the next ones are like those in the tetrachord *hypatōn*.[46] As a class they are

[39] The number of notes listed by our various sources depends (a) on whether they put in the same series notes proper to each of the three genera, and (b) on whether they assign different names to the moveable notes corresponding to one another in different genera. Aristides gives only one name for the lower moveable note in each tetrachord, but three for the higher one: it is the latter which he treats as determining or 'revealing' the genus of a melody (8.10–12 below). His names for the notes fall within a common range of variants: notice particularly '*paramesos*', a form that appears at 10 Nicomachus *Ench.* 258.25 and often later, but is *paramesē* in all earlier sources except 8 Eucl. *Sect. Can.* proposition 19, whose date is a matter of controversy.

[40] Cf. 10 Nicomachus *Ench.* 258.2. The name *proslambanomenos* means 'taken in addition', since it is an 'extra' note beyond the lowest tetrachord.

[41] Whereas Aristides' talk of 'semitones' (e.g., 8.21) is casually Aristoxenian, this treatment of intervals as ratios is Pythagorean.

[42] Compare the fanciful explanation given at 10 Nicomachus *Ench.* 241.18ff.; see n. 20 on that passage, and contrast Plut. *Quaest. Plat.* 1008f.

[43] In most sources it is the size of the highest interval in the tetrachord, and that of the lower two taken together, that primarily determine the genus, the position of the lower moveable note being less important: cf., for example, 7 Aristox. *El. Harm.* 23.2ff., 11 Ptol. *Harm.* 49.9–13. The notations give different symbols for the three generic versions of the upper moveable note, only one for the lower. Only 10 Nicomachus *Ench.* ch. 12 distinguishes two *parhypatai*.

[44] The word occurs elsewhere (e.g., 9.4 Thrasyllus ap. Theon. Smyrn. 88.18), but rarely, and means 'beyond *hypatē*': cf. the use of *hypermesē* for *lichanos* at 10 Nicomachus *Ench.* 272.5. Other sources use *lichanos hypatōn* as the common name of the three notes mentioned here. But *lichanos* ('forefinger') is a practical musician's word. Perhaps some theorists preferred the more scholarly-sounding terms, which brought these notes' names into line with those of the others. See also the Appendix to chapter 1.

[45] *Mesōn* means 'of middle (notes or strings)'. Aristides seems to mean that it is the central tetrachord of the LPS.

[46] The Greek uses the adjective *hypatoeidēs*, a term that sometimes means 'belonging to the tetrachord *hypatōn*' (as here and at 81.21–2), sometimes 'having the position of *hypatē* in any given tetrachord' (as at 9.21, cf. 10.15, Bacchius *Eisagoge* 302.8), sometimes merely 'low in the vocal range' (as at 28.12, 30.4, 10, cf. 10 Nicomachus *Ench.* 245.12, 256.3).

called *lichanoi*, named synonymously with the finger which plucks the string that sounds them.[47] The note which follows them is called *mesē*, because it lies in the middle of the series of notes, as they are set out in each of the *tropoi*.[48] If you go up a semitone you reach *tritē synēmmenōn*, so called because in the tetrachords after *mesē* we count from the last note, now that we have arrived at the higher *systēmata*.[49] The ones which follow it are called *enharmonios*, *chrōmatikē* and *diatonos*[50] for the reasons given above: they are also called *paranētai*, because they lie immediately before *nētē*. Next to them is *nētē*, that is, 'last', since '*neatos*' was the word used by the ancients to refer to what is
9 last.[51] This whole *systēma* is called *synēmmenōn* because it is conjunct with the complete *systēma* which precedes it, the one ending with *mesē*.[52] If we return to *mesē* and go up by a tone, the note which lies beside it is called *paramesos*. The ones that follow it are given their names on the same basis and for the same reasons as those in the tetrachord *synēmmenōn*. This *systēma* is called *diezeugmenōn* because it lies upon different parts, and not equally with the *systēmata* that precede it.[53] Then there is *tritē hyperbolaiōn* and the notes which follow it, which are given their specific names on the same basis and for the same reasons as are the preceding ones. The name for the class generally is *hyperbolaiōn*, because it is in them that the power of the human voice finds its limit and comes to a halt.[54]

Of these notes some are fixed, others moveable; and some are *barypyknoi*, some *mesopyknoi*, some *oxypyknoi*, some *apyknoi*.[55] A *pyknon* is a disposition

[47] For the explanation compare 10 Nicomachus *Ench.* 258.16. Aristides is unusual in restricting the term's application to notes in the tetrachord *mesōn*. Most authors also apply it to the equivalent notes in the tetrachord *hypatōn*. See n. 44 above.

[48] Here, as often in Aristides, *tropos* is synonymous with *tonos*: see chapter 10 below.

[49] This explains the word *tritē* ('third'); the note is third from the top of the tetrachord, in a scale of any one genus. *Systēmata* here are tetrachords: cf. 9.1.

[50] This note was called *paranētē* in Aristides' original list.

[51] Compare 10 Nicomachus *Ench.* 241.22.

[52] *Synēmmenōn* means 'of conjoined (notes or strings)'. This tetrachord, the highest of the LPS, is conjoined with its predecessors in the sense discussed at 7 Aristox. *El. Harm.* 58.14ff. The preceding *systēma* is 'complete' because it is an octave: cf. 14.15–18.

[53] 'Upon different parts' represents the Greek literally; its sense is obscure, and there is no parallel to this explanation of the term *diezeugmenōn* ('of disjoined notes'). Conceivably the expression might mean 'disposed differently in the arrangement of its parts' (see LSJ, under *epi*, c.2.e), but this would not be appropriate. Perhaps Aristides means no more than 'apart': cf. 113.28–114.3. His later explication of disjunction (13.13–14.2) is odd in a different way.

[54] *Hyperbolaiōn* means 'of (notes or strings) thrown beyond the others', i.e., 'additional', or 'extreme'. For the thesis that the voice has a two-octave range see 21.12ff.

[55] Cleonides also makes these distinctions, after his list of notes (*Eisagoge* 185.16). The four Greek terms indicate, respectively, the lowest, middle and highest note of a *pyknon*, and a note that is no part of a *pyknon*: cf. Cleonides 186.1ff. They do not occur in Aristoxenus, but compare 7 *El. Harm.* 70.15ff. In Aristoxenus the term *pyknon* is applied to the lowest pair of intervals in a tetrachord only in enharmonic and chromatic, not diatonic (see, for example 7 *El. Harm.* 24.11ff., 25.6ff.). Hence, for him the terms *barypyknos* etc. are insufficiently general to be of much use. Most sources follow him, and Cleonides 186.9 therefore adds *diatonoi* to complete his classification. But Aristides' definition of *pyknon* in the next sentence is vague enough to include diatonic groupings, and this usage is explicit at Bacchius *Eisagoge* 300.4, cf. 298.1.

of three notes that has a certain quality. The notes holding the first position in the *pyknon* are *barypyknoi*, those holding the middle position are *mesopyknoi*, and those holding the last position are *oxypyknoi*. Those which have no place in the organisation of the tetrachord with respect to the *pyknon* are *apyknoi*.[56] Of these the *apyknoi* and *barypyknoi* (which are also called *hypatoeideis*) are fixed notes, because they do not admit a variety of pitches; and the remainder are moveable, since they sometimes display a smaller and sometimes a larger interval, according to the quality of the way in which the tetrachords are put together. Of these moveable notes some are called *parhypatoeideis*, others *lichanoeideis*.[57]

Again, some notes are concordant with one another, some discordant, some in unison. Concordant notes are those such that when they are struck simultaneously, the melody is no more conspicuous in the higher than in the lower: discordant notes are those such that when they are struck simultaneously the character of the melody properly belongs to one or the other:[58] and notes in unison are those whose sounds have different functions [*dynamis*] but which have equal pitch.[59]

Notes have other distinctions as well; first in respect of their pitch in height or depth, secondly that in respect of their participation in intervals, when one

[56] The only *apyknoi* are those that are the outer limits of the Perfect Systems, *proslambanomenos*, *nētē synēmmenōn* (in the LPS) and *nētē hyperbolaiōn* (in the GPS): see Cleonides *Eisagoge* 186.5–7. *Mesē* is no part of a *pyknon* when it has the tetrachord *diezeugmenōn* above it, but is a *barypyknos* in relation to the tetrachord *synēmmenōn*.

[57] Compare n. 46 above on the term *hypatoeidēs*. The three such terms used here are found in this connection only in Aristides (cf. 10.15) and at Bacchius *Eisagoge* 302.8 (but cf. 7 Aristox. *El. Harm.* 26.17–18). As alternatives to *barypyknos* etc. they have the advantage of avoiding references to the *pyknon* (see n. 55 above).

[58] Aristoxenus offers no description or definition of concord or discord, taking them apparently as auditory 'givens'. Those writers who attempt definitions (by contrast with physical or mathematical explanations) almost always invoke the notion of a 'mixture' or 'blending' to describe concord, 'separation' to describe discord. Two notes in concord are heard as a single blended phenomenon, distinct from the mere sum of the two sounds, while two discordant notes do not blend, and are heard simply in their own characters. The idea goes back at least to 2.5 Plato *Tim.* 80b and 3.16, 3.12 Aristotle *De Anima* 426b, *De Sensu* 447a–b, 448a: see also e.g., 6 Theophrastus ap. Porph. *Comm.* 63.15ff., 8 Eucl. *Sect. Can.* 149.17ff., 9.7 Aelianus ap. Porph. *Comm.* 35.26ff., 10 Nicomachus *Ench.* 262.1ff., Cleonides 187.19ff., Bacchius 293.8ff., Gaudentius 337.8ff. Aristides seems to suggest a similar concept, but with different emphasis. His account suggests that when a concord is played the melody is as it were ambiguated, belonging no more to the one note than to the other – perhaps belonging properly to the 'third thing' that is the undifferentiated blend of the two. There are hints of such an idea in Aelianus, but the passages cited from Bacchius and Gaudentius give clearer parallels.

[59] *Homophōnos* means 'in unison' in most authors. The main exception is Ptolemy, who uses it for the octave. The distinction between pitch and *dynamis* is Aristoxenian (e.g., 7 *El. Harm.* 36.7ff.), though few writers use it in this context (cf. perhaps 10 Nicomachus *Ench.* 263.3, Gaudentius 350.12). Aristides is thinking of the different functions that may attach to the same pitch in different genera, in the different Perfect Systems, and in different *tonoi*. Each of the notes named in the preceding list has a different function, but some may share the same pitch (e.g., diatonic *parhypatē mesōn* may have the same pitch as enharmonic *lichanos mesōn*; without a change of genus, diatonic *tritē diezeugmenōn* has the same pitch as *paranētē synēmmenōn*).

is understood to belong to a single interval, another to several,[60] thirdly that in respect of their participation in *systēmata*, when one belongs to a single *systēma*, another to two, fourthly that in respect of range [*topos*] of sound, when one belongs to a larger, another to a smaller range (we shall explain later what ranges of sound are),[61] and fifthly that in respect of character: for the characters that flow upon the higher notes are different from those that flow upon the lower ones, and those that flow upon the *parhypatoeideis* are different from those that flow upon the *lichanoeideis*.[62]

Chapter 7 The term 'interval' is used in two ways, one general and one specific. In general, an interval is any magnitude bounded between limits: in the usage specific to music, an interval is a magnitude of sound circumscribed by two notes.[63] Of intervals some are composite, some incomposite, the incomposite being those bounded by successive notes, the composite those bounded by notes which are not successive, and capable of being divided in a melody into several intervals.[64] Of these intervals the smallest, so far as their use in melody is concerned, is the enharmonic diesis,[65] followed – to speak
11 rather roughly – by the semitone, which is twice the diesis, the tone, which is twice the semitone, and finally the ditone, which is twice the tone.[66] Again,

[60] The translation of the opening clause is a little generous to Aristides, since it probably means merely 'There are several distinctions between notes'. He is drawing on a new source, whose information he adds as though he had not already set out to discuss such distinctions. The first distinction is clear; the second is not. Probably it refers in effect to the distinction between moveable and fixed notes: each note 'belongs' to the interval that determines its precise relation to some fixed point, and in this sense, for example, *lichanos* 'belongs' to several different sizes of interval (the various sizes of the interval that separates it from *mesē*). Compare the expression 'ditone *lichanos*' at 7 Aristox. *El. Harm.* 23.3.

[61] Probably *systēma* here means 'tetrachord', as at 8.24, 9.1. Then any note at the junction of two conjoined tetrachords belongs to two, all others to one only (cf. 13.12–13). The 'ranges' should be those within which the pitch of a moveable note can lie: see 7 Aristox. *El. Harm.* 22.25ff. But then Aristides' promise to discuss the subject later is not kept. Possibly he misunderstood his source's reference to 'range' (*topos*), construing it as 'level of pitch': in that case he may be looking forward to 29.2ff. But that has nothing to do with larger and smaller ranges.

[62] On *parhypatoeideis* etc. see n. 57 above. That distinctive character, *ēthos*, attaches to different notes is a theme of Book II; see especially chs. 12–14, and cf. Book III ch. 10. See also *Anon. Bell.* 27 and Bacchius 310.14, 311.4, 24, cf. 292.11 (without Jan's emendations).

[63] Like most Greek definitions of 'musical interval' (*diastēma*), this is Aristoxenian, related to 7 *El. Harm.* 15.24–32. There are similar lists of distinguishing features of intervals at *El. Harm.* 16.16ff., Cleonides 187.3ff., *Anon. Bell.* 58. Aristides' rather disorganised discussion adds two more to the Aristoxenian list, the distinction between odd and even, and that between open-textured and dense.

[64] See 7 Aristox. *El. Harm.* 29.25–9, 60.10–61.4, cf. 58.14–60.9. Aristides' account is close to that of Cleonides 188.4ff.

[65] That is, it is the smallest interval used as a melodic step, but not the smallest used in theoretical analysis. See 7 Aristox. *El. Harm.* 14.21–5, cf. 25.11–25. By 'these intervals' Aristides presumably means 'incomposite intervals', though composite ones have been mentioned more recently.

[66] The list is indeed 'rather rough', in that it takes no account of sizes of interval occurring in several Aristoxenian 'shades' of chromatic and diatonic. But possibly this roughness is in the implication that tones and semitones can be exactly halved, which in Pythagorean theory they cannot (e.g., 8 Eucl. *Sect. Can.* propositions 16 and 18); compare 12.10–11.

some intervals are smaller, others larger; some are concordant, others discordant; some are enharmonic, some chromatic and others diatonic; some are rational, others irrational. Rational intervals are those of which it is possible to express the ratio (where by 'ratio' I mean the numerical relationship between one term and another), irrational intervals those for which no ratio between the terms is found.[67] Thus the ratio of the fourth is epitritic, that of the fifth is hemiolic, that of the octave is duple, and that of the tone is epogdoic.[68] Concordant and discordant intervals are to be understood in the way we explained when speaking of notes.[69] We shall discuss enharmonic intervals, and the rest, at the proper time.[70]

Some intervals, such as the diesis, are incomposite, others, such as the fourth, are composite, while others again, such as the semitone and the tone, are both composite and incomposite.[71] Further, some of them are even, others odd, the even being those which are divided into equal parts, like the semitone and the tone, the odd those which are divided into unequal parts, like the intervals of three, five and seven dieses.[72]

Intervals are put together in the following way. Two dieses are placed in succession, but not more: two semitones are placed in succession, but not more: two tones are placed together, but not more; otherwise the total becomes discordant.[73]

[67] Aristoxenus has a distinction between rational (*rhēton*) and irrational (*alogon*): see 7 *El. Harm.* 16.30 with note. It involves complexities which his followers may have misunderstood: cf. Cleonides 189.2, Gaudentius 328.17. Aristides seems to be using a source where the distinction is mentioned but not explained. The account offered here belongs to Pythagorean or 'mathematical' harmonics. To say that there is 'no ratio' between two terms probably means only that there is no ratio of a satisfactorily simple sort: see 3.11 Aristotle *De Sensu* 439b–440a, with Porphyry's comments at *Comm.* 152.7ff.

[68] These ratios are of course standard in Pythagorean sources. Their expression by adjectives rather than numerals is also usual: epitritic ratio is 4:3, hemiolic 3:2, duple 2:1, epogdoic 9:8. See n. 6 to 8 Eucl. *Sect. Can.* 149.14–16.

[69] See 10.2–5.

[70] See 15.21ff.

[71] This returns to the distinction made at 10.19–23. The last clause reflects the fact that, for example, the semitone is incomposite in diatonic scales, composite in enharmonic. Compare Aristoxenus' remarks on the ditone, 7 *El. Harm.* 60.16ff.

[72] This distinction rests on the assumption that the enharmonic diesis is the common measure of all melodic intervals. The assumption predates Aristoxenus (cf. 2.1 Plato *Rep.* 531a, 3.8 Aristotle *Metaph.* 1016b 18), who uses the distinction in a polemic against his rivals and predecessors at ps.-Plut. *De Mus.* 1145c. It is out of place in his own system, since his analysis of the shades of the genera involves such intervals as $\frac{1}{3}$ tone, which it cannot accommodate, and in one shade (hemiolic chromatic) divides the interval of three dieses, treated by Aristides as 'odd', into two equal steps of $\frac{3}{8}$ tone each (7 *El. Harm.* 51.1–4). It is presumably one of the distinctions he passes over as 'not being useful' (*El. Harm.* 16.32–4). It is alluded to by Ptolemy (11 *Harm.* 61.15–16).

[73] This paragraph depends ultimately on the arguments of 7 Aristox. *El. Harm.* Book III. For the proposition about dieses see *El. Harm.* 62.34ff., cf. 71.23ff.; that about semitones is not directly proved by Aristoxenus, but 62.34ff. and 65.8–9 cover all cases, and it plainly follows from his principles. That about tones does not: three consecutive tones occur in diatonic (see *El. Harm.* 65.3–4). Aristides is presumably thinking of sequences inside a single tetrachord. Then his last phrase probably means that three tones exceeds the boundaries of the concord of a fourth, not that such a sequence breaks Aristoxenus' 'law of fourths and fifths' (e.g., *El. Harm.* 54.2ff.), which it does not.

Some intervals are open-textured, others dense. The smallest, such as dieses, are dense; the greatest, such as the fourth, are open-textured.[74]

12 There are various divisions of the ditone. The first is that into twenty-four twelfth parts of a tone, the second that into eight dieses or fourth parts of a tone, the third that into six third parts of a tone, the fourth that into four semitones – that is, eight dieses.[75] This last was the way in which the ancients put together their *systēmata*, making each string the boundary of a diesis.[76] 'Diesis' was the name given to the smallest interval of sound, being as it were the vanishing-point of sound.[77] 'Tone' was the name given to the first magnitude that 'stretches' the voice;[78] and 'semitone' is the name of that which is half either of a tone, or of something approximating very closely to a genuine tone – for people deny that the tone can be equally divided, and this claim may perhaps be correct.[79]

I have set out below the *harmonia* divided by dieses which is found in the ancient authors: it makes the first octave through a total of twenty-four dieses, and proceeds in the second by steps of a semitone each.[80]

[74] 'Open-textured' translates *araia*, 'dense' *pykna*. The distinction seems not to be identical with that of Aristoxenus between *pykna* and *apykna systēmata*, which he applies only to the lowest two intervals of a tetrachord taken as a pair (e.g., 7 *El. Harm.* 24.11ff., 25.21ff., 29.1–2, 48.23ff.). He never describes a single interval as *pyknon* or the contrary. It is probably another pre-Aristoxenian distinction (perhaps connected with the procedure of *katapyknōsis*), rejected by Aristoxenus as uninteresting (cf. n. 72 above). The same distinction is applied to *systēmata* at 14.24.

[75] Aristoxenus gives an analysis of the tone (7 *El. Harm.* 21.23ff., cf. Bacchius 306.16, Gaudentius 330.6, *Anon. Bell.* 24), but mentions only divisions into melodic intervals (half-tones, thirds, quarters). No other author gives a list of divisions of the ditone, and the reference to twelfths makes it clear that the division is purely theoretical. The twelfth of a tone is the smallest interval used in Aristoxenus' analyses (*El. Harm.* 25.11ff.), but it is not a melodic step. But from a theoretical point of view the division is incomplete (why are eighths not mentioned?), and its purpose is not clear.

[76] A reference to the procedures of Aristoxenus' predecessors: see particularly 7 *El. Harm.* 2.8ff., 27.34ff., 53.4ff. 'String' (*chordēn*), here as often, has a sense closer to 'note'.

[77] This sentence suggests, fancifully, an etymological connection between *diesis* and *dialysis* ('vanishing-point', 'cessation'). For the idea see 2.1 Plato *Rep.* 531a, Aristotle *De Sensu* 445b.

[78] There is a genuine etymological link between *tonos* and *diateinein*, 'to stretch', and again *tasis*, 'tension' and hence 'pitch'. Cf. 15.29–16.1, and for the various meanings of *tonos*, 20.1–5.

[79] I suggest that *to...paraplēsion* (line 10) should be emended to *tou...paraplēsiou*. The claim referred to is the Pythagorean thesis proved at 8 Eucl. *Sect. Can.* proposition 16: cf. 9.9 Panaetius ap. Porph. *Comm.* 65.26ff. Aristoxenus standardly assumes the possibility of equal division. (He may have had some qualms, see 7 *El. Harm.* 28.13–15, and cf. 24.4ff., 55.3ff., 56.13ff., but there are other ways of interpreting these passages.)

[80] The source of the scheme of notation that follows is unknown. Nothing like it appears elsewhere. It evidently purports to record a notational system used by Aristoxenus' predecessors, who certainly did have a symbolism enabling them to map *systēmata* onto a table of consecutive dieses (see the references given at n. 6 to 7 *El. Harm.* 2.12).

The first line of Aristides' table (with the exception of the second symbol) is merely the series of numbers, one for each quarter-tone step; in the second line I have added their Arabic equivalents. The third and fourth lines give the notational symbols associated with each numbered step. Lines 5 to 8 continue the series to the end of the first octave. Lines 9 to 12 present, in the same form, a sequence of semitones covering the second octave.

A few points of detail. The numbers α (1) in the first line and β̄ (2) in line 9 indicate



13.4 *Chapter 8* A *systēma* is that which is constituted out of more than two intervals.[81] Some of the differences between *systēmata* are just like those we have spoken of in relation to intervals, but there are more, as follows.[82] Some

the beginnings of the first and second octaves respectively. Probably a second α, indicating the first note or step, has been lost, along with the associated symbols. The second symbol in line 1 is unexampled, and its significance is unknown. The first symbol in line 4 is probably an error, copied accidentally from line 5. I have reversed the two symbols under number 36.

If the notational symbols represent notes (as their similarity to the Alypian symbols suggests), there should be 25 pairs, not 24, in the first octave. Possibly the first pair stood below the mysterious ⋇, which marked the point of origin, and the second under the missing α, so that the numbers indicate the number of quarter-tones between each note and the starting-point; Winnington-Ingram (1973) argues against this suggestion. Just conceivably the symbols represent intervals rather than notes. The existence of an interval-notation has sometimes been thought to be indicated in 7 Aristox. *El. Harm.* 39.4ff.

In its adoption of two symbols for each note or interval, and in its use of rotated forms of the letters of the alphabet, the notation resembles the Alypian system, and might be supposed to be its ancestor. Powerful arguments against this supposition are marshalled by Monro ((1894), pp. 98–9), who bluntly calls the notation a forgery, and by Winnington-Ingram (1973), the most thorough modern discussion of the matter. For a more favourable assessment see Chailley (1973), and (1979), pp. 122–3.

81 Aristoxenus (7 *El. Harm.* 15.34–16.1) allows the title *systēma* even to sequences of two intervals, and most later writers follow him (e.g., 10 Nicomachus *Ench.* 261.19, Cleonides 180.2–3, Gaudentius 331.2, and the second part of *Anon. Bell.* 51). But others define by reference to notes, not intervals (e.g., *Anon. Bell.* 23 and the first part of 51, Bacchius 292.18), and 'more than two' would be correct. The expression I have translated 'constituted out of' normally means 'bounded by', and some editors propose to emend 'intervals' to 'notes'. But Aristides may mean what he says, deliberately treating the tetrachord as the smallest *systēma*: cf. 8.24, 14.3ff., 79.26. (Ptolemy at 11 *Harm.* 50.12ff. refuses the name *systēma* to anything smaller than the octave.)

82 Compare Aristoxenus' list of distinctions at 7 *El. Harm.* 17.1ff., Cleonides 193.3ff. Aristides' discussion is thoroughly disorganised. He begins with the three distinctions treated by Aristoxenus as peculiar to *systēmata* (continuous/transilient, simple/not

systēmata are continuous – those that proceed melodically through successive notes – while others are transilient – those which proceed melodically through notes that are not successive.[83] Some – those which are set out within a single *tropos* – are simple, while others – which are generated through the interweaving [*plokē*] of several *tropoi* – are not simple.[84] *Systēmata* differ also in that some are conjunct, some disjunct and some a mixture of both: those which share a common note are conjunct, and are also called 'corresponding', while
14 those between which there falls one note separating them from each other are disjunct, and are also called 'parallel'. Those which lie partly in conjunction and partly in disjunction are mixed.[85]

Then again, some *systēmata* are tetrachords, being delimited by four notes lying in their natural relations: some are pentachords and others octachords, for which a similar definition should be given.[86] Some of them are concordant, others discordant. Those bounded by concordant notes are concordant, and those which are not like this are discordant.[87] We have already said what

simple, conjunct/disjunct): the first two are repeated at the end of the passage in slightly different terms. He gives two distinctions drawn from the list of differences between intervals (concordant/discordant, dense/open-textured), but omits those in respect of magnitude, genus and rationality. He inserts further distinctions: between tetrachords, pentachords and octachords; in respect of form (repeated at the end of the passage, 14.27ff., under the title 'arrangement'); in respect of the status of the bounding notes (as fixed or moveable); in respect of completeness and incompleteness. He appears to have jotted down remarks from a number of sources without working them up into anything systematic.

[83] See 7 Aristox. *El. Harm.* 17.28–31, cf. Cleonides 199.8ff. 'Successive' notes are those between which, in a given form of scale, no note falls (cf. Aristox. *El. Harm.* 60.10ff.). But then each note in any *systēma* is by definition successive with its neighbour, and there is no room for 'transilience'. Aristides is probably treating the sequence of notes in a Perfect System (7.18ff.) as constituting 'succession'. If a *systēma* is conceived as the note-series forming the framework for a given style of melody, some *systēmata* may then be transilient, though it may be hard to distinguish transilience of a *systēma* from that of the note-series chosen by a composer from among the notes of a *systēma*; cf. 29.14ff., 79.24, 80.25ff., and the scale-systems of 18.5ff., ps.-Plut. *De Mus.* 1134f–1135b, 1137b–d.

[84] *Tropos*, here as often in Aristides, is a synonym for *tonos*. The distinction is equivalent to that between non-modulating and modulating, 14.24–6 (see n. 93 below). Modulation between *tonoi* is discussed at 22.15ff.

[85] Aristides' account of disjunction is again odd, though its peculiarity is different from that of 9.7–8. It would be correct to say that *systēmata* are disjoined if one interval, or one tone, lies between them (we might emend *phthongos* to *tonos* in 14.1); cf. 7 Aristox. *El. Harm.* 58.14–22, Cleonides 199.16–17. *Systēmata* that are 'mixed' are mentioned in these terms only by Aristides and Aristoxenus (*El. Harm.* 17.24–6). An example is the GPS itself.

[86] *Systēmata* of these three kinds are those of special interest in harmonic theory: Aristides' apparent implication that every *systēma* is tetrachordal, pentachordal or octachordal is therefore understandable, and of a piece with his use of '*systēma*' to designate a tetrachord (e.g., 9.1). With the phrase 'four notes lying in their natural relations' compare Bacchius' definition of 'tetrachord' at 298.19–20.

[87] No theoretically important tetrachordal, pentachordal or octachordal *systēmata* are discordant, though there are tetrachordal discords between notes in disjoined tetrachords, pentachordal discords between notes in conjoined tetrachords, and octachordal discords in the LPS. But the term 'tetrachord', at least, is standardly used to refer only to concordant systems. (Possibly the reference to tetrachords etc. is interpolated, and the reference here is to *systēmata* in general.) See also n. 91 below.

concordance between notes is.[88] *Systēmata* are set out in terms of various dissimilar intervals, such as the diesis, the semitone and the tone. They also differ in form [*eidos*]: some are bounded by fixed notes, others by moveable ones.[89] Some of them are complete, others are not, the tetrachord and the pentachord being incomplete, the octave complete, since every note which follows it is in all respects similar to one of its predecessors.[90] The tetrachord is called *dia tessarōn*, and consists of two tones and a semitone, or five semitones, or ten dieses. The pentachord is called *dia pente*, and is put together from three and a half tones, or seven semitones, or fourteen dieses. The sequence of eight notes is called *dia pasōn*, and is arranged out of six tones, or twelve semitones, or twenty-four dieses.[91]

Again, of all the *systēmata* some are dense, others open-textured;[92] some non-modulating (those which have a single *mesē*), others modulating (those which have several *mesai*).[93] Some proceed melodically by successive notes,

[88] See 10.2–5.

[89] That is, they differ in the internal arrangement of their intervals (7 Aristox. *El. Harm.* 74.9–17). Tetrachords, for instance, between fixed notes in a given *systēma* will be of a different form from those between any two moveable notes (if such structures are allowed the title 'tetrachords'), and there will be two forms of tetrachordal *systēma* between pairs of moveable notes. See also 14.27–15.2.

[90] The description of the octave as 'complete' is common in Aristides, but unusual in other Aristoxenian sources. It is natural enough, and probably older than Aristoxenus; cf. 11 Ptol. *Harm.* 50.18. In Cleonides (200.10), Gaudentius (333.19) and others, the title 'complete' or 'perfect' is reserved for the LPS and the GPS, both of which exceed the octave; Ptolemy restricts it to the double octave (*Harm.* 50.12ff.). But for the same idea as Aristides', put to different use, see Ptol. *Harm.* 13.3–7, cf. 58.21ff.

[91] Such analyses are common elsewhere: the completest is *Anon. Bell.* 71–73. The names cited are the standard ones for '(interval of a) fourth', 'fifth' and 'octave' respectively: literally 'through four (strings or notes)', 'through five', 'through all'. The passage is apparently inconsistent with the suggestion that tetrachords etc. may be discordant (14.7–10, and see n. 87 above). All the quantifications are of course Aristoxenian; Pythagoreans would deny them (e.g., 8 Eucl. *Sect. Can.* propositions 14, 15).

[92] For the application of this distinction to intervals see 11.21–3 with n. 74. Aristoxenus applies the term 'dense' (*pyknos*) to *systēmata*, but only to ones consisting of two small intervals. This usage is inconsistent with Aristides' treatment of *systēmata*. The sense here is not clear. If 'open-textured' *systēmata* are those that omit some available notes, the distinction is identical with that between successive and transilient. But perhaps Aristides' source used 'dense' of *systēmata* whose tetrachords contain *pykna* in Aristoxenus' sense, 'open-textured' of those lacking them (i.e., diatonic progressions).

[93] This repeats the distinction between simple and not simple (13.9–11): the contrast between 'simple' and 'modulating' is Aristoxenian (7 *El. Harm.* 38.6–7, 40.20–21, cf. 17.32–18.3; see also Cleonides 201.14ff.). A melody that modulates from one *tonos* to another thereby shifts the sequence of named notes to a new position, identified by the position of *mesē* (see 22.11ff. and General Introduction, pp. 17–27). If the notes used in a melody involving such modulation are written out in a single sequence, the result could be described as a 'modulating *systēma*', in which a *mesē* appears in two or more places. (The best example in the surviving musical fragments is in the First Delphic Hymn, bars 34–62.) A special case of a modulating *systēma* is that which combines a conjunction and a disjunction at the same point (i.e., includes both the tetrachords *synēmmenōn* and *diezeugmenōn*). A shift between these is called 'modulation in respect of *systēma*' by Cleonides (205.5): cf. also Aristides 16.24–17.2, 29.12–14. Ptolemy (11 *Harm.* Book II ch. 6) shows that change from a system involving disjunction above *mesē* to one involving conjunction at that point is equivalent to a modulation of *tonos*, at the interval of a fourth, between two systems involving disjunction. Cf. Bacchius 304.10ff., *Excerpt. ex Nicom.* 274.20ff.

others by transilient ones.[94] They have a variety of arrangements, understood on the basis of which interval comes first or what order they are in: for either the first is a semitone, or the second or the third or whatever.[95]

There are five tetrachords to be distinguished in each *tonos*, the tetrachords *hypatōn*, *mesōn*, *synēmmenōn*, *diezeugmenōn* and *hyperbolaiōn*.[96] There are three concordant pentachords, *mesōn*, *synēmmenōn* and *diezeugmenōn*,[97] and two octachords, *synēmmenōn* and *diezeugmenōn*.[98] They have several forms, constructed by the addition of each successive note. The ancient writers, however, called the fourth *syllabē*, the fifth *di' oxeian*, and the octave *harmonia*,[99] the last of which has also been given a variety of names

[94] The same distinction as at 13.7–9. 'Transilient', *hyperbatos*, is used of notes (rather than intervals or *systēmata*) also at 11 Ptol. *Harm.* 67.8.

[95] *Schēma*, 'arrangement', is equivalent to *eidos*, 'form' (7 Aristox. *El. Harm.* 74.11–13), and hence this is the same distinction as at 14.12–14. The last clause refers to tetrachords of the diatonic genus: in a tetrachord between fixed notes the arrangement is semitone, tone, tone; in one beginning from the second-lowest note of the same tetrachord and ending on the second note of one conjoined with it above, it is tone, tone, semitone; in one beginning and ending on the third note it is tone, semitone, tone. There are thus three forms or arrangements of the fourth (*El. Harm.* 74.71ff.), four of the fifth, seven of the octave (cf. *El. Harm.* 6.21–31). Different authors identify these arrangements in different ways: see 10 Nicomachus *Ench.* 249.17ff., 11 Ptol. *Harm.* 49.9ff., Cleonides 196.3ff., Bacchius 308.8ff., Gaudentius 345.17.

[96] These are the five tetrachords between fixed notes; the list is common, e.g., 10 Nicomachus *Ench.* 259.14, Cleonides 201.8, *Anon. Bell.* 77.

[97] The three pentachords reappear at 114.29, where the first is described as that 'through *hypatōn* and *mesōn*'. This fifth might lie between *hypatē mesōn* and *paramesē*, though the description is not wholly apt. Possibly it is the first of the three mentioned here. It has the form semitone, tone, tone, tone (in diatonic). Most sources treat this form as primary, just as semitone, tone, tone is the primary form of the fourth: e.g., Cleonides 196.9ff., Bacchius 308.8ff., *Anon. Bell.* 60. But there are only two such fifths in the system, the one mentioned and that running upwards from *mesē* through the tetrachord *synēmmenōn* and ending on the first note of the tetrachord *hyperbolaiōn* (which according to 10 Nicomachus *Ench.* 259.15ff. is separated from *synēmmenōn* by a disjunctive tone). At Porph. *Comm.* 162.11ff. the form tone, semitone, tone, tone is treated as primary, but there are five instances of this form in the system (cf. Mart. Capella 962).

Perhaps the point is not that these are pentachords of a single privileged form, but that they are those lying between fixed notes. This might be the intention behind their description as 'concordant' (and cf. Bacchius 300.11–13). Then Aristides' three might be (a) from *hypatē mesōn* to *paramesē*, (b) from *mesē* through the tetrachord *synēmmenōn* to the first note of *hyperbolaiōn*, (c) from *mesē* through *paramesē* and the tetrachord *diezeugmenōn*. But there is still another that fits the description, that from *proslambanomenos* to *hypatē mesōn*. Why is it excluded? Again, it turns out that the *mesoeides* fifth mentioned at 81.21–3 is the fifth downwards from *mesē* to *lichanos hypatōn*. This does not lie between fixed notes, though it has the form preferred by Porphyry, and it has no structural importance in the system. If it is the first of the pentachords mentioned here, it remains unclear how we are to identify the others (but see nn. 132, 142 to Book III). The purpose and nature of Aristides' classification is thoroughly obscure.

[98] One would expect the two octaves to be those from *proslambanomenos* to *mesē* and from *mesē* to *nētē hyperbolaiōn*. The name assigned to the first seems to count against this. There are a number of other possibilities, but none that is very appealing. At 115.8ff., 116.9ff. there are perhaps hints that the octave *synēmmenōn* is lower than the octave *diezeugmenōn*.

[99] *Syllabē* indicates something taken together, a union or combination. There may be a link with its use to mean 'syllable', the tetrachord being conceived as the first intelligible

corresponding to its forms [*eidē*]. The octave from *hypatē hypatōn* was called Mixolydian, from *parhypatē* Lydian, from *diatonos* Phrygian, from *hypatē mesōn* Dorian, from *parhypatē* Hypolydian, from *diatonos* Hypophrygian, and from *mesē* Hypodorian.[100] From this it is clear that if we always put the same sign in first place, but name it each time in accordance with a different note-function [*dynamis phthongou*], the quality of the *harmonia* will be revealed through the sequence of consecutive notes.[101] Let that be enough about *systēmata*, which the ancient writers also called the origins of the characters.[102]

Chapter 9 A genus is a distinctively qualified division of a tetrachord.[103] There are three genera of melody, the enharmonic [*harmonia*], the chromatic [*chrōma*] and the diatonic, which acquire their distinctions from the narrowness or largeness of their intervals. The one which has several of the smallest intervals is called 'enharmonic' because it is closely fitted together: that which has several tones is called 'diatonic' because in it the voice is more intensely stretched: the one whose tension is increased by semitones is called 'chromatic'. For just as what is between white and black is called 'colour' [*chrōma*], so the

combination of notes, as the syllable is of letters. *Di' oxeian* (or *di' oxeiōn*) means 'through the high-pitched (strings or notes)'. The names were thought to belong to Philolaus and the early Pythagoreans (see 10 Nicomachus *Ench.* 252.4–10, where explanations of the usages are given: cf. Porph. *Comm.* 96.21ff.). But *harmonia* for 'octave' (the 'attunement' of a full complement of strings) seems to have been in quite general use in the early period (cf. 7 Aristox. *El. Harm.* 36.30–2): *di'oxeian* is used, apparently quite colloquially, in 4.25 ps.-Ar. *Probs* XIX.41 (see also 34): and Porphyry adds an account of *syllabē* in terms of the 'grasp' of a performer's hand across the strings (*Comm.* 97.2ff.). Probably all three terms were originally used by practising musicians, not theorists.

100 These are the cyclically arranged species of the octave, enumerated by Eratocles (see 7 Aristox. *El. Harm.* 6.21ff., cf. 2.16–18 and 36.30–2). They are also the basis of Ptolemy's system of *tonoi*. There are corresponding lists of the names at Cleonides 195.7ff., Bacchius 308.17ff., Gaudentius 347.6ff., and in every case except Gaudentius (where perhaps the text should be emended) they are spoken of as belonging to the distant past. All four authors seem to rely on a single source, perhaps a summary of Aristoxenus' essay on his predecessors (see e.g. 7 *El. Harm.* 2.28–30). But Aristides' use of the term *diatonos* suggests that he is thinking of the diatonic genus, which conflicts with Aristoxenus' account of earlier practice (e.g., *El. Harm.* 2.7ff.).

101 The first description identifies the octave-species by their positions in the GPS. This sentence describes a procedure for locating all seven in a single octave of pitch, first giving the lowest note the name *hypatē hypatōn*, next calling it *parhypatē hypatōn*, and so on, and in each case setting out above it the sequence of intervals that runs upwards from the named note. See General Introduction, pp. 14–17. This is perhaps the 'compression of the diagram' (*katapyknōsis*) spoken of by Aristoxenus at 7 *El. Harm.* 28.1–2 and elsewhere. 'Sign' (*sēmeion*) here must refer to a notation, presumably one that simply identified a note as the first, second, etc. available pitch in a series (cf. 12.12ff.), a task performed by the Ptolemaic system of *theseis* (11 Ptol. *Harm.* Book II ch. 5).

102 This is the only clear statement we have that explicitly makes the *harmoniai*, conceived as species of the octave, the basis of *ēthos*, though of course the word *harmonia*, and various named *harmoniai*, are often spoken of in this connection, e.g., at Plato *Rep.* 398d ff. *Ēthos* is discussed at length in Books II and III; on its relation to *harmoniai* see particularly Book II chs. 12–14.

103 Cf. 7 Aristox. *El. Harm.* 46.19ff., 50.19ff. If he gave a definition parallel to this one, it is lost. For comparable versions see Cleonides 180.1, Bacchius 298.3, Gaudentius 331.7.

genus located between the other two is called '*chrōma*'.[104] The melodic series of each of them is as follows. The enharmonic proceeds upwards by diesis, diesis and incomposite ditone, downwards in the reverse order. The chromatic proceeds upwards by semitone, semitone and an interval of three semitones, downwards in the reverse order. The diatonic proceeds upwards by semitone, tone and tone, downwards in the reverse order. Of these the diatonic is more natural, since it can be performed by everyone, even the wholly untutored: the chromatic is more technically sophisticated, being performed only by those who have been trained: and the enharmonic demands stricter precision, being accepted only by the most outstanding musicians, while for most people it is impossible. Hence some people refuse to recognise the melodic form which proceeds by dieses, supposing, because of their own incapacity, that this interval is not capable of being melodically performed at all.[105]

We perform each of these genera both in consecution and in interweaving. Consecution occurs when we create a melody by means of successive notes, interweaving when we do so by means of notes reached by transilient motion.[106] Again, one kind of melody is called 'direct', one 'returning' and one 'circular'. The direct moves from low to high, and the returning in the opposite direction; while the circular is that which modulates, as for example when one
17 moves up a tetrachord in conjunction and then descends it in disjunction.[107]

Some of the generic types of *systēma* are divided into species,[108] while others are not. The enharmonic, since it is constructed out of the smallest dieses, is indivisible.[109] The chromatic may be divided into as many divisions as there are

[104] These uses of *harmonia* and *chrōma* go back to Aristoxenus. The etymologies link *harmonia* with the verb *synharmozein*, 'to fit closely together', and 'diatonic' with *diateinein*, 'to stretch'; see n. 78 above. Different derivations are given in the interpolated passage at 92.19ff.: see also 9.2 Theon Smyrn. 54.12, 55.4, 15, 10 Nicomachus *Ench.* 262.11ff., *Anon. Bell.* 26.

[105] These remarks on the relative difficulty of the genera derive ultimately from Aristoxenus: see 7 *El. Harm.* 19.17–29. The thesis that the enharmonic is especially difficult may reflect *El. Harm.* 23.4–22, but whereas Aristoxenus links the rejection of the 'true' enharmonic with an intolerance of a ditonal interval between *lichanos* and *mesē* (cf. also 11 Ptol. *Harm.* Book I ch. 13), Aristides traces it to the 'impossibility' of singing quarter-tones. It seems that Aristoxenus also took note of this point, however: see ps.-Plut. *De Mus.* 1145a–c.

[106] 'Consecution' translates *agōgē* (cf. 7 Aristox. *El. Harm.* 29.32–4, Cleonides 207.2–3); 'interweaving' translates *plokē*, lit. 'twisting', 'weaving' (cf. 11 Ptol. *Harm.* 67.7–8, Cleonides 207.3–4). The subject is dealt with again at 29.7ff. below, under the heading of 'melodic composition': it is not clear why it is mentioned here.

[107] On the last clause see note 93 above. One would perhaps expect the third type to be defined simply as movement both up and down, without reference to modulation; the definition does not seem to be cognate with the other two. But 'circular' movement may fairly be conceived as proceeding upwards by one route, downwards by another, and this account is repeated when these distinctions are discussed in the passage on composition. There they are treated as species of *agōgē* (29.8ff.).

[108] The *chroai* or 'shades'. They are described as species falling under genera by many authorities, e.g., Cleonides 190.6, *Anon. Bell.* 52–4, but not in any surviving passage of Aristoxenus.

[109] No source admits genuinely distinct species of enharmonic, though various authorities compute its intervals differently (see especially 11 Ptol. *Harm.* Book II ch. 14), but compare 7 Aristox. *El. Harm.* 49.10ff.

rational intervals to be found between the semitone and the enharmonic diesis, while the diatonic may plainly be divided into as many as there are rational intervals between the semitone and the tone.[110] There are thus three species of the chromatic and two of the diatonic, so that with the addition of the enharmonic there are six species of melody in all. The first is characterised by quarter-tone dieses, and is called enharmonic; the second by dieses of one third of a tone, and is called the soft chromatic; the third by dieses one and a half times the enharmonic diesis, and is called the hemiolic chromatic; the fourth has as its peculiar feature a tone constructed out of two incomposite semitones, and is called the tonic chromatic; the fifth consists of a semitone, an interval of three dieses, and a remainder of five dieses, and is called the soft diatonic; the sixth contains semitone, tone and tone, and is called the tense diatonic. To make what we are saying clear, we shall set out the divisions in numbers, starting from the assumption that the tetrachord consists of sixty units.
18 Enharmonic, 6, 6, 48: soft chromatic, 8, 8, 44: hemiolic chromatic, 9, 9, 42: tonic chromatic, 12, 12, 36: soft diatonic, 12, 18, 30: tense diatonic, 12, 24, 24.[111]

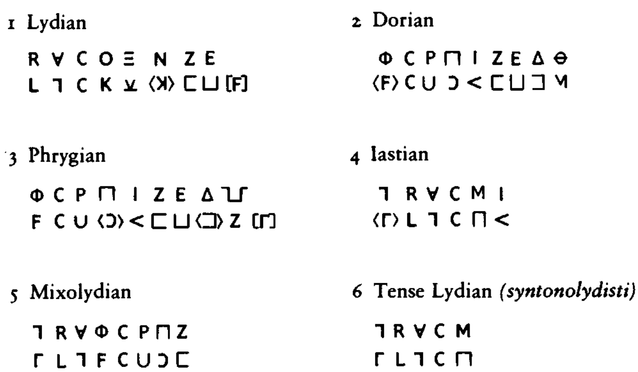
There are also other divisions of the tetrachord, used for the *harmoniai* by people of distant antiquity.[112] Sometimes they covered the span of a full octachord, and there are cases where they exceeded a six-tone *systēma*. Often they fell short of it. Nor did they always include all the notes, for a reason

[110] On the Aristoxenian concept of 'rationality' see 7 *El. Harm.* 16.30 with n. 63 to that passage. Neither it nor its Pythagorean counterpart (cf. n. 67 above) can be used to limit the shades of chromatic and diatonic to just the ones listed below. Probably *rhētos*, 'rational', has no technical sense here, and is being used as an equivalent to Aristoxenus' phrase 'noteworthy and familiar' at *El. Harm.* 50.20–1. Cleonides echoes this passage with the sentence 'There are six rational (*rhētai*) and familiar shades' (190.7). Aristoxenus never seeks to establish by argument that there are just these shades and no more. They are merely the ones in commonest use, and any number of others could in principle be allowed. See especially *El. Harm.* 26.13–27, 49.8ff., 50.19–21.

[111] These divisions follow 7 Aristox. *El. Harm.* 22.24ff., 50.22ff. Aristoxenus is said to have divided the tone into twelve for these purposes, the fourth into thirty (Porph. *Comm.* 125.24ff., 137.25), and Cleonides followed him; Aristides doubles the figures, as does Ptolemy sometimes (e.g., 11 *Harm.* 29.21ff.) so as to make them all whole numbers (cf. Porph. *Comm.* 138.10–13). Various interpolations, omitted in my translation, have crept into this passage. One of them might be taken to suggest the division 8, 28, 24, which would correspond to the 'mixed' tetrachord of *El. Harm.* 27.8–12. But almost certainly it does not belong in Aristides' text.

[112] It is not obviously appropriate to describe the scales that follow as giving new 'divisions of the tetrachord'. When Aristides says that they were used for the *harmoniai*, he probably means that they were schemes of analysis used to describe methods of 'attunement' used in different melodic styles. Like the notation at 12.12ff., this set of scales is unknown elsewhere, and we cannot be sure what Aristides' source was. It may have been a commentary on Plato's *Republic*, but the non-Platonist terminology counts against this suggestion. It may have been a summary of Aristoxenus' work on his predecessors (see 7 *El. Harm.* 2.29–30). Just possibly it was a source that recorded analyses made (or believed to have been made) in the school of Damon: see 80.29ff. below. Detailed discussion of the scales and their provenance is impossible here. The issues should be considered in the context of passages from ps.-Plut. *De Mus.*, particularly 1134f–1135b, 1136b–1137e. See also Winnington-Ingram (1936), pp. 21–30, *GMW* vol. 1, pp. 163–8.

which we shall explain later.[113] Thus they constructed the Lydian *systēma* from diesis, ditone, tone, diesis, diesis, ditone, diesis, and this was a complete *systēma*; the Dorian from tone, diesis, diesis, ditone, tone, diesis, diesis, ditone, and this exeeded the octave by a tone; the Phrygian from tone, diesis, diesis, ditone, tone, diesis, diesis, tone, and this was a complete octave; the Iastian from diesis, diesis, ditone, an interval of three semitones, tone, and this fell short of an octave by a tone; the Mixolydian from two successive dieses, tone, tone, diesis, diesis, tritone, and this was a complete *systēma*; and the so-called 'tense Lydian' was diesis, diesis, ditone, interval of three semitones.[114] In all
19 these cases the diesis is to be understood as being the enharmonic diesis. For the sake of clarity, let a diagram of these *systēmata* be written out below. It is these that the divine Plato mentions in the *Republic*, where he says that the Mixolydian and the tense Lydian [*syntonolydisti*] are suitable for laments, while the Iastian and Lydian are appropriate for drinking parties, and excessively unrestrained. And he goes on to say: 'You seem to be left with the Dorian and the Phrygian'.[115] These, then, were the kinds of exposition of the *harmoniai* which they used to give, fitting the qualities of the notes to their respective moral characters. Those, however, are matters of which we shall speak precisely later.[116]



113 That is, they are 'transilient', in the sense mentioned at 13.8, 14.26–7. The reference to a later explanation may point to 80.28ff., as is perhaps hinted at 19.7–9: see n. 116 below.

114 'Tense Lydian' translates *syntonos Lydios*: cf. the *syntonolydisti* of Plato *Rep.* 398e, mentioned below.

115 See *Republic* 399a.

116 The suggestion is that the ethos of a scale-system, and that of the piece of music based on it, is ultimately derived from the ethos of each individual note it contains. This would explain why some scales omit certain notes (cf. 18.9–10). The idea is not found in Plato or any other of the older sources, but is attributed by Aristides to the school of Damon in Book I ch. 14, to which the last sentence refers. Philodemus (*De Mus.* IV.23.27ff.) mentions a comparable view in connection with Heraclides.

The tables give the scales described at 18.5ff. in the 'Alypian' notation, which is certainly later than the period to which Aristides assigns the scales. The original source may have provided only lists of intervals, in which case their arrangement around notes of the Perfect System is the work of a later hand. The relative pitches of the notes in the

Chapter 10 Let us now discuss *tonoi*. We use the term '*tonos*', in music, in three ways. It can mean the same as 'pitch': it can indicate a certain magnitude of sound, such as that by which the fifth exceeds the fourth: or it can mean the *tropos* of a *systēma*, such as Lydian or Phrygian. The last is the one which we are now to discuss.[117]

According to Aristoxenus there are thirteen *tonoi*, whose *proslambanomenoi* are bounded by the octave, while according to more recent writers there are fifteen, whose *proslambanomenoi* are bounded by an octave and a tone, since they incorporate in addition the tone of disjunction.[118] Aristoxenus names them as follows. Hypodorian; two Hypophrygians, one low, which is also called Hypoiastian, and one high; two Hypolydians, one low, which is also called Hypoaeolian, and one high; and Dorian; two Phrygians, one low, which is also called Iastian, and one high; two Lydians, one low, which is also called Aeolian, and one high; two Mixolydians, one low, now called Hyperdorian,
21 and one high, now called Hyperiastian; one Hypermixolydian, which is also

tables can be represented in modern notation as follows, where the symbol + indicates that a pitch is raised by a quarter-tone.

(i) Lydian: B+, c, e, f#, f#+, g, b, b+.
(ii) Dorian: d, e, e+, f, a, b, b+, c′, e′.
(iii) Phrygian: d, e, e+, f, a, b, b+, c′, d′.
(iv) Iastian: B, B+, c, e, g, a.
(v) Mixolydian: B, B+, c, d, e, e+, f, b.
(vi) Tense Lydian: B, B+, c, e, g.

[117] Porphyry gives the same three-fold classification in similar but more elaborate terms at *Comm.* 82.1ff. Cleonides 202.6ff. adds a fourth sense, in which *tonos* is equivalent to *phthongos*, 'note'. The first two are commonplace: Aristides' definition of the interval of a 'tone' goes back at least to Aristoxenus (7 *El. Harm.* 21.20ff., 45.34–46.1). It is implied in 1.12 Philolaus frag. 6, and is effectively common ground between Aristoxenians and Pythagoreans.

The term *tropos* has many uses, in the general sense 'form', 'style', 'variety'. Thus, Aristides speaks of *tropoi* of melodic and rhythmic composition (e.g., 30.1, 40.12), of modulation (40.2), and of the harmonic and educational *tropoi* of musical science (31.1, 53.7–8). In the present passage *tropos* is initially qualified by the adjective *systēmatikos*, to mean 'variety of *systēma*'; subsequently *tonos* and *tropos* are used interchangeably. *Tropos* for *tonos* in this sense is quite common, but the history of the usage is unclear.

No definition of *tonos* in the sense intended here is preserved in Aristoxenus, but Porphyry (*Comm.* 82.3–5) attributes to him the statement that a *tonos* is 'a range, without breadth, capable of receiving a complete (or 'perfect') *systēma*': cf. Cleonides 180.5, 'a range of the voice, without breadth, capable of receiving a *systēma*'. See particularly 7 *El. Harm.* 7.22–8.

Aristides' discussion of the *tonoi* contains numerous obscurities, the most important of which relate to the question whether he conceives *tonoi* simply as transpositions of identical sequences to different pitches, or as systems projecting different interval-sequences on to one abstract range. In the following notes I shall use the word 'key' to identify the former conception, and refer to the latter as 'modal'. Both terms need to be treated with caution, and neither will have quite its modern connotations. See General Introduction, pp. 17–27.

[118] Cleonides 203.5 also ascribes thirteen *tonoi* to Aristoxenus. Not all modern commentators have accepted the ascription: see General Introduction, pp. 22–5. Alypius gives notations for fifteen. To relate *tonoi* by their *proslambanomenoi* is reasonable if they are understood as keys. If they are conceived modally it leads to some awkwardness (see 21.13ff.). Ptolemy (11 *Harm.* 64.16ff.) generates a more elegant account by reference to their *mesai*.

called Hyperphrygian.[119] To these, more recent writers have added the Hyperaeolian and the Hyperlydian, so that each *tonos* might possess depth, intermediacy and height.[120] Each of them will exceed its predecessor by a semitone, if we decide to begin from the lowest, or fall short of it by a semitone, if we make a start from the highest. Thus their *proslambanomenoi* are bounded, as I said, by an octave and a tone. For this reason they can also be found by means of concords: if I begin from the lowest, and set about raising and then lowering the pitch through various intervals, I shall inevitably land on some one of their *proslambanomenoi*.[121]

Melodies employ some of these *tonoi* throughout their range, but not others.[122] Thus the Dorian is melodically employed in its entirety, since our voice extends to a limit of twelve tones, and the *proslambanomenos* of the Dorian lies in the middle of the Hypodorian octave.[123] Of the remainder, those which are lower than the Dorian are used as far down as the note which is in unison with the *proslambanomenos* of the Dorian, and those which are higher

[119] If the thirteen *tonoi* are Aristoxenian, the names he used are probably those cited first in each case (e.g., 'low Hypophrygian'). The others (e.g., 'Hypoiastian') belong to the triadic system which includes the extra two. In the General Introduction (pp. 23–4) I suggest reasons why Aristoxenus might have conceived some of the *tonoi* as 'high' or 'low' variants of one another.

[120] The suggestion is that there should be five basic *tonoi*, Dorian, Iastian, Phrygian, Aeolian and Lydian, each with a *Hypo-* version and a *Hyper-* version. Each basic *tonos* stands at the interval of a fourth from each of its relatives, providing melodically acceptable modulation between them (see 22.15ff.). This consideration is significant no matter whether *tonoi* are considered modally or as 'keys', but within the modal conception there is no other serious warrant for the triadic grouping, any more than there is for the addition of the fourteenth and fifteenth *tonoi*. The date of these additions is uncertain, but compare Athenaeus *Deipn.* 625d.

[121] 'For this reason': that is, because each is a semitone from its neighbour, and semitones can be constructed by the 'method of concordance'. For the method see 7 Aristox. *El. Harm.* 55.3ff., cf. 8 Eucl. *Sect. Can.* proposition 17. Bryennius 476 is evidence that Aristoxenus used it for the purpose discussed here; so too did Ptolemy (11 *Harm.* 63.12ff.). By 'various intervals' Aristides of course means the fourth, fifth and octave, and combinations of them.

[122] At first sight this paragraph suggests that *tonoi* are the keys of structurally identical two-octave scales. They are not determined to absolute pitches. One, the Dorian, is defined as that whose *proslambanomenos* is the lowest note a given voice can reach, and the others are found through their relations to it. Then (vocal) melodies can use other *tonoi* only in part of their ranges. A similar idea is indicated in the next chapter (23.1ff.), but this conception of *tonos* would at best be limited and awkward in application, and seems to be contradicted at the end of the paragraph. For a different suggestion see n. 125 below.

[123] Most ancient writers give two octaves as the range of the voice (e.g., *Anon. Bell.* 94, Porph. *Comm.* 152.20–5, 10 Nicomachus *Ench.* 255.25ff.): according to others it is two octaves and a fifth (7 Aristox. *El. Harm.* 20.23–30, Cleonides 194.20ff.). The different estimates are related to the practices of identifying a total of either six or eight concords (see particularly Porph. *Comm.* 96.7–12). 'Hypodorian octave' must refer to the lower octave of the Hypodorian *tonos*, conceived as a key whose starting point is a fourth below the Dorian *proslambanomenos*. So conceived, the *proslambanomenoi* lie in the Hypodorian octave. Dorian lies, in fact, only roughly in the middle; it is exactly central only to the Ptolemaic system of seven *tonoi*. Aristides gives a similar characterisation of Dorian at 81.17–23.

are used as far up as the note which is in unison with its *nētē hyperbolaiōn*.[124] We can then assign songs or instrumental pieces [*kōla*] to their *tropoi* in the following way. We designate the lowest of the notes of the *systēma* as one of the *proslambanomenoi*, and sing downwards from it. If we can go no lower, it will be Dorian, since the lowest audible note is defined as the
22 *proslambanomenos* of the Dorian. If we can hear still lower notes, we shall try to determine how great the difference is between this note and Dorian *proslambanomenos*, which is by nature the lowest; and we shall define the *tropos* as the one which is higher than the Dorian *proslambanomenos* by the same amount as the lowest note of the melody was found to be higher than that which is lowest by nature. If the lowest note of the song falls outside and above the Dorian octave, we shall take the note an octave lower, and shall then find no difficulty in discovering its *harmonia* by the method we have just described.[125]

[124] For the probable sense see n. 125 below. There is a gap in the text of this sentence, and I have translated the supplement suggested by Bellermann. 'In unison' translates *symphōnountos*, which usually means 'in concord'. One would expect *homophōnountos* on the analogy of 10.5–6. The usage perhaps reflects the technical sense of *symphōnia* explained at 24.2–6.

[125] It may just be possible to construe this paragraph on the basis of the relativised notion of 'key' suggested in note 122 above, but there are difficulties. In the last sentence, the 'Dorian octave' is evidently the lower octave of the Dorian *tonos*. If the 'lowest note of the song' is a *proslambanomenos*, it cannot be the notion of *tonos* as key that Aristides intends, since in this conception no *proslambanomenos* lies above that octave. If it is not a *proslambanomenos*, the procedure of taking the note an octave lower would work only if the note is a *mesē*, which Aristides has not said or implied. Even if it was a *mesē*, the procedure would work only for *tonoi* above the Dorian, and we are left with no method for finding the others.

An alternative interpretation would treat *tonoi* 'modally', as different structural orderings of intervals in the same range. In that case the phrase at 21.19–20, 'the lowest of the notes of the *systēma*', designates what Ptolemy would call the 'dynamic' *proslambanomenos* of the system implied in a given melody, and the phrase 'the lowest note of the song' in the last sentence must be taken in the same sense. If it lies at the bottom of the two-octave space under consideration, the *tonos* is Dorian. If it lies a little higher, it will be one of the *tonoi* 'higher' than Dorian, and we can find which by discovering the interval between it and the bottom of the two-octave range. If it lies high, in the upper octave of the two-octave space, this is because, on the modal conception, the *proslambanomenos* of a *tonos* 'lower' than Dorian disappears from the bottom of the range, as the intervals are cyclically reordered, and reappears at the top. There are difficulties here too. (a) The reference to the lowest 'audible' note at 21.23 is misleading. What should be meant is only the lowest note of the two-octave range that is being treated as the melodically usable space. (b) The 'method' that will be appropriate in the context of the last sentence will not be identical with that described above. If it is to identify *tonoi* lower than the Dorian, it should do so by counting upwards from the note in question to Dorian *mesē*, not by counting downwards to *proslambanomenos* (and it is not clear why we need to take the note an octave lower at all, when we could simply count upwards to Dorian *nētē hyperbolaiōn*). (c) The identification of the relative positions of *tonoi* by reference to their *proslambanomenoi* is a little awkward within a conception of 'mode', since the *proslambanomenoi* will fall into two groups, one high in the range and one low. (This problem does not arise within the conception of *tonos* as key.) Ptolemy's procedure of relating *tonoi* by their *mesai*, all of which fall into the central octave, is simpler and more elegant.

On the two notions of *tonos* see the General Introduction, pp. 18–27. Despite its

Chapter 11 Modulation is an alteration of the underlying *systēma* and of the character of the sound.[126] For if each *systēma* carries with it a sound-type of a particular quality, then plainly when the *harmoniai* change, so will the species [*eidos*] of the melody. There are various kinds of modulation between *tonoi*, corresponding to each of the composite and incomposite intervals, but those which proceed by concordant intervals are the more pleasant, the others not altogether so.[127] Their arrangements and interweavings [*plokai*] can be found if we modulate from a note by a tone or a semitone, or indeed by any interval, whether odd or even, upwards or downwards.[128] Some *tonoi* also have tetrachords in common, since some of them are higher than others by a semitone, some by a tone, and others by intervals greater than these, so that the *mesai* of the lower become the *hypatai* of the higher or vice versa, and so on.[129]

23 *Tonoi* fall into three generic kinds, the Dorian, the Phrygian and the Lydian. Of these the Dorian is useful for the lower-pitched activities of the voice, the Lydian for the higher, the Phrygian for the intermediate. The others are found

difficulties, and the inconsistency between it and the fairly clear notion of 'key' at 23.1ff., I suggest that a 'modal' interpretation gives the most coherent reading of this difficult passage.

[126] This definition of modulation (*metabolē*) focusses on shifts between different species of *systēma*, i.e., on the *tonoi* conceived in their 'modal' guise. In this Aristides probably follows Aristoxenus (see 7 *El. Harm.* 7.22–8.3, 38.6–15, cf. ps.-Plut. *De Mus.* 1142f). In Cleonides 204.19ff. and *Anon. Bell.* 65 we find a distinction between modulation of *tonos* and modulation of *systēma*, the latter being defined as occurring 'when a shift takes place from conjunction to disjunction or conversely'. Bacchius 304.6ff. mentions the same distinction, but defines modulation of *systēma* more broadly: 'when the melody moves from the existing *systēma* to another, adopting a different *mesē*': cf. 14.25–6 above with n. 93. Ptolemy, arguing that modulation of *tonos*, properly understood, must involve change of 'mode', attacks both those who envisage such a modulation as involving only the transfer of the same interval-series to a different pitch (11 *Harm.* 54.12ff.), and those who give a special status to the *systēma* which contains a tetrachord in conjunction above *mesē* (the LPS): see *Harm.* Book II ch. 6. This is just what we find in Cleonides. At some stage modulation of *tonos* (in the sense 'key') has become detached from modulation of *systēma* ('mode'), and among modulations of the latter sort, one has assumed special importance. Aristides' present conception may be closer to that of Aristoxenus, and agrees broadly with Ptolemy's. His talk of 'character', 'type' and 'quality' of sound has its counterpart in Ptolemy's insistence that a true modulation of *tonos* must 'impose on the senses an impression of difference in respect of *dynamis*, by which the ethos is altered' (*Harm.* 55.7–8).

[127] The thesis that modulation at concordant intervals is melodically most acceptable is common to both conceptions of *tonos*. See particularly Cleonides 205.10–206.2.

[128] 'Arrangements and interweavings': the phrase has no clear technical meaning here. Aristides may be thinking of the pattern of 'jumps' up and down by which we get from a note in one *tonos* to its counterpart in another, or perhaps of the pattern of intervals formed by the combination of the original *tonos* with that to which the modulation takes place, cf. 13.10–11. 'Odd or even': presumably not in the sense of 11.14ff., but meaning 'an odd or even number of semitones'.

[129] Aristides is thinking of modulation at the fourth, where, for example, the tetrachord *mesōn* of the lower *tonos* becomes the tetrachord *hypatōn* of the higher. Cleonides (205.16ff.) also mentions 'commonness' (*koinōnia*) in modulation: every modulation must involve some *koinōnia*, and the greater the *koinōnia* the more melodious the modulation. This turns out to mean that a modulation is melodious if notes with the same position in the tetrachord coincide in pitch in the two *tonoi*, and the more so the more such coincidences there are. This is why both authors describe modulations at concordant intervals (Cleonides adds 'or at the interval of a tone') as 'more pleasant' or 'more melodious'.

mostly in instrumental compositions, since they are constituted by the *systēmata* which have the most extreme ranges.[130]

The complete set of *tonoi* is put together from the series of the twenty-four letters of the alphabet, taken in reverse order.[131] Descending a tone from the lowest of all, the Hypodorian, we adopt the sign ᘐ as the first of the symbols: then we take the one which comes after it, in the enharmonic standing to it in the ratio of a diesis, in the diatonic and chromatic that of a semitone: and then the next.[132] The fourth we define as standing at the distance of a tone, and this we designate as the starting point of the lowest of the *tropoi*. Rising by a semitone, we position the *proslambanomenos* of the next *tropos*, and by adding its successors at distances of the same size we complete the tally of the fifteen *tropoi*.

I have set out below the diagram of the letters in the sequence of semitones, and that in the sequence of tones, together with the *tropoi* which can be represented by them.[133] Our diagram sets out the symbols in two series, so that we may grasp the sequence of those in the upper series from their similarity to those in the lower; so that we may represent instrumental pieces [*kōla*], and interludes in songs, for *auloi* or for stringed instruments without the voice, by the lower series, vocal pieces by the upper; and so that we may readily keep hidden the mysteries of music by replacing the letters in everyday use with the symbols written below in the diagram setting out the *tonoi*.[134] The diagram of

[130] Here the *tonoi* are unequivocally keys, and we may suspect that Aristides is drawing on a different source. In describing Dorian, Phrygian and Lydian as the 'generic' *tonoi*, he probably means only that they are the most important. They are assigned to vocal music and distributed over vocal types again at 81.17ff., where there is also a reference to 'intermediate' *tonoi*, presumably Aeolian and Iastian (81.26–7). He cannot mean that every *tonos* is a 'species' of one or another of these 'genera', since he plainly implies that the 'others', used on instruments, are neither Dorian, Phrygian nor Lydian. (The others are again assigned to instrumental pieces at 81.27ff.) Bacchius 303.3–6 contrasts those 'who sing three *tropoi*', Dorian, Phrygian and Lydian, with those who sing seven. Contrast Heraclides at Athenaeus *Deipn.* 624c.

[131] The Alypian vocal notation is based on the letters of the alphabet. In each octave (approximately), the sequence of letters runs from the top downwards, different octaves being distinguished by different rotations or distortions of the symbols. The tables of the *tonoi* are set out with the lowest note first. Hence, the alphabetical order is reversed.

[132] The lowest note in the system of *tonoi* is Hypodorian *proslambanomenos*, whose symbol is -•, used as a form of the letter phi (Φ), the twenty-first letter of the Greek alphabet. Aristides adds three further symbols below it, the lowest being ᘐ, to be read as an omega lying on its side. His tables give the one next to it as Ӿ, clearly a form of the letter chi (X). These are the twenty-fourth and twenty-third letters of the alphabet. The remaining symbol is missing, but must have been a form of psi (Ψ), the twenty-second letter, and it was probably ⋟-. It is not clear that the addition of these symbols had any purpose beyond the completion of the alphabet. The use of the term 'ratio' suggests a Pythagorean source.

[133] There were three tables, giving (a) a series of semitones, (b) a series of tones, and (c) the complete notation of the fifteen *tropoi* or *tonoi*. Our MSS preserve (a) and (b), with a good deal of corruption; (c) is missing, but is readily restored on the basis of the Alypian tables.

[134] Aristides gives three reasons for setting out two systems of notation in parallel. The point of the first is probably that the symbols of the 'instrumental' notation appear in groups of three; in most groups each symbol is a variant of the same sign. The first symbol of a group represents a note in the continuous diatonic series upwards from Hypodorian *proslambanomenos*. The second and third represent the boundaries of two

the *tropoi* looks like a wing,[135] displaying as it does the amounts by which the *tonoi* exceed one another. They are set out in the three genera, and include the consonances [*symphōniai*]: there is a 'consonance' when just one or the other of two symbols which in the enharmonic bound two intervals, in the other genera indicates the two intervals taken together.[136]

Diagram of the series of tones[137]

1	4	7	10	13	18	21	25	28	31	34	39	42	46	49	52	55	60	63	67	70
E♭	F	G	A	B	cx	dx	f	g	a	b	c′x	d′x	f′	g′	a′	b′	c″x	d″x	f″	g″
⊐	⊸	ɜ	♀	W	ɥ	∇	Ω	φ	C	O	K	H	Γ	Ʊ	θ	O′	K′	H′	Γ′	Ʊ′
⊏	⟜	ε	H	h	Ǝ	⊣	ʏ	F	C	K	Λ	>	N	Z	Ϻ	K′	Λ′	>′	N′	Z′

3	6	9	12	16	19	22	27	30	33	37	40	43	48	51	54	58	61	64	69
E♭x	Fx	Gx	Ax	c	d	e	fx	gx	ax	c′	d′	e′	f′x	g′x	a′x	c″	d″	e″	f″x
⚹	⊣	⊔	Ϻ	–	7	⌉	X	T	Π	M	I	Z	A	✱	⊥	M′	I′	Z′	A′
⋇	T	ɜ	Ⲡ	E	⊢	Γ	ч	ꟻ	Ↄ	ℸ	<	⊏	∖	⍀	⋋	ℸ′	<′	⊏′	∖′

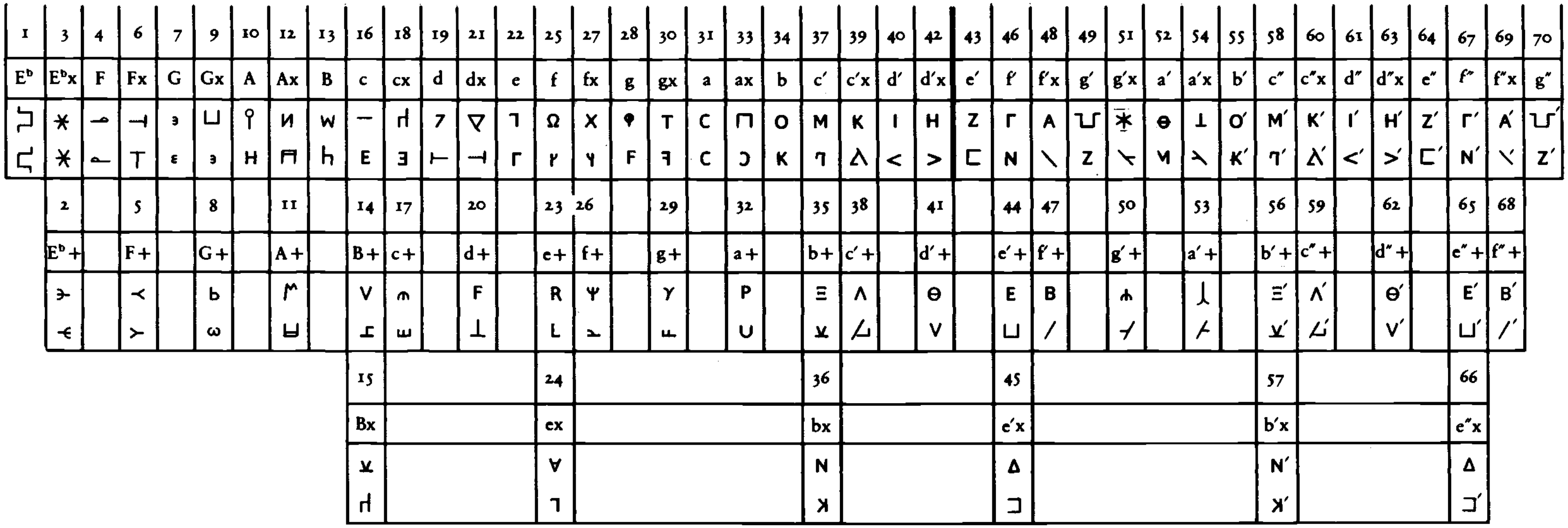
small intervals above the first notes, and the three together can be thought of as indicating the notes of a *pyknon* (see also n. 136 below). These structurally significant groupings are not revealed by the vocal notation. Aristides' second reason is that usually given elsewhere (e.g., Gaudentius 350.9ff., Alypius 367.22, *Anon. Bell.* 67, 68). The objective was apparently to distinguish vocal notes from instrumental ones where they differed in the same piece (see particularly *Anon. Bell.* 68). Aristides' third reason depends on the fact that the vocal notation uses the familiar alphabetical symbols, while those of the instrumental notation are more arcane, less intelligible to non-specialists. The attitude it reflects is Neopythagorean: cf. 3.32–4.1, with the references given in n. 9.

135 The fifteen sequences fit together in the form of a parallelogram.

136 On the word *symphōniai* cf. n. 124 above: its effective sense here is 'unisons' (Gaudentius 350.11ff. calls them *homotona*). The notes in each group of three symbols (see n. 134 above) may be treated as in principle a quarter-tone apart, the scheme being designed initially to accommodate enharmonic *pykna* (though in fact the intervals are variable, depending on genus and shade, and on matters to do with intonation). The third symbol of a group (e.g., symbol 9 in the table of semitones below) is thus a semitone above the first (e.g., symbol 7), and symbols 7 and 9 are used, *inter alia*, to indicate the *proslambanomenoi* of adjacent *tonoi* (Hypophrygian and Hypoaeolian). But the second symbol of the group is always used where the intention is to indicate the second note of a tetrachord, even in diatonic and in those forms of chromatic where the interval between the first and second notes is a semitone. Then, for example, the second note of the lowest tetrachord of Hypodorian will be represented by symbol 8, regardless of genus, and in diatonic etc. this note will be in unison with that represented by symbol 9 (which will not of course appear in the notation of Hypodorian diatonic, but does appear in Hypodorian enharmonic, and also as Hypoaeolian *proslambanomenos*). A further complication is introduced by the fact that the basic symbols (i.e., the first in each group) follow a diatonic sequence, and are therefore separated sometimes by semitones, sometimes by tones. See further note 137 below.

137 In each diagram the top line contains numbers attached to the notes for ease of reference. (They appear as a continuous sequence in the table of *tonoi*.) The second

Diagram of the series of semitones[137]



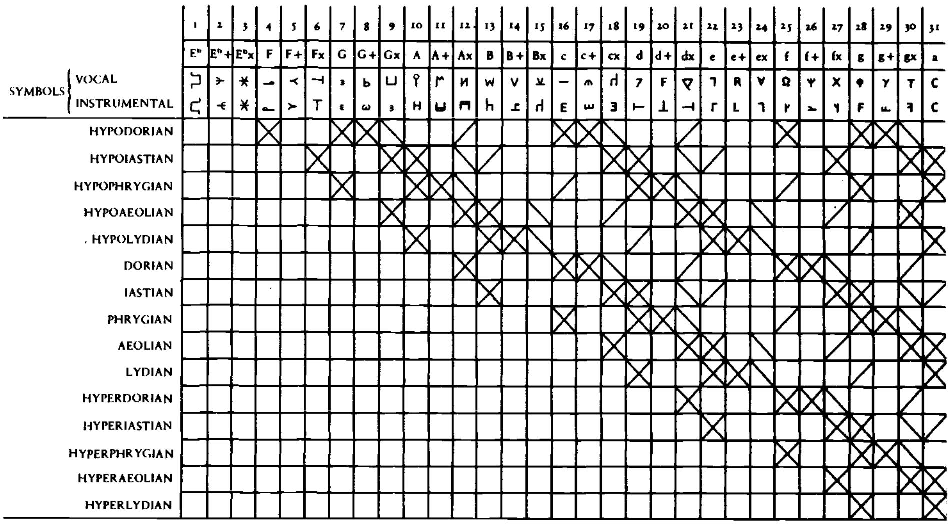


Diagram of the *tonoi* (1) The Greater Perfect System, comprising the tetrachords ***hypatôn***, ***mesôn***, ***diezeugmenôn*** and ***hyperbolaiôn***, together with the

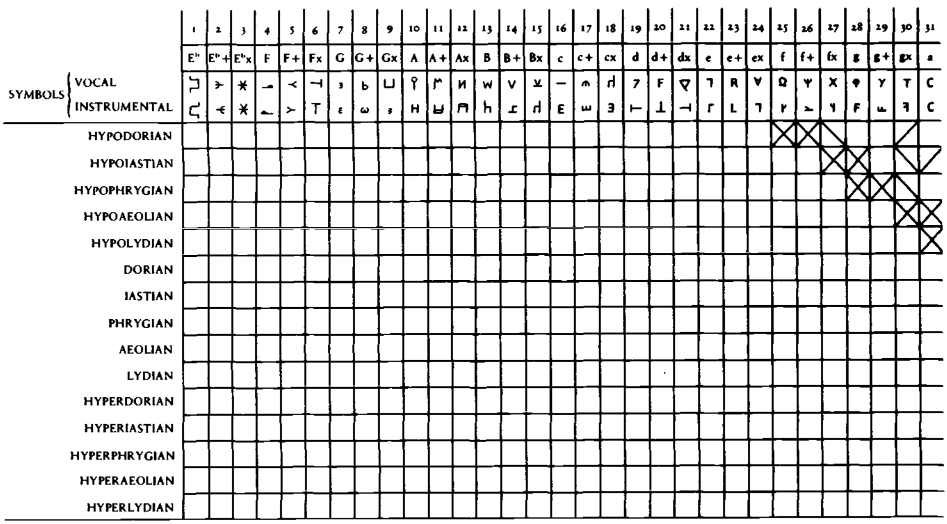
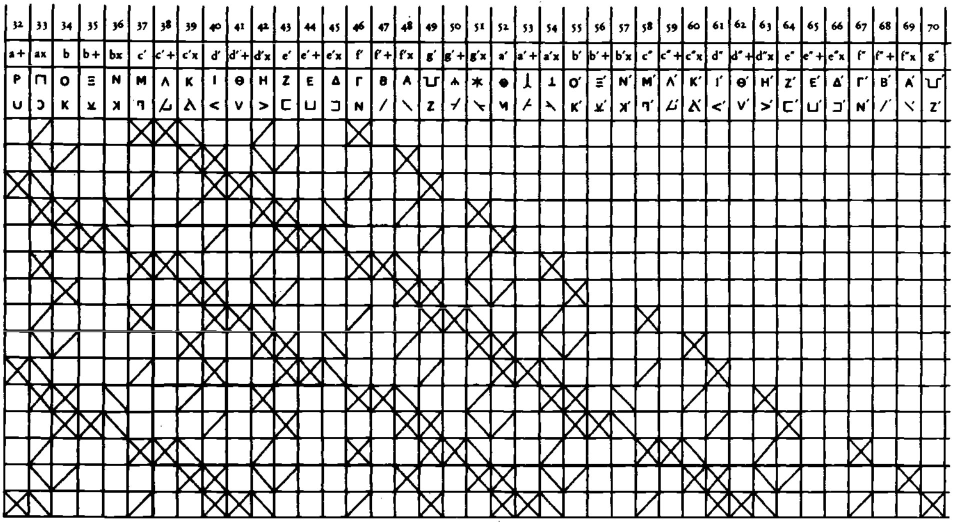
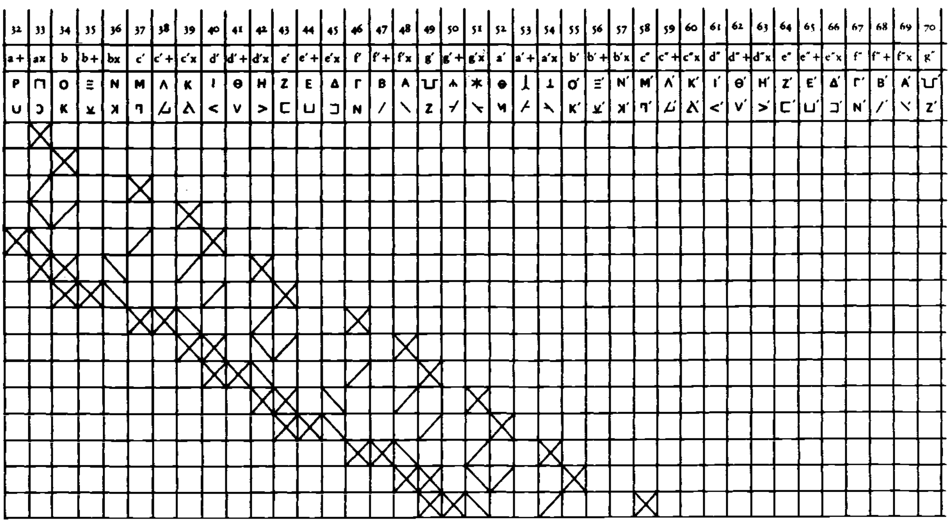


Diagram of the *tonoi* (2) The tetrachord ***synêmmenôn***



initial note *proslambanomenos*



28 Our remaining task is to speak of *eklysis*, *spondeiasmos* and *ekbolē*, since the ancients also adopted the use of these intervals to mark distinctions between the *harmoniai*. A descent of three incomposite dieses was called *eklysis*, an ascent by the same interval was called *spondeiasmos*, and an ascent by five dieses *ekbolē*. These were also called 'modifications' of intervals, since their use was uncommon.[138]

Chapter 12 'Complete' melody is that which is composed of *harmonia*, rhythm and diction. In the more specific sense in which it is used in harmonics, a 'melody' is an interweaving [*plokē*] of notes which are unlike in respect of height and depth of pitch.[139]

Melodic composition is the capacity for constructing melody. It is divided
29 into *hypatoeidēs*, *mesoeidēs* and *nētoeidēs*, corresponding to the characteristic

gives the conventional modern equivalents (following the practice of Chailley (1979) I have indicated the three members of a triadic group (see notes 134, 136 above) as for instance G, G+, Gx). The third line gives the Greek vocal notation and the fourth the instrumental; only these, of course, appear in the MSS.

The first and second diagrams are preserved in the MSS. I have followed Winnington-Ingram's printed text, which restores the few missing symbols and deletes accretions on the basis of Alypius' tables. The third has dropped out of the MSS; the reconstruction I give rests again on Alypius' authority. The fourth may or may not have been given by Aristides, but is added here for the sake of completeness.

The two parts of the first diagram are not continuous with one another. They display parallel sequences of tones, each note in the second part being a (theoretical) semitone above its counterpart in the first.

In the second diagram the first four lines represent a series of semitones. The next four give the 'consonances' or unisons discussed in note 136. The last four display the additional unisons generated by the fact that some notes in the basic diatonic sequence are themselves only a semitone apart, while each note forming the lower boundary of such a semitone (modern E's and B's) is provided, like the others, with its pair of partners. Thus not only does, for example, symbol 23, if used as a sign in diatonic Hypolydian, form a unison with symbol 25, so also does symbol 24, if used in this *tonos* in an enharmonic context.

In the third and fourth diagrams I have indicated the notation of each *tonos* by oblique lines in a grid lying below the sequence of symbols. (No doubt Aristides wrote out the notation in full for each *tonos*.) A note in an enharmonic or chromatic sequence (these are notated identically) is represented by the sign ╲, and a diatonic note by the sign ╱.

The MSS give various other sets of symbols in the spaces following the first and second diagrams. They are certainly no part of Aristides' work, and they might perhaps be a copyist's exercises. Winnington-Ingram rightly rejects them, and I have not reproduced them here.

[138] This paragraph seems a little out of place: it might have been more appropriate as an appendix to chapter 9. Compare especially the way Aristides introduces the 'ancient scales' there, 18.5–6, with the first sentence of the present paragraph. The source of both passages is probably the same, and is likely to derive from Aristoxenus. Little is known about the uses of the intervals described. They were apparently felt to be characteristic of particular musical styles, each associated with its own system of attunement (*harmonia*). Other writers agree with Aristides' evaluations: see Bacchius 301.20–302.6 on *eklysis* and *ekbolē*, and ps.-Plut. *De Mus.* 1135a–b on *spondeiasmos* (1141b also mentions *eklysis* and *ekbolē*, but without analysing them). For a modern discussion see Winnington-Ingram (1928).

[139] On 'melody' and 'complete melody' (*teleion melos*) see n. 13 above, and n. 1 to 7 Aristox. *El. Harm.*

properties of sound which we mentioned previously.[140] Its parts are selection, mixture and use. Selection is that through which the musician discovers the range of sound from which he should construct his *systēma* – the *hypatoeidēs* range, or some other. Mixture is that through which we fit together with one another either notes, or ranges of sound, or melodic genera, or the *systēmata* of the *tropoi*.[141] Use is the production of a melody in a particular way.[142] It in turn has three forms, consecution, distribution and interweaving. There are three forms of consecution [*agōgē*], direct, returning and circular. Direct consecution is that which makes an ascent through successive notes, returning consecution is that which brings about depth of pitch through notes that follow each other, while circular consecution is that which proceeds upwards in conjunction and downwards in disjunction, or conversely, and is also treated as a type of modulation.[143] Interweaving [*plokē*] projects a single sound across two or more intervals or notes by leaps, whether it places the lower or the higher of these first, and in this way produces a melody.[144] Distribution [*petteia*] is that through which we understand which notes should be omitted and which used, and how often in the case of each of them, and from which we should begin and on which we should end. It is distribution that determines the character of the melody.[145]

140 On the inclusion of melodic composition in harmonics see n. 36 above. 'Mentioned previously': the direct reference may be to 23.1ff. The three terms designate respectively a low, intermediate and high range (cf. n. 46 above). It is uncertain whether anything very precise is intended by this division, or what its source is. It is used below at 30.2–4, 10. A similar division, but into four ranges, is given with some detail at *Anon. Bell.* 63, 64.

141 'Selection', 'mixture', 'use' translate *lēpsis*, *mixis*, *chrēsis*. This division of the subject is not mentioned in other Aristoxenian sources, who standardly define melodic composition in general as the 'use' (*chrēsis*) of the items studied by harmonics: e.g., 7 Aristox. *El. Harm.* 38.22–3, Cleonides 180.8, 206.19, *Anon. Bell.* 66. This may be because such authors overlook the tasks of *lēpsis* and *mixis*, rather than because they deliberately include them under *chrēsis*: see n. 142 below. Aristides' *mixis* is apparently the composer's preliminary strategy in laying out the composition, the planning of its structure in terms of genera, *tonoi*, etc. Once this has been done, and the general pitch-range determined (*lēpsis*), he can proceed to the details of melodic construction (*chrēsis*).

142 Writers who define melodic composition as *chrēsis* (n. 141 above) seem to understand it in the same way as Aristides, since they give a similar list of its subdivisions. See, for example, Cleonides 207.1ff., which adds a fourth division, *tonē*, 'extension', referring to a composer's instruction that a note be held for a relatively long time.

143 On these types of *agōgē* see 16.19ff. and nn. 93 and 107 above; cf. n. 126.

144 *Plokē* has already been briefly defined at 16.20–1. It is characterised by melodic leaps. Possibly the term refers specifically to a sequence with regularly alternating upwards and downwards steps (cf. Cleonides 207.3), and may be the same as Ptolemy's *anaplokē*. Ptolemy mentions a number of other kinds of melodic figure (11 *Harm.* 67.6–8), unfortunately without defining them. (There are some grounds for suspecting corruption in the text of the present sentence.)

145 One would expect the third term of the classification to be 'repetition', and this is indeed the sense given to *petteia* at Cleonides 207.4–5. Aristides' *petteia* is quite different and its reappearance in the same sense at 81.3–6 confirms that this is not a textual error. *Petteia* was a game involving the movement of pieces on a board in accordance with a strategy. It seems more plausible to transfer the name metaphorically to 'distribution'

Melodic composition is distinct from melodic performance, in that the latter is the expression of a melody, the former a creative capacity. Styles of composition fall into three generic classes, dithyrambic, nomic and tragic. The nomic style is *nētoeidēs*, the dithyrambic *mesoeidēs*, the tragic *hypatoeidēs*.[146] We also find several species, which can be grouped, on the basis of similarity, under the generic classes. Some are called 'erotic', of which wedding songs form a special category, others 'comic', and others 'encomiastic'.[147] They are called 'styles' [*tropoi*] because through their melodies they reveal, in one manner or another, the character of a state of mind.[148]

Compositions differ in the following ways: in genus (enharmonic, chromatic or diatonic), in *systēma* (*hypatoeidēs*, *mesoeidēs* or *nētoeidēs*),[149] in *tonos* (for example Dorian or Phrygian), in style (nomic, dithyrambic), in character (for instance we call one through which we arouse mournful emotions 'depressing', one through which we lift up the spirit 'exalting', and one through which we bring the soul round to peacefulness 'intermediate').[150] These are called

than to 'repetition'. However, 'distribution' assorts oddly with *agōgē* and *plokē*; they are kinds of melodic figure, and it is not. Aristides may have taken his account of *petteia* from a non-Aristoxenian source (the school of Damon is mentioned in the context of 81.3–6), and fitted it, ineptly, to the term as it appeared in the Aristoxenian lists of melodic figures. Aristides holds that the moral character of a melody is founded in that of the notes it contains. A good composer will therefore 'distribute' his notes in accordance with the moral effect he intends to produce (Book II chs. 12–14).

[146] This classification of styles (*tropoi*) of melodic composition may be Aristoxenus' (cf. 7 *El. Harm.* 40.22, and the probably Aristoxenian division of rhythmic *tropoi* at 40.14–15 below), though attempts at classification are found much earlier: cf. Pindar frag. 126 (which distinguishes paeans, dithyrambs, laments), Plato *Ion* 534c (dithyrambs, *enkōmia*, *hyporchēmata*, epics, iambics), and especially *Laws* 700b (hymns, laments, paeans, dithyrambs, *nomoi*). Aristides is evidently suggesting that the genre of a proposed piece will govern the aspect of composition that he calls *lēpsis*. The three *tropoi* mentioned (linked below with three kinds of ethos) are best construed generically, rather than with exclusive reference to *nomoi*, dithyrambs and tragedies. *Nomoi* were solo songs sung by professionals, noted for highly coloured effects. The nomic *tropos* probably includes all emotionally affecting solo pieces. Dithyrambs were choral, and scarcely distinct in form from other choral lyrics. It may be the fact that they were performed by a group of amateurs that links them with medium pitch (cf. ps.-Ar. *Probs* XIX.48), and the designation may embrace a wide range of choral music. Tragedy included both solo and chorus: probably solos are intended here, and specifically those of a solemn and serious character. On all these matters see note 150 below.

[147] 'Erotic' refers to the broad genre of love-songs. 'Wedding-songs' translates *epithalamioi*; many early lyrics are of this type. 'Comic' (*kōmikoi*) may mean 'related to comedy' or 'related to revelry': a *kōmos* was a revel, or a procession of revellers, or a song sung in such a procession, e.g., Pindar *Pyth.* 8.20, 70. It is not clear under which generic *tropos* Aristides intends each to fall.

[148] *Tropos* can mean 'style' quite generally, and is commonly used to designate a habit, character or way of life. On the connection between *tropos* and *ēthos* ('character') see also n. 150 below.

[149] It is a little unusual to describe difference in pitch-range as difference of *systēma*, which normally refers to the organisation of intervals: but compare 81.7ff.

[150] 'Style' translates *tropos*, 'character' *ēthos*. In the parallel classification of rhythmic *tropoi* at 40.14–15, the types here listed as kinds of *ēthos* reappear (with 'peaceful', perhaps 'calming', *hēsychastikos*, for 'intermediate') as kinds (genera) of *tropos*. 'Depressing' translates *systaltikos*, lit. 'contracting'; 'exalting' translates *diastatikos*, whose related verb usually means 'separate', 'divide', but sometimes 'open' or 'expand'. Cleonides 206.3–18 gives the same classification of the *ēthē* of melodic

'characters' because it is primarily through them that conditions of the soul are diagnosed and put right.[151] But this is not done by them alone: rather, while they assist, as parts, in the remedial treatment of the emotions, we have established that it is 'complete' melody which offers, in addition, an education that lacks nothing. For just as in the case of medical drugs no one substance has the natural capacity to cure the afflictions of the body, but full recovery is brought about by a mixture of several, so also in our own field melody alone makes only a slight contribution towards putting things right, while a complete combination of all the elements is fully sufficient.[152]

31 We should now have an adequate grasp of the harmonic aspect of musical science. Let us pass on to the study of rhythm.

Chapter 13 The term 'rhythm' is used in three ways. It is applied to bodies that do not move, as when we speak of a statue having 'good rhythm', to anything that moves, as when we speak of someone walking with 'good rhythm', and it has a specific application to sound. The last of these is the one we are to discuss.[153]

Rhythm, then, is a *systēma* of durations put together in some kind of order.[154]

composition, with useful comments. 'The *diastatikon* (or possibly *diastaltikon*) *ēthos* of melodic composition is that through which there are signified magnificence and a manly elevation of soul, and heroic deeds, and emotions proper to these. They are used especially by tragedy, and by such other things as are bound up with the character of this *ēthos*. The *systaltikon ēthos* is that through which the soul is drawn together into dejection and an unmanly condition. Such a condition accords with the emotions of love, and with dirges and laments and the like. The *hēsychastikon ēthos* of melodic composition is that upon which follows calmness of soul and a free and peaceful condition. In accord with it are hymns, paeans, encomia, *symboulai*, and things like that.' (*Symboulē* means 'counsel', 'advice'. Its musical sense is unclear, but something didactic and morally improving is evidently intended.) The 'styles' of 30.7–8 are probably to be linked with these 'characters', the nomic style with the *systaltikon ēthos*, the dithyrambic with the *hēsychastikon* or intermediate, the tragic with the *diastatikon*. See further the classifications in Plato *Rep.* 398–9, Aristotle *Pol.* 1340b, 1341b, ps.-Ar. *Probs* XIX.15, 30, 48: cf. also the quotations from Theophrastus at Plut. *Quaest. Conv.* 623a.

[151] On the diagnostic and curative powers of music see Book II, especially chs. 5, 6, 9, 10, 16.

[152] On 'complete' melody see 4.20 with n. 13. Aristides is fond of medical analogies: they abound in Book II (where these matters are discussed at length) and in Book III.

[153] The ultimate source of much of the chapters on rhythm is Aristox. *El. Rhythm.* The notes that follow refer extensively to the surviving parts of Book II, translated in the Appendix to chapter 7: passages are identified by reference to the numbered paragraphs. With the present passage compare *El. Rhythm.* II.1. Aristoxenus himself seems to have drawn on Damonian theory: see Plato *Rep.* 400a–c. The basic sense of *rhythmos* is probably to do with form and order. It was used in the sense 'shape', without reference to movement, by the early Atomists (Aristotle *Metaph.* 985b 16): cf. also Herodotus V.58, Xenophon *Mem.* III.10.10.

[154] I use 'duration' throughout to translate *chronos*, lit. 'time'. Rhythms divide temporal continua into *chronoi*, whose relative lengths determine rhythmic type: see especially chapter 14. Bacchius (313.1–10) collects six definitions of rhythm, two of which (attributed to Nicomachus and Leophantus) have affinities with this one. The one he ascribes to Aristoxenus ('time divided up in relation to each of the things that are capable of being rhythmically organised', cf. *El. Rhythm.* II.6, 9) is not like Aristides', but cf. *El. Rhythm.* II.7. Bacchius' definition emphasises an important characteristic of Aristoxenus' view as expressed in these passages: time cannot be divided up and

The modifications of these durations we call arsis and thesis, and sound and silence.[155] Notes as such, because of the lack of differentiation in their movement, leave the interweaving [*plokē*] of the melody obscure and confuse the mind: it is the elements of rhythm that make clear the character of the melody, moving the mind part by part, but in an ordered way.[156] Arsis is the upwards movement of a part of the body, thesis the downwards movement of the same part. Rhythmics is the science of the employment of the things we have mentioned.

Rhythm in general is perceived by three senses, which are these: sight, as in dancing; hearing, as in melody; and touch, by which we perceive, for instance, the pulsations of the arteries. Musical rhythm, however, is perceived by two of them, sight and hearing. Rhythm is imposed in music upon the movement of the body, upon melody, and upon diction:[157] and each of these appears both alone and in combination with the others – with either of the others on its own or with both together.[158] Melody is apprehended by itself in diagrams and in tunes which are rhythmically unstructured;[159] in combination with rhythm

rhythmically organised except through the temporal division of some 'material' (the *rhythmizomenon* of *El. Rhythm.*), whose parts are then organised in a rhythmic structure.

[155] Arsis is 'rise', 'up-beat'; thesis is 'placing', 'down-beat'. The terms relate directly to the movements of dancers' feet (and perhaps arms, see 31.15–16). Thesis is heavier, more emphatic: Aristoxenus sometimes calls it *basis*, 'step', though his usual terms are *anō chronos* and *katō chronos* ('upwards duration', 'downwards duration'). Cf. Bacchius 314.10ff. For Damon's use of these terms see Plato *Rep.* 400b6 (*anō* and *katō*), and 399d10, 400a5, b2 (where *basis* seems to refer to the pattern of movement of a whole foot or rhythmic sequence). Aristides usually describes arsis and thesis as 'parts' of the foot or rhythm, not as modifications (*pathē*) of durations. But the sense seems clear, each duration, in addition to being of a certain length, has the quality of an up-beat or a down-beat. Similarly, it is characterised either by sound or by silence (at a 'rest' in the melodic line: see 38.28–39.2).

[156] The 'elements' here are arsis and thesis. The reference to 'notes as such' is to a melodic or scalar sequence abstracted from any associated rhythm: cf. 31.24–5. If any such sequence is sounded, the durations of the notes must have some definite temporal relations, but there may be no differentiation of arsis and thesis, and hence no intelligible rhythmic sense. Aristides' reference to the function of rhythm in clarifying the character (*dynamis*) of a melody is acute, and scarcely paralleled in ancient sources. But compare 40.20ff., 77.5ff., and ps.-Ar. *Probs* XIX.49.

[157] Paralleled at *El. Rhythm.* II.9. These are the *rhythmizomena*, the 'materials' to which rhythmic form is given: see n. 154 above.

[158] In most of the rest of the paragraph, *rhythmos* is conceived as this sentence suggests: it is something imposed on movements, melody and diction. But a different notion is also involved, where *rhythmos* is identified with rhythmic bodily movement. (There would otherwise be no point in limiting the combination of *rhythmos* and diction to cases where poetry is read in a 'dramatised' way, lit. 'with acting'.) Similar juxtapositions of the two conceptions are found in Plato (throughout *Laws* II) and Aristotle (*Poetics*, 1447b 24ff.). Aristoxenus (*El. Rhythm.* II.3–6, 9) carefully distinguished them, confining *rhythmos* to its first sense, and treating bodily movements just as one of the 'materials' on which rhythm may be imposed. He is presumably the source of the first part of the present sentence. The sequel may be drawn from an earlier source, or one independent of Aristoxenus. It has considerable affinities with the first chapter of Aristotle's *Poetics*.

[159] 'Diagrams' are representations of scales, such as the tables of notes at 24–7 above, used and called by this name since before Aristoxenus (e.g., 7 *El. Harm.* 2.15). The context requires that the items mentioned be actually sung or played. We do not know whether

alone in instrumental pieces [*kroumata*] and interludes [*kōla*];[160] and with diction alone in so-called 'melted songs'.[161] Rhythm appears on its own in unaccompanied dance, with melody in instrumental pieces [*kōla*], and with
32 diction alone in dramatised recitations of poems, such as those of Sotades and others like them.[162] We have already mentioned the ways in which diction is combined with each of the others. The combination of them all makes song. In diction, rhythm is differentiated by syllables, in melody by the ratios of arses to theses, and in movement by the figures and their boundaries, which are also called 'points'.[163]

The science of rhythmics has five parts. We shall discuss (i) primary

the Greeks practised 'scales', but Book II chs. 13–14 quite strongly suggest something of the sort, as does 11 Ptol. *Harm.* 105.10–12. On 'rhythmically unstructured tunes' (*ataktoi melōdiai*) see n. 161 below.

[160] Our sources are inconsistent in their uses of *kōlon* and *krouma*. *Krouma* and its correlates are derived from a verb meaning to strike, and are proper to stringed instruments, but are sometimes used of wind instruments too (e.g., Plut. *Quaest. Conv.* 638c, Pollux *Onomastikon* IV.83, 84). One would expect the term to embrace instrumental pieces generally. *Kōlon* means a part or limb, and is used in poetics of (roughly) what we would call a 'line' of verse. In the present context a sense such as 'instrumental phrase' (in a mainly vocal piece) seems likely: cf. *Anon. Bell.* 68. But the reverse usage seems indicated at 23.21, and the fact that *kōlon* appears alone in contrast to vocal music at line 29 below and at 77.30 suggests a broader sense.

[161] *Kechymena asmata. Anon. Bell.* 3, 85, says that sequences in scores written without rhythmic symbols are called *kechymena* in the case of song, *diapsēlaphēmata* in that of instrumental music. The latter may correspond to Aristides' 'unstructured tunes' above. (*Anon. Bell.* assumes that rhythmic symbols are always used where rhythmic structure is intended; this is not borne out by the surviving scores.) But Aristides must be referring to something actually performed, not merely notated. *Anon. Bell.* 95 defines *kechymenai* songs and melodies as ones 'irregular in respect of time, and performed without order (*chydēn*) in respect of time'. (Plato *Laws* 811d, and Aristotle *Rhet.* 1409b, use *chydēn* of prose by contrast with verse.) Something rather like recitative seems intended: cf. the references to *parakatalogē* at ps.-Ar. *Probs* XIX.6, ps.-Plut. *De Mus.* 1141a. 'Melted' also recalls Aristophanes' allusion to the rhythmic 'flexibility' (amounting in his view to total disorder) of Agathon's verse: the poet has to bring his strophes out into the sun to be softened, so that they can be bent (*Thesm.* 68–9).

[162] 'Dramatised' renders *meta peplasmenēs hypokriseōs*, lit. 'with carefully moulded acting'. The reference must be to formal gestures in the dance: cf. 6.22–4, 84.11ff. On the scurrilous mime-writer Sotades see particularly Strabo *Geog.* XIV.1.41, which confirms that he used speech rather than song.

[163] The original source seems again to be *El. Rhythm.* II.9. Aristides' divergences from Aristoxenus suggest that he was using an intermediate authority. In Aristoxenus it is time (*chronos*), not rhythm that is 'differentiated' or 'divided', and he implies that the ratios of up-beat to down-beat are involved in all three cases (cf. *El. Rhythm.* II.16ff.), as indeed they are. It is not clear whether Aristides means that 'points' (*sēmeia*, alternatively meaning 'signs') is a name for the boundaries of figures (i.e., the points of change between them), or for figures (*schēmata*) together with their boundaries. In the next paragraph the geometrical usage suggests 'boundaries' (which may have been marked by 'signs' from the chorus-master); cf. Plut. *Quaest. Conv.* 747c–e, Psellus 6 (= Westphal 75.10–23). *Schēma* is often the word for a dance-position or gesture; *sēmeion* is freely used in this context by Aristoxenus, but appears rarely elsewhere. Where it does, neither the sense 'boundary' nor the geometrical analogy seems intended. *Schēma* and *semeion* seem to be interchangeable at Aristotle *Poetics* 1462a, and compare the uses of *sēmeion* in *El. Rhythm.* with Aristides' use of *schēma* at 32.18. Athenaeus *Deipn.* 628d says that *schēmata* were treated as *sēmeia*, 'signs' of what was being sung, which may indicate the origin of the usage. In the sequel Aristides' focus is on units of duration, not their boundaries (despite the geometrical analogy).

durations, (ii) the genera of feet, (iii) rhythmical tempo [*agōgē*], (iv) modulations, and (v) rhythmical composition.[164]

Chapter 14 A primary duration is one that is indivisible and minimal: it is also called a 'point'. By 'minimal' I mean minimal in relation to us, that is, the first duration that can be grasped by perception.[165] It is called a point because it has no parts, just as the geometers have used the term 'point' for what in their science has no parts.[166] This duration without parts has the role, as it were, of a unit: in words it is located in the syllable, in melody in the note or the single interval, in bodily movement in a single figure.[167] It is called 'primary' in relation to the movement of each of the singers, and in relation to the way it is combined with the other notes: for each of us might select a different one of them, until we have come across the magnitude which contains two intervals. It can most accurately be judged, as I have said, from the magnitude of the notes which follow.[168]

A composite duration is one which may be divided. One composite duration is double the primary duration, another is triple, another quadruple. Rhythmical durations increase up to the quadruple unit, being analogous to the number of dieses in the tone, and having a natural affinity to intervallic sound.[169] Some of these durations are called 'rhythmical', some 'un-

[164] (i) is discussed at 32.11–24: the sequel, concerning composite durations (32.25–33.11) and feet (33.12–28) is also relevant; (ii) at 33.29–38.14; (iii) at 39.26–30; (iv) at 40.1–27; (v) at 40.8–25.

[165] This suggests that the primary duration is an absolute, determined by the sense of hearing. Porphyry (*Comm.* 78.21–79.28) quotes a long passage of Aristoxenus *On the Primary Duration* defending the doctrine that the size of the duration to be accounted 'primary' (*prōtos*) depends on the rhythmic tempo (*agōgē*) of each piece. He argues that this does not make rhythmic science lapse into indeterminacy. Its main business is with the ratios of durations in a given rhythm, which are independent of the absolute length of the duration treated as primary, so long as this role is assigned to the same length throughout the piece under analysis. Perhaps this essay was written to defend and elaborate the views of *El. Rhythm.*, where the issues do not seem to have been fully clarified (see *El. Rhythm.* II.10–12; Aristides could easily have been misled by 11). See also 7 *El. Harm.* 34.11ff., cf. 68.13ff. The view implied by the present sentence is replaced by that of Aristoxenus at lines 19ff. below.

[166] See note 163 above. For the geometrical usage see, for example, Aristotle *Phys.* 240b 3, Euclid *Elements* Def. 1; as a point of time Aristotle *De Caelo* 283a 11, *Phys.* 262b 2.

[167] Compare *El. Rhythm.* II.12, which confirms the general sense. The reference to an interval is odd; the passage from one note to another in a continuous melody has no duration (e.g., 7 *El. Harm.* 8.25–9.1). But compare the parallel oddity of *El. Rhythm.* II.9.

[168] The main point of the last two sentences is to state the Aristoxenian position outlined in note 165 above. The primary duration is identified by its relation to others in the same piece, which will be equal to it or multiples of it, never smaller. The clause beginning 'for each of us might' is awkwardly expressed. The sense is probably that we cannot tell whether a given note in a piece constitutes one or more than one primary duration by considering it alone. We might take it to be a primary duration, and later find that the same magnitude elsewhere in the piece was divided into two durations (or more, though Aristides does not say this). It is not clear whether 'two intervals' is a mistake for 'two notes' (cf. n. 167), or whether Aristides means 'two intervals (i.e., durations) of time'.

[169] Aristides enjoys displaying numerical affinities of this sort (cf. 42.8ff.) and uses them abundantly in Book III (see especially chapter 6). The notion 'composite duration' is quite different in Aristoxenus, as can be seen from *El. Rhythm.* II.14–15. A duration's

rhythmical', some 'quasi-rhythmical'. Those which maintain an order in some definite ratio to one another are rhythmical: examples are the double, the hemiolic, and so on (a ratio being the relationship of two commensurable magnitudes with one another). Those which are altogether disordered and linked together in no ratio are unrhythmical; while the quasi-rhythmical are those intermediate between the first two, possessing in some respects the orderliness of the rhythmical, in others the confusion of the unrhythmical. Some of them run on more quickly than they should, and are called 'abbreviated', while the composite notes of others make them proceed more slowly than is normal, and these are called 'expanded'.[170] Again, some durations are simple, others multiple, the latter also being called 'foot durations'.

A foot is the part of a complex rhythm through which we understand the whole.[171] It has two parts, arsis and thesis. Feet differ from one another in seven ways:[172] in magnitude, as feet of three units differ from those of two;[173] in genus, for instance the hemiolic and the duple;[174] in composition, in respect of which some, like the two-unit foot, are simple, while others, like the twelve-unit foot, are compound (the simple being those divisible into durations, the compound those analysable also into feet).[175] The fourth distinction is that between rational and irrational, rational feet being those the ratio between whose arsis and thesis we shall state below, irrational ones those of which we cannot specify the relationship between the temporal parts as the same ratio throughout.[176] The fifth is in respect of types of division, because when

character as composite or incomposite has nothing to do with its relation to the primary duration. It depends only on whether, in the individual case, it is or is not divided between several notes, syllables or figures, or is divided in one of these ways and not in another. It is not clear why Aristides treats the quadruple as the largest multiple of the primary duration. Elsewhere he mentions, for example, feet of twelve durations (33.18), feet with a thesis of eight durations (36.5), etc. Perhaps he has misunderstood *El. Rhythm.* II. 17–18.

170 This sentence is best taken as referring to the 'quasi-rhythmical' (*rhythmoeideis*) sequences, not as an additional classification of rhythmical sequences. 'Abbreviated' represents *strongyloi*, lit. 'round', but in rhetoric 'terse', 'laconic'. 'Expanded' translates *peripleōi*, lit. 'over-full'. The sense is probably that some durations in an 'abbreviated' rhythm fall short of their proper length, while some in an 'expanded' rhythm exceed it, in each case by less than a measurable unit (i.e., a primary duration). Compare the distinction between rational and irrational feet, 33.19–22, 34.15–18, and the discussion of a special example, 37.24ff. See also Dion. Halic. *De Comp. Verb.* 17 and 20. But at Aristides 84.5–10 *strongyloi* may merely designate rhythms composed of quick durations, *peripleōi* those composed of slow ones.

171 Compare *El. Rhythm.* II.16.

172 The list of seven kinds of distinction is derived from *El. Rhythm.* II.22.

173 Compare *El. Rhythm.* II.23. The two-unit foot reappears at 35.5–6. Aristoxenus treats the three-unit foot as the smallest (*El. Rhythm.* II.31).

174 Compare *El. Rhythm.* II.24.

175 Compare *El. Rhythm.* II.26. Aristides has reversed the order of Aristoxenus' third and fourth distinctions.

176 Compare the account of rational and irrational intervals, 11.4–7, and see 34.15–18, 37.24ff. For Aristoxenus' approach to the matter see *El. Rhythm.* II.20–21, and compare the notes to Aristides' two later passages. The gist of Aristides' conception of irrationality here should be that in an irrational foot the ratio between arsis and thesis is not one of the regular kinds 'mentioned below' (i.e., at 33.29ff.). 37.24ff. suggests that

compound feet are divided in different ways, different simple feet arise from them.[177] The sixth is in respect of the arrangement created by the division.[178] The seventh is in respect of antithesis, when one of two given feet has the longer duration in the first place and the shorter in the second, the other the opposite way round.[179]

There are three genera of rhythm, the equal, the hemiolic and the duple (to
34 which some people add the epitritic), constituted out of the magnitude of their durations.[180] Thus the number 1 conjoined with itself creates the ratio of equality, 2 related to 1 creates duple ratio, 3 to 2 hemiolic, and 4 to 3 epitritic. Equal ratio begins from the two-unit magnitude, and increases up to that of sixteen units since we lack the power to recognise larger rhythms in this genus. Duple ratio begins from the three-unit magnitude, and its limit is that of eighteen units: beyond that we cannot grasp the nature of this kind of rhythm. Hemiolic ratio begins from the five-unit magnitude and increases up to that of twenty-five units: our sensory faculty recognises this sort of rhythm only up to this size. Epitritic ratio begins from the seven-unit magnitude and continues up to that of fourteen units; and it is rarely used.[181] There are other genera besides, which are called 'irrational', not because they have no ratio, but because they do not fit comfortably into any of the ratios specified above, and because it is with respect to numbers rather than with respect to rhythmic forms that they preserve their proportions.[182]

he is thinking of cases where, in a foot that normally exhibits one of the regular ratios, the arsis or thesis is expanded or shortened by less than a primary duration (cf. Bacchius 313.19ff.). These may correspond to the 'quasi-rhythmic' cases of 33.1ff. But his form of expression is odd (and non-Aristoxenian). He is probably envisaging such a foot in the context of a series of similar but 'rational' ones; the sequence as a whole does not then exhibit the same ratio throughout.

177 Compare 34.19ff., 38.15ff., and *El. Rhythm.* II.27: see also the next note.

178 Compare *El. Rhythm.* II.28. In this and the previous paragraph, Aristoxenus speaks of divisions of a temporal magnitude, not a compound foot. Such a magnitude will be defined for these purposes as so many primary durations. The same magnitude may be divided into elements differing in length, number or both (difference of 'division'); the same elements of a given division may be reorganised into different patterns (difference of 'arrangement', *schēma*: cf. 7 *El. Harm.* 74.9ff.).

179 Compare *El. Rhythm.* II.29. Aristoxenus makes it clear that this is not merely a reversal of longs and shorts, but of up-beat and down-beat. Cf. Aristides 45.26–9.

180 More exactly, out of the ratios between the magnitudes of the durations of arsis and thesis. Aristoxenus discusses the genera at *El. Rhythm.* II.30–6; he expressly rejects the epitritic in 35. (But compare the metrical analysis given at Hephaestion *Ench.* 3.3, and see Aristides 34.13–15, 38.20, 39.18–20, 45.5–11.)

181 These points reappear at Psellus 12 (= Westphal 85.8ff.). Neither author clearly identifies the rhythmic units whose maximum recognisable lengths these are. The numbers of durations given do not correspond either to those of the largest compound feet mentioned by Aristides in chs. 15–17, or to those of the metres discussed in chapter 23, nor can they be derived from Aristoxenus' doctrine (*El. Rhythm.* II.17–18) that the analysis of some larger feet involves reference to two up-beats or two down-beats or both, but no more. But Aristides' emphasis on perceptual recognisability, not merely numerical proportion, as a crucial feature of rhythm is Aristoxenian: see the next sentence, and cf. *El. Rhythm.* II.21, 7 *El. Harm.* 48.15–49.7.

182 Compare note 176 above. Aristoxenus' account of rhythmic rationality and irrationality is at *El. Rhythm.* II.20–1. Aristides' distinction between rhythmic form and numerical proportion is probably derived from the latter part of that passage: a set of durations may at the same time be numerically rational and rhythmically irrational; cf. Bacchius

Some rhythms are composite, some incomposite and some mixed. Those which are put together from two or more genera, such as those of twelve units, are composite, those which use just one genus of foot, like those of four units, are incomposite, while those which are anlaysed sometimes into durations and sometimes into rhythms, like those of six units, are mixed.[183] Some composite rhythms are coupled, others periodic. A combination of two simple and dissimilar feet is coupled: a combination of more than two is a period.[184]

Chapter 15 Of the genera of feet the dactylic is first, because of its equality. Let us then discuss it first. In the dactylic genus there are six incomposite rhythms, (i) the simple prokeleumatic (short thesis, short arsis), (ii) the double prokeleumatic (two shorts for the thesis, two shorts for the arsis, and the converse), (iii) anapaest *a maiore* (long thesis, arsis of two shorts), (iv) anapaest *a minore* (arsis of two shorts, long thesis), (v) simple spondee (long thesis, long arsis), (vi) greater spondee, also called double spondee (thesis of four units, arsis of four units).[185] Two rhythms are produced by coupling, of which one is called the ionic *a maiore*, the other the ionic *a minore*. The former is composed of a simple spondee followed by a prokeleumatic of two units, while the other has their order reversed.[186]

Dactylic rhythm was so called because the arrangement of its syllables is analogous to the parts of the finger, anapaestic either because it is arranged in reverse order, or because the sound runs quickly across the short syllables and pauses when it comes to the long one.[187] The prokeleumatic is also called

313.19–20, 315.17–18. For analogous contrasts between quantifiability and perceptible form see 7 *El. Harm.* 39.4ff., 47.8ff. See further 37.24ff. below.

183 'Incomposite' rhythms involve only one genus of foot, 'composite' more than one. 'Mixed' rhythms are sequences divisible in different ways, into feet or into durations; in the latter case the whole sequence will count as a single foot. (The distinction corresponds to that between 'compound' and 'simple' feet at 33.16–19 and elsewhere.) It is not immediately obvious why Aristides connects such 'mixture' with considerations of genus. But a sequence of six durations, e.g., ⏑–⏑–, if conceived as two feet, is of the iambic genus, in the ratio 1:2, whereas if conceived as a single foot, divided 3:3, it is of the dactylic genus, in equal ratio. Such rhythms are discussed at 38.3ff., where they are again called 'mixed'; cf. *El. Rhythm.* II.34.

184 'Coupled' represents *kata syzygian*, 'periodic' *kata periodon*. Coupled rhythms are mentioned at 35.14–17, 36.6–8, 37.19–23, periodic ones at 36.8–24. Metrical couplings are mentioned at 45.15 and elsewhere in chs. 23–7.

185 The foot usually called 'dactyl' is –⏑⏑ (called 'anapaest *a maiore*' here by Aristides, but compare 35.18, 37.16); calling the genus 'dactylic' indicates only the equality of arsis and thesis. The usage may be Damonian: see Plato *Rep.* 400b5–7. Of the feet mentioned, the second and sixth are not 'incomposite' or 'simple' in the sense of 33.16ff. (see n. 183 above). The first is the foot Bacchius calls *hēgemōn* (315.4ff.); it is rejected by Aristoxenus (*El. Rhythm.* II.31).

186 Coupled rhythms are 'composite', and composite rhythms were said in this connection to combine different genera (34.19–35.2). The present examples do not: their components are – – and ⏑⏑, both of the genus defined by equal ratio. Different genera are involved, perhaps, in the sense that equal ratio characterises each major component, but not the relation between the components, which is 2:1. More probably Aristides is now using 'coupling' to refer to a combination of components that are dissimilar in any way, not necessarily in genus; cf. 36.6ff.

187 The account of 'dactylic' is familiar: the finger (*daktylos*) has one long section and two short. It relates to the dactyl standardly so called, rather than to anything characteristic of the 'dactylic' genus in general. Aristides' derivations of 'anapaestic' are unconvincing

'pyrrhic', because it is used in war-dances and at the games. The spondee gets its name from its use in libation songs, and the ionic from the vulgarity of its rhythm – vulgarity being something for which the Ionians used to be made fun of.[188] So much for the dactylic genus.

36 *Chapter 16* Within the iambic genus fall the following simple rhythms: (i) the iambus (arsis half the length of the thesis, thesis double the arsis),[189] (ii) the trochee (double thesis and short arsis), (iii) the orthian (arsis of four units, thesis of eight), (iv) the semantic trochee (thesis of eight units, arsis of four).[190]

The composite rhythms produced by coupling are the two bacchii, of which one has an iambus first and a trochee second, and the other reverses the order.[191] There are twelve that are periodic. Four are made up of one iambus and three trochees (the one with the iambus first is called a trochee *ab iambo*, the one with it second a trochee *a bacchio*, the one with it third a bacchius *a trochaeo*, the one with it fourth an epitritic iambus). Four have one trochee, the remainder being iambi. (The one with the trochee first and the iambi after it is called an iambus *a trochaeo*, the one with the trochee second an iambus *a bacchio* or an intermediate bacchius, the one with it third a bacchius *ab iambo*, the one with it fourth an epitritic trochee.) Four contain two trochees and an equal number of iambi: the members of each pair may be successive, or those of one pair may lie in between those of the other. (The one with its iambi first and its trochees following is called a simple bacchius *ab iambo*, the one with its trochees in the lead and its iambi following is a simple bacchius *a trochaeo*, the one with its iambi in the central position an intermediate iambus, and that with its trochees in the centre an intermediate trochee.)[192]

The iambus is so called from the verb '*iambizein*', which means 'to abuse', and is itself based on the word '*ios*' (the rhythm is appropriate for this use

('in reverse order' is *anapalin*; 'to pause' is *anapauesthai*). The real derivation is probably from *ana*, 'backwards', and *paiein*, 'to strike'.

188 'War-dances' translates *pyrrhichai* (see, for example, Athenaeus *Deipn.* 630d–631c). 'Libations' are *spondai*, an important part of many Greek ceremonies (on related music see Xenophon *Symp.* 2.1, Plato *Laws* 799b, ps.-Plut. *De Mus.* 1134f ff., 1137a, 1137b–c). On the character of the Ionians see particularly Athenaeus *Deipn.* 625b–d, cf. 620e and, for example, Aristoph. *Peace* 46, *Thesm.* 163.

189 Lit. 'arsis of a half and thesis of a double'.

190 The solemnity of the last two rhythms is noted at 83.4–6, cf. 36.29ff. They are mentioned together also at ps.-Plut. *De Mus.* 1140f. The 'orthian' rhythms of Athenaeus *Deipn.* 631b are plainly different: so too is the *orthios* foot described by Bacchius, said to consist of 'an irrational arsis and a long thesis' (315.17–18. He defines 'irrational' as 'longer than a short and shorter than a long', 313.19–20; his example of a word in this rhythm is *orgē*.) Usage was evidently variable.

191 The two bacchii are then ⏑––⏑ and –⏑⏑–. Elsewhere the name 'bacchius' is usually given to ––⏑ (e.g., Dion. Halic. *De Comp. Verb.* 17) or ⏑–– (e.g., Hephaestion *Ench.* 3); Bacchius (316.1–2) assigns it to ⏑⏑––. Aristides himself uses it for ⏑–– at, for example, 44.22, cf. 50.23–26. On 'coupling' see n. 186 above.

192 On the sense of 'periodic combination' see 35.1–2. The 'bacchii' whose names are involved here are again –⏑⏑– and ⏑––⏑. The scheme excludes rhythms in which trochees and iambi alternate (–⏑ | ⏑– | –⏑ | ⏑– and ⏑– | –⏑ | ⏑– | –⏑), but each of these might be read as a sequence of two identical bacchii.

because of its similarity to ordinary speech, and the inequality of its parts).[193] The trochee is so called because it makes a running step;[194] the orthian because of the dignity of its gestures and steps;[195] the semantic because, being slow in
37 its durations, it employs artificial time-indications for the sake of intelligibility, and reduplicates its thesis.[196] The bacchius gets its name from the fact that it is suitable [*harmozein*] for bacchic melodies.[197] The various species of arrangement assigned to these rhythms take their names from the orders of their component feet.

In the paeonic genus there are two incomposite feet,[198] the *paiōn diaguios* (thesis long and short and long arsis),[199] and the *paiōn epibatos* (long thesis, long arsis, thesis of two longs, long arsis). The *diaguios* is so called as if the term were '*diguios*' (since it uses two time-units), the *epibatos* because it has four parts, and consists of two arses and two distinct theses.[200]

Chapter 17 Several species of rhythm are generated by mixtures of these genera. There are two dochmiacs, of which the first is composed of an iambus and a *paiōn diaguios*, the second of an iambus, a dactyl and a paeon. These mixtures were found to be particularly elegant. (They are called dochmii because this form of rhythmic composition appears complex, varied and indirect.)[201] There are also the rhythms called 'prosodiacs'. Some of these are

[193] *Ios* can mean 'arrow', 'poison' or 'rust'. *Iambizein* and its cognates certainly could convey the sense 'abuse' (e.g., Aristotle *Pol.* 1448b 32, cf. 1449b 8); *iamboi* are poems in iambic metre, but especially lampoons, e.g., Plato *Laws* 935e; cf. also Athenaeus *Deipn.* 181c, 622b. But the derivation is fanciful.

[194] 'Running', *epitrochon*, related to *trechein*, 'to run', 'to move quickly'. 'Step', *basis*: the reference is probably to dancing.

[195] On this and the 'semantic trochee' see n. 190 above. *Orthios* can mean 'upright' (also 'high-pitched', a common usage in musical contexts). 'Gestures', *hypokriseis*, lit. 'actings'.

[196] The thesis is so long (eight durations) that a second, 'artificial' down-beat is felt (perhaps on its fifth duration). 'Time-indications', *sēmasiai*, is related to *sēmeion* etc., and so used to explain *sēmantos*, 'semantic': cf. n. 163 above. The reference is conceivably to a rhythmic notation: see *Anon. Bell.* 1 and 83, and *El. Rhythm.* II.18: cf. n. 200 below.

[197] Compare, for example, Xenophon *Symp.* 9.3.

[198] The paeonic genus is in the ratio 3:2. Aristides does not mention here the four feet usually called 'paeons' (–⏑⏑⏑, ⏑–⏑⏑, ⏑⏑–⏑, ⏑⏑⏑–), but discusses them in his study of metre, 45.1–5.

[199] The Greek is ambiguous: 'and short' might go with 'thesis' or with 'arsis'. The former is rhythmically more likely, the latter slightly preferable linguistically.

[200] *Diguios*, lit. 'two-limbed'. Its thesis is neither equal to nor a factor or multiple of its arsis. Hence, if either is taken as a unit, the other cannot be represented as so many such units. 'Time-units' renders *sēmeia*; there may again be a reference to a notation. See n. 196 above. (An alternative explanation might relate *diaguios* to *guios*, 'lame': cf. the use of *chōlos* at 48.24, 49.10.) *Epibatos* is derived from *epibainein*, 'to tread', 'to walk on'. The word has no obvious connection with the number 4, except that *epibatēs* may refer to a rider on horseback etc. (Aristotle *Eth. Nic.* 1106a 20, cf. Plato *Critias* 119b, Arrian *An.* 5.17.3). Probably the rhythmic sense of *epibatos* is something like 'walkable'.

[201] *Dochmios* means 'aslant'. Aristides' two dochmiacs are ⏑– | –⏑– (e.g., Soph. *O.T.* 1329, *O.C.* 118) and ⏑– | –⏑⏑ | –⏑– (e.g., Aesch. *Ag.* 1157). Bacchius 316.3–5 defines the dochmiac as iambus plus anapaest plus paeon *kata basin* ('by step'). He has defined the paeon as –⏑ | ⏑⏑ (315.21–2). The qualification might indicate the combination of the last

composed of three elements, a pyrrhic, an iambus and a trochee, others of four, with an iambus added to the three-foot complex just mentioned, and others again from two coupled feet, a bacchius and an ionic *a maiore*.[202]

There are also two irrational choreii. One is iambic in species: it is composed of a long arsis and two theses, and resembles the dactyl in rhythm, but the
38 iambus in respect of the number of elements of its diction. The other is trochaic in species, consisting of two arses and a long thesis, reversing the arrangement of the previous one.[203]

There are other mixed rhythms too, six in number: the cretic, consisting of a thesis of a trochee and an arsis of a trochee; the iambic dactyl, consisting of a thesis of an iambus and an arsis of an iambus; the baccheic (*a trochaeo*) dactyl, which is made up of a thesis of a trochee and an arsis of an iambus;[204] the baccheic (*ab iambo*) dactyl, which is arranged in the opposite way to the preceding one; the choreic (iambic species) dactyl, containing one of them as its thesis, another as its arsis; and the choreic (trochaic species) dactyl, whose construction is analogous to that of its predecessor.[205] The word 'cretic' comes from the race of that name: the other rhythms take their names from the feet mentioned previously.

Chapter 18 That, then, is the kind of technical analysis given by those who combine the science of rhythm with that of metre. Those who treat it separately proceed in a different way.[206] Beginning from a rhythm of two durations, they

into a long, making the foot equivalent to Aristides' *paiōn diaguios*. Then his dochmiac will be like Aristides' second, except that the dactyl is replaced by an anapaest. (The example Bacchius gives is corrupt in our MSS and therefore unhelpful.)

202 'Prosodiac' has a wide range of reference. It usually describes rhythms which are expansions of the core –⏑⏑– (trochee plus anapaest, Aristides' second bacchius, 36.6–8).

203 On irrationality see nn. 176 and 182 above. Aristoxenus (*El. Rhythm.* II.20) describes as an 'irrational choreios' a rhythm whose *basis* (thesis) is of two durations, and whose arsis is intermediate between one duration and two. 'Choreios' is commonly defined as equivalent to 'trochee', –⏑, e.g., Bacchius 315.12–14. Aristides defines it as ⏑⏑⏑ in his discussion of metre (44.17), and the present passage shows that it may be basically either trochaic or iambic (i.e., in the ratio 2:1 or 1:2). If the theses of the iambic variety are short, this choreios will indeed resemble the dactyl in rhythm (–⏑⏑), but the 'number of the elements of its diction', if this means 'syllables', is not like that of the iambus (⏑–). Perhaps the text is confused, and we should follow Burette and Boeckh in transposing 'iambus' and 'dactyl'. Then if the theses are long, we have the foot –′––, like the iambus in rhythm (1:2), and with syllables equal in number to those of the dactyl. We can best make sense of the description 'irrational' if the arsis of the iambic version and the thesis of the trochaic are 'intermediate' in length in an Aristoxenian sense, though Aristides does not say this. In that case the two rhythms might be represented musically as ♩♩′♩♩ and ♩♩′♩♩ (Aristoxenus' irrational choreios might be notated as ♩′♩♪).

204 Commonly called the choriambus (choreios plus iambus, where 'choreios' is equivalent to 'trochee').

205 Aristides' descriptions are unclear, but he seems to mean that the first choreic dactyl contains a baccheic (*ab iambo*) dactyl followed by a baccheic (*a trochaeo*) dactyl, while the second reverses the order (⏑––⏑ | –⏑⏑– and –⏑⏑– | ⏑––⏑). On the reason why these are called 'dactyls', and their designation as 'mixed rhythms', see n. 183 above.

206 The chapter that follows sets out the procedures of a rival school. With perhaps two exceptions (see n. 207 below) their results are similar to those of the Aristoxenians; it

put together combinations of numbers which extend to the compound rhythms; and they arrange them according to the ratios mentioned previously,
20 the equal, the duple, the hemiolic and the epitritic.[207] Their constructions of some rhythms begin from the thesis, others from the arsis, some from longs and others from shorts. Some they construct out of shorts alone, others from longs alone, others again from a mixture of the two, either in equal quantities or with a preponderance of one or the other. They make arses correspond to theses either by equal or by unequal durations. Some of their rhythms are complete, while others involve pauses or additions, where they incorporate empty durations. An empty duration is one without sound, adopted to fill up the
39 rhythm. A 'pause', in the context of rhythm, is the smallest empty duration, and an 'addition' is a long empty duration, twice the size of the smallest.[208]

They then construct the compound rhythms as follows. They set out the complete number, and divide it into rhythmic arrangements. If these stand to one another in one of the ratios which the durations of the simple rhythms maintain, they assert that this arrangement is rhythmical. If not, they change the arrangement, until the division of the number achieves rhythmical ratios. For instance, let us take the number 10, and determine its arrangements with
10 a view to the generation of rhythm. From a 2 and an 8 no rhythm will result, since the quadruple ratio is not rhythmical. Hence a rhythm of ten durations will not be constituted of two durations plus eight. I now divide the 8 into a

is their method that differs. The most obvious sense in which they separate rhythms from metres is that they do not analyse the former in terms of the familiarly named metrical feet, but only through numbers and ratios. This suggests that (like the Pythagorean harmonic theorists) they begin from mathematical principles, treating as rhythmically legitimate those patterns that fit mathematically simple or satisfactory ratios (as in harmonics, the equal, duple, hemiolic and epitritic, 1:1, 2:1, 3:2, 4:3), rather than from what 'perception' finds acceptable. Secondly, in constructing compound rhythms they do not build up from single feet to greater lengths, but start by positing, as so many durations, the greater length which is to be analysed or constructed. They then demonstrate which divisions of this length are rhythmically legitimate and which are not. The procedure seems completely detached from the business of composing poetry and music, where the materials deployed are never mere undifferentiated lengths of such and such a size. As a method of analysing poetic and musical rhythms it is crudely mechanical and aesthetically unsatisfactory. In this respect too it has some affinities with aspects of Pythagorean harmonics, belonging to the realm of mathematical theory rather than musical reality. All this is un-Aristoxenian (though it might have been encouraged by *El. Rhythm.* II.31ff.). Aristides, however, utters no criticisms; he merely reports it as an alternative approach. Chapters 20–9 offer an account of the methods and results of a distinct science of metrics, which would complement the present separation of rhythmics. Some of its procedures are quite close to those of the present passage (see n. 231 below). The immediate sources of Aristides' account of metrics are Dionysius of Halicarnassus and Hephaestion; that of the present section is unknown. On differences between metre and rhythm see also 45.18ff.

207 Neither rhythms of two durations, if these are construed as 'primary' durations, nor rhythms in epitritic ratio are accepted by Aristoxenus (*El. Rhythm.* II.31, 35). Notice that what are combined are 'numbers', rather than durations (or syllables, notes, etc.).

208 'Empty' durations (equivalent to 'rests' in modern music) are mentioned at *Anon. Bell.* 1, 3 (= 83, 85). A notation for them is set out at 102, where rests of one, two, three and four durations are given their own signs: the author does not use Aristides' terms *leimma* ('pause', lit. 'remainder', as in Pythagorean harmonics) and *prosthesis* ('addition').

3 and a 5: no rhythmical ratio will result in this way either. I divide the 5 again into 3 and 2; and I assert that the 3 stands to each of the units of two durations in hemiolic ratio, so that the rhythm of ten durations has been constructed by means of these. Once again, if I were to divide the same number into a 3 and a 7, the numbers will stand in no rhythmical ratio. I divide the 7 into 3 and 4; and this preserves the epitritic ratio, from which, I assert, the rhythm of ten durations is constructed. Again, I constitute the same number out of a unit of four durations plus one of six, and a rhythmical hemiolic ratio has been constructed. I divide it again into two units of five durations. If each of these is simple, they will stand in equal ratio, which is rhythmical. If they are compound, I shall use the method of division described above to construct the rhythm of ten durations.[209]

Chapter 19 Rhythmical tempo [*agōgē*] is the quickness or slowness of durations. It is exemplified when, while maintaining the ratios between thesis and arsis we perform the magnitudes of each duration differently. The best indication of rhythmical tempo is the size of the interval between one thesis, or one arsis, and the next.[210]

Rhythmical modulation is alteration of rhythm or of tempo. Modulations occur in twelve ways: one is in tempo, the others in respect of the ratio of the foot – when it changes from one ratio to another one, from one to several, from incomposite to mixed, from rational to irrational, from irrational to irrational, from one to another of those differing by antithesis, or from mixed to mixed.[211]

Rhythmical composition is the technique of constructing rhythm.[212] Complete rhythmical composition is that which includes all rhythmic figures.[213] Its divisions are the same as those of melodic composition: selection, through which we understand what kind of rhythm we should use; use, through which we relate the arses appropriately to the theses; and mixture, by which we

209 The language and argument of this paragraph might fairly be described as 'sub-Euclidean', perhaps the work of an author seeking to do for rhythmics what 8 *Sectio Canonis* had done for harmonics. (At line 22 I supply *merizō*, 'I divide', to fill the lacuna.)

210 For uses of *agōgē* in harmonics see 16.19, 29.8ff., 7 Aristox. *El. Harm.* 29.31. The sense 'pace', 'tempo', appears at *El. Harm.* 12.14, 34.15, the latter in a rhythmic context. Aristides discusses the emotional effects of rhythmic tempi at 84.3ff.; see also Plato's tantalising hints at *Rep.* 400c.

211 If the MSS 'twelve' is correct, four items are missing from the list, which is in any case unsystematically arranged ('mixed to mixed' seems to be in the wrong place). On the designations 'incomposite' and 'mixed' see n. 183 above. The list might be completed in various ways, none of which seems entirely satisfactory. Probably we should add 'several ('composite') to several' and 'several to mixed', but after that the choice seems arbitrary.

212 The definition is analogous to that of 'melodic composition' at 28.10. The language, here and in the immediate sequel, is Aristoxenian, but Aristides may be trying to fit Aristoxenus' terminology to a new subject, rather than reporting his doctrine (see n. 215 below). There are rather few references to rhythmic composition in the surviving passages of Aristoxenus: see 7 *El. Harm.* 34.22, *El. Rhythm.* 11.13–14, 19, Psellus 8 (Westphal 85.31ff.).

213 'Complete', *teleia*, but the usage is not parallel to that of *teleion melos* at 4.20 and elsewhere.

interweave rhythms with one another, where the need arises.[214] As with melody, the styles of rhythmic composition fall into three genera, 'exalting', 'depressing' and 'peaceful': and we divide each of them into species in the same way as those mentioned under the heading of melodic composition.[215] The best rhythmic composition is that which results in virtue, the worst that which produces vice. We shall explain how each of these comes about in our discussion of education.[216]

Some of the ancients described rhythm as male, melody as female, on the grounds that melody is inactive and without form, playing the part of matter because of its capacity for opposite qualifications, while rhythm moulds it and moves it in a determinate order, playing the part of the maker in relation to the thing made.[217]

We have now completed our account of rhythm. Our remaining task is to touch briefly on the science of metre.

Chapter 20 The starting point of metrics is an account of the elements. Next comes the study of syllables, then that of feet: next, as we proceed in this
41 manner, comes that of metres; and last comes that of the poem, added to display the final objective of metrical science.[218]

An element is the smallest part of articulate sound.[219] Those elements which project a clear and fully audible sound are called vowels, while those which

[214] 'Selection', *lēpsis*; 'use', *chrēsis*; 'mixture', *mixis*: the terms are those of 29.2ff., but again the uses are not parallel.

[215] 'Melody' at the beginning of the sentence represents *harmonia*. The three types are referred to as kinds of *ēthos*, not genera of styles (*tropoi*), at 30.12ff. The generic *tropoi* of melodic composition are dithyrambic, nomic, tragic, 30.1–2 (though there are certainly connections between the two classifications: see n. 150 above). Aristides' continual backwards references in this paragraph, including the vague gesture at further levels of classification at the end of this sentence, together with the characteristically Aristidean remark that follows, strongly suggest that the passage as a whole is his own original work, not culled from an earlier source.

[216] See Book II ch. 15.

[217] The terms of this distinction between 'matter' and 'agent' are Peripatetic, and their use here recalls Aristotle's account of the male and female contributions to reproduction (e.g., *De Gen. An.* 730b). Similar conceptions of the roles of melody and rhythm are suggested by ps.-Ar. *Probs.* XIX.49. We cannot confidently identify the origin of the application of sexual distinctions to music (used extensively by Aristides in Book II), and specifically of the equation of rhythm with male, melody with female; such ideas may possibly be connected with the school of Damon (see 80.25ff.). Aristides returns to the present point at 77.5ff.

[218] 'Elements', *stoicheia*, the smallest significant or identifiable parts of anything, and specifically 'letters', treated either as written symbols or (more appropriately here) as elementary sounds. Elements are discussed at 41.3–17, syllables at 41.18–44.10, feet at 44.11–45.17, metres at 45.18–52.7, the poem at 52.8–21. The programme, beginning from an account of the smallest units and showing how they may be built into larger ones, is analogous to that suggested by the incomplete third book of 7 Aristoxenus *El. Harm.* On the differences between its principles and those of Aristoxenian rhythmics see n. 231 below. It seems clear that the immediate sources of Aristides' metrics are Dion. Halic. *De Comp. Verb.* and Hephaest. *Ench.*, but we do not know whether these authors relied to any great extent on earlier writings in the musical tradition.

[219] Cf. Plato *Crat.* 424d, *Theaet.* 202e, Aristotle *Metaph.* 998a 23, *Poetics* 1456b 22. With the remainder of the paragraph cf. Dion. Halic. *De Comp. Verb.* 14.

come to the hearing indistinctly are called semivowels. Those which make a slight, faint sound are described as altogether 'mute', as though this meant 'of little sound'. Those of the vowels which can be pronounced in the least duration of time are called short, those which demand a greater duration, long. Vowels which are ambiguous in their durations are called *dichrona*.[220] Of the semivowels, those which, in metre, are equivalent to two consonants are called double. Those whose value, in combinations, is less than that of one consonant are called liquids. The one which has neither of these distinguishing features, the sigma, is called 'peculiar'.[221] Of the mutes, those which move the breath only at the surface are called 'simple', those which bring it up vigorously from within 'aspirated', and those whose action falls between these 'intermediate'.[222]

Chapter 21 Combinations of elements produce syllables, which have the same names as the classes of vowel which they contain. Some take their values [*dynameis*] from a single element, others from several. Of these latter, some take their values from vowels: these are the diphthongs which, we say, are produced either by blending or by combination or by predominance.[223] Others do so through consonants too: these are the ones which are shaped by their position. Syllables containing a long element, a lengthened *dichronon*, a short element in combination with a *dichronon*, or two *dichrona* combined with one another, are known as 'long'. Those containing a short vowel, or a shortened *dichronon* either alone or with a single consonant are described as 'short'.
42 These are the distinctions natural to syllables. Some syllables also become long through the addition of consonants, being lengthened either by the syllable's own constitution, when a syllable ends in two consonants or a double consonant, or by the elements which follow, when a syllable has after it two

[220] Greek has seven vowels. Epsilon (ĕ) and omicron (ŏ) are short; eta (ē) and omega (ō) are long; alpha (a), iota (i) and upsilon (u or y) are 'ambiguous'. That is, whereas syllables whose only vowel is epsilon or omicron can be metrically long only if made so 'by position' (i.e., if the vowel is followed by several consonants: see 42.1ff.), alphas, iotas and upsilons are sometimes 'naturally short' in this sense, sometimes 'naturally long' (i.e., long regardless of position, for reasons to do with, for example, the etymology of the word in which they occur, or its grammatical role).

[221] The double semivowels are zeta (z or dz), xi (x) and psi (ps); the liquids are lambda (l), mu (m), nu (n), rho (rh). Sigma (s) is called 'peculiar', *idiazon*, presumably because it is the only one of its kind (i.e., the only spirant). Aristides' description of the liquids reflects the fact that whereas a syllable containing a short vowel generally becomes long if the vowel is followed by two consonants, this lengthening does not always occur if one of the consonants is a liquid. Issues of this sort are discussed at length in the next chapter.

[222] 'Simple', *psila*, lit. 'bare'; the adjective is also used of 'bare' instrumental music without the voice. 'Aspirated', lit. 'rough', 'shaggy'. The simple mutes are kappa, pi, tau (k, p, t), the corresponding aspirates are chi, phi, theta (ch, ph, th), and the 'intermediates' are gamma, beta, delta (g, b, d). All twenty-four letters of the Greek alphabet have now been classified. For Aristides' special uses of these classifications see Book II chs. 11–14, and cf. 130.1ff.

[223] 'Blending' (*krasis*) occurs where a vowel ending a word is fused with one beginning the next. The distinction between 'combination' (*symplokē*, 'weaving together') and 'predominance' (*epikrateia*) seems to be that between cases where the two individual vowel sounds in a diphthong are given roughly equal weight, and those where one is largely swamped by the other. See further 43.8ff.

consonants or a double consonant, or by a combination of both, when a syllable ends in a single consonant and is followed by another.[224]

Since these things are so, it has been shown that the magnitudes of the elements are equal in number to the intervals constituting the tone. The smallest of them is a quarter of the greatest, just as the diesis is a quarter of the tone: the intermediate one is half the greater and double the smaller. For a short syllable is half a long, and a single consonant is half a short. This is clear from the fact that a short syllable becomes long by the addition either of a double consonant or of a single vowel.[225]

Again, some syllables display the same quality when what follows them is added as they do when taken alone, while others change, as we can see from those which are long by position and those which are common. (The different attributes attached to the so-called 'common' or 'intermediate' elements in feet undoubtedly arise from the distinctions between syllables which were mentioned above.) They are called 'intermediate' and 'common' because they sometimes fill the role of a short syllable and sometimes that of a long. Some of them are found among syllables that are naturally long, when one syllable ends in a long vowel and the next one begins with a vowel. Because they have no linking consonant between them, they leave a gap between the sounds and destroy the vocal tension, while our eagerness to grasp the second syllable without breaking vocal continuity cuts short the length of the preceding sound [*tonos*] before the first syllable has completed its duration. Others are found
43 among syllables that are naturally short, when a syllable ends at the end of a word. The gap between the end of the first word and the beginning of the second lengthens the syllable. Others again are found among syllables long by position, when they are followed by two consonants, the first a mute, the second a liquid. Because the first of the elements in the combination is thicker, the lighter sound of the second is cramped and compressed by it.[226]

Some of these display the characteristic of being 'common' to a smaller degree. This is true of syllables long by nature when the syllable where two vowels are juxtaposed ends at the end of a word, and becomes a diphthong of the type involving combination. (I mean the type where vowels are placed side by side: these give the sounds greater distinctness, by enunciating both the vowels clearly.)[227] It is true of naturally short syllables when a very weak syllable which ends at the end of a word is involved in a juxtaposition of

[224] This last sentence describes lengthening 'by position', further discussed at 42.15ff. The distinction between long, short, etc. 'by nature' and 'by position' is applied to syllables, not to vowels. Cf. Hephaest. *Ench.* 1.

[225] For the harmonic parallel see 10.23ff. Fortunately for writers of quantitative verse, these calculations are largely misguided. If every consonant really counted as half the length of a short syllable, the consequences for metre would be alarming. In most cases, the addition of a single consonant to a syllable ending in a short vowel, or of any number of consonants to one containing a long vowel, adds nothing to the metrical length of the syllable.

[226] This classification of three types of 'common' syllable follows Hephaest. *Ench.* 1.

[227] On 'combination' see n. 223 above. 'Placed side by side': the text is corrupt, and Winnington-Ingram indicates a lacuna. I suggest *tōn dia tōn syntithemenōn*, 'of those (diphthongs that are constituted) through (vowels) placed together'.

vowels, acquiring a length greater than that of a common syllable if the element which begins the next syllable is aspirated.[228] It is true of those which are long by position, when what immediately succeeds them is a mute in combination with one in particular of the 'unchangeables', the mu. The other liquids are sounded by an expulsion of breath, whereas we have to pronounce this one, uniquely, with our passages closed. Since our vocal organ is simultaneously subject to contrary forces, it is not surprising that it makes the sound less smooth. Thus even if we combine it with another 'unchangeable', and try to pronounce both clearly, we shall make the syllable not common, but long by position. Indeed, some of the ancients, who often wished to make the syllable
44 preceding this combination short, slide over the sound of the mu and pronounce audibly only that of the nu.[229]

It is important to recognise that if we want to assess a syllable taken on its own, we must work out its magnitude from the evidence of its own parts: but if we want to assess it within the arrangement of a foot, we must also take into account the syllable which follows, if our investigation of the foot is to yield a thorough understanding of it. Thus we treat the last syllable of every metre as indifferent, since it is succeeded by no syllable which would justify us in assigning to it some one determinate magnitude.[230]

Chapter 22 When syllables are combined with one another, feet are produced: hence they are also called 'combinations [*systēmata*] of syllables'.[231]

[228] Aristides is best understood, here and in the previous description, not as identifying something else that can make a syllable 'common' (though to a lesser degree), but as pointing to conditions that reduce the 'commonness' of a syllable which would be 'common' by the preceding criteria. Thus, (a) we have been told that 'when one syllable ends in a long vowel and the next one begins with a vowel', the first becomes 'common' (i.e., may be metrically either long or short). But if the syllable 'where two vowels are juxtaposed' comes at the end of a word, and becomes a certain sort of diphthong, this commonness will be less evident. That is, apparently, the two syllables together will usually count unambiguously as a single long syllable (cf. 41.20ff.). Secondly, (b) a short syllable at the end of a word has been said to be 'common'. Now we are told that such a syllable, when it is both weak and involved in a juxtaposition of vowels, becomes longer than a common syllable if the next element is aspirated. In both cases Aristides seems to be describing forms of hiatus, but his language is awkward and unclear. Quite possibly the text at this point is imperfect.

[229] Here the third rule of the previous paragraph is modified. Syllables that are long by position have been said to be 'common' if the first consonant following them is a mute, the second a liquid. But if the liquid (here, oddly, called an 'unchangeable', *ametabolon*) is mu (m), this 'commonness' is reduced and the syllable tends to be long. Further, even though it is the liquid that is suppressed in these combinations, a vowel followed by the sequence of two liquids mu, nu (as in, for example, *hymnos*) can only be treated as 'common', and used metrically as a short syllable, if the mu is not in fact pronounced. (The first syllable of *hymnos* is usually, perhaps always, scanned long, but the equivalent syllable in related verbs is scanned short, for example, at Aesch. *Ag.* 990, Eurip. *Bacch.* 72.)

[230] 'Metre' or 'metron' here means roughly a 'line' of verse. The principle Aristides states is true of, for example, epic hexameters and the iambics of tragic dialogue, but applies in much lyric only to the last syllable of a complete strophe or 'stanza'.

[231] This conception of the foot is entirely different from that of Aristoxenian rhythmics, where a foot is defined by the ratio of arsis to thesis, and the subdivision of these parts is a matter of 'rhythmic composition', not of rhythm as such. Relations between arsis

If two syllables are combined, feet of four kinds are produced: when a foot has both syllables short it is a pyrrhic, also called a pariambic, when it has both long it is a spondee, when it has a short first and a long second it is an iambus, and when these are reversed it is a trochee. If three syllables lie together eight kinds are produced: either it has three shorts and makes a choreios, or three longs and makes a molossus, so called from the race of that name, or one long and the others short, distributed over the three positions, making a dactyl, an amphibrach or an anapaest, or one short and two longs, shifted round in sequence, yielding a bacchius, an amphimakros and a palimbacchius.[232]

If four syllables are set out, sixteen feet are produced, which can be discerned by the same methods. Either it has four shorts and is called prokeleusmatic, or four longs and is called a dispondee; or it has two shorts leading and two longs following and is called an ionic *a minore*, or it has the reverse arrangement and is called an ionic *a maiore*; or it has the longs on the outside and the shorts in the middle and is called a choriamb, or it is the other way round and is called an antispastos; or it has the longs in the odd-numbered places and the shorts in the even and is called a ditrochee, or it is the other way round and is called a diiambus; or it has one long and the others short and makes the four paeons, which take their individual names from the position of the long. The one with the long in first place is called the first paeon, the one with it in second place the second paeon, and so on. If it has one short and the others long it produces the epitrites, named after the position in which we place the short, on the same principle as the paeons. (The arrangement is called 'epitrite' because it is constituted out of feet standing in epitritic ratio, that is, the ratio 4:3. One of its component disyllables is of three durations, the other of four.)[233]

If disyllables and trisyllables are in turn combined, thirty-two pentasyllabic feet are produced, and if trisyllables are combined with one another, sixty-four hexasyllabic feet are produced. These are also called 'metrical couplings'.[234] The syllable, the foot and the metre are increased only up to the number six,

and thesis play no part in Aristides' metrics: the shape of a foot is determined only by the number and sizes of the syllabic elements, and the order in which they appear. Further, the primary classifications of metrical feet, below, group together those that give different ways of dividing up the same number of syllables, not those displaying the same ratio. There seem to be affinities with the kind of rhythmics that builds up its feet additively, and seeks to identify the mathematically legitimate divisions of given numbers of durations (see chapter 18 above), which is said to be that of people who pursue rhythmics and metrics independently. The same mechanical reshufflings of the mathematical possibilities are evident in both passages. But cf. also n. 237 below.

232 Most of these descriptions are unambiguous. The dactyl is –⏑⏑, the anapaest ⏑⏑–; the name 'amphibrach' means 'short at both ends', ⏑–⏑. An 'amphimakros', 'long at both ends', is –⏑–. The bacchius is plainly not that of 36.6ff., but one of those more usually so called (⏑–– and ––⏑); the other will be palimbacchius ('reversed bacchius'). At 48.25 'bacchius' refers to ⏑––, and probably does so here.

233 The language of the parenthesis, with its reference to ratios and its use of *trisēmos*, *tetrasēmos* ('of three durations', 'of four durations'), belongs to Aristoxenian rhythmics, and marks it as Aristides' own reflection on the term 'epitritic', not drawn from the source of the rest of the chapter (for which see Hephaest. *Ench.* 3).

234 *Syzygiai*. Contrast the use of the term in rhythmic contexts, 35.1–2, 14, 36.6, 37.22. For its metrical usage see also 46.5, 47.17–19, 48.10–11, 49.15–17.

because it is a perfect number and includes within itself all the ratios of concord.[235]

Chapter 23 Out of feet are constructed metres. A metre is a combination [*systēma*], extending to a well-balanced length, of feet which are constituted out of dissimilar syllables. As to how metre differs from rhythm, some say that they are related as part to whole: they say that metre is a division of rhythm, in accordance with their derivation of 'metre' from '*meirein*', which means 'to divide'.[236] According to others, the difference lies in their material. Everything that comes into being, they say, is generated out of at least two dissimilar entities; and while rhythm has its being in arsis and thesis, that of metre lies in syllables and their dissimilarity.[237] Thus a rhythm may be constituted even out of equal syllables or antithetical feet, whereas a metre is never constituted out of syllables which are all equal, and seldom out of antithetical feet.[238]

235 Another remark made by Aristides in person. On the perfection of the number 6 see 102.10–12, 112.4ff. It is also of special importance in the cosmological computations of Book III. The sense in which it contains all the ratios of concord is not clear. They would have to include at least 2:1 (octave), 3:2 (fifth) and 4:3 (fourth). The passage at 112.4ff. emphasises the fact that 6 is the sum of its proper parts, 1+2+3. These elements can be said to 'contain' 2:1 and 3:2, but hardly 4:3. Perhaps Aristides means that the number 6 includes, as a part, the number 3, to which it stands in the ratio 2:1, and the number 4, giving 3:2; and the 4 stands to the 3 in the 'epitritic' ratio of the fourth. But in that case the number 4 would 'contain' these ratios in a precisely similar way.

Metres are increased up to the number 6 in the sense that a metre contains up to six feet. Feet are so increased in the sense that a foot contains up to six syllables. But nothing has been said to suggest that syllables are composed of up to six smaller elements. Perhaps the remark about them is just another way of saying that there are up to six syllables in a foot, and Aristides' statement has rather less content than it pretends.

236 The two words are perhaps etymologically related, but the use to which this fact is put is a little naive. It may suggest a conception of rhythmics which deals, like Aristoxenus', with several kinds of 'material' on which rhythm can be imposed. The term 'metre' is then reserved for the rhythmic organisation of sequences of words (contrasted with notes and bodily movements). Such a conception is possible (though it is not the one Aristides has been using), but the terms 'metre' and 'metrics' are of course used with reference to a process of 'measuring', not because metrics is a part (*meros*) of rhythmics.

237 Rhythm is defined by ratios of arsis to thesis (each of which may include several syllables, and may relate to notes or gestures rather than words). Metre is defined by sequences of syllabic lengths. The generalisation in the first clause of this sentence, and the doctrine concerning rhythm, may be distantly related to Aristox. *El. Rhythm.* II.3–6; metre, however, is not mentioned there. Metre and rhythm had been distinguished as early as the fifth century (Aristoph. *Clouds* 637), and the distinction is made several times by Plato (e.g., *Gorg.* 502c, *Rep.* 601a), but without being explained. Its basis may have been similar to that suggested by Aristides. In the *Clouds* (645) metrics is apparently concerned with lengths defined quantitatively as so many feet, whereas rhythmics distinguishes such patterns of movement as the 'enhoplian' and the 'dactylic' (651–2). See also particularly *Rep.* 400a–c, where the rhythmic theory attributed to Damon deals at least in part, like Aristoxenus', with ratios between 'up' and 'down' beats (*anō* and *katō*, 400b6). Plato's remarks contain hints of a rich and complex system of analysis. (It is possible that Aristophanes' pastiche is also aimed at the researches of Damon.)

238 On antithetical feet see 33.26–8. The rhythmic pulses of arsis and thesis may be divided by 'rhythmic composition' into any available syllabic elements without losing their identity. In Aristides' metrics arsis and thesis play no part, the metre being determined only by relative lengths of syllables. It has, therefore, no way of identifying the metre belonging to a string of, for example, twelve equal syllables. Unless rhythmic

There are nine simple and fundamental metres: dactylic, anapaestic, iambic,
46 trochaic, choriambic, antispastic, two ionics, paeonic.[239] They may be extended with aesthetic propriety up to a maximum of four feet, all except the dactylic, which in its catalectic form extends to six: for the movement of the dactylic metre proceeds one foot at a time and extends up to twenty-four durations, or thereabouts, a number equal to that of the dieses in the octave, while the others proceed by doublets or couplings and extend up to thirty durations, or a little more.[240] Hence where metres exceed the quantity of durations specified, some people divide them in two and call them 'composite'.

Metres whose feet are completed simultaneously with the end of the syllables they contain are called acatalectic. Those which shorten the last foot by a syllable, for the sake of the solemnity produced by a longer termination, are called catalectic. Those which are short by a disyllabic foot are called brachycatalectic, while those which contain one extra syllable are called hypercatalectic. If there is an extra foot, the result is the same as if the metre were brachycatalectic.[241]

Some of them are dimeters, some trimeters, some tetrameters, and so on up to six. Some of them can be generated only by feet of equal duration, others also by feet that are longer in their durations. Some metres admit feet of equal duration, if they are still able to maintain their own proper character: some will not, those which are thereby caused to be altered so as to give the impression of a different metre.[242] Some, again, begin from a complete foot of the kind

considerations are invoked, such a sequence cannot be said to be 'in' any particular metre. It is less clear why antithetical feet are problematic. But Aristides wishes to describe each metron ('line' of verse) as being 'in' a particular metre (see chs. 24–7), and pairings of antithetical feet would disrupt the scheme (cf. the ban on trochees in iambic metres, 48.17). In some cases which might be treated as pairings of antithetical feet (e.g., –⏑⏑–), the whole grouping is treated as a single foot and is given its own name ('choriamb', 44.29). Exceptional cases where antithesis does occur are mentioned at 51.7–27, where Aristides identifies some of the problems they provoke.

239 Each is discussed in turn below: dactylic, 47.3–48.3; anapaestic, 48.4–15; iambic, 48.16–49.2; trochaic, 49.3–13; choriambic, 49.14–20; antispastic, 49.20–50.3; the ionics, 50.4–19; paeonic, 50.20–30.

240 'Catalectic', i.e., with the last foot incomplete. The dactylic hexameter of epic is treated as catalectic, since it never has a complete dactyl in the last foot, at 47.8–14. 'Doublets or couplings' (*dipodiai*, *syzygiai*): the latter is a pairing of trisyllabic feet (35.13–15, the usage being quite different from that of rhythmics, 34.24ff.), the former a pairing of two disyllables (48.10). Aristides' point is that in enumerating the feet in a metre, those of the dactylic are counted one at a time, those of the others in pairs. 'Or thereabouts': because the final syllable may be ambiguous in length. 'Or a little more': e.g., four pairs of anapaests will give a line of thirty-two durations.

241 Catalexis produces a 'longer termination' by assigning to the last syllable of an incomplete foot the duration proper to the relevant part of a complete foot. The term 'brachycatalectic' is apparently reserved for cases where the component feet are disyllables. In all such cases, metres proceed by pairs of feet, up to four pairs. In a context where most lines have three pairs, a line with three and a half will 'have an extra foot', but it is equivalent to the brachycatalectic version of a line with four pairs. On catalexis see Hephaest. *Ench*. 4.

242 Most metres containing sequences of similar feet admit feet of other types as variants. In some cases these must be equal in length to the feet proper to the metre; in other cases longer ones are allowed (e.g., –– for ⏑– in iambics). Shorter ones occur only in catalexis and in logaoedics (see the next sentence). But even those of equal length are avoided if they give the 'wrong' metrical impression; see, for example, 48.17–18.

from which they take their names, others, such as the logaoedics, from a smaller one.[243] Some admit synizesis between syllables to help with the metre: others do not. Synizesis occurs when, in order to maintain the proportions of the foot, we adopt two syllables not separated by a consonant in place of one syllable; either two shorts in the place of one short, or a short and a common syllable
47 in the place of a common syllable (but this is unusual), or again two shorts, a short and a long, or a common syllable and a long, in the place of a long.[244]

Chapter 24 Let us then say a little about each of the metres, beginning with the dactylic. The fact that it always has the long syllable in first place gives it a greater solemnity than all the other metres.[245] Dactylic metre admits the dactyl, the spondee, since it is of the same duration, but never the prokeleusmatic, which is unsuitable because of its plethora of shorts. Its smallest length is the dimeter and its largest the hexameter. It may be acatalectic or catalectic, and in the latter case comes to admit also a trochee at the end, and has the specific title 'heroic'. This name is given only to the hexameter, which is more solemn than the others both because of its length, and because it begins with a long syllable and has an ending of substantial dimensions.[246]

Its acceptable caesurae are as follows.[247] The first is a syllable beyond two feet. (If this length is taken twice, it gives the elegiac, of which it is a special feature that the extra syllable attached to the first coupling is necessarily long, while the second coupling must consist of two dactyls, admitting no variation.)[248] The second is a trochee beyond two feet, and the third a syllable after three feet. The fourth, according to some authorities, is four dactyls: alternatively and more properly, it ends with the fourth trochee. (Division into similar parts is to be called 'diairesis' rather than 'caesura'. A caesura is the first division of a metre to complete a word after the second foot, while dividing the metre into dissimilar parts.)[249]

[243] 'Logaoedic' seems to mean 'between prose (*logos*) and verse or song (*aoidē*)'. Logaoedics occur mainly where trochees are combined with dactyls: Aristides' description fits a sequence of dactyls prefaced by an initial trochee. Cf. 47.29–48.3, 48.11–15, and Hephaest. *Ench.* 7–8.

[244] Aristides, like Hephaestion (*Ench.* 2), uses the term *synekphōnēsis*: *synezēsis* occurs as an equivalent in a schol. to Hephaest. 2.1. Common examples are the contraction of *poleōs* (⏑⏑–) into a disyllable (⏑–), and of *mē ou* (– –) into a single long.

[245] Metrical (as opposed to rhythmic) considerations are not prominent in the aesthetic and ethical analyses of Book II, except in ch. 11: see particularly 76.18ff., and cf. the remarks on rhythms at 82.4ff. The dactylic metre is discussed at Hephaest. *Ench.* 7.

[246] Because of the catalexis: see n. 241 above. The 'heroic' hexameter is the metre of epic. Each of the first two lines of the *Iliad*, for instance, ends in a trochee.

[247] 'Caesura' renders *tomē*, defined at the end of this paragraph. Aristides seems to conceive the *tomē* as a segment of the line, ending at the end of a word, not as a boundary between such segments; cf. 51.4–5. But the term 'caesura' seems unavoidable in translation, though its different implications lead to some awkwardness of expression.

[248] This is the familiar 'pentameter' of the elegiac couplet, scanned –⏖ | –⏖ | – || –⏑⏑ | –⏑⏑ | ⏓; cf. Hephaest. *Ench.* 15.

[249] 'Dissimilar' in the sense that they are not constituted by feet of similar form. A caesura, properly speaking, cannot come at the end of a foot in a line made up entirely of similar feet.

The species within the dactylic genus have been said by some people to be as follows: dimeter, trimeter, tetrameter, pentameter. Others scan it also on the basis of couplings, producing the catalectic tetrameter.[250] Some even replace
48 the dactyl, in the first position only, and put there a disyllable of a different duration, producing the so-called logaoedics.[251]

The anapaestic metre, the antithesis of the dactyl, admits the following feet: dactyl (which gives it added solemnity), spondee, whose duration is the same, and prokeleusmatic. Its shortest length is the dimeter, its largest the tetrameter. When it is simple it proceeds by single feet: when it is compound it proceeds, for reasons given above, by couplings or by doublets. (A doublet is a foot of four syllables: we have already explained what a coupling is.)[252] It admits all the species of catalexis and the methods and practices associated with logaoedics, not only accepting disyllabic feet of shorter duration in first place, but also allowing the bacchius in last place.[253]

Chapter 25 The iambic metre admits dactyl, tribrach and anapaest, but never the trochee, which will turn it into a different metre.[254] It admits the spondee in odd-numbered feet, but never in even, since what keeps this metre distinct from the dactylic is the iambus in the even-numbered positions.[255] For this reason it also allows feet of the same duration as the spondee very sparingly, and only in odd-numbered positions. In its acatalectic version it admits a pyrrhic in the final position, where in the so-called 'lame' iambic it also admits a spondee.[256] In its catalectic version it admits an amphibrach or a bacchius, since the last syllable is indifferent.[257] Its shortest length is the dimeter, its longest the tetrameter, and it proceeds by doublets.[258] It admits all
49 the species of catalexis, and has the following as its proper caesurae: a syllable

[250] The hexameter is oddly excluded from the first list, perhaps because it can only occur in catalectic form (46.2–3). The notion of a catalectic tetrameter 'by couplings' is no doubt intended to give a way of describing very long lines (here seven dactyls plus a trochee) without exceeding the limit of six units to a metron.

[251] See n. 243 above.

[252] The 'reasons given above' are presumably at 46.1–8. Aristides explains 'metrical coupling' (*syzygia*) as a pairing of trisyllabic feet (45.13–15); cf. 49.15–17.

[253] This bacchius must correspond to a shortened anapaest or pair of anapaests. It cannot therefore be ⏑––⏑ or –⏑⏑–, as in the rhythmical analyses of 36.6–8, nor ⏑––, as apparently at 48.25. The name 'bacchius' is sometimes given elsewhere to ⏑⏑–– (e.g., Bacchius *Ench.* 316.1–2). This corresponds to a shortened anapaestic coupling, and is commonly found at the end of anapaestic lines (e.g., Aesch. *Ag.* 96, Aristoph. *Clouds* 999). Anapaestic metre is discussed at Hephaest. *Ench.* 8.

[254] Cf. 46.19–21. It is indeed rare for a trochee to occur in iambics, but it is occasionally allowed in the first foot, e.g., Aesch. *Choeph.* 1049. Iambic metre is discussed at Hephaest. *Ench.* 5.

[255] Cf. 49.4ff. The exception is the 'lame' iambic mentioned immediately below.

[256] The 'lame' (*chōlon*) iambic, also called 'choliambic' or *skazōn*, is an iambic line with a spondee in last place; cf. 49.10–11.

[257] Catalexis gives ⏑–⏑ (amphibrach) in the place of the last two iambi. The 'indifference' of the last syllable makes ⏑–– equally possible. This must be the 'bacchius' referred to here, as at 44.22 (contrast line 15 above, and 36.6–8).

[258] *Kata dipodian*, by pairs of disyllabic feet (48.10); cf. 46.1ff. Thus the iambic 'hexameter' of tragic dialogue etc. is more properly called a trimeter; cf. Herodotus 1.12.

beyond two feet (this is called 'penthemimeric'), and a syllable beyond three feet (this is named 'hephemimeric').[259]

The trochaic metre, the opposite of the iambic, admits tribrach, dactyl and anapaest. The spondee is accepted only in even-numbered positions, for the reasons mentioned above in connection with the iambic,[260] and also because where there are many long syllables in succession the continuity of the voice is interrupted by the sequence of their magnitudes. If it is catalectic, it also admits the amphimakros or the dactyl.[261] Its shortest length is the dimeter, its longest the tetrameter. It also becomes 'lame', when a spondee falls in the last of the odd-numbered positions.[262] Its more elegant caesura is at three trochees, though others are also possible.[263]

Chapter 26 The metres compounded out of these ones are the choriambic and the antispastic.[264] The choriambic admits a pure iambic doublet, and an iambic doublet of seven durations, but only rarely a coupling equal to itself in duration.[265] In its simple form its shortest length is the dimeter, its longest the tetrameter. It admits the various kinds of catalexis. The antispastic metre is measured by the foot of the same name, but the first disyllable often changes into other disyllables (most elegantly into the spondee, but occasionally into the others). Its shortest length is the dimeter, its longest the tetrameter. It adopts the above-mentioned variations of the first foot in all its positions, except that it makes its ending in a pure iambic doublet, for the sake of its pleasing effect. When it is catalectic it admits the amphibrach or the bacchius.[266] Sometimes it follows an antispast in an odd-numbered position with an iambic doublet, sometimes the other way round: similarly, it sometimes turns the first foot of an iambic doublet into a different disyllable. Occasionally it turns it into a tribrach or an anapaest, because of their affinity with the iambus.[267]

Chapter 27 Of the ionic metres, the ionic *a maiore* consists of a spondee and a pyrrhic.[268] It often turns the spondee in the first doublet into an iambus, in all the lengths of this metre. Its shortest length is the dimeter, its longest the tetrameter; and it admits all the species of catalexis. It admits a pure trochaic

259 'Penthemimeric', 'of five halves'; 'hephemimeric', 'of seven halves'.

260 See 48.18ff.

261 That is, –⏑– and –⏑⏑, which count as equivalent in this position (cf. n. 257 above).

262 The only occasion when a spondee in an odd-numbered foot is allowed; cf. 48.22–4.

263 Such a division would be diairesis rather than caesura, by the criteria of 47.22ff. Trochaic lines are, in fact, most commonly broken by a diairesis after four trochees. Hence, Harmann proposed to read *tettaras* ('four') for the MSS *treis* ('three'). Trochaic metre is discussed at Hephaest. *Ench.* 6.

264 The choriambic foot is –⏑⏑–, the antispastic ⏑––⏑ (44.29–30).

265 A pure iambic doublet is ⏑–⏑–. The iambic doublet of seven durations (*heptasēmon*) must be ––⏑– (not ⏑–––: cf. 48.18–19). Since a 'coupling' (*syzygia*) is a pair of trisyllabic feet (45.13–15), the one mentioned must be a pair of tribrachs, ⏑⏑⏑ ⏑⏑⏑.

266 That is, ⏑–⏑ or ⏑––; cf. n. 257 above.

267 Compare 48.16–17. 'Affinity', *syngeneia*, lit. 'kinship'. On these two metres see Hephaest. *Ench.* 9 and 10.

268 That is, ––⏑⏑ (the two ionic feet have been defined at 44.26–8).

doublet in odd-numbered positions, and a trochaic doublet of seven durations in the even ones.[269] It often adopts the molossus, blending the short syllables into a long, or by resolving longs into shorts it adopts a five or six syllable iambic coupling.[270] The ionic *a minore* is measured by the foot of the same name, often replaced by a ditrochee.[271] When that is adopted, we make the preceding doublet a third paeon, to avoid the poem's becoming harsh through a sequence of three longs.[272] It admits all the species of catalexis, and is varied by blending the short syllables into long ones and resolving the longs into shorts.

The paeonic metre is also called 'cretic', because it is measured sometimes by pure paeons, sometimes by cretics.[273] Its upper limit is the tetrameter: some people have even written pentameters. These can also be constructed out of the pure fourth paeon, which people often either make baccheic by constructing it out of pure bacchii, blending the two shorts in the middle into a long,[274] or else by resolving the final long syllable into two shorts they write it in a complete, unvaried series of shorts, keeping only the ending as a fourth paeon, because the long is appropriate to a pause.

These, then, are the simple and fundamental metres.

51 *Chapter 28* Duplications of identical metres produce composite metres, while combinations of dissimilar ones produce compounded metres.[275] Some of these make one *kōlon* out of two metres, others out of a metre followed by a segment or segments, or out of nothing but segments, or the other way round, out of a segment or segments followed by a metre. Their use and their precise technical analysis are complex, but can readily be understood by an expert.[276]

[269] That is, –⏑–⏑ and –⏑––; cf. 49.4–8 and 49.16 with n. 265.

[270] The molossus is ––– (44.18). Since a coupling is a pair of trisyllables, the term is not quite properly used of a five-syllable complex. If the duration of the whole remains the same, the six-syllable coupling will have to be a pair of tribrachs; the five-syllable coupling could be ⏑–⏑⏑⏑ or ⏑⏑⏑ ⏑–. But other iambic lengths (often of five syllables) are commonly found in basically 'ionic' contexts, e.g., Aesch. *P.V.* 128, 143 (⏓–⏑–⏓).

[271] Ionic *a minore*, ⏑⏑––; ditrochee, –⏑–⏑ (e.g., Aesch. *Suppl.* 1021, 1029, ⏑⏑–– | –⏑–⏓).

[272] The third paeon is ⏑⏑–⏑ (45.1–5). A third paeon followed by a ditrochee makes the 'Anacreontic' of, for example, Eurip. *Bacch.* 531. This is common, but the ionic foot is sometimes retained in first place (see n. 271 above). Ionics are discussed at Hephaest. *Ench.* 11–12.

[273] The paeons are –⏑⏑⏑, ⏑–⏑⏑, ⏑⏑–⏑, ⏑⏑⏑– (45.1–5). The first and third are rarely used as the basis of a metre, though their feet occur as variants in other settings (cf. n. 272 above). Aristides is thinking initially of metres based on the first; he mentions the fourth below. The cretic intended here is presumably –⏑–, as at Hephaest. *Ench.* 3 and usually elsewhere. But at 38.3–5 Aristides defines cretic rhythm (not metre) as trochaic thesis and trochaic arsis, –⏑ | –⏑.

[274] This bacchius must be ⏑––, as at 48.25.

[275] 'Compounded', *asynartēta*, lit. 'disconnected', 'disjointed'.

[276] A *kōlon* (lit. 'limb') is treated here as a length intermediate between the unit of metre (normally a 'doublet' or a 'coupling') and the stanza (or the poem, in non-stanzaic verse). Cf. 52.3. Hephaest. *Poem.* 1 defines it as a unit containing fewer than three couplings, without catalexis. Elsewhere the term is used rather vaguely of something like a 'line', an element in the strophe (e.g., Dion. Halic. *De Comp. Verb.* 19). A lyric stanza is often a sequence of similar *kōla* (themselves commonly 'compounded', in Aristides' sense, from units belonging to different metres), perhaps followed by one of different

There are two metres which are constructed on the basis of antipathy. One is called epionic, and has an iambic doublet followed by an ionic doublet. The latter, as we have shown, has an affinity with the trochaic, and is thus properly described as antipathetic to the iambic.[277] The other is called epichoriambic, and has a trochaic doublet followed by a choriambic one, the latter having, as we said before, an affinity with the opposite of the trochaic, that is, the iambic.[278] It in turn is often followed by an ionic, whose character is related to the trochaic: it also follows the one opposed to it, by which I mean the antispastic.[279] These metres also extend to a limit of a tetrameter, some of them being acatalectic, others catalectic.

Metres are called 'intermediate' when between two antithetical feet there falls one which has affinities with both of them, making the movement of the metre difficult to determine. For instance, if we have a single dactyl and an anapaestic dimeter with a spondee falling between them, it is unclear whether we should say that there are two metres, one dactylic and the other anapaestic, each consisting of a dimeter, or that the whole complex is an anapaestic tetrameter.[280] The same phenomenon is found in other metres too.

Metres made up of composite feet are called 'confused' when the resolution of the long and the blending of the short syllables in a single doublet makes it hard to decide which of two kinds the doublet belongs to.[281] For instance, if one resolved the first long and blended the short syllables of an ionic *a maiore*, it
52 would not be clear whether it was really an ionic *a maiore* or an ionic *a minore*.[282] Such metres are understood on the basis either of the pure doublets with which they are juxtaposed, or of the *kōla* which follow,[283] or of those which correspond to them in an antistrophe.

form (e.g., the Sapphic stanza of three *kōla* each of the form –⏑–⏓ | –⏑⏑– | ⏑––, followed by the length –⏑⏑––). But many are more complex.

The term translated 'segment' here is *tomē*, rendered 'caesura' at 47.15ff.: see n. 247 above.

[277] 'Antipathy' is not the same as 'antithesis' (45.29, 51.19ff.), but is related to it as bringing together metres with opposed characteristics; cf. Hephaest. *Ench*. 14. The ionic doublet said previously to have affinities with trochaic metre might be the ionic *a maiore*, in which, as in the trochee, the long syllables precede the shorts (––⏑⏑), and trochaic variants are allowed in this metre (50.8–10). But 50.11–13 suggests that its predominant character is iambic (cf. also 50.5–7). In that case the ionic *a minore* (⏑⏑––) must be intended; cf. 50.13–14.

[278] Cf. 49.15–17. The whole length is –⏑–⏑ | –⏑⏑–.

[279] The antispastic is ⏑––⏑. This is 'opposed' (*antikeimenē*, cf. 49.3) to the choriambic (–⏑⏑–), but not strictly 'antithetical'. But Aristides has strongly implied that antispastic metre, like choriambic, is closely related to iambic (49.24–50.3).

[280] Antithesis is defined (in the context of rhythmics, but quite appropriately) at 33.26–8; cf. 45.26–9. The length that Aristides describes is –⏑⏑ | –– ‖ ⏑⏑– | ⏑⏑–. Since the spondee is a recognised variant of the dactyl (47.6), this might be a dactylic dimeter plus an anapaestic dimeter. Alternatively, since both dactyl and spondee are permissible variants of the anapaest (48.4–6), the whole length might be anapaestic; cf. Aesch. *Ag.* 53–4, –⏑⏑ | –– ‖ ⏑⏑– | ⏑⏑– | ⏑⏑⏓, and 55, ⏑⏑– | –– ‖ –⏑⏑ | ––.

[281] 'Confused' renders *synkechymena*, lit. 'melted together'; cf. 31.27 on *kechymena asmata*.

[282] The ionic *a maiore* is ––⏑⏑. If we resolve the first long into two shorts and fuse the two shorts at the end into a long, we get ⏑⏑––, the ionic *a minore*.

[283] On the term *kōlon* see n. 276 above.

Some metres are called 'inconsistent': they occur when a long syllable is used in a composite foot where there should be a short. Even in antiquity some poets were compelled to employ this device when using proper names.[284]

Chapter 29 An appropriate combination [*systēma*] of metres is called a poem. Some poems, such as Homer's, proceed line by line, others, such as elegiacs, by pairs of metres, others by groups of three metres, as when an iambic or other line is added to an elegiac, others again by larger groups. Some are free, as are the *parabaseis* of the comic poets,[285] others in correspondence, as are those involving strophe and antistrophe. Of the latter some have two parts, while others, which add an epode, have three. Some keep the order the same, others have it reversed; the same when the first metre of the antistrophe corresponds to the first of the strophe, the second to the second, and so on, reversed when the first corresponds to the last, the second to the last but one, and the rest according to the same principle.[286]

That is enough on the subject of metres and the poem: our technical discussion of musical science is now sufficiently complete.

Book II

53 *Chapter 1* After this we ought to investigate whether it is possible to educate by means of music or not; whether such education is useful or not; whether it can be given to all or only to some; and whether it can be given through just one kind of composition or through several. We must also enquire whether the kinds thought unsuitable for education have no use at all, or whether even these can sometimes be found beneficial. The educational side of our subject includes all these questions.[1] But first we must give some account of the soul. Just as

[284] The 'rules' are quite often broken for this purpose, especially in iambics. A trochaic opening to an iambic line is usually restricted to proper names, and the anapaest, which otherwise occurs only in the first foot, may appear elsewhere if a recalcitrant proper name is involved (e.g., Soph. *Antig.* 11).

[285] In the *parabasis* ('digression') of Old Comedy, the chorus addressed the audience in the poet's own name. *Parabaseis* were 'free' in that the metrical scheme of one part did not necessarily correspond to that of another; cf. Hephaest. *Poem.* 8.

[286] Strophe and antistrophe are paired 'stanzas' in choral lyric, particularly in drama, usually identical in metre. A choral 'ode' often contains several such pairs, and is sometimes, not always, completed by a metrically distinct single stanza, the epode.

[1] The odd-sounding claim that non-educational uses of music fall under 'educational' studies reflects the use of *paideutikos*, 'educational', as a synonym for *praktikos*, 'practical', at 6.13–14. Aristides' first three questions are considered in chapters 3–6; cf. Book I ch. 1. The fourth underlies most of the rest of Book II. The fifth is addressed briefly in chapter 6 (58.14–61.3), the generalisations made there being amplified in the course of chapters 12–19. The overall context of the enquiry is set by Plato, especially in *Republic* and *Laws* (but his educational ideas are already adumbrated in earlier dialogues such as *Gorgias* and *Protagoras*). Aristides' fifth question reflects Aristotle *Pol.* 1338a 30–7, 1339a 11ff.; cf. Aristoxenus' comment on Plato, quoted at ps.-Plut. *De Mus.* 1136f. Related issues were discussed by Aristoxenus himself (see especially 7 *El. Harm.* 30.9–32.9, ps.-Plut. *De Mus.* chs. 27 and 31ff.). Behind these fourth-century sources stand fifth-century assumptions about the role of music in education, reflected in Aristophanes' lampoons (e.g., *Clouds* 961ff., *Knights* 985ff.), and given a theoretical basis by Damon (see particularly Plato *Rep.* 400b with context). It

none of the other arts can be understood until we have grasped the nature of the object at which its energies are directed, so too we cannot understand musical education until we have a clear grasp of the soul, to which it directs all its concern. What the soul is, and from what it is composed, we shall explain in due course. For the present let us give a brief account, sufficient for our immediate purposes.[2]

Chapter 2 Inspired men of ancient times, it seems, strongly maintained the following thesis, among many others: that the soul is not one of the simple entities, nor one of those whose nature and capacities are unitary. Things in this world, they argued, if they were to be equipped with rhythm and order,
54 needed the leadership of soul.[3] But the soul cannot be present and actively engaged among the things on earth without being confined in the fetters of the body, which, borne downwards as it is towards the heaviness whose nature it shares, drags down the soul and prevents it from flying away. Nor can the soul accomplish its designs for things in this world rightly and in concord [*symphōnōs*] with the whole, if it does not also have an understanding and perception of the beauties of the other. Thus it needed a double nature, equipped with wisdom, and yet also, because of its affinity with the body, not disposed wholly to reject the things of this world. For he who orders the universe, so they say, when he constructed the soul to take charge of bodies, gave it, out of what is proper to the divine, an instantiation of reason, by which it was to order things on earth; and out of what belongs to the irrational he attached to it desire, through which it would seek after things in this world. And to ensure that its long stay here would not make it forget completely the beauties of the other world, and that it would not be shackled by its sympathy for things less worthy than itself, he providently instilled in it memory, as an antidote against irrationality, and sent the sciences, in their indescribable beauty, to be its companion in its downward journey. To this beauty it could

has often been surmised that Damon, who is mentioned at 80.29 below, was a primary influence on the development of Aristides' ideas. The general thesis that music has power in the education of character was not uncontroversial; Aristides has to argue the point. Plato and Aristoxenus plainly viewed their own positions as running counter to popular assumptions. The contrary idea, that music can give nothing but enjoyment, is suggested by the (fourth-century) writer of Hippocratic *De Victu* 1.18; music's educational capacities are disputed by the (fourth or third-century) author of Hibeh Papyrus 1.13. Later, the opposition is most notably represented by the Epicurean Philodemus in his *De Musica*, and the Sceptic Sextus Empiricus in *Adv. Math.* Book VI. Their arguments are directed against a conglomeration of Platonic and later ideas put together by Stoic writers, particularly Diogenes of Babylon. Aristides was certainly familiar with Stoic compilations, and also with some of their critics (not necessarily Philodemus and Sextus themselves).

2 The theme is Platonic: see, for example, the argument of *Alcibiades* I, 128–33. Aristides offers three progressively more detailed accounts of the soul (chapters 2, 8, 17), each related in emphasis to the chapters that follow it. The third also points forward to Book III (especially chapters 24ff.); cf. also 1.13–14.

3 The reference to 'inspired men' points primarily to Plato. Aristides later draws heavily on *Phaedrus* and *Timaeus*, as well as *Republic* and *Laws*. For the soul as 'leader' see, for example, *Phaedr.* 245c, *Tim.* 34b–c. On the sense of 'rhythm' here see 31.3–7 above.

direct the love that its nature gave it, so living its life here in holiness, adorned with virtuous impulses and deeds, and making as happy an end as is possible.[4]

These, then, are its two aspects, the rational, through which it accomplishes the works of wisdom, and the irrational, through which it engages in the business of the body. The activities of the irrational aspect gave it, in turn, a two-fold nature: that which is addicted to much slackness they called 'appetitive', and that which is displayed in excessive tension they named 'spirited'.[5]

Chapter 3 There have also come to be two different kinds of branches of learning. Some maintain the rational part in its natural freedom, making it sober and keeping it pure by gifts of wisdom; while others take the irrational
55 part, as though it were some savage beast that is moved without order, and heal and tame it, allowing it neither to pursue excesses nor to be wholly subdued. Of the first kind philosophy is the leader and high priest. The ruler of the others is music, which from earliest childhood moulds the character with its *harmoniai*, and makes the body more melodious with its rhythms.[6] The very young could not be educated through unadorned [*psilois*] words, which contain instruction but no pleasure, nor could they be wholly neglected. For them there remained an education which did not stir up the rational part before its time, this being left in peace because of their youth, and which gave benefit along with pleasure to the other part, training it by habituation.[7] Nature itself taught the means through which education should be applied: for it is not through things of which we know nothing, but through those with which reason and experience have made us familiar, that some of us are led to be convinced and others work to produce conviction. Song always comes readily

[4] Many of these themes and images are Platonic, and a major source is *Tim.* 41–7, 69–72; cf. 87–90. Aristides' words for 'the universe' (*to pan*, *to sympan*, elsewhere also *ta hola*) may be Pythagorean in origin: *to pan* occurs in this sense at *Tim.* 28c, Arist. *De Caelo* 268a 11. The reference to memory alludes to Plato's 'theory of recollection': see especially *Meno* 81a ff., *Phaedo* 72a ff., *Phaedr.* 249c ff.

[5] For these subdivisions of the 'irrational' soul see *Tim.* 69–70 (where it is called the 'mortal' soul); cf. *Phaedr.* 246a ff. The theory of psychic tripartition elaborated at *Rep.* 436ff. does not explicitly treat the two 'lower' parts as subdivisions of one larger element. 'Slackness' and 'tension' are the musical terms *anesis* and *epitasis* (e.g., 6.1 above); for the idea cf. *Rep.* 399e, 410–12. Philodemus comments critically on the practice of associating the words' moral, physical and musical senses at *De Mus.* I.22.12–14, III.8.1–6, 22.7–20.

[6] 'High priest', *mystagōgos*, properly of one initiating novices into a cult, but also 'priest' and 'teacher' more generally. The word is late, but compare, for example, Plato *Gorg.* 497c, *Euthyd.* 277d–e, *Meno* 76e. On the roles assigned to philosophy and music see further Book III ch. 27, cf. Book I ch. 1 (2.3ff.). They are presupposed in the educational programme of the *Republic*, Books III–VII (note especially 401d–402a); cf. *Tim.* 47c–e, 90c–d, Aristoxenus quoted at ps.-Plut. *De Mus.* 1142d. On the contrasted roles of *harmonia* and rhythm see 2.8–9, but it is not only the body that rhythms affect: see chapter 15 with n. 168, and cf. *Rep.* 400d–401a, 410b ff. Aristides' sentence may reflect *Laws* 660a. 'More melodious', *emmelesteron*: the mild paradox is no doubt deliberate, and a pun may be intended (*melē* means alternatively 'melodies' or 'limbs'), but *emmelēs* is often used of character etc., e.g., Plato *Theaet.* 174a, Aristotle *Eth. Nic.* 1128a 1.

[7] A recurrent theme of *Laws* Book II: see especially 659–60, cf. *Rep.* 536–9.

to all children, as we can see, and so do patterns of joyful movement: nor would anyone in his senses forbid them the pleasure they get from such things. It may be that the delightfulness of the activity enchants their minds; or perhaps the soul, released from the torpor which enfolded it in infancy because of the softness of its coverings, springs with enthusiasm into its natural movements as soon as it senses that its body is becoming firm.[8]

Chapter 4 Since all this is so, we have a reply to those who doubt whether everyone is moved by melody.[9] To begin with, they have failed to realise that learning is for children, all of whom, as we can see, are naturally overcome by this kind of delight. Secondly, they have not noticed that even if it does not at once capture those whose way of life makes them less amenable to it, nevertheless it enslaves them before long. Just as one and the same drug applied
56 to the same kind of complaint in several bodies does not always work in the same way, depending on the slightness or severity of the condition, but cures some more quickly, others more slowly, so music too arouses those more open to its influence immediately, but takes longer to capture the less susceptible.[10]

The reasons why music is effective are clear. Our earliest learning comes from similarities, which we assess by bringing them to the attention of our senses. Though painting and the plastic arts teach through sight alone, they nevertheless arouse and startle the soul: how then could music fail to capture it, given that it creates its imitation [*mimēsis*] through several of the senses and not just through one?[11] Poetry, with the medium of words alone, uses only the sense of hearing: but without melody it does not always arouse the emotions, and without rhythms it cannot bring the emotions into conformity with its subject matter.[12] Here is some evidence: if we are to arouse an emotion in the course of a performance, this cannot be done without inclining the voice in some way towards melody.[13] Only music teaches both by words and by images of actions, and through agents that are not static or fixed in a single pattern, but are alive, and alter their form and their movement to fit every detail of what

[8] Compare *Laws* 653d–e, 664e, *Tim.* 44a–c.

[9] See the references to critics of theories about music's educational effectiveness, n. 1 above; cf. Plutarch *Non Posse Suav.* 1095c ff. The latter part of ps.-Plut. *De Mus.* (1135e ff.) may also be conceived as a response to arguments from the Epicurean and Sceptic camps.

[10] Aristides argues at length in this book that different people are differently affected by the same music. Philodemus may have disputed the claim, on the grounds that a given musical sound affects everyone's hearing in the same way (*De Mus.* IV.2.9–15, cf. I.22.4–6).

[11] Aristides derives his mimetic conception of the arts from Plato (e.g., *Rep.* Books II–III, X, *Laws* Book II). Philodemus seems to have attacked both a Damonian-Platonic notion of mimesis, and a theory of musical works as 'non-mimetic ethical likenesses', which he associated with Diogenes of Babylon (see Philod. *De Mus.* IV.3.2ff., especially 23ff.; cf. 32.30ff.). On the comparison between music and the other arts see 1.15 above, cf. ps.-Ar. *Probs* XIX.27. The word 'nevertheless' translates *homōs*, Winnington-Ingram's tentative suggestion in place of the MSS *homoiōs*, 'similarly'.

[12] Compare Plato *Rep.* 601a–b.

[13] Compare 7 Aristox. *El. Harm.* 9.30–3, where, however, the point is not that we must use a melodic form of utterance in order to express emotion, but that emotion sometimes causes us to deviate from the vocal movement of speech towards that of song.

the words express. This is clear both from the dances of the choruses of ancient times, whose teacher was the science of rhythm, and also from what has been written by most authors about performing.[14] The other arts, which work with their own specialised materials, cannot bring us quickly to a conception [*ennoia*] of the action they represent, for some things are based upon colours, others on solids, others on words, all of which are alien to the reality.[15] Music,
57 however, persuades most directly and effectively, since the means by which it makes its imitation [*mimēsis*] are of just the same kind as those by which the actions themselves are accomplished in reality. For instance, in actual trains of events deliberation comes first and speech follows, and after them comes the performance of the action: music imitates the characters and emotions of the soul through its conceptions [*ennoiai*], speech through *harmoniai*[16] and the moulding of the voice, action through rhythm and bodily movement.[17] Hence education of this sort should attend most especially upon children, so that through the imitations and likenesses they encounter when they are young they may come, through familiarity and practice, to recognise and to desire the things which are accomplished in earnest in adult life.

Why then are we surprised to find that it was mostly through music that people in ancient times produced moral correction? – for they saw how powerful a thing it is, and how effective its nature makes it. Just as they applied their intelligence to such other human attributes as health and bodily well-being, seeking to preserve one thing, working to increase another, limiting to what is beneficial anything that tended towards excess, so also with the songs and dances to which all children are naturally attracted. It was impossible to prohibit them without destroying the children's own nature: instead, by cultivating them, little by little and imperceptibly, they devised an activity both decorous and delightful, and out of something useless made something useful.[18]

No human activity is complete without music. Music beautifies hymns to the gods, and their worship; it brings good cheer to private celebrations and public festivities; it gives vigour and support to those at war or on journeys; it takes away the hardship from sailing and rowing, and from the hardest kinds of

[14] *Hypokrisis*, usually 'acting', as in the drama, but here as, for example, at 73.30ff. it is the 'performance' or 'delivery' of a work (whether dramatic, rhetorical or musical), by contrast with its composition. Aristides' limitation of his remarks about rhythm to 'ancient' choruses might suggest that a dancing chorus was rare in his time. Alternatively it may reflect sources who note the shift in emphasis from rhythm to melody that took place in the fifth century B.C. (e.g., ps.-Plut. *De Mus.* 1138b–c). The special excellence of the ancient forms of dance is noted at Athenaeus *Deipn.* 629b.

[15] *Ennoia* is close to being a technical term, it refers specifically to the moral or emotional content of what the artist seeks to convey: see particularly 65.22ff., 67.15ff., 68.14ff.; cf. 70.18, 72.7, 85.21. It is one of Philodemus' main themes that such 'conceptions' cannot be expressed or conveyed by strictly musical means, but only through words (e.g., *De Mus.* IV.14.4ff.).

[16] Perhaps just 'melodies' here, as at 55.5.

[17] Yet it is still a matter for an expert to determine just what a given musical work 'imitates', and how well, or how a given 'conception' is to be successfully presented in musical form: see chs. 9ff., and cf. Plato *Laws* 668c ff.

[18] The themes of this paragraph are developed at length in Plato *Laws* Book II, but are found frequently elsewhere (e.g., ps.-Plut. *De Mus.* chs. 26ff.).

manual labour, easing their toil; and among some foreign peoples it is used even in mourning, where it blunts with melody the sharpness of grief.[19] It has also been observed that there is no one single cause that makes us turn to making melody. For those who are happy it is pleasure, for those in sorrow it
58 is grief, and for those possessed by the impulse of a divine breath it is inspiration:[20] and in certain conjunctions of circumstances these causes are mixed, when children, because of their age, or their elders, because of some weakness in their nature, are subjected to such emotions.

Chapter 5 Even though not everybody is stirred up by these emotions, as the wise, for instance, are not,[21] and though not all emotions make us sing, as untempered ones do not,[22] nevertheless it was found expedient to use this treatment on the emotions which do so arouse us, and the people whom these emotions affect, thus making citizens who were useful when the time for serious activity arrived.[23] No cure could be found in reason alone for those who were burdened by these emotions; for pleasure is a very powerful temptation, captivating even the animals that lack reason (as is shown by the shepherd's *syrinx* and the goatherd's *pēktis*),[24] and grief which remains unsolaced casts many people into incurable illnesses; while inspired ecstasies, if not kept in moderation, do not advance to the right end, but bring on superstition and irrational fears.

These emotions were understood on the basis of their derivation from the different parts of the soul. People saw that pleasure is rife in its appetitive part, grief and its offspring anger in the spirited part, and inspired ecstasy in the rational. But for each of these there was a fitting style [*harmottōn tropos*] of treatment through music, which brought the sufferer gradually, and without his knowledge, into a proper condition. A person who is under the moderate

[19] The power of music to ease toil is a commonplace. The similarity of the examples used here and in the critical writings of Philodemus (especially *De Mus.* IV.8) and Sextus (*Adv. Math.* VI.24) suggests the existence of a more or less standard compendium of instances: cf. n. 1 above. For a list of working-songs giving much more detail see Athenaeus *Deipn.* 618d-620a. Aristides' apparent implication that the Greeks did not use music in mourning is of course misleading, but funeral music was particularly associated with Eastern peoples: see, for example, Aesch. *Persae* 935–40, *Choeph.* 423–8, Plato *Laws* 800e.

[20] This directly reflects Theophrastus, quoted at Plut. *Quaest. Conv.* 623a and elsewhere; compare Aristotle *Pol.* 1341b 32ff. The threefold division is important to Aristides, and is associated with the tripartition of the soul (see especially ch. 5, and Book III ch. 25). For related uses of the idea see 30.12ff., cf. 40.15, Cleonides *Eisagoge* 206.3ff.

[21] The imperturbability of the philosopher is a Stoic theme, though developed from Platonic roots. See particularly 133.11ff. below.

[22] Compare 59.8ff.

[23] This picks up the contrast between 'imitated' and 'real' action made at 57.10: the theme is thoroughly Platonic.

[24] The *syrinx* is any kind of pipe played without a reed mouthpiece, sometimes a single pipe (*monokalamos*) with finger-holes, often the familiar Pan-pipe (*polykalamos*): traditionally a shepherd's instrument (e.g., Homer *Il.* XVII.525–6, Plato *Rep.* 399d), but not always (e.g., *Il.* X.13, ps.-Plut. *De Mus.* 1136a–b). *Pēktis* was usually the name of a sophisticated kind of harp (e.g., Plato *Rep.* 399c, cf. Ath. *Deipn.* 635e). Occasionally in late authors, as here, it indicates an instrument similar to the *syrinx* (e.g., *Anth. Pal.* 9.586), but in these cases its exact reference is not known.

influence of any of these emotions makes music of his own accord, while one who has succumbed to untempered emotion may be taught through his hearing. For a soul subject to excesses of disorder cannot be benefited except through the means by which it acts itself when it is affected only moderately [*symmetrōs*]. The susceptibility of souls[25] to different types of melody varies also with their sex and age: the souls of children are led to sing by pleasure, those of women, for the most part, by grief, and those of old men by divine possession, by breaths of inspiration during festivals, for example.[26]

59 *Chapter 6* In view of all this, they required everyone to cultivate music from childhood throughout their lives, and made use of properly authorised melodies, rhythms and dances.[27] They prescribed certain melodies, which they called *nomoi*, for customary usage both in private revelries and in public religious festivals, employing their role in religion as a device to ensure their stability – even the name they gave them was a promise that they would remain unchanged.[28] Again, they tried to bring calm, in one way or another, to the movements which the soul at times performs within itself when seized by untempered desires, and to transport it into that sweetness of mood which comes through ear and eye – just as if they were diverting a stream, which was rushing through impassable crags or dispersing itself in marshy places, into an easily trodden and fertile plain.

Their anxieties about musical matters were two-sided. They saw that those who neglected music, melody and unaccompanied poetry alike, were utterly crude and foolish; while those who had involved themselves in it in the wrong way fell into serious errors, and through their passion for worthless melodies and poetry stamped upon themselves ugly idiosyncracies of character.[29] Accordingly, they employed for the most part those melodies which are suitable for education,[30] and only very occasionally those that are more relaxed,[31] either to discover men's characters, in the same way that drunkenness was often used as a test (and just as the great Plato tests the young people in

[25] The text is corrupt. I read *eisi de tais psychais*, following the hint in Winnington-Ingram's note.

[26] This chapter begins to expound the idea of musical 'therapy', developed more fully in the sequel and decked out with medical analogies (cf. also 55.30 above). The idea seems to have interested Theophrastus: see Athenaeus *Deipn.* 624a-b, Apollonius *Hist. Mirab.* 49.1, Aulus Gellius *Noctes Atticae* 4.13.1–2, and compare the last paragraph of the passage translated in our chapter 6.

[27] 'They' are the 'people of ancient times' mentioned at 57.11, whose practices are the notional subject of discussion through much of this book. But Aristides is drawing much less on historical sources than on the writings of social theorists, in this chapter the Plato of *Republic* and *Laws*. Plato presents many of his own recommendations as reconstructions of ancient custom, often quite implausibly.

[28] See, for example, *Laws* 656b–657b, 660b, 700a ff., 799e–800b; outside musical contexts, *nomos* means 'law' or 'custom'. For its musical uses see especially ps.-Plut. *De Mus.* 1132e ff., and *GMW* vol. 1, pp. 249–55.

[29] The primary source here is Plato *Rep.* 410c ff.

[30] See *Rep.* 398c ff.

[31] 'Relaxed', *aneimenois*, see n. 5 above, and cf. the usage of Aristotle *Pol.* 1290a 20, equivalent to Plato's use of *chalaros* at *Rep.* 398e.

the *Republic* by means of pleasures),[32] or else to divert towards education anything that was over-excited by desires, in the way I have already described. All education influences people either by imposing something on them, as the laws do, or by pressure of persuasion, such as that of the company we keep.[33] Music has both kinds of power, both enslaving the hearer by words and melodies, and inveigling him, by subtle variations of sound and posture, into conformity with what the words express. They assigned educational music to as many as a hundred days, the relaxing kind to no more than thirty.[34] Through the serious songs and dances they educated persons of the better sort – audiences and performers alike – while through the pleasurable kind they gave recreation to the common people.[35] Where all the guardians of the state were men of serious worth, as in the wise Plato's Republic, use was made only of melodies conducive to education. But where there was diversity among those who welded together the whole complex of the state, as in all other cities, amusements suited to each one of them were also necessary.[36]

Many people have failed to grasp these points, and have misunderstood the reasons for what is said in the *Republic*. As a result, some of them have treated melodies that are conducive to pleasure as completely worthless, without distinguishing the people for whom they are suitable and the manner in which they are used.[37] Others have prohibited melodic composition as a whole, on the grounds that it can produce nothing but melodies of that sort.[38] They have not realised that it is more useful than many other things for its educational value alone: besides, our nature demands melodies of that sort too, and it is

[32] For the test of drunkenness see *Laws* 637a ff., 649d; the *Republic* reference is to 413d–e.

[33] This distinction between two modes of education is implicit in, for example, Plato *Protagoras* 325c ff. On the importance of the company we keep compare *Rep.* 538a ff.

[34] The reference is certainly to the institution of festivals, to each of which appropriate music was assigned. Plato emphasises the importance of such festivals in the *Laws* (see particularly 653c ff., 803c ff.). He requires that a calendar of festivals be fixed, and suitable music allotted to each (799a–b), but no definite number of days is allocated. In any case the suggestion of thirty days' specifically non-educational music sounds un-Platonic. Aristotle, who might have favoured it (see next two notes) is unlikely to have prescribed a definite number of days, but may possibly have recorded existing or past practices, either in a missing section of the *Politics* or in one of the lost books of *Constitutions*. Mathiesen (n. 72 ad loc.) notes Meibom's association of thirty days with the Roman Saturnalia, and evidence for at least one hundred and twenty festival days in ancient Athens.

[35] An echo of Aristotle *Pol.* 1342a.

[36] This last remark is historically plausible, but if Aristotle is a source for this paragraph it may also reflect the pragmatic character of his political proposals, by contrast with the ideological purity of Plato's *Republic*. For the rigorous restriction of musical form in *Rep.* see especially 398e ff.

[37] The interpretation of the *Republic* to which Aristides objects seems in fact to be broadly correct. Though there and in the *Laws* (especially 802e) different types of music are prescribed for different people and occasions, it is always its educational value that is its passport to acceptability. No doubt Aristides has in mind such passages as *Laws* 658e–659a, where it is agreed that the pleasure music gives is the criterion of its excellence; but this pleasure is that of the educated and morally upright judge, and its occurrence is primarily a sign of the music's ethical rectitude. A similar position is adopted by Aristides in the final sentence of this paragraph.

[38] Who these critics are is uncertain. Possibly they based their view on the very radical remarks about art made by Plato in *Rep.* Book x.

impossible to thwart it (the wise man's words about the bow were well said);[39] though, on the other hand, we should select the kind of relaxation that is profitable. I say this without condoning those melodies which are altogether discreditable (since they are no part of the province which music controls: its task is rather, like that of the other arts in their own fields, to separate out the worst from the best), and without commending those who abuse the whole of the art because of the existence of bad melody: exhortations in unadorned prose are not to be silenced, after all, just because some of them incite us to virtue and others to vice.[40] Just as these exhortations have a dual nature, whose better side we choose, so also in melodic composition the better kind should be preferred, and we should not avoid song altogether just because it gives
61 pleasure. Not all delight is to be condemned, but neither is delight itself the objective of music. Amusement may come as it will, but the aim set for music is to help us towards virtue.[41]

This has escaped the notice of many people, including the character who speaks against music in the *De re publica* of Cicero the Roman.[42] I would not say, myself, that it was Cicero who said such things;[43] for how could one maintain that it was he who abused the art of music and censured it as worthless – the art, that is, which distinguishes excellences from vices in melody and rhythm – when he was the man who admired the contemporary dancer Roscius (who performed in rhythms alone, second-rate and vulgar ones at that) so enthusiastically as to say that he had come to mankind through divine providence?[44] If anyone were to argue that what he wrote in the *De re publica* expressed Cicero's own true opinions, while his remarks about Roscius were said for the sake of making a case, nothing stops us from reversing the argument; and in any event my opponent would unwittingly be undermining rather than reinforcing the orator's status so far as the present enquiry is concerned. A man who is bound to a point of view imposed on him by someone else's whim,[45] or adopted out of his own, rather than to that dictated by the nature of the case, cannot be relied on in the pursuit of truth or of unbiassed

[39] The reference may be to Plato's citation of Heraclitus at *Symp.* 187a. The words are then those of the Heraclitean epigram 'There is a back-turning *harmonia* like that of the bow or the lyre'. Plato's discussion (extending to 187e) concerns the harmonising of opposites in music by the principle of love, introducing also the distinction between 'heavenly' and 'vulgar' love. Aristides' purpose will then be to underline the essential duality in our nature. (But note that the ideas propounded by Eryximachus, the speaker in that part of the *Symposium*, are far from coinciding with Plato's.) Alternatively, the 'words about the bow' may have been to the effect 'Bend it as you will: it will spring back', emphasising the futility of ignoring an aspect of our natural motivation.

[40] A characteristically Platonic thesis: see, for example, *Euthyd.* 306d–307e, and Gorgias' defence of rhetoric at *Gorg.* 456d ff., cf. ps.-Plut. *De Mus.* 1146a.

[41] See n. 37 above.

[42] The work survives only as a fragment, and the main section on music is lost, but with this passage compare pp. 114–15 of the Teubner text (ed. K. Ziegler, Leipzig 1960).

[43] Cicero's *Republic*, like Plato's, is a dialogue in which the author does not appear as a character.

[44] Possibly a reference to Cicero's *Pro Quinto Roscio Comoedo*, now fragmentary. The passage recalled here is lost.

[45] The text is corrupt. I suggest (and translate) *kat' allou*, or perhaps *kat' allēn*, for the MSS *kat' aulēn*.

judgement. And I do not suppose that my opponent would condemn the art of rhetoric just because there are corrupt orators. In the same way, if some artists perform despicable melodies because these please the crowd, it is not the art that is to blame.[46]

In their native land at the time of Numa and a little later (as Cicero himself tells us),[47] the citizens, who were still relatively barbaric, were already being
62 civilised by music, which both accompanied their private merry-making and assisted in the celebration of their public rituals. As to war, in which Rome was and is outstandingly glorious (and, let me add, may it so remain), the fact that drill exercises 'in pyrrhic style'[48] are done to music hardly needs mentioning, since everyone knows it quite well. What most people do not know is that in the perils of battle itself they often avoid the use of verbal commands, since damage would be done if they were understood by those of the enemy who speak the same language. Instead they signal by musical means, using that martial and rousing instrument the *salpinx*, and each command is assigned a specific tune. Thus frontal attacks and flanking advances, for instance, have each been given their own particular melodies: another sounds the retreat: there are special calls for wheeling to left or to right. Thus they can go through all these manoeuvres one after another, using signals which are incomprehensible to the enemy but perfectly clear to their own side, and which are understood the moment they are given. The signals are not heard first by one section, then by the next: the whole army acts at a single sound.[49]

Most remarkable of all, when at various times Rome had, as leaders of the state, men deficient in musical cultivation, it experienced in practice what Plato in his *Republic* had prophesied, and saw its citizens murdering one another most brutally in the middle of its streets and among the beauties of its temples.[50]

[46] Aristides' uncharacteristic foray into vigorous polemic suggests that the attack on music in the *De Re Publica* was extreme. If the present reply is to be trusted, it will have been based on a survey of examples of musical vulgarity and corruption. On the manoeuvre made in Aristides' last two sentences see n. 40 above.

[47] Cicero *Tusc.* IV.1.3. Numa, the second king of Rome, is traditionally dated around the beginning of the seventh century B.C.

[48] That is, to the steps of the *pyrrhichē* or war-dance: see, for example, Aristoph. *Frogs* 161, Plato *Laws* 815a–b, Athenaeus *Deipn.* 630d–631c; and cf. 35.22–3 above, 82.19–20 below.

[49] The *salpinx* was a form of trumpet, normally of metal, and according to Pollux its mouthpiece was of bone. It was used from early times and in most Mediterranean cultures, though it is often said to be an Etruscan invention, generally for martial purposes or at the games, seldom to make music (but for its use in merry-making see 5 *De Audib.* 803a). Among military signals Pollux (*Onomastikon* IV.85–7) distinguishes the arousal to action, the encouragement in battle, the recall from battle, and the signal to halt and make camp; such uses are recorded, for example, at Thucydides VI.69.2, Xenophon *Hell.* V.19, *Anab.* IV.4.22.

[50] The reference is to Plato's account of the degeneration of states and of characters in *Rep.* Books VIII–IX. The neglect of musical education is identified as a root cause of such degeneration in the introductory passage at 546e–547a. The allusion to events at Rome does not make it clear whether Aristides has in mind just one among the many bloody episodes of Roman history. The civil wars at the end of the Republican period are an obvious candidate.

It can be shown both that music is the most powerful agent of education, rivalled by no other, and that our characters commonly deteriorate if they are left undisciplined, lapsing into base or brutal passions. My argument will not deal with facts about the individual, which are hard to discern, but with facts about cities and entire races, since research is always easier when its objects are
63 on a large scale.[51] So far as education is concerned, there are two undesirable conditions, lack of culture and corruption of culture.[52] The first comes from absence of education, the second from poor teaching. Now the soul is found to contain two generic varieties of emotion, spirit and appetite.[53] Hence of the races that have never tasted the beauties of music at all, those that truckle to their appetites are insensitive and bovine,[54] as are the peoples of Opicia and Leucania, while those that encourage their spirited side are savage, like wild beasts, as are the peoples of Garamantis and Iberia.[55] Of those among whom music has been perverted against its nature into depravity and cultural corruption, the peoples that cultivate the appetites have souls that are too slack, and improper bodily affectations, like those who live in Phoenicia and their descendants in Africa;[56] while those that are ruled by the spirited part lack all mental discipline – they are drunkards, addicted to weapon-dances [*enhoplia*] no matter whether the occasion is right, excessive in anger and manic in war, like the Thracian peoples and the entire Celtic race.[57] But the races that have embraced the learning of music and dexterity in its use, by which I mean the Greeks and any there may be who have emulated them, are blessed with virtue and knowledge of every kind, and their humanity is outstanding. If music can delight and mould whole cities and races, can it be incapable of educating individuals? I think not.

Again, no other activity has so great a capacity both for establishing a community and for sustaining it once it is established. When a constitution has already begun to incline in one direction or the other, the other activities follow in its wake: but music directs all such changes.[58] Of all branches of learning it is first in order and in power, and with its melodies it moulds the will of each man in suitable ways from his earliest youth.

64 Let me also give you an indication of the way it serves to detect character. Whatever the qualities of the melodies and rhythms in which each group of

[51] Aristides echoes the strategy announced by Plato's Socrates at *Rep.* 368c ff.

[52] The terms are *amousia* and *kakomousia*.

[53] See 54.23–6 and 58.18ff. above.

[54] *Boskēmatōdeis*, translated by LSJ as 'bestial', but meaning 'like animals that merely feed', i.e., cattle. It is contrasted with *thēriōdeis*, 'like wild beasts'.

[55] Opicia and Leucania are in southern Italy, Garamantis in northern Africa; Iberia is Spain, specifically its northern sector.

[56] That is, the Carthaginians.

[57] On *enhoplia* and their associated rhythms see, for example, Xenophon *Anab.* v.1.11, Aristoph. *Clouds* 651, Plato *Rep.* 400b, Athenaeus *Deipn.* 184f, 630f, Dio Chrys. II.61, Bacchius *Eisagoge* 316.7–8. Aristides' account of the Celts, and perhaps of the other races too, is probably derived from Posidonius; cf. also Plato *Rep.* 435a–436a.

[58] See *Rep.* 424c, where the thesis is attributed to Damon, and is enthusiastically endorsed by Plato: see also *Laws* 701a–b.

people takes pleasure, publicly and privately, the qualities of character which they find agreeable will turn out to be the same. From lack of concern about the fact that bad things are promoted by song, there arises conformity of habit, and from habit springs character, the source of impulses towards action. Pleasure joins them, to confirm and intensify the evil, treating postures, styles and even words quite indiscriminately, till in the end it turns upside down the whole of private and public life.[59] The reverse is also true: from fine song arise good words, characters and habits, noble impulses and most excellent deeds.

Thus it was that in the earliest times, when political institutions were nowhere firmly established, the cultivation of music in association with virtue corrected civil discord and put an end to hostilities with neighbouring cities and races. It specified set times for communal assemblies, and through the celebrations and revels customary at such occasions it restrained their aggressiveness towards one another, replacing it with kindliness[60] – just as skilful farmers first clear their fields of weeds and barren scrub, making a safe place to sow seeds of the highest value.

If in these days every city and virtually every race has come to love good order and human friendship, music has not therefore become useless. Medicine is not something to be called upon when we are ill and to be treated with ingratitude when we are healthy: we should acknowledge that we owe to it our good health, and make it our guide through the rest of our lives, so that we do not fall back through idleness and self-indulgence into our original condition. It is the prerogative of one and the same art both to instil an excellence when it is missing, and to preserve and augment that same excellence when it is already
65 present.[61] That is how we should speak of music too. We should acknowledge our debt to it for bringing each of us into friendship with himself and with one another in the community; and we should practise it always, looking for a safeguard of our mutual fellow-feeling. The task should be pursued privately as well as communally. Just as there is a kind of health that belongs to the community and one that belongs to the individual, so there is a concord [*symphōnia*] that holds between a whole city and itself, and one that holds between a single soul and its own parts.[62]

No one, I believe, could now deny that we should educate the young through music, and that we should ourselves cultivate it throughout our lives wherever it is possible.[63]

[59] The last clause is quoted almost verbatim from *Rep.* 424e; the major source for the whole paragraph is plainly *Rep.* 424b–425b.

[60] In addition to his Platonic sources, Aristides is probably thinking of such tales as those about Terpander and Thaletas at Sparta: see ps.-Plut. *De Mus.* 1146b–c, Plut. *Lyc.* 4, Pausanias I.14.4, Philodemus *De Mus.* I.30.31ff., IV.18.33–20.27 (Philodemus refers to comparable stories about other musicians too).

[61] Compare 80.25ff.

[62] Compare, for example, *Rep.* 431e–432b, 441e–442a, 443c–444a.

[63] This completes Aristides' answers to the first three questions of chapter 1 (53.1–3). He turns now to his fourth question (53.3–4), that concerning the kinds of music that are educationally effective. After an introductory passage (chapter 7) and a second disquisition on the soul (chapter 8), he discusses the materials available to the musical

Chapter 7 It is now time to explain what kinds of melody and rhythm will discipline the natural emotions. I shall set out what some ancient writers said, and also some things which have not previously been discussed – not, however, because the writers were ignorant or malicious. That would be a quite improper thing to say of philosophers initiated in the mysteries of music. The fact is rather that while they expounded some things in their writings, they reserved the more esoteric secrets for their discussions with one another.[64] The reason lay in the enthusiastic affection of the men of those times for all that is finest. But now, when indifference to music (to put it politely) is so widespread, we cannot expect people with only a mild interest in the subject to tolerate being faced with a book in which not everything is explicitly spelled out.

The four main objectives at which the musical educator should aim are these: suitability of idea [*ennoia*], of diction, of *harmonia*, and of rhythm.[65] The idea is in all respects the leader, since without it there can be no choice or rejection of anything. The diction is an imitation [*mimēma*] of it, and diction is of the first importance if an audience is to hear and be convinced. When diction adopts high and low pitches with intervals between them, but still lacks
66 definite pattern, it generates *harmonia*;[66] when ordered in concordant ratios,[67] it generates rhythm. But since music is a treatment for the passions of the soul,[68] we should first investigate the ways in which they arise in it, and where they come from. If these points are not established, our discussion of subsequent matters will fail to be clear.[69]

Chapter 8 It seems to me that when the soul is far away from this region and associates with that which is greater, it lives with reason and is pure from desire:[70] but when it turns towards earthly things and seeks to learn by experience about life here, it comes to need a body, and seeks one that is

educator under the headings of conception (*ennoia*) in chapter 9, verbal expressions in chapter 10, diction in chapter 11, *harmonia* and melodic form in chapters 12–14, and rhythm in chapter 15. The remainder of the book is concerned principally with the uses of instruments, into which a further discussion of the soul (chapter 17) is inserted.

64 Compare Book I, 3.12ff. Many ancient thinkers were said to have reserved their most important doctrines for discussion in their immediate circle. Pythagoras is the prime example. Plato too is credited with 'unwritten doctrines', and he himself attributes a 'secret doctrine' to Protagoras, probably ironically (*Theaet.* 152c ff.). Later writers were often anxious to avoid accusing wise men of the past of ignorance. The tendency is vividly exemplified in ps.-Plut. *De Mus.* chs. 17ff.

65 On the word *ennoia* see n. 15 above. The classification and sequence of topics (see n. 63 above) follows Plato's *Republic*, which considers first the proper subject matter for poetry (377–92), then diction (392–8), then *harmonia* (398–9), and finally rhythm (399–400).

66 Compare Book I, 31.24–7.

67 That is, in the ratios of the primary harmonic concords (including the unison), 1:1, 2:1, 3:2 and according to some authorities 4:3; see 33.29ff.

68 See the Theophrastan passages cited at n. 26 above.

69 Compare 53.8ff., with n. 2.

70 This conception of the soul derives ultimately from Plato (particularly *Symposium*, *Phaedo*, *Republic*, *Phaedrus*). Aristides' accounts of the soul are not strictly Platonic in all their details, however. They contain traces of Stoicism, and of the Pythagorean and Platonist revivals of the first centuries A.D. But it is uncertain whether, as Mathiesen argues (see his references), Aristides depends also on the writings of Plotinus.

suitable for it. It has the capacity to perceive the duality that exists among them (maleness and femaleness, I mean),[71] a duality which is present not only in those that have souls, but also in those that are directed by nature alone – all the varieties of plants and minerals and spices. There is a clear duality in these things too: they display one nature through softness, smoothness, freshness of colour or sweetness of scent, or the other through their opposites.[72] In itself the soul is simple and undifferentiated in form, but when it clothes itself in this human shape it conceals its own natural beauty, and is moulded, partly of its own will and partly by necessity, into the space and the conformation of its outer shell.[73] It does not then merely desire a body, but a body of one definite kind: it is attracted either to the male or to the female, sometimes to one or the other unmixed, sometimes to a body with a strange, truly peculiar mixture of the two.[74] And then, if the body they light on is not naturally of the kind they sought, they reorganise it to suit their own tendencies, and transform it into something like themselves. Thus it is that upon males there is sometimes superimposed a female form, which can be seen to make their way of life female too; and a male form may be imposed upon women, an indication that
67 their character is also male. Men may be beardless and women grow beards: some men have a languishing look and some women an aspect of ferocity: and you will find that in every case the character is in accord [*synōida*] with the appearance.

Passions arise in the soul out of its affinity with the male or the female or with both. Thus the female is seriously lacking in restraint, and with it the appetitive part is in accord [*synaidei*], while the male is violent and energetic, and the spirited part resembles it.[75] In the female – both the female type of soul and the female branch of humanity – griefs and pleasures are rife, anger and recklessness in the male. Couplings [*syzygiai*] of these passions arise too: of griefs with pleasures and of anger with recklessness, of recklessness with pleasure and grief, of anger with both, and indeed of each with any one or more of the others. One could find a thousand different varieties of these emotions if one studied them in all their complexity.

Since the emotions are disposed differently in each individual according to his nature, there arise also differences of conception [*ennoia*]. Thus one man enjoys looking at whiteness while another admires black: one likes sweetness

[71] The male–female dichotomy is central to what follows: its extension into metaphysical contexts was characteristic of early Pythagoreanism. Its use in musical analysis (see particularly chs. 10, 12–14, 16, 18–19) may be due to Damon: see 80.25ff., and compare particularly Plato *Laws* 802e (see also *GMW* vol. 1, pp. 168–9 with references). There is a hint in Philodemus (*De Mus.* IV.23.27–24.9) that the idea was adopted by Heraclides.

[72] Compare Soph. *Trach.* 1195–7, Theophrastus *Hist. Plant.* III.9.3, VII.4.3 (of plants), *De Causis Plantarum* II.6.3 (of water), VI.15.4 (of a scent).

[73] See Plato *Rep.* 611b–612a, cf. 588d–e, and compare also the imagery at 88.6 below.

[74] Comparable ideas are expounded by Eryximachus and Aristophanes in Plato's *Symposium* (187ff.).

[75] Compare 54.23ff. and 58.18ff. 'Lacking in restraint' renders *aneimenon*: see nn. 5 and 31 above, and cf. 73.19.

for its pleasure, while another likes bitterness.[76] The contrariety I have spoken of is found, as I said, both in the emotions and in external objects. When the soul encounters these, we are told, it obtains an impression of them, and each of us takes pleasure in those which resemble his own emotions. The two natures are to be observed, first, in visible objects. We distinguish as belonging to the female those colours and shapes that are vivid and decorative, and to the male those that are subdued and conducive to mental reflection. Secondly, among the objects of hearing, we associate [*harmottonta*] sounds that are smooth and gentle with the female, rougher ones with its opposite.[77] To avoid mentioning everything individually, one may assert that it is quite generally true, in all
68 cases, that those objects of perception which naturally invite us to pleasure and to the gentle relaxation of the mind are to be adjudged female, those that stimulate us to thought and arouse activity are to be assigned to the province of the male, while those that do neither, or a mixture of both, are to be treated as intermediate. We may speak in the same way of all those varieties of skill and virtue and knowledge which, while it is admittedly through reason that we grasp them, nevertheless derive their mutual similarities and differences from the quality of the subject matter with which each is concerned.[78] This is the way in which virtue is related to virtue and science to science, vice to vice and ignorance to ignorance, virtue to vice and knowledge to ignorance, and vice to any of the others. The interweaving and mingling that this subject presents is again exceedingly complex.

Chapter 9 It is through conceptions [*ennoiai*], then, that approval of things in life first comes, conceptions that have either grasped each thing successfully through an inherent capacity for learning, or have later been persuaded to change. Here the common talk of the majority has as much power to instil character as have the doctrines of the sciences. For everyone's soul constantly remodels the impressions and representations that it contains in the light of ideas [*ennoēmata*] aroused through words; and then by this kind of familiarity and practice it imperceptibly and gradually establishes a fixed disposition, which may be a blessing or a curse.[79]

Moral education as a whole is divided into two types. One kind, that through which we correct vice, is therapy: it also has two forms. One is

[76] Indicating, as does the sequel, that an *ennoia* is not merely a 'concept', but one with an evaluative or emotional loading: cf. n. 15 above. Hence the expression of an *ennoia* in words or music will seek primarily to convey the moral or affective character of what it portrays. See especially chapter 9.

[77] These remarks are developed in chs. 11–14.

[78] Aristides means that though the necessary role of reason in them might lead us to adjudge them all 'male', they are in fact to be distinguished as male or female in correspondence with the quality of their subject matter.

[79] This emphasises again the guiding moral dimension of *ennoiai*: they are not evaluatively neutral concepts used merely for the descriptive classification of sense experiences. On the influence of everyday social contacts on moral dispositions see, for example, Plato *Rep.* 537e ff., among many other passages. On the interaction between changes of moral attitude and shifts in vocabulary see Thucydides III.82.3ff.

amelioration, which we use to diminish little by little an emotion that we cannot persuade all at once, to the point where it is no longer felt: the other is eradication, used when we achieve a complete transformation in the hearer from the start. The second main kind is cultivation, of which one form is conservation, used when we strengthen the best habits and secure them in the same condition with our teaching, and the other is supplementation, used when we attempt to bring a moderate degree of excellence [*kalokagathia*] by gradual
69 additions to the pinnacle of virtue.[80]

There are two techniques of teaching through ideas [*ennoiai*]. If one can find things useful for influencing the soul already manifested in the materials of which the subject in hand consists, we shall use them. If we are in difficulties, we shall employ various devices to track down what is needed.[81] Of such devices the following are the most useful: epithets, substitutions, metaphors, similes, synecdoches, circumlocutions, allegories; and there are many more.[82]

Consider now how the poet tries to describe the slow rising of the sun in a way that complements what he will say about the burial of the heroes in the twilight.[83] He uses epithets expressing slowness: but since they cannot properly be applied to the swiftest of the heavenly bodies, he first makes his statement about the sun's rays –

'Then the sun was just beginning to touch the fields' (*Il.* VII.421) –

and then applies his armoury of epithets to the slow-moving element of water. First he indicates the peacefulness of its flow (he says 'from the soft-flowing'), and next he reveals the cause of the slowness of its motion. 'Of deep-flowing Ocean' is what he says (*Il.* VII.422); for shallow waters, rolling about on the convexity of the earth below them, are always swift-moving, while the deeps are slow and steady, fragmenting the impulse to movement into many parts of the concavity beneath them. But this has also expressed the leisureliness of the sunrise, for it is to be expected that an emergence from the deep will be rather

[80] On the distinction between corrective and cultivatory arts compare Plato *Gorg.* 464b ff., where Plato draws parallels between arts of both sorts concerned with the body (medicine, physical training) and with the soul (administration of justice, legislation). The words 'of which one form is conservation' are missing from the MSS, but must evidently be supplied.

[81] 'Influencing the soul', *psychagōgia*, sometimes 'amusement', as at 60.8, 61.2, but that is plainly the wrong sense here. An educational meaning is required also at 80.19 and at 82.2–3. In some cases a direct description of some subject will serve to affect the hearer appropriately; in others, special literary or rhetorical techniques must be adopted. (Aristides is not referring, as Mathiesen supposes, to the need for interpreting obscure texts by the devices of literary criticism. He means that devices identified by literary theorists must be assessed for their suitability and then *used* by the poetic educator.)

[82] The devices listed are exemplified in the remainder of the chapter. They are discussed by various ancient theorists (e.g., Aristotle *Rhetoric*, Dion. Halic. *De Comp. Verb.*), but Aristides seems not to base his treatment directly on any one of them. 'Substitutions', *metalēpseis*: the word can mean 'translations'. Here it might indicate the substitution of one word for another, more probably the transference to one subject of a description normally applied to another; see 70.15–20, where the figure is called *metastasis*. Synecdoche is reference to a whole by an expression literally designating a part, or conversely.

[83] All the examples that follow are from Homer.

prolonged. When, by contrast, he is expressing the swiftness of a sunrise, he ascribes to the sun an ascent full of vigour and imbued with life. He says

'The sun sprang up, leaving the lovely lake' (*Od.* III.1), giving added sweetness to the words by the feminine name he uses for the Ocean.[84] And
70 again, when he wants to depict the delights of a sunrise in their entirety, he adorns his words with the charms of colour and with the sweetness of perfume at the same time, applied not now to the sun, but to the female character, Dawn:

'Dawn in her saffron robe was scattered over all the earth' (*Il.* VIII.1),

and again

'When rosy-fingered Dawn, daughter of morning, appeared' (*Il.* I.477).

In another place, when he wishes to depress our spirits, he speaks of the armies drawn up for battle as

'Dark with shields and bristling with spears' (*Il.* IV.282). Here he achieves by metaphorical expressions what he could not have done through the terms whose use would have been natural. The word 'black' produces almost the same effect when heard as does the colour when seen,[85] and the word 'bristle' expresses vividly through its fearfulness the agony of war.[86]

Again, for the sake of vividness, he says

'All the feet and heads of Ida with the many springs were shaken' (*Il.* XX.59–60);

and he leads us to a better conception [*ennoia*] of the disturbance by transferring to things that are immoveable a description drawn from our own bodies, which are naturally subject to motion.[87]

Further, he recounts the story of Ares and Aphrodite in coarse terms, using the words 'they coupled', 'surreptitiously', and 'shamed' (*Od.* VIII.268–9), of which the first expresses the pollution which pleasure naturally breeds, the second the culpability of the act, the third their disgraceful wrongdoing; but on the very same subject in the case of Odysseus he beautifies his narrative with dignified words, since he is recounting behaviour that was right and lawful:

'Joyfully, as in past times, they came to the hallowed custom of the bed' (*Od.* XXIII.296).

71 When he speaks of love-making that is neither blameworthy under the law nor a subject for praise, he achieves an intermediate effect through a mixture of opposed significations:

[84] The word is *limnē*, 'lake', a feminine noun: Aristides himself adds extra pungency to the description by giving the sea the designation 'Okeanos', the name of a masculine divinity.

[85] It is curious that Aristides uses a different word in his comment (*melas*, 'black') from the one used by Homer in the quotation (*kyaneos*, 'dark'). See the next note.

[86] Homer's verb is *phrissein*: like the cognate noun used by Aristides (*phrikē*) it has not only the sense 'bristle', but also that of 'shudder' or 'shiver', especially with fear. Aristides is drawing attention to the metaphorical aspects of this usage and that of *kyaneos*. There is nothing to indicate that he is concerned here with onomatopoeia (as Mathiesen suggests), and the point made in n. 85 above seems decisively against it.

[87] *Koryphē* can mean 'head', but is often used for 'peak', 'summit'. On 'transference', *metastasis*, see n. 82 above.

'I never went to bed with her and coupled, as is the proper custom of mankind, of man and woman' (*Il.* IX.133–4).[88]

Where we cannot effect changes by differences of style [*tropai*][89] we shall use similes. Thus Homer distresses our mind when he says

'As when from a hill-top a goatherd sees a cloud' (*Il.* IV.275),

and relaxes it when he speaks of

'A flower like milk' (*Od.* X.304),

for of all the white things that there are he has selected the one which also carries the suggestion of sweetness.

Again, when describing the laments of Achilles, he uses many words that trouble the mind, 'black cloud', 'sooty dust', 'black ashes', and not so many that are cheerful (*Il.* XVIII.22–5). But he wants to give an attractive account of the fate of Euphorbus, to release us from the emotion surrounding Patroclus, so that there cheerful expressions abound, such as 'tresses like the Graces', and materials that delight us, such as gold and silver; while the sequel has a flourishing olive, a broad plain, gushing streams of water, the sweet breath of winds, and blossoms, shining and precious (*Il.* XVII.51–6).

To bring weight and dignity to his words he uses synecdoche:[90]

'They ran with a loud war-cry to the well-built wall, raising dried oxen above them' (*Il.* XII.137–8).

72 Here he has used immensity of dimension[91] to elevate the statement. Since the word 'shield' was inadequately powerful, he used the word 'ox', which suggests bulk. But since this is discordant [*asymphōnos*] with the word 'raising', he added 'dry'. Through the lightness suggested by the dryness he attached plausibility to the action, and delighted our minds [*ennoiai*, 'conceptions', 'ideas'] with a picture that is beyond criticism.

Elsewhere he says

'And all around trumpeted [*salpinxen*] the great heaven' (*Il.* XXI.388).

This adds weight to his account of the battle, but as applied to heaven it is inappropriate. Hence he uses two devices to produce the appropriate effect: with the epithet[92] he magnifies the melody of the *salpinx*, and by the addition of the prefixed expression[93] he makes the sound come from every quarter of heaven. It is also clear that humble words give plainness to the account, as in

'Putting beside him a poor stool and a small table' (*Od.* XX.259).

Notice also how the complexities of circumlocution can simultaneously give

[88] Agamemnon is speaking of Briseis, the girl he had taken from Achilles, whose war-prize she was.

[89] Compare the uses of this term in reference to styles of melodic and rhythmic composition, 30.1ff., 40.12.

[90] See n. 82 above. Here a word for the whole ('oxen') is used for the part (their hides).

[91] *Megethei tōn diastēmatōn. Diastēma*, the usual word for 'interval' in harmonic contexts, commonly means 'dimension'. It has nothing to do here with rhythmic lengths, as Mathiesen holds, arguing that the lines are 'dominated by long syllables'. The second line quoted has in fact the greatest number of short syllables that an epic hexameter can, nor is *diastēma* used in this sense in the rhythmic analyses of Book I.

[92] That is, the word 'great'. On the *salpinx* see n. 49 above.

[93] That is, 'all around'. The term *prothesis* can have the grammatical sense 'preposition'.

clarity to character and sweetness to the words. Thus when he wants to describe the personalities of the young heroes, each with his own kind of excellence, he speaks sometimes of 'the might of Eteocles' (*Il.* IV.386), and elsewhere of 'the sacred power of Telemachus' (e.g., *Od.* II.409, XVI.476), indicating the preeminence of the one for his strength and the divine favour bestowed on the other for his self-control.[94]

Allegories use many words lying in succession to convert the mind towards
73 a different quality. Notice, for example, how when he has decided to speak without passion or grief about those who had died in the war, he writes as if about the gathering of the wheat:

'Men are quickly sated with battle, in which the sword scatters most straw on the land, but the harvest is leanest' (*Il.* XIX.221–3).

Chapter 10 From the interweaving of these conceptions [*ennoēmata*] with one another arise verbal expressions [*logoi*] and their species,[95] just as medicines and sauces are made from a mixture of juices and flavours. Conceptions which lead to relaxation and cheerfulness engender simple and charming expression, while those that stir up thought and impulses to action display one suitable for politics and the law courts. The characteristics of political expression, in conformity with the nature of the male, are conciseness and brevity, dignity and nobility, harshness and vehemence, bulk and concentration, which express the natural great-mindedness of the male, the bulk of his body, the power of his intelligence, and his swiftness in the accomplishment of the greatest of deeds. The features of the simple kind are slightness and charm, and things of beauty and sweetness, of which the former display the frivolous side of the female,[96] the latter her passion for adornment. Some things may be produced by an interweaving of these. Thus acerbity manifests the swiftness of the male and the triviality of the female, while diligence shares the parsimony of the female and the hard-working intensity of the male.[97] By using these species of expression, either singly or in combination, on each soul, you will sometimes convince the soul with similarities, sometimes overcome it with dissimilarities.[98]

Concerning the nature of the figures[99] by means of which these conceptions should be put forward, I do not think I need to say much. Their actual delivery will be sufficient to show what they are. Each of them either abases the mind,

[94] These are 'circumlocutions' because Homer uses them as roundabout ways of referring to the people themselves: 'the might of Eteocles' is equivalent to 'the mighty Eteocles'.

[95] *Ennoēma* seems little more than a variant for *ennoia*. It occurs only twice in this work, here and at 68.19 above. But in both cases it refers specifically to a conception aroused by or expressed in words, and Aristides may intend to reserve it for this specialised use. *Logoi* are usually (as here) complex verbal expressions – sentences or clauses rather than individual words.

[96] 'Frivolous' here renders *aneimenon*, 'slack', 'relaxed'; cf. 67.5 and nn. 5 and 31 above.

[97] 'Intensity', *parektetamenon*, another version of the metaphor of tension. With the whole paragraph, compare 67.3–68.13.

[98] See especially 80.10ff., cf. 85.21ff.

[99] *Schēmata*: often the position adopted by dancers (cf. 32.6, 18). Here, more generally, the bodily postures associated with utterances with different social or persuasive functions.

74 as do supplications, or relaxes it, as do expressions of consent, and makes people either humble, as do apologies, or exalted, as do expressions of advice or explanation. Their character can best be understood from the posture which each of them imposes on the body at the moment of delivery, which is why they have come to be called 'figures'.[100]

In my *Poetics*[101] I have given a precise account of the assumptions and the major categories on the basis of which we should organise our imitations [*mimēseis*] and expositions.[102] Poetry is divided into imitation and narrative,[103] and each of these is in turn divided into two: creations of character, and narrations. The latter frequently occur on their own, and are called 'forms'[104] or 'poems'. That these too provide a useful musical education is adequately attested by Homer: for when Achilles in the *Iliad* wants to distance himself from his grief over Briseis, he is not introduced singing a love-song, but summons his soul to courage by recounting to his *kithara* the great martial deeds of men of old (*Il.* IX.186–9). Of the characters in the *Odyssey* who use music to give education, one is seeking to restrain the suitors from their insulting treatment of Penelope and their impiety towards Odysseus' guardian goddess. He sings of the disasters which were brought upon the Greeks by the crime of the Locrian:[105]

'He sang of the dismal homecoming of the Achaeans from Troy, laid upon them by Pallas Athene' (*Od.* I.326–7).

The other, reproving [*paideuōn*, 'educating'] the self-indulgence of the Phaeacians and seeking to forestall the outcome to which it naturally leads, sings of the binding of Ares and Aphrodite,[106] to show how not even they could escape punishment for their offences (*Od.* VIII.266–366). Odysseus himself, furthermore, did not reveal who he was to the Phaeacians until Demodocus had taught them, through music, Odysseus' noble qualities, and through the persuasion of melody had instilled in them a longing to know so wise a man. He says to Demodocus

75 'Come now, change your song, and sing of the devising of the wooden horse' (*Od.* VIII.492),

and he adds

[100] 'Delivery', *hypokrisis*, the delivery of an orator or the performance of an actor. Aristides apparently means that the 'figures' appropriate to each kind of *ennoia* are revealed in those that we naturally adopt when 'delivering' them. On *hypokrisis* see further 84.11–19.

[101] The work is lost, and nothing is known of its contents.

[102] 'Assumptions' translates *hypotheseis*; 'major categories' translates *kephalaia*. In Aristotelian usage, paralleled in Aristoxenus, these would refer respectively to principles, grounded in perception and induction, from which subordinate propositions are demonstrated, and to the general headings into which the field of a science is to be divided. Compare 6.10: on *hypotheseis* see also 11 Ptol. *Harm.* Book I ch. 2 with n. 15.

[103] Compare Plato *Rep.* 392d ff.

[104] *Eidē*, 'forms', 'shapes', 'kinds', sometimes specifically of literary genres. There is no perfect parallel to the usage suggested here, though the word is used by a scholiast to refer to Pindar's Odes.

[105] The Locrian is Ajax son of Oeleus. At the fall of Troy he dragged Cassandra from the statue of Athena ('Odysseus' guardian goddess') and raped her. The singer is Phemius.

[106] On this story see also 88.9ff.

'The snare which the noble Odysseus once brought into the citadel' (*Od.* VIII.494).

This passage is related to the one we looked at before, since it describes the punishment of someone who had interfered unlawfully with another man's marriage.[107] But we can also see how the subtle Odysseus, by speaking as if of someone else, seeks first to receive the minstrel's praises in a way appropriate to his worth, without offending his hearers. This makes him both more convincing and more welcome to them later, when they wish to learn who he is, and he replies

'I am Odysseus, son of Laertes, renowned among men for all kinds of cunning; and my fame reaches to heaven' (*Od.* IX.19–20).

Chapter 11 Let us now change the subject, 'pulling on the reins in the Muses' mouths', as the merriest of the comic poets puts it (Aristoph. *Wasps* 1022), and talk about diction, of which the first and most crucial aspect to be discussed concerns the letters of the alphabet.[108] Our task is to map their individual characters accurately on to the opposition of genders discussed above. Some of them produce smooth sounds (these are the vowels), others harsher ones (the mutes), others again sounds that are intermediate (the semivowels). Of the vowels, those that give out an unimpeded sound are naturally more sonorous and thus more dignified (these are the long vowels), while those that are quickly curtailed are less so (these being the short vowels). The sonority possessed by the intermediate vowels depends on the magnitude[109] of their durations. Of the semivowels, those that project a thin hissing sound from the surface of the lips (the double ones and the 'peculiar')[110] are rougher,
76 the rest more sonorous. This is especially characteristic of those in which the tongue strikes both on the breath and on the mouth, such as lambda and rho, less so of those in which the breathing-channels are closed, or which are projected only through the nose, such as mu and nu. Of the mutes, some are sounded only through the lips, the breath forcing its way through the middle of the obstruction they create (these are beta and those on either side of it); others with the cheeks shaped into something of a grin, and the breath propelled vigorously over a broad front (these are gamma and the extremes in each direction from it); others again with the tongue shooting the breath all in one piece like a sling-shot, between teeth that are more or less clenched (these are tau and theta and what lies between them). Of these, the ones that project the air gently, and do so from around the area of the teeth, are called 'pure', and are more sonorous, while those that project it from inside, from the pharynx, are called 'aspirated', and are very rough. Those projected from the

[107] In the earlier passage Ares 'interferes' with the marriage of Hephaestus and Aphrodite. Here the reference is to Paris' disruption of the marriage of Menelaus and Helen.

[108] On the different kinds of vowel and consonant see also 41.3–17.

[109] The MSS read *poiotēta*, 'quality'; I adopt the emendation *posotēta*, 'quantity', as suggested by Winnington-Ingram.

[110] That is, the sigma (s): see 41.14. The double semivowels are xi (x) and psi (ps).

middle of the vocal region are called 'intermediate', and share the nature of both the others.[111]

The mixtures constituting syllables are put together out of the quality of the letters, and in their characters they resemble those of the letters which occur most frequently, or which predominate in pronunciation. Long syllables create magnificence in diction, short ones the opposite; and out of the combination of these there arise feet. Feet in which long syllables come first, or cannot be resolved, or form the foot's boundaries, or are in the majority, are the most elegant and dignified, and so are the phrases, clauses, periods and metra which are made up of them. Those in which short syllables predominate in any of the ways mentioned are plainer and less elevated.[112]

That, then, completes our account of conceptions [*ennoiai*], of diction, and of combination, so far as I have been able to achieve it.

Chapter 12 Let us now speak of matters most specific to music – melodies,
77 rhythms and instruments.[113] These are things whose capacities we must discuss at full length, as befits experts in music.

The relation which we described earlier as existing between rhythm and *harmonia* – that of male to female – must now be clarified further.[114] That is, we must explain that while their extremes are understood on the basis of an opposition of quality, things intermediate between them share the character of both. For *harmonia* bears to *harmonia*, and again rhythm to rhythm, the same difference of species. Just as what is called 'rhythmic melody' lies, in its essence, mid-way between *harmonia* and rhythm, so also, among the qualities of each of the two, the intermediate ones display a complex mixture, depending on the distance of the means and the extremes, while to those that stand in complete isolation has been allotted the nature of being opposed to one another.[115]

[111] The triadic groupings of this paragraph are characteristic of Aristides. In his examples, beta (b) is intermediate between the pure pi (p) and the aspirated phi (ph); gamma (g) between pure kappa (k) and aspirated chi (kh); between pure tau (t) and aspirated theta (th) there lies delta (d). On the 'pure' (*psila*) and the 'aspirated' (*dasea*) see 41.15–17.

[112] Syllables are discussed in Book I ch. 21 (41.18ff.), feet and metres in chapters 22–8. On the 'magnificence' or 'dignity' of the long syllable, particularly in first place, see 47.4–14. 'Phrases, clauses, periods, metra' translates *kommata*, *kōla*, *periodoi*, *metra*. *Komma* is not used in Book I, and the rhythmical sense given to *periodos* at 35.1–2 is not relevant here. A possibly appropriate sense for *kōlon* is given at 51.3; *metron* is defined at 45.18–20. In the present passage the likely senses for *komma*, *kōlon* and *periodos* are grammatical or rhetorical, not to do with rhythmics or metrics: a *periodos* is a complete sentence; a *kōlon* is a segment of a period; and a phrase, *komma*, is a unit smaller than a *kōlon*. On clauses and periods see, for example, Aristotle *Rhet.* 1409a–b, on phrases Demetr. *De Elocut.* 9.

[113] Issues to do with melody occupy chapters 12–14, rhythm chapter 15, instruments chapters 16–19 (chapter 17 being concerned with an account of the soul designed to match the discussion of instruments).

[114] The relation is described, and attributed to 'some of the ancients', at 40.20–5.

[115] Aristides' point is that while rhythm has a male character in relation to the female *harmonia*, nevertheless some kinds of rhythm and of *harmonia* are more male or more female than others. In each case there are extremes and a graded series of intermediates: the subsequent chapters explain the details. The example of 'rhythmic melody' (*rhythmoeides melos*) seems confused. A combination of rhythm and melody will no doubt be 'intermediate' in character, but in Book I, 32.30–33.10, *rhythmoeideis chronoi*

Our account of *harmonia* must begin from the smallest elements, which are called 'notes' [*phthongoi*].[116] Notes, too, differ from one another in the way which we defined at the outset as a difference of character.[117] Some of them are hard and male, others relaxed and female. Others again lie between them, and are mixtures of both, but belong to a greater or lesser degree to one class or the other. It is from notes that the intervals take their qualities, and from notes that *systēmata* are filled out: the nature of these is readily understood from a consideration of the greatest of them, the octave, which we also call *harmonia*.[118] From here the difference between instruments is also understood; for the kind of tune suitable for the *kithara* is not appropriate for the *aulos*. If it were, the existing diversity of instruments would have been unnecessary, nor would they charm their hearers in different ways. But I shall speak of instruments a little later.[119]

Chapter 13 Since the character of melody, both in song and in instrumental pieces [*kōla*], is grasped through its similarity to the sounds produced by our
78 vocal organs, I have made a selection of the letters that are suitable [*harmottonta*] for use in vocalising melodies.[120] There are seven vowels, and we can see the distinctions mentioned previously both in the long ones and the short.[121] In general, the ones that extend the mouth vertically have a more dignified sound, appropriate to the male, and those that pull it out horizontally give out inferior sounds of a more female sort. To be specific, among the long vowels the sound of the omega [ō] is male, since it is rounded and concentrated, and that of the eta [ē] is female, since the breath is somehow dispersed and filtered in producing it. Among the short vowels, the omicron [ŏ] displays a male sound, since it compresses the vocal organ and snatches away the sound

('durations') are made intermediate between rhythmic and unrhythmic. If a reference to that distinction is intended here, *harmonia* must mean 'melodic sequence without rhythmic differentiation' (cf. 31.24–7), and 'rhythm' must mean 'rhythmically articulated melody', not merely 'rhythm without melody' as at 31.28. But Aristides is probably conflating two ideas.

[116] Following Schäfke I transfer this sentence from its position in the MSS at the end of the first paragraph of this chapter. On the note as the smallest element see 7.15–16. Aristides' discussion of harmonics in Book 1 also begins from notes and builds up to larger structures (chapters 6ff.).

[117] Compare Book 1, 10.13–15. Different characters are assigned there to notes with different positions in the tetrachord, but the characters are not explained. Aristides now develops the resources needed to describe them, and these are brought to bear on the tetrachordal positions at 79.26ff.

[118] For this 'ancient' terminology see 15.9–10.

[119] In chapters 16, 18–19. That melodies suitable for one instrument are unsuited to another is stated surprisingly rarely in our sources, but compare Plato *Rep.* 399c–e, ps.-Plut. *De Mus.* 1141c.

[120] As the sequel shows, Aristides has in mind a notational system of solmisation. A nearly identical system is set out at *Anon. Bell.* 77: see n. 131 below. There is some evidence, recently brought to light, that such a notation was in practical use as early as the sixth century B.C.: see A. Bélis, 'Un Nouveau Document musical', *Bulletin de Correspondance Hellénique*, 108 (1984), pp. 99–109. But the project of attaching the solmisation to a theory of the 'characters' of notes, and using it as a sort of 'ethical notation', seems to be Aristides' own (unless, as is just conceivable, it stems from Damon: see 80.25ff.).

[121] Compare 41.3ff. The seven vowels of Greek are alpha (a), epsilon (ĕ), eta (ē), iota (i), omicron (ŏ), upsilon (u or y), omega (ō).

before it is fully uttered, while the epsilon [ĕ], which has a way of making the mouth gape as it is pronounced, is female. Of those which may have either long or short duration,[122] the alpha [a] is best for melody, since the breadth of its sound makes it easy to prolong. The others are less so, because of their thinner sound.

Here again an intermediate character may also be detected. Thus alpha displays both affinity and contrast with eta: so far as it is adopted for a use opposite to that of the latter it is male, while so far as it produces a similar signification it is female. This is shown by the contrasting dialects, Doric and Ionic, whose differences correspond to the opposing characters of the two races. Doric avoids the female quality of the eta, and in practice generally converts it into a male alpha, while Ionic shrinks from the hardness of the alpha and settles on the eta.[123] The epsilon is female for the most part, as I said before, but because when it is prolonged it produces a sound similar to the diphthong written 'alpha-iota' ['ai'], it acquires through the alpha a very small trace of the male. Further, if you scrutinise the letters used in the articles and terminations in all their cases, you will see clearly that masculine nouns are
79 preceded and ended by masculine letters, feminine ones by feminine letters and sounds, and those that are neither by intermediate ones.[124]

Chapter 14 Four of the vowels, those that are readily prolonged by the singing voice, turned out to be useful for representing the notes. Since a consonant had to be added to them, to avoid the hiatus which would be produced by a sound consisting of vowels alone, we adopt tau [t], the most attractive of the consonants.[125] Tau is the letter used to pick out the article placed before a noun.[126] It is also the only one that sounds like the strings of an instrument,[127] and it is smoothest in its sound. It is not roughened by a quantity of breath, like the aspirates, and it does not leave the tongue

[122] These are the *dichrona* of 41.10–11: they are alpha, iota and upsilon, but the solmisation uses only the alpha.

[123] What Aristides says of the Ionic and Doric uses of eta and alpha is, with some qualifications, correct. The Dorian Greeks were commonly considered tougher and more 'manly' than those of Ionian race.

[124] 'Articles', 'terminations' and 'cases' are to be understood here in their grammatical senses. 'Preceded and ended', because the article precedes the noun and the grammatical termination ends it. What Aristides says about the masculine and the feminine is broadly true of the article and of first and second declension nouns. Here, all the masculine forms include an omicron or an omega, while feminine ones have in some instances an alpha, in others an eta (except in the genitive plural, which uses omega in all genders). 'Those that are neither' are presumably grammatical neuters, but these in fact use omicron or omega in all cases except the nominative and accusative plural, which end in alpha.

[125] The letter tau is used as the initial sound for each of the elements of the solmisation. Aristides' explanation of this fact indicates plainly that he construes it as a system in which notes can be sung, not merely written.

[126] The tau is the initial letter of every form of the (grammatical) article except the masculine and feminine nominatives.

[127] The reference is presumably to the initial, percussive sound of a string plucked with a plectrum. For the symbolism of the tau and a comparison of its shape with that of a plectrum see Book III, 130.9–15.

motionless, as does each of the other pure mutes: it does not emit a crude and vulgar hissing noise [*syrigmos*], as do the double consonants and the 'peculiar', nor is it thin and weak like the liquids.[128]

Given this starting point, the notes sung to the letter eta are liquid, and in general emotional and of a female character. Those sung to the omega are active and manly: and of the intermediates those sung to the alpha have more maleness, those sung to the epsilon more femaleness. The intervals constituted out of them are similar, and the *systēmata* bounded by them correspond to the intervals, those bounded by notes that are similar in their sounds being extreme in character, those bounded by dissimilar ones being intermediate. They may take their quality from the sequence [*agōgē*] of notes in the *systēma*:[129] alternatively, where the melodic composition is transilient, they are assimilated to the sounds that occur most frequently.[130]

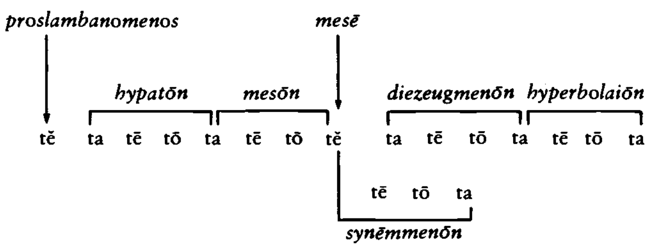
In the primary *systēma*, the tetrachord, the first note is sung to the letter epsilon, while the remainder follow the order of the vowels. Thus the second note is sung to alpha, the third to eta, and the last to omega.[131] For the most part these notes can be worked out directly from one another in an acceptable

[128] Compare chapter 11 above. The 'vulgarity' of the sigma (the 'peculiar': see 41.14, 75.27) is suggested by the practice of writing 'asigmatic' poems (ones not using the letter sigma) attributed particularly to Lasus of Hermione: see Athenaeus *Deipn.* 455c, cf. Pindar frag. 61.

[129] The reference of 'they' must apparently be to actual melodies, though these have not yet been mentioned.

[130] 'Transilient' is *hyperbatos*. On transilient *systēma* see 13.8–9, cf. 14.26–7; on transilient forms of melodic sequence see 16.20–1, 29.14–17.

[131] It would be natural to take the first part of this paragraph as meaning that the first (lowest) note of each tetrachord is sung to epsilon, the second to alpha, the third to eta, the fourth to omega. But this cannot be right. The latter part of the paragraph indicates that only *proslambanomenos* and *mesē* have the letter epsilon (that this applies to the former as well as the latter is confirmed, indirectly, at Book III, 121.3–15). In any case, where tetrachords are conjoined in the system, the highest note of the lower is the same as the lowest of the higher. The system set out at *Anon. Bell.* 77 assigns alpha to the lowest note of each tetrachord, eta to the second, omega to the third, except that *mesē* (above which there lies a disjunction) is given epsilon. The best MSS of *Anon. Bell.* give an omega to *proslambanomenos*, but this is anomalous, and most commentators emend to epsilon. Aristides almost certainly means to describe the same system. Possibly the text of the first two sentences has been corrupted, his intention being to say that the first note of each octave *systēma* is assigned epsilon, and that in each of the tetrachords lying above this note the symbols follow the order of the vowels; cf. also *Anon. Bell.* 9–10. If we accept that *proslambanomenos* must have an epsilon, the system recorded by the Anonymous (with a letter t attached to each vowel) will read as follows.



way.[132] Those which follow on from the three last mentioned are found by means of concords.[133] The note attached to epsilon, which stands alone at the beginning both of the first octave and of the second, gives us *mesē* on the basis of the octave-unison it makes with *proslambanomenos*.[134] We shall explain later why this is so.[135]

Now *systēmata* derive their qualities either from the notes between which they lie, or from those occurring most often, or from those which are predominant in both of these respects, or from two different groups, each having one of these features, and exerting their influence jointly as a mixture; and from *systēmata* arise the *harmoniai*.[136] If you use *harmoniai* in the ways we have explained, applying them to each soul on the basis either of their similarity or of their opposition to it,[137] you will disclose the bad character that lurks within it, and cure it, and replace it with a better. If the underlying disposition is coarse and stubborn, it is through what is intermediate that you will generate persuasion and bring it into the opposite state; while if it is fine and good you will use what is similar to it, and thereby augment it to the right proportion.[138] Further, if the disposition is plainly apparent, you will need just the one appropriate style of melody.[139] If it is obscure and hard to diagnose, you should begin by applying whatever melody comes to hand. If this is effective in influencing the soul,[140] you should persist with it, but if the patient remains

[132] The text is in doubt: I follow the readings of Meibom and Winnington-Ingram. Aristides seems to mean that the identities of notes within a given tetrachord are implied directly by their order, no reference being required to a relation with notes elsewhere in the system: contrast the next sentence.

[133] 'Those which follow on' are apparently those in tetrachords other than the one from which we have started. If we can pitch the notes in the original tetrachord and assign each its vowel, then we cân attach the right vowel to a note lying above or below this tetrachord, by discovering to which of our original notes it stands in which concordant relation. (For example, if our original tetrachord is *hypatōn*, a note standing a fourth above its 'tē'must be the 'tē' of the tetrachord *mesōn*.)

[134] The text is certainly corrupt, and this seems to be the best sense that can be wrung from it.

[135] The reference is probably to the astrological speculations of Book III ch. 21.

[136] The sense of *harmonia* in this passage is not altogether clear, but Aristides evidently intends a fairly sharp distinction between it and *systēma*. It is unlikely that *systēmata* are merely small scalar groupings (e.g., tetrachords), *harmoniai* larger ones compounded from them (octaves, double octaves, etc.). Possibly the *systēma* is the framework of notes and intervals (e.g., the GPS taken in one genus), the *harmonia* the particular scheme of tuning that determines a species of the octave or double octave (cf. the usage of Book I, 15.18). But perhaps both the abstract overall framework and the species are determined by the *systēma*, as the main part of the present sentence strongly suggests, and *harmonia* is a conception more closely allied to practice, indicating the tuning-system required to produce those notes that the melody to be performed will actually use, and no others. These will be selected from those in the *systēma*, but need not include them all. This interpretation is supported by the references to melody at 80.16ff., and more strongly by the following paragraph, especially the remark about composition at 81.3–6: see n. 145 below.

[137] Compare 73.25–7.

[138] Compare 68.22–69.1.

[139] *Tropou kai melous*, lit. 'style and melody', but probably a hendiadys of a common type. *Tropos* might have the semi-technical sense of, for example, 30.2, 7, but may be intended quite vaguely.

[140] *Psychagōgoumenou*: see n. 81 above.

unaltered you should introduce a modulation;[141] for it is likely that someone who is resistant to one sort of melody will be attracted by its opposite.

The *harmoniai*, as I was saying, resemble either the intervals which are commonest in them, or the notes that bound them: and the notes in turn resemble the movements and emotions of the soul.[142] The fact that it is through similarity that the notes – even the notes of a continuous melody – both instil a character previously absent, in children and in older people too, and draw out a character that lay hidden within, was demonstrated by the followers of Damon.[143] Certainly, in the *harmoniai* which he handed down, we can see that
81 of the moveable notes it is sometimes the female and sometimes the male that are in the majority, or else are used less or not at all. The reason is clearly that the usefulness of a *harmonia* depends on the character of each individual soul.[144] Hence the most important part of melodic composition is that known as 'distribution' [*petteia*], which consists in the selection of the notes most appropriate on each occasion.[145]

Of the *systēmata*, the lower are suited by their nature to the male, and in education to the training of character.[146] They are roughened by the copious and violent expulsion of breath from deep within, and the impact of a larger body of air, due to the width of the channels, makes them give an impression of aggressiveness and weight. The high *systēmata* are suited to the female, and

[141] *Metabolē*, probably in a technical or semi-technical sense: see 22.11–26, and cf. 130.1–3, 25ff.

[142] See chapter 8 above, and compare Book III chapters 16ff. The character of intervals depends in turn on that of their bounding notes, 79.20.

[143] On these two functions of music, rather differently presented, see 68.22ff. On Damon see nn. 1 and 71 above, with references given there, and for attempts to reconstruct Damon's theories see particularly Lasserre (1954), pp. 53–95, Anderson (1966), pp. 38–42. Much has been made of this sentence and the next, but few conclusions can confidently be drawn. If Aristides is reliable, we can infer (a) that Damonians thought music a powerful educational force, and this is well known from elsewhere; (b) that they distinguished the two educational functions mentioned here; (c) that they associated the influence of music on character with similar characters attributed to notes; (d) that something describable as 'Damon's *harmoniai*' were available to Aristides; and (e) that these presented 'gapped' structures, in which one or other of the two moveable notes between tetrachordal boundaries was omitted. What these *harmoniai* were we cannot say (though the description suggests tuning-structures rather than actual melodies): the grounds for identifying them with the scales of Book I, 18.5ff., are very slim. We do not know what notation, if any, was used. The solmisation system is the only one that can be traced back with any confidence to a time before the fourth century B.C. (see n. 120 above), but compare also the method referred to at 7 Aristox. *El. Harm.* 39.4ff. It is also at best uncertain that Aristides intends to attribute the male–female classification to a Damonian source.

[144] Compare 67.9–16.

[145] On *petteia* see 29.8 and 18–21, with the relevant note. The reference to composition indicates that the structures considered here are not simply those described by scientific harmonics, but are those abstracted from individual melodies or classes of melody. Their components are then treated (by Aristides, not necessarily by Damon) as 'selections' from the sets of notes that the theoretical harmonic systems made available: cf. n. 136 above.

[146] As the sequel shows, the *systēmata* in question are the *tonoi* or *tropoi* discussed in Book I chs. 10–11, and see notes. Aristides seems to treat them here straightforwardly as pitch-keys, as at 23.1ff.

the narrowness of the channels, when the impact on the air occurs only at the surface and around the lips, makes them mournful and shrill.[147] Those that proceed by successive notes are even and steady, while those proceeding by leaps are harsher and chaotically agitated, and impose a violent strain on the mind by their sudden change [*metabolē*, perhaps 'modulation'] from one region to its opposite.[148] Of the *tropoi*, then, the Dorian is the lowest, and suits the character of the male. (It was the Dorian, and its starting point, that we made the initial boundary of the vocal register.[149] It lies in the middle of the lowest octave, and forms the upper boundary of the *hypatoeidēs* tetrachord and the lower boundary of the *mesoeidēs* fifth in the Hypodorian *harmonia*.[150]) The one that lies a tone higher is intermediate in character, and that which is higher by the greatest of the incomposite intervals, the ditone, is more female.[151] Those that lie in between them are to be thought of as having a tendency towards both sides.[152] In the case of instruments, whose range of notes is wider,[153] the *tropoi* that extend below the Dorian *proslambanomenos* are deep and male, those lying above it up as far as *diatonos* are intermediate, and those added above them are highest, and more female.[154]

82 That completes our discussion of melodies, of which those that are decorous and male in character are needed for education, while those that are otherwise are required for influencing the soul in various ways.[155]

Chapter 15 Of the rhythms, those that give an initial calm to the mind by beginning from the thesis are more peaceful: those that transmit the beat

147 These accounts of the physiological causation of high and low sounds are based on Peripatetic sources: cf. 3.17 Aristotle *De Gen. An.* 786b 7ff., 4.4–4.7, 4.13 ps.-Ar. *Probs* XI.13–16, 34, 6 Theophrastus ap. Porph. *Comm.* 63.1ff.

148 The distinction between *systēmata* proceeding 'by successive notes' and ones proceeding 'by leaps' (transilience) is drawn in the same terms at 14.26–7: cf. 16.19–21. In the present passage *hyperbatōn*, 'transilient (notes)', represented in the phrase 'by leaps', is Winnington-Ingram's emendation for the MSS *hyperbolōn*, 'extremes'. In line 14 the MSS have *houtō prochōrountōn*, which is certainly corrupt; I follow Jan in reading *prochōrounta*, and omit *houtō*.

149 On this sentence and the remainder of the paragraph see 21.13ff.

150 For the forms *hypatoeides* etc. see 9.21ff., 28.12ff., 30.4ff. Here the *hypatoeides* tetrachord cannot be simply the tetrachord *hypatōn*, since the note being identified is Dorian *proslambanomenos* (see 21.15–16), which lies a tone below the upper limit of the Hypodorian tetrachord *hypatōn*. The '*hypatoeidēs* tetrachord' must be simply the lowest assemblage of four notes, from Hypodorian *proslambanomenos* to *lichanos* (*diatonos*) *hypatōn*: the *mesoeidēs* fifth is then that taken downwards from *mesē*. On Aristides' classification of fifths see 15.4–5 with n. 97.

151 These are the 'higher' Phrygian and Lydian of 20.12–14: cf. 23.1–4.

152 Between Dorian and higher Phrygian is lower Phrygian or 'Iastian'; between higher Phrygian and higher Lydian is lower Lydian or 'Aeolian' (20.12–14). Each is separated from its neighbour by a semitone.

153 Compare 23.4–6.

154 The *tropoi* are located by reference to their *proslambanomenoi*, as at 21.7ff., 23.6ff. *Diatonos* here is diatonic *lichanos hypatōn*. The low instrumental *tropoi* are then those from Hypodorian to Dorian, the intermediate those from lower Phrygian (Iastian) to lower Mixolydian (Hyperdorian), and the high are higher Mixolydian (Hyperiastian) and Hypermixolydian (Hyperphrygian). The later system mentioned at 21.1–4 would add Hyperaeolian and Hyperlydian at the top.

155 'Influencing the soul', *psychagōgia*, see n. 81 above. Reasons why non-educational music might nevertheless have therapeutic and diagnostic uses are suggested by such passages as 80.10ff.

[*krousis*] to the voice by beginning from the arsis are restless.[156] Those whose periods contain complete feet are more elegant, while those including empty durations are less so: those in which the empty durations are short are simpler and slighter, and those in which they are long are more impressive.[157] Those organised in equal ratio are more graceful because of their evenness: those organised in epimoric ratio are agitated, for the opposite reason. Those in duple ratio are intermediate, having a share of unevenness because of their inequality, and of evenness because of the purity of their numbers and the perfection of their ratio.[158]

Of the rhythms in equal ratio, those composed only of short syllables are most swift and more passionate [*thermoteroi*, lit. 'hotter'], those composed only of long syllables are slower and calm, and those that are a mixture have the qualities of both.[159] If the feet are constituted out of very long durations, the calming effect on the mind is greater. Hence we can see the value of short durations in war-dances,[160] of mixtures in intermediate forms of dance, and of the longest in sacred hymns. In using these extended durations people displayed their concern for things sacred, and their attachment to them,[161] and by the equality and length of the durations they also brought their minds into a state of good order, believing that this was what constituted the health of the soul. That is why, in the movements of the pulse too, the healthiest people are those in whom contraction and dilation answer to one another through movements of these kinds.[162]

Rhythms that we understand as hemiolic in ratio are more ecstatic, as I said.
83 The most agitated of them is the *epibatos*: it disturbs the soul with its double thesis, and lifts the mind upwards with its long arsis.[163]

[156] Compare 76.22ff. The word *krousis* commonly means the striking or plucking of the string of an instrument; on the importance of instrumental accompaniment in emphasising rhythm see *GMW* vol. 1, p. 204. There may be an implicit reference to the *kroupezai*, wooden shoes or attachments to shoes, used for marking the tempo, to keep performers in time (see, for example, Pollux *Onomast.* VII.87). The discussion of rhythms that follows should be read in conjunction with Book I chs. 14–17.

[157] On the *periodos* see 34.24–35.2, cf. 36.8ff. But the sense here may be more general, roughly equivalent to 'line' or 'metron'. On 'empty durations' see 38.26–39.2. The clause 'while those including empty durations are less so' is not in the MSS: a lacuna was noted by Meibom. I translate on the basis of Boekh's supplement, emended by Winnington-Ingram's note.

[158] On these ratios of rhythms see 33.29ff., and the detailed analyses in the following chapters. Epimoric ratios are those of the form n + 1 : n. Of these the ratio 3:2 is treated as acceptable, 4:3 with some qualifications: see 33.30, 34.13–15.

[159] On the rhythms in equal ratio see Book I ch. 15. Those including only short syllables are the pyrrhics or prokeleumatics (35.5–8, 22–3, cf. 47.7–8); those with only longs are spondees (35.11–13).

[160] *Pyrrhichai*: cf. nn. 48 and 159 above.

[161] I omit the word *mian* in line 23, which is agreed to be corrupt.

[162] That is, in slow, equal durations. On the relation of rhythm to the pulse compare 31.19–20, and see 83.21–5, 89.10–22; this latter passage cites medical authority for the thesis that the pulse is a movement of the soul, not of the body. The pulse had been studied in connection with the rhythmic ratios by medical scientists of the third century B.C.: see, for example, Lloyd (1973), pp. 79–80.

[163] 'Hemiolic', i.e., 3:2. These rhythms are the paions, discussed at 37.5–12. The *paiōn epibatos* is described there: cf. ps.-Plut. *De Mus.* 1143b.

Of the rhythms in duple relation,[164] simple trochees and iambi express swiftness, and are passionate [*thermoi*] and suited to dancing, while the orthians and semantics[165] have a dignifying influence, because they contain so many of the longest sounds. That, then, is what the simple rhythms are like.

Compound rhythms are more emotional,[166] because for the most part the rhythms from which they are constituted are unequal; and the impression they give is very tempestuous, because the number from which they are constructed does not keep the same order of its parts in each position,[167] but sometimes begins from a long and ends in a short, sometimes the reverse, or sometimes uses the thesis to begin a period and sometimes begins the other way round. The most emotional are those that consist of more than two rhythms, since their unevenness is greater. Hence by imposing a diversity of movements on the body they lead the soul into considerable confusion.[168] Those that remain within a single genus are less disturbing, while those that modulate to others pull the soul violently in opposite directions, forcing it through their multiplicity to follow and assimilate itself to every variation.[169] Thus it is also true that those movements of the arteries which keep the same form, though varying slightly in their durations, are disordered but not dangerous, while those that alter too much in their durations, or go so far as to change from one genus to another, are frightening and deadly.[170]

Again, if we consider styles of walking, we shall find that people whose steps are of good length and equal, in the manner of the spondee, are stable and manly in character: those whose steps are long but unequal, in the manner of trochees or paions, are excessively passionate: those whose steps are equal but too short, in the manner of the pyrrhic, are spineless and lack nobility: while those whose steps are short and unequal, and approach rhythmical irrationality,
84 are utterly dissipated. As to those who employ all the gaits in no particular order, you will realise that their minds are unstable and erratic.[171]

Again, rhythms whose tempo [*agōgē*] is swifter are passionate and active, those in which it is slow and held back are relaxed and peaceful. 'Abbreviated' and 'running' rhythms are vigorous and compressed and inspire us to action, while 'expanded' rhythms, in which several sounds are coalesced, are supine

164 'Relation', *schesis*, is here a mere variant for 'ratio', *logos*. These rhythms, those of iambic genus, are discussed in Book I ch. 16.

165 Described at 36.3–6, cf. 36.29–37.2.

166 'Compound' (*synthetoi*) rhythms are defined at 34.19ff. as putting together more rhythmic genera than one. Such rhythms are discussed in Book I ch. 17. In chapter 16 the term is also used of rhythms compounded out of several different simple feet within just one genus.

167 This use of the term 'number' may hint at the alternative form of rhythmic analysis described in Book I ch. 18.

168 If the effect of rhythm on the soul is mediated by movements of the body, we may have some explanation of the apparent anomaly at 55.5–6, remarked on in n. 6 above.

169 On rhythmic modulation see 40.1–7.

170 On the reference to the pulse see n. 162 above; cf. also 106.8ff.

171 On rhythmical irrationality see 33.21–2, 34.15–18, 37.24–38.2, cf. also 32.30–33.11. On character and styles of walking compare the Aristotelian *Physiognomica* 813a3–9.

and flabby. Intermediate rhythms are compounded out of both, and are moderate [*symmetroi*] in their constitution.[172]

Chapter 16 Concerning the art of delivery the following must be said.[173] Of the bodily movements in which delivery consists, those which imitate ideas, diction, melodies and rhythms of a reverent and male character, and which incite us to manliness, should be seen and copied by everyone. Those whose nature is the opposite may be watched and imitated by the common people – but not all of them, and not by everyone. At any rate, people of noble nature and sound character should refrain from imitating and watching them altogether.[174]

I said that I would also discuss instruments, whose use can easily be understood from their name. We use the word 'instrument' either for that which is the only means of doing something (as, for instance, we see through our eyes), or for that through which something can be done best (as we cut twigs with a pruning-knife); and it is in the second of these senses that musical 'instruments' are required.[175] Though songs and melodies can be performed with the voice alone, nevertheless instruments also have their use. Just as no one kind of voice or *harmonia* pleases every listener, but one can give delight to some people, another to others, so it is with instruments. For whatever notes
85 an individual's character resembles, he will love and admire the instruments that are suited to them.[176]

Among the wind instruments, we may say, the *salpinx* is male, because of its vehemence.[177] The Phrygian *aulos* is female, since it has a mournful and dirge-like sound. Of the intermediate kinds the Pythian *aulos* is more male, because of its low pitch: the choric *aulos* is more female, because of its facility in the high range.[178] Of the stringed [*katateinomenois*, lit. 'stretched'] instruments

[172] Rhythmic *agōgē* is defined at 39.26–30. On 'abbreviated' (*strongyloi*) and 'expanded' (*peripleōi*) durations see 33.8–10. The word 'running', *epitrochos*, is used in the derivation of the term *trochaios* at 36.28–9.

[173] On delivery (*hypokrisis*) cf. 73.28–74.6.

[174] These minor concessions to the 'common people' are un-Platonic, and probably stem from Aristotle *Pol.* 1342a.

[175] The Greek *organon*, like its English counterpart, means in the first place 'instrument' in the general sense 'tool'; its musical usage is a specialisation of this meaning. Aristides' analysis echoes Plato's account of 'function', *ergon*, at *Rep.* 352e–353a.

[176] This sentence seems to restrict the ethos of an instrument to that implicit in the notes it can produce (cf. 77.25–8). The distinctions made in the rest of this chapter, and in chapters 18–19, are a good deal wider in scope. On differences in individual tastes cf. 67.15ff.

[177] On the *salpinx* see n. 49 above.

[178] On the Phrygian *aulos* see Athenaeus *Deipn.* 176f, 185a, Pollux IV.74, 9.7 Aelianus ap. Porph. *Comm.* 34.11–16; cf. Eurip. *Bacchae* 120–34, *Iph. Aul.* 578. Note that Aristides does not link its femaleness with high pitch; Athenaeus and Aelianus say that its pitch was deep. One of its pipes was curved, and ended in a bell. The Pythian *aulos*, also called *teleios*, 'complete', was that used in professional solo auletic, specifically in the *Pythikos nomos* (Pollux IV.84). Pollux says (IV.81) that it was of medium range, and was also used to accompany paeans. What Aristides means by the 'choric' *aulos* is less clear. In Pollux (IV.81) it is the *aulos* that accompanies any chorus, especially the dithyramb, but there was not just one type. Our sources commonly classify such *auloi* by the range of voice they were designed to accompany. Thus Aristoxenus distinguishes *parthenioi*,

one can see that the *lyra* corresponds to the male, because of its extreme deepness and roughness,[179] the *sambykē* to the female, since it lacks nobility and incites people to abandonment because of its very high pitch, caused by the shortness of its strings.[180] Among the intermediate types, the kind of *kithara* with many strings is more female, while the kind of *kithara* that does not differ greatly [*apaidon*] from the *lyra* is more male.[181] If there are other instruments to be found, falling between these groups, their nature will not be difficult to discern, now that we understand the general classes of character under which we are to classify the individual types. Thus each of the *harmoniai* and each rhythm, according to its own special nature, is appropriate to a specific kind of instrument, and would not produce its effect so well on an unsuitable one.

The total activity of music then involves an appropriate idea [*ennoia*], suitable diction, and equally suitable *systēma*, and the corresponding *harmonia* of notes,[182] quality of rhythm and use of instruments. When extreme measures will do no harm, all the resources of the craft of music should be applied complete. Sometimes, however, we should use a mixture, incorporating something of a different quality, in case without being aware of it we drive the patient's character into the opposite condition, with our extreme measures. A wise doctor does not always give the strongest drugs but will respect the weakness of the patient's constitution. The mixture should not, however, be
86 made up of pure opposites (for such a mixture is unsuitable and its elements are hostile to one another), but of intermediates blended harmoniously together [*synharmottomenōn*] with extremes. Thus we should link with a male *systēma* not a female rhythm, but an intermediate one, or with a more female rhythm an intermediate kind of *systēma* and instrument rather than the opposite kind alone, and so on, making the mixture out of the male and the intermediate or the female and the intermediate, either with one or two elements of one kind and several of the other, or with several of each.

paidikoi, *kitharistērioi*, *teleioi*, *hyperteleioi* (at Athenaeus *Deipn.* 634e–f), *parthenioi* ('maiden-pipes') being the highest; cf. Aristotle *Hist. An.* 581b, 7 Aristox. *El. Harm.* 20.32–4.

[179] If Aristides means the familiar tortoiseshell *lyra*, of which this term is most often used, it was commonly quite small and its pitch will not have been especially deep. But the terminology was flexible, and he may have intended its larger relative, sometimes called *barbitos*; see *GMW* vol. 1, pp. 4 and 14.

[180] On the *sambykē* see especially Athenaeus *Deipn.* 182e–f, 633f–634b, 635a. It is said to be high pitched at 633f. It has usually been identified as a triangular harp (see 9.7 Aelianus ap. Porph. *Comm.* 34.29), but Landels (1966) argues that it was an instrument of the lute family.

[181] The latter kind of *kithara* is presumably the 'concert *kithara*', a large, solid and resonant instrument, elaborately constructed of wood. It was the principal stringed instrument of public performance. Its strings may have been higher in pitch, length for length, than those of the *lyra* and *barbitos*, since its strong structure could support greater tension: cf. Athenaeus *Deipn.* 637f. The apparent reference to a *kithara* with many strings is odd. It can hardly mean a concert *kithara* to which extra strings had been added (as by composers of the later fifth century B.C.), since that is still similar in form to the *lyra*. But *kithara* sometimes refers to stringed instruments quite generally, and Aristides probably means to indicate the various kinds of harp that were often designated as *polyphthonga*, 'many-noted': see, for example, Plato *Rep.* 399c–d.

[182] On the sense of *harmonia* here see n. 136 above.

Chapter 17 Are not those who have heard all these things filled more than ever with a desire to discover why it is so, and to find out what it is that makes the soul so easily captivated by the melody of instruments?[183] The account I shall give is an ancient one, but its exponents were wise men, and it is to be trusted. Even if other considerations might lead us to doubt it, its truth is indisputably attested by experience; for the fact that the soul is naturally stirred by the music of instruments is one that everybody knows. Given this starting point, if another and better explanation for it can be found, then the one I am about to set out should be rejected. But if that is impossible, surely we must put our faith in the conclusions which follow from the evident facts.

The account states that the soul is a *harmonia*,[184] and that *harmonia* exists through numbers: that musical *harmonia* is constituted out of the same proportionalities: and that when one group of things is set in motion, things with similar properties are moved with them. This is a doctrine we shall investigate thoroughly later.[185] But there is a second doctrine, which says something like this. There is an analogy between the matter and nature of instruments, and the former constitution of the soul, the constitution by means of which it made contact with its present body. For while the soul lives in the purer region of the universe and is unmixed with bodies, it is unblemished and undefiled, and goes round in company with the lord of this universe.[186] But when, as a result of its declination towards things in this world, it acquires
87 impressions derived from things in the earthly region, it gradually forgets the beauties of the other place, and sinks downwards. And the further it travels from the things above, the more it addresses itself to things in this world, and is filled with greater ignorance, and turns its path into the darkness of the body. The diminution of its former excellence makes it unable any longer to range in its mind over the whole universe, while its forgetfulness of the beauties of the other place and its passion for earthly things carry it down towards the things that are most solid and bound up with matter.[187]

[183] Aristides now launches into the third phase of his account of the soul (cf. chapters 2 and 8), this time specifically designed to explain its affinities with instruments.

[184] An 'attunement' or 'fitting together'. The thesis is indeed old: see 1.15–1.18. Though it is often attributed to Pythagoreans, it is incompatible, as Plato notes, with the doctrine of immortality that they espoused. Plato himself rejects the theory, when stated in terms that identify the soul with a balance and proportion between bodily elements, but he uses it as the basis of his account of the proportionalities inherent in an independent, immaterial soul: see especially 2.3 *Tim.* 35b ff., on which Aristides draws heavily in Book III (particularly chapter 24). Here, however, he is not thinking of the soul's constitution when separate from the body, but of the partly physical structure it acquires during its descent to the earth. This is the 'former' constitution of the soul mentioned later in this paragraph: see also 88.4–5. Plainly the destructibility of that *harmonia* need not imperil the soul's own immortality.

[185] Aristides is thinking of sympathetic vibration, which is discussed in the next chapter. The relevant numbers and proportionalities are elaborately deployed in Book III.

[186] In this paragraph Aristides draws on Plato *Phaedrus* 246a ff., here particularly 247a–248a. The details of the paragraphs that follow are not Plato's, and may be taken from Neoplatonist sources. The term *neusis*, 'declination', in the next sentence is itself almost a technical term in Neoplatonism: see, for example, Plotinus *Enn.* I.6.5.

[187] Compare *Phaedrus* 246b–c, 249e–250a.

Thus in its desire for a body, they say, it takes and draws to itself from each of the upper regions some elements of its bodily constitution. As it passes through the circles of the aether it gathers everything that is radiant and suitable for warming the body and giving it its natural cohesion;[188] and from these circles and the lines constructed by the interplay of their movements it weaves bonds for itself, like a net, as it goes on its disorderly course.[189] As it passes through the regions around the moon, which are vaporous and share the resistant character of breath, its natural movement makes it give out a loud and vehement whirring sound, and it is filled from below with the breath of which the region is composed: and it stretches the surfaces and lines of the circles, dragged down as it is by the ponderous masses of breath, while still through its own nature clinging to the things of the other world; and thus it loses its spherical shape, and changes into that of a man.[190] Its surfaces, which are constituted out of the radiant matter of the aether, it alters into the form of membranes, while the lines, which it brought down from the region of divine fire[191] and are tinted with the redness of flame, it transforms into the
88 appearance of sinews. Finally, from these nearer regions it takes moist breath.

This, then, is to be its first natural body, welded together out of surfaces that have taken the form of membranes, lines that have taken the form of sinews, and breath. They say that this is the 'root' of the body, and they call it a *harmonia*.[192] By it, they say, this oyster-like organ of ours is nourished and held together.[193] The poet himself ascribes to it a constitution of this kind, when he says

[188] On the *aithēr* compare the Platonic *Epinomis* 981c, Aristotle *De Caelo* 270b. It is conceived by Aristides as a distinct fifth element, the bright substance of the divine region; cf. 90.10, 92.10, 111.30–112.3.

[189] The text is in some doubt, but some sense can be made of the MSS. The aetherial region contains substance arranged in moving circles, travelling at various angles to one another, and hence, their movements 'interplay' with one another. The soul travels a 'disorderly' course among them, gradually weaving a web or net to clothe itself from their material. It is like an irregularly moving shuttle, but one that gathers material rather than dispensing it. The interacting motions of the aetherial circles may be derived by Aristides from an acquaintance with attempts to explain the movements of the stars, but the details, such as they are, are thoroughly idiosyncratic.

[190] Here the soul-net behaves rather like a parachute. The distortion of its original shape may be an echo of Plato *Tim.* 43d ff., though the intention is quite different.

[191] *To empyrion*: the word has the literal sense 'fiery', but its usual form is *empyros*. *Empyrios* is found only in late writers (e.g., Proclus, Iamblichus), always in a theological context. The contrast between the dry and fiery constitution of the membranes and sinews, and the moistness of breath (see the next sentence) is related to differences in the characters of instruments in chapters 18 and 19.

[192] See n. 184 above. The word *rhiza*, 'root', is used by poets and cosmologists of a thing's source or foundation in any of several senses. What has been constructed is not the whole body, but its structural basis, sinews, membranes and tubular vessels, the parts which provide its cohesion and transmit to it the soul's vitality; cf. especially 89.10–22. Aristides' view that these parts are more closely associated with the soul than is the rest of the body reflects the ancient idea that it is the presence of soul that makes the body a unity. These are the means by which the soul binds the body together, cf. Anaximenes frag. 2 (DK 13 B2), Aristotle *De Anima* 411b 7ff.

[193] The 'oyster-like organ' is the body as a whole (the word *ostreon* refers primarily to the creature's shell); cf. 66.20 with n. 73. The same image is used at Plato *Phaedrus* 250c.

'The sinews hold flesh and bones no longer' (*Od.* XI.219).

Elsewhere he calls the soul 'Aphrodite', and its bodily nature, because of its instantiation in blood, he calls 'Ares'; and he says that it has been bound by the divine craftsman,[194] whom he calls 'Hephaestus', in fetters of just this kind. This is what he says:

'Then he hung the fetters in a circle all around the bed-posts, and many were hung down from above, from the roof-beam, like fine spiders' webs' (*Od.* VIII.278–80). We shall not go far wrong if we equate the bed-posts [*hermines*], which bear the same name as Hermes, god of eloquence [*logios*], with the ratios [*logoi*] and proportions [*analogiai*] through which the soul has been fastened to the body;[195] the spiders' webs with the surfaces and shapes by which the human form is bounded; and the house,[196] naturally enough, with the dwelling-place crafted for the soul. That his tale concerns the soul is made clear by what follows. When he recounts how they part and go each to his own place, he sends Ares to the natural home of irrationality, the land of the barbarous Paeonians,[197] and says no more about him; while Aphrodite he sends to the original place of her birth and of her happy life, Cyprus,

89 'Where is her sacred ground and her fragrant altar' (*Od.* VIII.363),

and he purifies and sanctifies her, now that she is withdrawing from ignoble things:

'There the Graces washed her and anointed her with oil' (*Od.* VIII.364).[198]

The wise Heraclitus also speaks in the same vein somewhere, not out of tune [*apaidōn*] with these doctrines. It is the happy experience of the soul in the aether that he is describing when he says 'A soul parched dry is wisest' (DK 22 B118): and when he says 'It is death for souls to become wet' (DK 22 B77, cf. 36) he is referring to a soul saturated by the rains and exhalations of the lower air.

Medical men also give evidence of this. They say that the most fundamental parts of the body, which are proportional to the natural masses and do most to hold the body together, the parts in which, so they tell us, even a slight alteration puts the creature at risk, are membranes and tubular vessels: and these are nothing other than sinewy films, like spiders' webs shaped into pipes, enclosing breath inside them. Through them the soul, not the body, is set in motion, stretching out as they expand, subsiding as they contract. They demonstrate this also from the beats of the pulse, whose orderly movement they identify with the creature's health, and whose disordered and chaotic movement they diagnose as a warning of death: and they maintain that a

194 *Demiourgos*: Aristides has in mind the Craftsman God of Plato's *Timaeus*.

195 The word *hermines* has no genuine etymological connection with Hermes. *Logios*, 'eloquent', is one of his regular titles. Aristides' engagingly fanciful interpretation is facilitated by the multiplicity of senses attached to words based on the noun *logos*. This Homeric passage was a favourite among allegorising commentators.

196 *Melathron*, but in Homer's line the word designates a roof-beam.

197 Homer says merely that Ares went to Thrace, *Od.* VIII.361.

198 With this whole concoction compare Aristides' previous references to the passage at 70.20–5, 74.25–8.

complete cessation of its movement indicates that the soul has finally departed.[199]

Chapter 18 It is therefore not surprising that the soul, having acquired through its nature a body similar to things that set instruments in motion, which are sinews and breath, is moved with them when they are moved; that when a breath gives out a melodic and rhythmic sound it is affected with it, in the breath which it itself contains; and that when a sinew is harmoniously [*enharmoniōs*] plucked it responds with a similar sound and tension in its own sinews. It is still less surprising when we notice that a similar phenomenon occurs in the *kithara*. If on one of two strings tuned in unison you place a light little piece of straw, and then strike the other, which is strung at some distance from it, you will see that the one which carries the straw is quite visibly set in motion with it. The divine craftsmanship, it appears, has a marvellous skill to bring things about and to act, even through inanimate objects. How much greater must be the power of similarity to cause activity in things that are moved by soul?[200]

Instruments that are fitted [or 'attuned', *hērmosmena*] with sinews resemble the region of the universe and the part of the soul which are made of aether, dry and simple; for these instruments are not readily affected or changed, and are hostile to moisture, since damp air puts them out of their proper condition. Wind instruments resemble the region of the universe and the part of the soul that are made of breath, moist and changeable: they make the hearing excessively female; they are adept at sudden modulation; and their constitution and powers depend on moisture.[201] Thus instruments resembling better things are better, the others less good. This, so people say, is what is shown by the myth in which Apollo's tunes and instruments were preferred to those of Marsyas. The Phrygian Marsyas, who was hung up like a sack over the river at Celaenae, is the region of the lower air, shadowy and full of breath, which hangs suspended below the aether and above the water; while Apollo and his instruments are the pure, aetherial essence and its master.[202]

[199] For other references to the pulse see n. 162 above: the idea that the pulsating arteries contain breath, not blood, (or a mixture of breath and blood) is a common one in ancient medical writings (see Lloyd (1973), pp. 81–2). Aristides is not identifying the soul with breath, though the movements of breath in the arteries are the soul's movements; breath is part of the soul's 'first natural body', along with membranes and sinews (87.28–88.4). Breath played an essential role in the Stoic theory of cosmic sympathy (see next note).

[200] The phenomenon of sympathetic vibration had often been studied: cf. 4.20, 4.26 ps.-Ar. *Probs* XIX. 24 and 42. In Aristides' hands it is linked with wider conceptions of affinity and sympathy between parts and elements of the universe, developed in Book III. The Stoic notion of cosmic sympathy appears, for example, at *SVF* vol. 2, 473, 475, 532, 534, 546, 912, 1,013. Aristides' conception of it may be derived from Posidonius.

[201] Aristides here adapts Plato's criticisms of the *aulos* (*Rep.* 399d) to the framework of his own theory: cf. Aristotle *Pol.* 1341a–b, 1342b.

[202] For the story of the contest between Apollo with his *kithara* and Marsyas with the *auloi* see Diod. Sic. III.59.2–5, cf. Herodotus VII.26, Xenophon *Anab.* I.2.8, Plato *Rep.* 399c, *Symp.* 215b–c. It is related to the tale of Marsyas and Athena (see 91.17ff. below), Athenaeus *Deipn.* 616e ff., Plut. *De Cohib. Irae* 456b–d, cf. Aristotle *Pol.* 1341b. On the generalised 'conflict' between *aulos* and *kithara* see, for example, ps.-Plut. *De Mus.* 1135f ff.

Chapter 19 In what they say about the use of instruments, the ancients have uncovered for us the following points. The kind of melody that is damaging and to be avoided, because it leads on to vice and ruin, they ascribed to the Sirens, mortal women in the shape of beasts: they are conquered by the Muses, and the wise Odysseus flees from them precipitately (*Od.* XII.154–200). There are two useful forms of music-making, one valuable for the benefit it brings to the best of men, the other for the harmless relaxation it gives to the common
91 run of mankind, and to anyone there may be still less exalted than they.[203] The educational kind of music for the *kithara* they attributed to Apollo, since it is of the male sort: the kind which must pursue pleasure, because it aims at the majority, they assigned to a female divinity, Polhymnia, one of the Muses.[204] Of the music for the *lyra* they ascribed to Hermes, as being appropriate to men, the kind that is useful for education: the kind conducive to relaxation they linked with Erato, since it often soothes the female and appetitive part of the soul.[205] Again, in the case of *auloi*, the kind of melody that panders to the pleasure-seeking majority of men and the pleasure-seeking part of the soul they assigned to Euterpe, who, as her name suggests, urges us to pursue pleasure as well as goodness.[206] The kind which can give benefit, though rarely and by means of much knowledge and self-control, and yet has not altogether abandoned its female nature, they ascribed not to a male divinity but to Athena, who is female in sex but self-controlled and warlike in character. However, to show that the benefit it gives does not last, and to warn the wise that they should on the whole avoid the indulgence of auletic music, they say that the goddess threw away the *auloi*, on the grounds that the pleasure they gave was unsuitable for those who desire wisdom, though of value to those who are wearied and worn out by the exertions of constant physical labour.[207] This is the guise in which they introduced Marsyas, who was later punished because he gave his own music a dignity beyond its real worth. His instruments were as inferior to those of Apollo as are ignorant craftsmen to the wise, or Marsyas himself to Apollo.[208] This was also the sense of the advice Pythagoras is said to have given his disciples: that if they heard the *aulos* they should wash out their ears because the breath had defiled them, but that they should use well-omened melodies sung to the *lyra* to cleanse their souls of irrational
92 impulses. The *aulos*, he said, serves the thing that is master of our worse part, while the *lyra* is loved and enjoyed by that which cares for our rational nature.[209]

203 Compare 60.3, 84.16–17.

204 Apollo's association with the *kithara* is universal. Polhymnia's name means 'of many hymns' and she was the Muse of sacred song, and of learning. Aristides is probably drawing on Plato *Symp.* 187d–e.

205 On Hermes and the *lyra* see especially the Homeric *Hymn to Hermes*. Erato was the Muse of lyric poetry; her name means 'lovely'.

206 The first syllable of her name means 'well'; the second is related to *terpsis*, 'pleasure'. She is the 'well pleasing', 'delightful', and is linked with the *aulos* at *Anth. Pal.* 9.504–5.

207 The story referred to is that of Athena and Marsyas; for references see n. 202 above.

208 See 90.18ff. and n. 202.

209 This reflects the tradition in which music is treated by Pythagoreans as a means of intellectual improvement, e.g., 1.19 Archytas frag. 1. Other tales about Pythagoras

Learned men of all nations also bear witness for me that it is not our souls alone that are constituted in this way, but also the soul of the whole universe.[210] Some of them worship the region below the moon, which is full of breaths and has a moist constitution, and yet derives its activity from the life of the region of aether: these people make propitiation to it with both kinds of instruments, wind and stringed. Others worship the pure and aetherial region: they reject all wind instruments as defiling the soul and tempting it towards earthly things, and sing their hymns and praises with the *kithara* and *lyra* alone, because these are purer. Wise men imitate and emulate the aetherial region. Even though they are in the body, they separate themselves, at any rate in their acts of will, from the disordered variousness of things below; and they cling to the unbroken simplicity and mutual concord [*symphōnia*] of the beauties of the other world, by making themselves resemble it in respect of virtue.[211]

[At the end of Book II the MSS add a paragraph which is clearly out of place. We can in fact be confident on stylistic grounds, if for no other reason, that it is not from the hand of Aristides at all. But whatever its origin, it is not without interest.]

The chromatic genus is the diatonic augmented and densely populated with semitones. The enharmonic is the diatonic with a tone doubled and a semitone divided in half. The diatonic is so called because it is densely populated with tones among its constituent intervals. It is of male aspect, and rather severe in character. The chromatic is so called because it colours the other intervals, but has no need of any of them in particular. It is very sweet and mournful. The enharmonic is so called because it is comprised within the whole range of harmonic sound: perception will not allow us to adopt intervals greater than the ditone or smaller than the diesis. It is both stimulating and temperate.[212]

Book III

94 *Chapter 1* Let that be the end of our remarks about education and the use of things with music. My next task is to discuss what can be said about music from the point of view of physics;[1] but first I must set out in terms of numbers

ascribe to him a belief in the efficacy of the *aulos* for emotional therapy: see Sext. Emp. *Adv. Math.* VI.7–9, 23.

210 The idea of the soul of the universe comes from Plato: see especially 2.3 *Tim.* 34b ff., cf. Aristotle *De Anima* 404b. It is discussed by Aristides, on the basis of Plato's treatment, at 125.29ff.

211 See the final chapter of Book III. The ideal of the separation of the soul from the confusion of the perceptible world is Platonic, a theme, most notably, of the *Phaedo*.

212 On the characters of the genera see 16.10–18, and cf. 7 Aristox. *El. Harm.* 19.23–29, 9.2 Adrastus ap. Theon Smyrn. 54–6, ps.-Plut. *De Mus.* 1142d, *Anon. Bell.* 26. A passage interpretable as an attack on the notion of generic ethos is *Pap. Hib.* 1.13 (see *GMW* vol. 1, p. 184).

1 The branches of musical study are set out in Book I ch. 5. For the subject matters of the 'physical' branch see 6.15–17. The 'arithmetical' part extends from here to the end of chapter 8, though a shift towards the specifically 'physical' part has already begun in chapter 7.

those of the intervals that are concords.[2] Those who first came to understand the natural fluidity of the body and its total lack of stability (which can be seen in its gradual change from youth to old age), and who were led by this fact more than any other to conclude that the senses are unreliable, had the idea of using the precise mental grasp that comes through numbers to make clear each of the intervals in music.[3] So they took two strings, and starting from the unit, they constructed the series of numbers.[4] Thus they hung one unit of weight on one string and two on the other, and when they plucked both together they found the concord of the octave, so demonstrating that the octave is in duple ratio. Then from another they hung three units of weight, and when they plucked the string they found that it sounded the interval of a fifth in relation to the second string, and an octave and a fifth to the first. Thus they found that the latter was in triple ratio, the former in hemiolic [3:2]. Next they added another string, subjecting it to the tension of four units of weight; and they found that it sounded the interval of a fourth in relation to the third string, in epitritic ratio [4:3], an octave to the second, and a double octave to the first. They therefore defined the fourth as consisting in epitritic ratio, the fifth in hemiolic ratio, and the octave in duple ratio, this being put together out of the other two (since 3:2 displays the hemiolic ratio, 4:3 the epitritic, and 4:2 the duple). The larger intervals, again, are found by combining the ratios we have specified.[5]

Since they also wanted to find the ratios of the intervals that are smaller than the ditone, of the tone, that is, and the semitone and the diesis, they proceeded as follows. They knew that the fifth exceeds the fourth by a tone.[6] Hence they put together a sequence of three numbers, of which the first stands to the second in epitritic ratio and to the third in hemiolic. The numbers are these: 6, 8, 9. Now 8 is in epitritic ratio with 6, 9 is in hemiolic ratio with 6, and 9

[2] Concordance is discussed at 9.26–10.6, and is referred to in the classifications of intervals in Book 1 ch. 7. Aristides' statement that he will set the concords out 'in terms of numbers' marks a definitive shift to the Pythagorean treatment of intervals, whereas most of Book 1 relied on Aristoxenian formulations.

[3] On the obstruction to knowledge caused by the 'fluidity' of phenomena see especially Plato *Theaet.* 179d ff.: the doctrine of flux was standardly linked with Heraclitus, as it is by Plato. Plato often emphasises the central role of number in what is knowable (e.g., *Philebus* 55d ff.): cf. 1.11 Philolaus frag. 4. Aristides evidently also has in mind the controversy about the claims of reason and of perception to be the proper criterion of judgement in harmonic science: see especially 9.11–9.14 Ptolemaïs and Didymus ap. Porph. *Comm.* 23.24ff., 25.3ff., 27.17ff., cf. 7 Aristox. *El. Harm.* 32.18ff., 11 Ptol. *Harm.* Book 1 chs. 1–2. Aristides means to imply that a purely 'rational' approach was characteristic of Pythagoreans from the start, but this may be misleading: see 2.1 Plato *Rep.* 531b–c, and the Appendix to chapter 1.

[4] For a more elaborate version of this alleged experiment see 10 Nicomachus *Ench.* ch. 6, with notes (Nicomachus may possibly be Aristides' source). As commentators never tire of remarking, the supposed results are impossible, since the pitch of the sound will not vary directly with the tension of the string, but with its square root.

[5] These ratios have been mentioned in Book 1 at 11.7–9. They are common to the Pythagorean and Platonist traditions, and are used by the acoustic theorists of the Lyceum. That the octave ratio is put together from the other two means that if A:B is 3:2 and B:C is 4:3, then A:C is 2:1. See 8 Eucl. *Sect. Can.* propositions 6 and 12.

[6] This is assumed on all sides: see, for example, 7 Aristox. *El. Harm.* 45.34–46.1, 8 Eucl. *Sect. Can.* proposition 13.

is in epogdoic ratio with 8. But it was agreed that the fifth exceeds the fourth by a tone: and they therefore concluded that the ratio of the excess of the fifth over the fourth, which is a tone, is in epogdoic ratio.[7]

They wanted to know also the ratio of the semitones. Since there is no number between 8 and 9, they doubled the original terms to make 16 and 18, and found that between them there lies the number 17. By this number, they said, the tone is divided into semitones. They found, however, that this was not a division into equal parts, but into a larger and a smaller, since 18 stands to 17 in a ratio which is not equal to that of 17 to 16, but is smaller than it. This is why in the notation by semitones there is set out a double series of symbols,
96 so that when the smaller semitone is required to sound, we ascend or descend to the nearer symbol, and to the further one when the larger semitone is needed. For this reason, people of ancient times called this interval the *leimma*, because its exact value [*isotēs*, lit. 'equality'] is hard to determine.[8]

Next, they doubled once again the terms we have mentioned, for the same reasons, and found that the dieses do not divide the semitone in half, either. The numbers they produced were 32, 34 and 36. Between these fall other terms, 33 between 32 and 34, and 35 between 34 and 36. Thus between 32 and 34 there are two intervals, one in the ratio 33:32, the other in the ratio 34:33; and similarly there are two between 34 and 36, one in the ratio 35:34, the other in the ratio 36:35. Thus we see that it is in the nature of dieses to be divided to form unequal units.[9]

Since this is so, it is easy to see that the fourth is not composed of exactly two tones and a semitone.[10] If we take two epogdoic [9:8] ratios in succession, and then add a fourth term which will stand to the first in epitritic [4:3] ratio, it will not stand to the third term either in the ratio 17:16 or in the ratio 18:17, but will be close to 20:19 plus 505:504.[11] The four successive numbers are as follows: 192, 216, 243, 256. The divine Plato mentions these numbers in the *Timaeus*, when he is proving that the music that is perceived is far less accurate than the music grasped by reason.[12]

[7] See 8 Eucl. *Sect. Can.* propositions 8 and 13. Epogdoic ratio is the ratio 9:8.

[8] The subdivision of intervals of a tone or less had been a problem for ratio-theorists since the time of Philolaus (see n. 36 to 1.12 Philolaus frag. 6). It was Archytas who proved that ratios of the form n+1:n (of which the tone, 9:8, is an example) cannot be halved: see Boethius *Inst. Mus.* III.11 = DK 47 A19, and 8 Eucl. *Sect. Can.* propositions 3 and 16, and cf. Aristides 12.9–11. With the reasoning, and the use of the numbers 16, 17, 18, cf. 9.3 Adrastus ap. Theon Smyrn. 69.14–15. For the sense of the reference to notation see 24.2–6 and the tables that follow. The word *leimma*, 'remainder', is more commonly used of the interval remaining in a fourth when two tones have been subtracted (i.e., 256:243); see the last paragraph of this chapter. Aristides does not even try to address the question of the sizes of the other ratios in a tetrachord when one of them is 18:17 or 17:16; cf. 11 Ptol. *Harm.* Book I ch. 15.

[9] Compare 8 Eucl. *Sect. Can.* proposition 18.

[10] See 8 Eucl. *Sect. Can.* proposition 15. But Aristides' point here is not that a fourth is not the sum of two tones and exactly half a tone. He means that it is not the sum of two tones and either of the 'semitones' mentioned earlier (i.e., 18:17 and 17:16).

[11] But not exactly. Aristides is trying to express all his ratios as epimorics (n+1:n); cf. 11 Ptol. *Harm.* 16.12ff., 33.5ff.

[12] The reference is to 2.3 Plato *Tim.* 35–6, though only the last two numbers are actually mentioned there. The complete set is found in a similar group of calculations attributed

Chapter 2 Given these results, if we were to stretch a string across a plane surface of corresponding dimensions, into which all the numbers can be fitted,
97 and if we plucked it according to the proportions we have set out, we would find the whole collection of notes, some of them possessing concordance through the numbers, others being inferior because of their lack of concord.[13] This is why Pythagoras, so they say, when he was on the point of leaving this world, exhorted his disciples to work at the monochord,[14] explaining that the pinnacle of musical excellence is to be achieved intellectually, through numbers, rather than perceptually, through the hearing.[15] In our attempt to understand this we shall use the following terms. *Proslambanomenos* will be 9,216, *mesē* 4,608, and *nētē hyperbolaiōn* 2,304.[16] We shall work out the ratios previously specified on the basis of these numbers, and mark them on the *kanōn* according to the number of units, like points on a line. Where the distances are smaller, we shall be striking a smaller quantity of air with smaller sections of the string, and will thus make the higher sounds; and we shall make the lower sounds, obviously, by the reverse of this principle.[17] But since the

to Philolaus by Nicomachus (*Exc. ex Nic.* 267–71). The four numbers give the ratios 8:9, 8:9, 243:256, i.e., two tones and a Pythagorean or Platonic *leimma*. These ratios comprise the diatonic tetrachord of Philolaus, Plato, Euclid, Adrastus, Thrasyllus, Nicomachus, Eratosthenes, and Ptolemy's ditonic diatonic, but not the diatonics of Archytas or Didymus (see 11 Ptol. *Harm.* ch. 14). On the inferiority of perceptible music see chapter 7 below, and notice that Plato's concern in the passage cited is nothing to do with the 'proof' Aristides attributes to him.

13 On the 'inferiority' of the non-concordant scalar intervals compare 11 Ptol. *Harm.* 15.3ff. With Aristides' account of the use of the monochord compare 8 Eucl. *Sect. Can.* particularly propositions 19–20, 9.4–9.5 Thrasyllus ap. Theon Smyrn. 87.4ff., 10 Nicomachus *Ench.* ch. 10, and more fully 11 Ptol. *Harm.* Book I ch. 8, Book II chs. 1–2, 12–13, Book III chs. 1–2.

14 The invention of the monochord (as a technical device for theorists) is attributed to Pythagoras by Diog. Laert. VIII.12, Gaudentius 341.12ff., and others, but it is unlikely to have been in use before the fourth century B.C. For relevant references see Burkert (1972), p. 375 n. 22. Ptolemy gives evidence for its occasional use as an instrument of practical music (11 *Harm.* Book II ch. 12, cf. Pollux *Onomast.* IV.60), but there are no grounds for assigning this practice an early date.

15 Compare 91.27ff. and n. 3 above.

16 The numbers are such that whole-number values can be assigned to every note in the diatonic double octave. The series of ratios from *nētē hyperbolaiōn* down to *mesē* is 9:8, 9:8, 256:243 (tetrachord *hyperbolaiōn*), 9:8, 9:8, 256:243 (tetrachord *diezeugmenōn*), 9:8 (disjunctive tone). The corresponding numbers are 2,304, 2,592, 2,916, 3,072, 3,456, 3,888, 4,096, 4,608. (In the lower octave the ratio series is repeated, and each number is twice its counterpart in the higher octave.) But the numbers are not in their lowest terms: all are divisible by 4, and all except 4,096 are divisible by 12. Dividing by 12 gives 192 for the first number, and generates for the highest tetrachord the familiar numbers mentioned above at 96.26. The multiplication by a factor of 12 leads one to suspect that the source may be one who sought to combine Pythagorean and Aristoxenian methods. It ensures that the difference between any two numbers bounding a tone is divisible by 12, and thus that the 'space' between them can be deemed (quite improperly) to be divisible into Aristoxenian twelfths of a tone (cf. 7 Aristox. *El. Harm.* 25.11–26.5). This conjecture is supported by the procedure of Ptolemy at the end of 11 *Harm.* Book II ch. 13 (see n. 112), and of Boethius at *Inst. Mus.* IV.5ff., where Aristides' numbers reappear. In computing the numbers for certain notes in chromatic and enharmonic he resorts, quite anomalously, to additive procedures rather than the manipulation of ratios.

17 This does not necessarily commit Aristides to any particular theory of the causation of pitch. See 8 Eucl. *Sect. Can.* 148.3–149.6 with notes.

magnitude of the numbers involved makes this way of describing the string awkward, let us do it by a geometrical method instead.

If a string is set out over a *kanōn*, and if its sound is *proslambanomenos*, by taking half of it we shall make the sound of *mesē*, by taking a quarter *nētē*
98 *hyperbolaiōn*, and by taking three quarters *diatonos hypatōn*. If we divide that three-quarters in half, we shall get *nētē synēmmenōn*. If we take two thirds of the half-string, which is one third of the whole, we shall make *nētē diezeugmenōn* sound: if we take two thirds of the whole, we shall get *hypatē mesōn*. If we subtract one third from two thirds of the whole, *paramesos* will sound: and if we take two thirds of the two thirds twice over, we shall sound *hypatē hypatōn*.[18] To construct the smaller intervals we shall proceed as follows. We take a quarter of the whole, and divide it into eight equal parts. We then take off another segment, equal to one of these parts, and thus find the difference of a tone. If we next divide this last length similarly into eight equal parts, and then take off another, equal to one of the eight, we shall find *tritē hyperbolaiōn*. We shall use the same procedure in the other quarters of the string, and by dividing these in the same ratios we shall produce the sounds of the various notes.[19] Thus: (i) We place an unmarked *kanōn* alongside the apparatus. (ii) We mark divisions on it first. (iii) We draw parallel lines from the marks on it to the *kanōn* that lies under the string. (iv) Having marked the parts of the second *kanōn* in this way, we divide the string in accordance with the marks. This is what the so-called 'division of the *kanōn*' is like.[20]

Chapter 3 There are also those who have demonstrated the concords by
99 means of several strings. They stretch four strings at equal tension across a square device which they call a *helikōn*.[21] Then they mark the *kanōn* under the fourth string at the half-way point, and extend a little *kanōn* [*kanonion*] across from the top of the first string to the point marked on the fourth. Next they construct a diagonal, running from the foot of the first string to the top of the fourth; and they then demonstrate all the ratios of the concords, since the divided strings will give out notes related to one another in the ratios of the numbers that divide them. If the device is set up in this way it contains the concords in its four strings, but it will sound all the notes if further strings of equal tension are added.[22]

With this we should compare[23] the views of those who say that they take

[18] In all essentials this follows 8 Eucl. *Sect. Can.* proposition 19.

[19] This is a simplified paraphrase of 8 Eucl. *Sect. Can.* proposition 20. With the whole division compare also the more elaborate treatment of 9.4–9.5 Thrasyllus ap. Theon Smyrn. 87.4–93.7.

[20] With this description of the *kanōn* compare 11 Ptol. *Harm.* Book I ch. 8.

[21] The name comes from that of Mount Helikon, home of the Muses (Porph. *Comm.* 157.15–16).

[22] Aristides' description is a little sketchy; Ptolemy explains the device in detail. See 11 *Harm.* Book II ch. 2.

[23] The MSS have *paraitēteoi*, 'we should reject'. But Aristides offers no reasons for rejecting this approach. He makes use of this sequence of numbers in the calculations of chapter 4, and in chapter 18 (cf. also chs. 6, 23, 24). They are commonly found in Pythagorising authors, e.g., 10 Nicomachus *Ench.* ch. 6: compare the diagrams of 8

numbers by way of examples, unequal ones since the amounts by which the heights of pitch exceed one another are themselves unequal,[24] and who – because of the concords formed by the first note with the fourth and by the fourth and fifth with the eighth – pick out the numbers 6, 8, 9, 12. In these the proportions are as follows: as the first is to the second, so is the third to the fourth (i.e., as *proslambanomenos* is to *diatonos hypatōn*, so is *hypatē mesōn* to *mesē*), and again, as the first is to the third, so is the second to the fourth (i.e., as *proslambanomenos* is to *hypatē mesōn*, so is *diatonos hypatōn* to *mesē*). The ratio displayed by the second to the third is epogdoic, the ratio by which the fifth exceeds the fourth.[25] So much for these matters.

Chapter 4 A task which follows naturally is to enquire why, though many intervals are found, the only concordant intervals recognised within the complete *systēma* are the epitritic, the hemiolic and the duple.[26] Since the complete *systēma* exhibits duple ratio, and this is resolved into hemiolic plus
100 epitritic, we take the following numbers: 3, the first hemiolic number, which is the first of the numbers to be complete; and 4, the first epitritic number, which is in geometry the first plane.[27] We construct lines equal to these numbers, intersecting with one another at right angles, one of them standing to its own part in hemiolic ratio, the other in epitritic. If we take points on these lines on the basis of the underlying units, and then draw parallels, the whole

Eucl. *Sect. Can.* Aristides' intention is apparently to compare this arithmetical approach with the geometric one of the preceding passage, and to show that their results are equivalent. I tentatively read *parablēteoi*, 'we should compare', or perhaps *paralēpteoi*, 'we should apply'.

24 There is some confusion here. Aristides conflates the points (a) that the numbers must be unequal, since they represent unequal pitches (cf. 11 Ptol. *Harm.* 11.8–10), and (b) that the arithmetical difference between numbers bounding an interval of given size will vary with the size of the numbers chosen (cf. 11 Ptol. *Harm.* 14.1ff., Porph. *Comm.* 107.15ff.).

25 It is curious that Aristides chooses these notes to exemplify the relations, since *diatonos hypatōn* is not a fixed note (cf. the similar anomaly at 8 Eucl. *Sect. Can.* proposition 19, with n. 65). Presumably he does so in order to make the analysis parallel to those of chapter 2, where the units analysed are the lower and higher octaves of the GPS. The numbers can be attached to a set of fixed notes if they are identified with *hypatē mesōn*, *mesē*, *paramesē*, *nētē diezeugmenōn*, the framework of the system's central octave: see 10 Nicomachus *Ench.* 248.18ff. But Aristides does not (as Ptolemy does) make the central octave the focus of his account of the *tonoi* (see Book I chs. 10–11), and he has a metaphysical interest in treating the higher and lower octaves separately (see especially 115.8ff.).

26 The complete *systēma* is the octave (e.g., 14.16): these intervals are in the ratios 4:3, 3:2, 2:1. For the question cf. 8 Eucl. *Sect. Can.* 149.11ff.

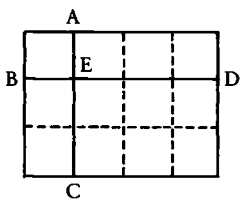
27 'Hemiolic' means 'half and whole': 3 is the first number of the form $n+\frac{1}{2}n$. It is 'complete' in the sense specified at 100.17–19. 'Epitritic' means 'a third in addition': 4 is the first number of the form $n+\frac{1}{3}n$. The description of 4 as the first plane seems anomalous: in Platonist and perhaps in Pythagorean theory, 1 represents a point, 2 a line, 3 a plane, and 4 the first solid (a pyramid). (For the Aristotelian evidence see, for example, *De Anima* 404b 20ff., *Metaph.* 1028b 24, 1085a 32, 1090b 20, etc. One of the Platonists, Speusippus, attributes these equivalences directly to the Pythagoreans. See Nicomachus *Theolog. Arith.* 82.10ff. = DK 44 A13.) This common idea is reflected in chapter 6 below: see particularly the characterisation of the number 4 at 102.5ff. The sense of the present description appears in chapter 5, at 100.20–3.

area will be twelve units if we set out our calculations geometrically, and if we do it arithmetically the sum of the numbers will turn out to make thirty-five segments.[28] Each of the parallelograms bounded by the parts of the straight lines in turn, and by whole and part, will display the ratios mentioned above, those belonging to the concordant intervals, and neither greater ones than these nor smaller. For the sake of clarity let me add the diagrams.[29]

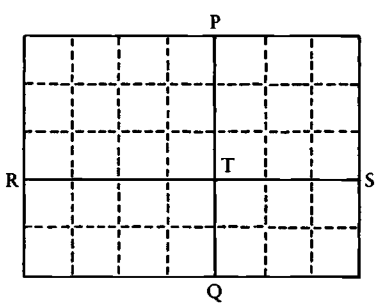
Chapter 5 We have taken two straight lines of which one incorporates the first perfect number (3 is the first number to display beginning, middle and end, an opposition and a mean), while the other is the first to display the nature of the plane in geometry. The number 1 applied to itself geometrically cannot generate a plurality, whereas twice two generates the first plane and square, the number 4.[30] We have set out their products in two ways, arithmetically and geometrically, because harmonic proportion arises from a combination of arithmetic and geometric proportion. The terms of an arithmetic proportion exceed and are exceeded by one another by equal numbers: in geometric

[28] The text is corrupt. For the unsatisfactory *telōn ta* of most MSS I suggest *tomōn to.*

[29] The diagrams are missing, but they can be reconstructed from the text, and from Plut. *De Anim. Procr.* 1018a–d.



The two lines AC and BD are of 3 and 4 units respectively. AC stands to its part EC in hemiolic ratio, BD to its part ED in epitritic ratio. The dotted lines are the parallels drawn from the points marking the 'underlying units'. The whole area contains 12 unit squares. 'Doing it arithmetically' involves a different diagram, in which lengths equivalent to the part-lines EC and ED are added to their related lengths AC and BD.



Here PT corresponds to the original AC, RT to BD, TQ to EC, and TS to ED. Now the four areas are of 6, 8, 9 and 12 unit squares respectively (cf. 99.17–18), summing to 35. The importance of the number 12 is brought out especially at 103.9ff. The number 35 underlies the discussion of procreation in chapter 18.

[30] See n. 27 above. The number 4 is not only the first square, it is also the first number to be 'generated' geometrically (i.e., by multiplication) from numbers other than itself. This allows it to count as the first plane, the first number generated as the area of a plane from other numbers representing sides.

proportion the differences between the terms stand in the same ratio to one another as do the terms themselves: harmonic proportion is produced by
101 taking the first difference from arithmetic proportion, the second from geometric.[31]

Quantity is of two kinds. Of the continuous kind geometry is the master: it constructs its ratios by magnitude, and compares wholes with their own parts. Arithmetic has discrete quantity as its sphere of judgement: it divides the whole into parts, and compares the parts with one another.[32]

There are numbers underlying each kind of proportion. In the arithmetical they are 2, 3 4, in the geometrical 2, 4, 8, and in the musical 3, 4, 6: for if we want to display in sequence the ratios constituting the three concords, we must first put together one number which exceeds the first in epitritic ratio, and another that does so in duple ratio.[33]

In general, if you insert arithmetic means between terms in geometrical proportion, you will produce either a plane or a solid harmonic proportion. Suppose we have a duple geometrical proportion, 2, 4, 8. If in each of the ratios you insert an arithmetic mean, you will produce a plane harmonic proportion, 2, 3, 4, 6, 8. Again, let there be a proportion on the basis of the triple: 3, 9, 27. If you insert two means into each of these ratios, 4 and 6 between 3 and 9, and 12 and 18 between 9 and 27, you will produce a solid musical proportion.[34]

[31] In arithmetic proportion, A–B = B–C. In geometric proportion, A:B = B:C, but Aristides expresses it differently: as A is to B and B is to C, so is A–B to B–C (e.g., 27:9 = 9:3 = 27–9:9–3). 'Harmonic' proportion is usually described elsewhere as being such that A–B is the same fraction of A as B–C is of C (i.e., A:A–B = C:B–C. On all these see especially 1.20 Archytas frag. 2, 2.3 Plato *Tim.* 35b–36a; cf. Theon Smyrn. 113.18–115.4, 116.8–119.16.) Then in the case of the numbers Aristides has in mind, the series 12, 9, 6 is in arithmetical proportion, 12, 8, 6 in harmonic. For Aristides' rather subtle derivation of harmonic proportions from geometric and arithmetic ones see 101.6ff. below. The derivation is recalled at 128.8–11. (Note that this is quite different from the 'musical' proportion of Nicomachus and Iamblichus, which combines arithmetic and harmonic proportion, and comprises four terms: it describes the complete series 12, 9, 8, 6. Aristides does not identify this as a form of proportion in its own right, and does not discriminate the terms 'harmonic proportion', as here, and 'musical proportion', at 101.8.)

[32] Aristides has in mind the differences between the diagrams set out above. He seems to mean that in working out a geometrical relation we ask 'What fraction of x is y?', while in working out an arithmetic relation we ask 'By how many units does x exceed y?' Compare the descriptions of harmonic and rhythmic ratios at 125.15–20.

[33] The three number-series are the first in which the proportions are exemplified (ignoring those beginning from the unit). The last sentence explains Aristides' description of the harmonic proportion as 'musical': the sequence 3, 4, 6 yields the ratios of the fourth (4:3), the octave (6:3 = 2:1), and, though Aristides does not explicitly say so, the fifth (6:4 = 3:2).

[34] The argument is based on the construction of 2.3 Plato *Tim.* 35b–36b (see also chs. 24–5 below), where the two geometric progressions 2, 4, 8 and 3, 9, 27 form the framework into which other means are inserted. When arithmetic means (3 and 6) are placed between the terms of the first series, the harmonic proportion 3, 4, 6 is generated. In the second series, 6 is the arithmetic mean between 3 and 9, and 18 is that between 9 and 27, and 6, 9, 18 is in harmonic proportion. The number 4 is added as being the harmonic mean between 3 and 6 (as shown in connection with the first progression), and 12, similarly, is that between 9 and 18. All this confirms Aristides' statement at 100.30–101.2. The series 2, 3, 4, 6, 8 divides a double octave into the concords fifth, fourth, fifth,

Chapter 6 Now that we have had to mention numbers, a further discussion of them will be welcome, since their investigation is a rather complex matter: at the same time, there will also be demonstrated their very great concordance [*symphōnia*] with the things that have to do with music.[35]

102 The unit was considered by men of ancient times to be the first principle [*archē*] and efficient cause of the concord of the whole universe, since all things come into being by being held together through *harmonia* into one.[36] The number 2 they cast in the role of matter, since it is the first to display contrariety.[37] They called the number 3 'the whole', because it is filled out by contrariety and mean.[38] The number 4 they called 'solid', for if you begin from the point and increase the number of dimensions one at a time, the solid will be found to come fourth, and it appears in the first number capable of admitting three dimensions.[39]

They called the number 5 'sense perception', for obvious reasons.[40] The number 6 they called 'completion of body', since it is put together geometrically from the first odd and the first even: for that reason they also called it 'marriage'.[41] The number 7 they called 'purity', since it is the only number

fourth. The series 3, 4, 6, 9, 12, 18, 27 divides a triple octave plus a tone into the concords fourth, fifth, fifth, fourth, fifth, fifth. (This latter series has no genuine musical significance and its place in the tradition depends wholly on its role in the *Timaeus*.) The two series are called 'plane' and 'solid' respectively on the basis of their affinities with the numbers 2 and 3: see 110.10–27.

[35] In this chapter and much of what follows Aristides is probably drawing on neo-Pythagorean writers who brought together ideas from two main sources. One begins from the (differing) accounts of early Pythagoreanism given by Aristotle, Aristoxenus and Theophrastus. The second originates in the Platonist absorption and reformulation of Pythagorean concepts (especially by Plato himself, and by Xenocrates and Speusippus). The numerological speculations present in these writers were in turn supplemented and embroidered by the neo-Pythagoreans themselves, and by interpreters of Plato. On the sources see Burkert (1972), pp. 15–96, and on numerology especially pages 465ff. Aristides' treatment has much in common with that of Theon Smyrn. 99ff. (see references below). There must be shared sources, but the differences are enough to show that the one is not derived directly from the other.

[36] *Harmonia* plays an important role in Philolaus' cosmology: see 1.12 frag. 6, cf. 1.9–1.10 frags. 1 and 2, and more generally 1.5 Aristotle *Metaph.* 985b 23ff. Philolaus also says that the One was the first thing to be harmonised, 1.13 frag. 7. Its function in unifying all things may be hinted at in Stob. *Anth.* 1.18.1c = DK 58 B30. Compare Aristotle *Metaph.* 987b 22, 1091a 13ff., *Eth. Nic.* 1096b 5. But the idea was developed by Platonists: see especially Theophrastus frag. 12 (Wimmer, p. 154), and cf. Theon Smyrn. 99.24–100.8.

[37] The designation of 2 as 'matter' may depend ultimately on the Pythagorean identification of even number with the 'unlimited', e.g., 1.5 Aristotle *Metaph.* 986a 15ff., but more directly on the Platonist 'indefinite dyad', e.g., Aristotle *Metaph.* 988a14, Theophrastus frag. 12: cf. Theon Smyrn. 100.9–12.

[38] Compare 100.18–19, Theon Smyrn. 100.13–17, with elaborations to 101.10.

[39] Compare n. 27 above, Theon Smyrn. 101.11–13. On the numbers 1, 2, 3, 4 see also the report of Plato's views in Aristotle *De Anima* 404b.

[40] That is, because there are five senses. Theon's discussion (101.14–102.3) is on quite different lines.

[41] See Theon Smyrn. 102.4–6, elaborated to 102.18; cf. 101.6–10, and, for example, Plut. *De Anim. Procr.* 1018c. In Aristides the importance of the number 6 appears especially at 112.3–13 and in chapter 18.

within the decad which is neither generated geometrically nor generates another.[42] The number 8 they called 'material body', because it is constituted from the first even number multiplied cubically.[43] They called the number 9 'music', because it is the first that is composed of numbers displaying the three concordant ratios (for 2, 3 and 4 make up 9); and also because the *harmonia* and rotation of the universe come together to make this number, since people name seven planets and two spheres, the 'unmoving sphere' and the 'fixed sphere'.[44] The number 10 they called the 'first concord', since if you divide the smallest concordant musical *systēma*, the fourth, into the smallest intervals (by which I mean dieses), you will find that there are ten of them. If you replace a diesis with a unit, and put together the remaining numbers in a way corresponding to the size of the intervals, you will see that the same number results in all three of the melodic genera.[45] 10 is also the first composed of two
103 numbers, 2 and 8, between which there falls a single mean proportional (2, 4, 8); and some people called it the 'first *harmonia*', since this was the name given by older writers to plane geometric proportion.[46] 10 also arises from the number 9, which signifies the *harmonia* of the whole universe, if you reckon it together with its dependence on the first whole, and its synthesis into the unity by participation in which it is bound together.[47] They also have something to say about the number 11. For the tone displays in relation to the first diesis, taken in ascending order, the ratio which bears this number's name.[48]

12, however, is the most musical of the numbers. None of those before it display the harmonic concords in relation to the majority of those that precede them, though when divided into their smallest elements they may exhibit

[42] That is, it is neither a product of other numbers within the decad, nor a factor of any. See Theon Smyrn. 103.1–104.19, which includes much more detail (such as the claim that the Pythagoreans called it 'Athena', since she neither had nor was a mother).

[43] The first even number, 2, was 'matter'. Theon notes briefly that 8 is the first cube; the rest of his discussion is independent (104.20–106.2).

[44] None of this is in Theon (his brief treatment of 9 is at 106.3–6). The seven 'planets' are Cronos (Saturn), Zeus (Jupiter), Ares (Mars), Hermes (Mercury), Aphrodite (Venus), Sun and Moon (see chapter 21). The two spheres are identified at 112.16–17.

[45] Here Aristides reverts, without explanation, to an Aristoxenian division of intervals; there are ten enharmonic dieses (quarter-tones) in the fourth.

[46] On plane geometric proportion see 101.16–19. The identity of these older writers is obscure.

[47] Here again Aristides is independent of Theon, who merely celebrates the fact that the decad 'contains the whole of number', and refers to its treatment by Archytas and Philolaus (106.7–11). The decad, conceived as the sum of the first four numbers (the *tetraktys* of the decad) was an object of veneration among Pythagoreans (see 1.2 Sext. Emp. *Adv. Math.* VII.94ff., 9.4 Theon Smyrn. 87.4–7, cf. 93.17ff.). Its bringing together of the first four numbers allows the ratios of the concords to be found in it, as these authors show. It is odd that Aristides does not mention the fact here. Perhaps it is because he wishes to reserve his greatest enthusiasm for the number 12.

[48] The text is corrupt, but the sense is clear enough. The value assigned to the smallest diesis (at 96.4–16) was 33:32, and that of the tone is 9:8. The difference between them is 12:11. Words for epimoric ratios (n+1:n) were formed by placing the prefix '*epi*' in front of the ordinal adjective proper to the lower of the two numbers (when taken in their lowest terms). The ratio 12:11 is *ephendekatos*, based on the name of the number 11 (*hendeka*).

certain ratios between parts of different sizes.[49] But the number 12 is unique in standing to 9 in epitritic ratio, to 8 in hemiolic, to 6 in duple, and, in addition, to 4 in triple ratio and to 3 in quadruple. This is why our nature has limited to this quantity (I mean to twelve tones) our capacity to move our voice upwards in pitch.[50]

Chapter 7 It might be supposed that our discussion is inconsistent, when we
104 first set out our investigation of musical matters in terms of numbers, and then assert that the intervals are not capable of receiving these very same numbers precisely.[51] If we are to explain why this is so, we must bring into action a doctrine both sacred and secret.[52] Things in this world are constituted in imitation of higher things, and it is in relation to the movements and changes of these latter that they acquire their generation and provide themselves with being.[53] The two regions differ, in that while one is pure and incorruptible, the other is turbid and murky, so that while activity there is perfect and unimpeded, here it is defective, lame and difficult, not because of a fault in the agent, but because of the confusion and feebleness of the matter.[54] A sculptor, so the argument runs, can readily impose on a stone whatever shapes [*typoi*] he chooses, but on pumice he cannot, or only with difficulty; and this is not due to his own weakness or lack of skill, but to the unsuitability of the materials. In just the same way, the activity of the universe applies itself more successfully to things in the other region, which are well suited to it because of their greater divinity, but more faintly to things here, which have become muddy because of the great distance, and the bodily fog and sediment. For just as a ray of the sun, they say, is seen in its full purity in the air, but in the depths of the sea it is faint and faded, not because it is so itself, but because our senses are impeded by the surrounding medium, so also the emanations which extend from above are not

[49] As 9 exhibits the ratios of the concords between its parts, 2, 3, 4. Possibly Aristides is surreptitiously attacking the preeminence of the Pythagorean number 10 (see n. 47 above).

[50] Compare 21.13–14. According to 7 Aristox. *El. Harm.* 20.23–30, the human voice can span two octaves and a fifth. Note that the equation of the octave with six tones is Aristoxenian; in the theory of ratios it does not hold (e.g., 8 Eucl. *Sect. Can.* propositions 9 and 14).

[51] It is not immediately clear what Aristides has in mind. It might merely be the fact that no instrument and no method of practical tuning can guarantee absolute precision in the construction of intervals (cf. 94.5ff., 11 Ptol. *Harm.* Book I ch. 1). There is also the quite separate issue of the impossibility of exactly bisecting certain intervals, to which Aristides has alluded several times in chs. 1–3 above; cf. Book I, 12.10–11. The 'impossibilities' involved in these two kinds of case are quite distinct, but the truth is that Aristides seems to have conflated them: see 105.22–5 with n. 62.

[52] Compare 107.25ff. and Book II, 65.11ff.

[53] This theory of 'imitation' (*mimēsis*) is not that of Plato, whose realm of 'higher things' (Forms) admits no movement or change. Here the higher realm is that of the heavenly bodies, whose influences are described in chapter 21; cf. also chapters 19–20. The ideas involved are more Stoic than Platonist.

[54] There is an implicit reference here to Aristotle's theory of four 'causes' (e.g., *Physics* 194b 16ff.). A thing's formal and final causes are to be defined in terms of its perfect activity: its efficient or moving cause is the agent that initiated its coming into being; its material cause is the matter in which it comes to exist.

equally effective in every region, but their effect varies with the worth of each of the materials involved.[55] In our own case too, therefore, when we have conceived an attachment to the confusion and disorder of earthly things, we receive from the greater power only a little assistance, barely enough, through the unity and community of the universe. But when we understand ourselves
105 and discover our origin, and turn our wills and our lives towards the things that are of greater worth, we receive the universal providence in its purity and perfection, since our nature is then brought to an appropriate condition through likeness to what is most fine.[56]

As clear evidence of the sympathy of things in this world towards things in the other,[57] they point to the seasons, to changes of wind and movements of rainfall, to times of great heat and times of mild weather, all of which are brought about in accordance with the quality of the condition of things in the other region.[58] They mention also things which may come about at virtually any time, the growth and decay of plants and the development and decline of animals, whose qualifications and changes go along with the waxing and waning of the sphere of the moon.[59] They refer also to the ebb and flow of the sea, each of whose alterations corresponds to a change in the course or the phases of that same divinity, as is regularly exemplified in the strait that runs between the pillars of Hercules:[60] while in Egypt the rise and fall of the river Nile, in their regular annual seasons, maintain a correspondence with the paths and movements of the sun.[61]

It is therefore not implausible to suggest that music, like everything else, has its source in the nature of the universe, but that through its mixture with bodily matter it declines from its pinnacle of precision with respect to numbers: for in the regions above us it is accurate and incorruptible. Thus it is because we are impeded by the grossness of body that we are unable to divide intervals equally, and have *systēmata* whose concords are defective.[62]

[55] 'Emanations', *aporrhoiai*. The word or its variant *aporrhoē* is found quite early, e.g., in Plato *Phaedrus* 251b ('an emanation of Beauty'), and in a directly physical sense at Empedocles frag. 89. The term was adopted for their special purposes by Neoplatonists, but was already current in astrological speculation, referring to the influence of the planets, in the first century B.C. (Geminus 2.14). This may be the context in which Aristides found it.

[56] With this paragraph compare especially Book II ch. 17.

[57] On the doctrine of cosmic sympathy and its sources see Book II, 89.23ff. with n. 200.

[58] Compare 119.10ff.

[59] Compare chapter 18 and 121.3ff.

[60] That is, the Straits of Gibraltar. Posidonius is known to have studied these tides (Strabo *Geog.* III.5). He may be the direct or indirect source for this whole paragraph.

[61] The phenomenon of the Nile floods had intrigued the Greeks from early times: see especially Herodotus II.20, Aetius IV.1.1.

[62] The concordant fourth bounding a tetrachord will of course be defective if it is divided into two 9:8 tones plus either of the 'semitones' identified by Aristides in chapter 1 (18:17 or 17:16 instead of the *leimma* of 256:243): special problems arise in modulation. Some musicians are said to have deliberately flattened various fixed notes, thereby distorting concords (ps.-Plut. *De Mus.* 1145d). That intervals of epimoric ratio (which include all those central to Greek harmonic analysis) cannot be divided equally is a mathematical truth to which Aristides' form of explanation seems quite

Chapter 8 Anyone who looks at the other arts will see clearly how great a benefit they derive from numbers.[63] If we consider painting, for instance, we shall find that it does nothing without the help of numbers and proportions [*analogiai*]: it is through numbers that it hunts for the proportionate measures [*symmetriai*] of bodies and mixtures of colours, and from these it gives its pictures their beauty. We may also observe that this same art uses numbers to
106 imitate a thing's fundamental nature: for it is the kind of proportion [*analogia*] which brings beauty to natural bodies that painters are pursuing in the dimensions [*metra*, lit. 'measures'] of their shapes and the mixtures of their colours. Thus painters too have shapes and colours and postures which represent kinds of life and character; and their whole art is constituted in the opposite way to that of the physiognomists. In the latter case what is given is the form, and the way of life is then detected, while in the former it is the character that is observed and the form that is then modelled.[64]

Medicine also describes everything in terms of numbers – attacks of palpitations, for instance, or the proportions involved in periodic kinds of disease.[65] Some of these diseases are proportionate to concordant ratios, those occurring every day being proportionate to duple ratio, those which occur on the third day to the hemiolic, and those appearing every fourth day to the epitritic.[66] These types are not necessarily dangerous. Others are serious, but have certain similarities to the first group (for example, those of the semitertians that have affinities with those ones), and these are dangerous, but offer some little grounds for hope.[67] Others again are altogether discordant (continuous ones, for example), and these are deadly and to be feared.[68] Nor can medicine contrive the qualities and capacities of its drugs except through the proportions of the quantities used.[69]

In general, no matter what you are investigating, you will find concordant numerical relations holding even where they are least expected. Thus where there are approximate similarities between life-styles and characters, they bring

inappropriate (see nn. 8 and 51 above). But possibly the considerations in his mind were comparable to those of Adrastus in the last sentence of 9.3 Theon Smyrn. 72.

63 The theme is Platonic: cf. *Philebus* 55d ff.

64 That character, not appearance, is the true object of imitation in the arts, is a theme underlying much of Books II–III, X of Plato's *Republic*. The science of 'physiognomy' was that of construing a person's character from his features (as in the Aristotelian *Physiognomica*, Hippocr. *Epid.* II.5, 6).

65 'Periodic' diseases are those whose phases come and go in regular temporal patterns. They were much studied by Greek medical writers: see particularly the Hippocratic *Epidemics*.

66 'On the third day', i.e., on the first day and the third. These are 'hemiolic' because 3 is a hemiolic number (see n. 27 above), and perhaps because the ratio of the whole three days to the time between onsets is 3:2 (cf. nn. 29 and 32 above). The analogy with hemiolic rhythms, in which a 3-unit duration is followed or preceded by a 2-unit duration is scarcely direct (a similar account applies to the 'duple' and the 'epitritic').

67 'Semitertians' have strong crises on alternate days, weaker ones on intervening days. They thus share the characters of the 'daily' and the 'third day' types.

68 The first phrase exactly repeats 83.25, whose context is closely related to the present discussion. On the medical analogies see n. 162 to that passage.

69 Compare particularly 30.20–4, 85.28ff.

unanimity: as for men's gifts of good fortune, their possession of wisdom, the degree of their success in actions and enterprises, and the condition of all other aspects of their lives, you will find that when their ratios are concordant they bring friendship, and when they are discordant, the opposite.[70] Often their unharmoniousness [*anharmostia*] is brought into harmony through the agency of a mean proportional, just as an intermediate concord harmonises discordant intervals.[71] If you want to identify the means in the case of the soul, you will find in the spirited part a mean between reason and appetite that is
107 proportional to both.[72] If you investigate political organisations, you will be struck by the way in which the military constitutes an element intermediate between the deliberative body and the mass of the people, and in the best kinds of community never abandons its own specific function.[73] Of the military itself, the hoplites and the cavalry form the extremes, while a mean between them is constituted by the light-armed troops, who bear a resemblance to both extremes, in that they are infantry with a high degree of mobility.[74] Again, the entire complement of the senatorial and plebeian orders has as its mean the equestrian order, which through its part in the descending scale of honour has affinities with both extremes.[75] To suppose, then, that these things are so clearly constituted on a foundation of numbers and means, and yet that music is not, would be the mark of someone outstandingly stupid and uncultivated in his nature.[76]

Chapter 9 Let us now go through the things that are discussed in music, one by one, and explain the correspondence of each to the universe as a whole. Just as no other things of beauty can be constituted except in concordance with the universe, so music could neither be constituted, nor if it were could its activity be so powerful, if it had not acquired a stable and truly divine strength through the great resemblance it bears to things above us. Of the things I shall mention, a very few are shared with other arts, but most of them, and the most important ones, belong to music alone. What is especially peculiar to it is that it is constituted, like the world of natural generation, out of opposites, and bears the image of the *harmonia* of the universe.[77]

[70] This vague sentiment has its roots in Plato, e.g., *Gorgias* 508a and especially *Laws* 756e–757e; cf. 744b–c. Aristides might also have in mind passages of the *Republic*, e.g., 432a–b, 443d–e (cf. also *Rep.* 402b–d, *Laws* 689d, quoted by Theon Smyrn., 10.12–11.7).

[71] Thus the interval of five tones (ratio 16:9) is discordant, but the insertion of a (geometric) mean, giving the series 16, 12, 9, converts it into two concordant fourths. Aristides intends an analogy with two antipathetic persons, brought into harmony through the mediation of a third with whom each is 'concordant'.

[72] Drawing on the analysis in Book II, 54.21–6, and its Platonic prototype (*Rep.* 436–44).

[73] By the 'best' kind Aristides again means that of the *Republic*: see especially *Rep.* 433–4.

[74] Hoplites were heavily armed infantry. 'Light-armed', lit. 'naked' troops were those with little defensive armour.

[75] The reference here is to the Roman social system. Cf. Pliny *Hist. Nat.* 33.7–8.

[76] 'Uncultivated' *amousos*, lit. 'unmusical'. This completes the 'arithmetical' part of the 'physical' study of music (see n. 1 above).

[77] This paragraph introduces the second, the 'natural' or 'physical' part of 'physical' musicology (n. 1 above). Though some issues proper to physical acoustics will be touched on, the main topic is the correspondence of musical relations to ones holding

Once again we must call upon the same god that we invoked at the outset, the lord of the constitution of every body and of the *harmonia* of every soul. We must ask him, so far as we speak with true judgement in what we shall say, and so far as it may without impiety be publicly revealed, to preserve our words for ever and make them worthy to be heard and understood; but if any of it
108 is out of harmony [*ekemelōs echei*] with the cause of all being, or should not be spoken of and is improperly included in this book, may he forgive us for the enthusiasm of our labours and for our benevolence to our fellow men, and either consign our words completely to oblivion or put them into the hands of those to whom it is not blameworthy to transmit them.[78]

Chapter 10 Turning first to the movement of the voice, who can doubt that it is in concord with the universe? The universe is constituted from agency and matter. Matter is simple and without shape, lacking all form: what for the other arts is fundamental and primary counts as matter for their specific activities, but so far as nature is concerned counts much more as form.[79] But the movement of the voice is conceived without reference to change in bodies, and like the primary things, it is essentially bodiless.[80] Now matter has raised innumerable problems concerning whether it is continuous or discontinuous,[81] and here too music reveals the opposition involved in matter, since it exhibits its own matter, which is the movement of the voice, as both continuous and intervallic.[82] Thus just as the providential power of the universe divides the excessive continuity of matter into distinct forms, and brings together its

between elements of the natural world. The power of music is derived, on the basis of Platonist and Pythagorean ideas, from its reflection of relations that constitute the order of the universe and everything in it. Much of the subsequent discussion deals with the 'oppositions' that exist in music and their natural counterparts. The *harmonia* of the universe is created out of these oppositions, but is more directly identified with the 'soul of the universe' discussed in chapter 24. On the reconciliation of cosmic opposites through *harmonia* see 1.9–1.10, 1.12 Philolaus frags. 1, 2, 6. The general idea is much older, going back through Heraclitus (e.g., DK 22 B51) to the 'equilibrium' and 'justice' of Anaximander (DK 12 A11, A26, B1) and beyond.

[78] Aristides' repetition of his prayer to Apollo (see Book I ch. 3) emphasises the status of the remaining chapters as the climax of his work. On the relation between music and the mysteries, and the esoteric nature of its secrets, see also especially Book II ch. 7, Book III ch. 27.

[79] The notions of matter and form at work are Aristotelian, further developed by the Stoics. For a discussion see, for example, Guthrie *History* vol. 6, pp. 227ff.

[80] It is not a contradiction to speak of 'bodiless matter' in an Aristotelian context. 'Matter' is simply that on which form is imposed, and there is purely intelligible, incorporeal matter in the realms of logic and mathematics (e.g., *Metaph.* 1023b 1–2, 1036a 9–12, 1058a 23–4). But this conception of bodiless vocal movement, movement in which no body is moved, is not Aristotle's. It has some affinities with the audible motion of 2.1 Plato *Rep.* 530c–d, but its true source is Aristoxenus (7 *El. Harm.* 8.13ff., especially 9.3ff.), picked up previously by Aristides in Book I, 5.24ff. For another case where this conception is adopted by an otherwise 'Pythagorean' writer see 10 Nicomachus *Ench.* ch. 2.

[81] Issues about the continuity or discontinuity of matter go back to Zeno, Anaxagoras and Democritus in the fifth century B.C., but acquired central status in metaphysics only with the Stoics and their development of the concept of a continuum (in contrast to the extension of Democritus' atomism by the Epicureans). See Sambursky (1956), ch. 6.

[82] See the passages in Aristoxenus, Aristides and Nicomachus cited in n. 80 above.

discontinuities in suitable proportions,[83] so music dismisses an extreme continuity of sound as unusable, and rejects excesses of discontinuity as indeterminate, basing its melody on suitably proportioned distances.[84]

It is plain to us that a note is a simple movement of the voice.[85] Here we have a feature which music shares with the other arts, since all of them take their starting point from things that are indivisible, so far as these arts themselves are concerned.[86] Its peculiarity is that it is capable of admitting opposites, in the same way as is the matter of the universe as a whole: for the same note participates in both high pitch and low.[87]

109 Again, some notes are fixed, while others move;[88] and we may say just the same about the totality of things, for of the things that there are, some are fixed, others move. Each of these groups is again divided into two different kinds. Some, like the earth and its contents, are fixed in respect of position; and to them correspond the lowest notes of *pykna*, because of the heaviness of the element and its adhesion to the things that are continuous with it.[89] Others, like the things that are divine, are fixed in respect of their powers: and to them the notes which form no part of a *pyknon* bear a resemblance, since they express the existence which divine things have outside of any conjunction with the body.[90] They are the origins of all the other notes, just as divine things are of bodies, and they contain all those that come after them, in strings by the progressive diminution of numbers, and in *auloi* by the subtraction of magnitude.[91] In respect of the universe at large, similarly, men of wisdom and

[83] For the Stoic credentials of these clauses see *SVF* vol. 2, 439–49.

[84] 'Continuous' movement is that of speech (references as in n. 82 above). Excesses of discontinuity should, correspondingly, be very large intervallic leaps, which give no clear impression of their melodic form, but Aristides may be referring to intervals that do not correspond to properly harmonic ratios.

[85] For Aristides' previous definition of 'note' see Book I, 7.15–16 with n. 37. With the notion of a 'simple' movement cf. 6.25ff.

[86] Aristides may have in mind the discussion by Eratosthenes of the different ways in which things are developed out of simple starting points, recorded at Theon Smyrn. 82.22ff.

[87] The implied contrast with the other arts is unclear. Perhaps the sense is that while the character of a given vowel or consonant, or of a given colour, is determined for the art that uses it by its independently perceptible nature, any given note, defined as a point of absolute pitch, may be treated as high or as low, depending on the position it holds in a musical structure.

[88] See Book I, 9.13ff.

[89] On notes defined by their relation to the *pyknon* see 9.14ff. The lowest note of a *pyknon* (*barypyknos*) is heavy because it is low (*barys*, lit. 'heavy'). It 'adheres' to the other notes of the *pyknon* because this is a 'compressed' sequence of notes closely attached together.

[90] On notes not belonging to a *pyknon* (*apyknoi*) see 9.18ff. The contrast between notes fixed in position (here *topos*) and in power or function (*dynamis*) cannot be given a specifically musical sense simply through the contrast between *barypyknoi* and *apyknoi*. Whether a note is in the Ptolemaic sense defined by position or by function depends on the thetic or dynamic character of the scalar framework in relation to which it is named (11 Ptol. *Harm.* Book II ch. 5). Within any one defined scale or system, *barypyknoi* and *apyknoi* are 'fixed' in exactly equivalent senses.

[91] Aristides' *apyknoi* are here the *proslambanomenoi*. His description of them as the origin of the other notes reflects his treatment of *proslambanomenos* as the basis of each *tonos* (Book I chs. 10–11). The reference to strings and *auloi* relates to the division of the monochord (Book III chs. 2–3) and to comparable procedures with wind instruments (see e.g., 9.2 Theon Smyrn. 56.9–57.10; cf. 60.16–61.23).

inspiration have discovered that it is through reduction and diminution that all things in this world, which are the last things, fall short of the primary things in their goodness, their life and their movement; and that it is through diminution of goodness that they display evil, through decrease of life that they engender decay, and through the insignificance,[92] indeed absence, of movement in accord with ratio that there is total immobility at the absolute centre.

Movement, again, has two forms, since some movement is in a straight line, some in a circle. Circular movement is employed by the body of the aether, which, so some people say, is a plane surface.[93] This resembles the notes like *parhypatē*, since these display two intervals, the diesis and the semitone.[94] Things under the moon move in a straight line, these being things with depth, as is generally agreed, and with three dimensions: similar to them are the notes like *lichanos*, which exhibit three sizes of interval, the diesis, the semitone and the tone.[95] What then? Do not the differences between these consist in diffuseness and density, the very differences which make apparent the distinctions between material bodies, some of them being diffuse and therefore light and liable to rise, others dense and therefore heavy and tending to fall towards the earth?[96]

110 Further, the fact that so far as their own nature is concerned, the pitches of the notes extend upwards indefinitely, but are given boundaries by art, draws attention to the power of the divine craftsman and the nature of matter, of which the latter is indefinite and irrational, while the craftsman is limit and ratio.[97] By this are constituted the beauties of bodies, the virtues of souls, and

[92] Reading *brachytēti* with most MSS. Winnington-Ingram adopts the variant *bradytēti*, 'slowness'. Goodness, life and movement are dispensations of the divine, conceived as existing in its purity in the outermost heavens; the power of the dispensations is reduced as the distance from their origin increases (cf. 104.16ff.). See also the Aristotelian *De Mundo* (397b 25ff.). There are certainly echoes in Aristides of both this treatise and the *De Caelo*; cf. also *Meteorol.* 340b 6–10.

[93] The superiority of circular motion is an astronomical commonplace from Plato onwards (particularly *Tim.* 34a, 40b, 43b): it is the only kind that can be eternal (Aristotle *Phys.* Book VIII chs. 8–9); and it is that of the aether (especially *De Caelo* 269a–270b, cf. *De Mundo* 392a).

[94] 'Notes like *parhypatē*', *parhypatoeideis*, i.e., those second from the bottom of each tetrachord; cf. 9.25, 10.15. (The present paragraph concerns moveable notes, as the previous one had dealt with fixed notes.) Such notes stand at a diesis from the lowest note of the tetrachord in enharmonic, at a semitone in diatonic or chromatic (16.4–10). The 'resemblance' depends on the association of the number 2 with the two dimensions of a plane; cf. 101.12ff.

[95] On the rectilinear motion of things under the moon see especially Aristotle *De Caelo* (cited in n. 93 above). Notes like *lichanos* (*lichanoeideis*, cf. 9.26, 10.15) stand at a diesis from the *parhypatoeideis* in enharmonic, a semitone in chromatic, a tone in diatonic (16.4–10).

[96] Small intervals are dense, *pykna*; larger ones diffuse or open-textured, *araia* (see 11.21–4).

[97] The theoretical possibility of indefinite extension in pitch is mentioned at 7 Aristox. *El. Harm.* 15.6–8. More relevant may be the remarks of Adrastus at 9.3 Theon Smyrn. 63.25–65.9, which contrast this theoretical extension both with Aristoxenian and other estimates of practical usage, and with Plato's extension of the *Timaeus* scale to four octaves and a sixth. The latter is based on the requirement that the world-soul's *harmonia* should embrace 'solid' as well as 'plane' numbers; only so can it run through the whole solid body of the cosmos and grasp all existing things. The craftsman is that

the balanced proportions [*symmetriai*] of climates, which neither develop indefinitely in one direction nor change all at once into their opposites, but make their alterations by very gradual stages, providing mild temperatures in the universe as a whole, well-being for plants, and good health for animals.[98]

Chapter 11 What is more, the fact that the primary concordant *systēmata* are three in number[99] implicitly reveals the triadic nature of the universe. Some of them, we say, are incorporeal, and correspond to the complete *systēma* of the octave; some are bodies, and correspond to the fourth; and some have an intermediate nature, and correspond to the fifth.[100] Again, some beings are divine and ever-living, some perpetually lifeless, while others, such as mortal animals, fall between the two. This is why the melodic genera are divided at the highest level into three classes.[101] But the nature of the dimensions [*diastēmata*] found in bodies is also threefold, the primary kind being disposed in a single dimension, as a line, the next in two, as a plane, the third through three increments making depth. Accordingly, the enharmonic is ordered on the basis of the line, and is simple and undifferentiated, the diatonic is like a solid and partakes also in depth, while the chromatic is organised on the basis of the plane, since writers of the past called everything of the nature of a plane '*chroa*', because it displays the colour of every surface that is discerned.[102]

Again, the melodic genera are themselves divided into several species. While
111 the enharmonic is indivisible, the chromatic is divided into three and the diatonic into two.[103] Thus the total is six, which is equal to the number of tones in the

of the *Timaeus*: see chapter 24 below. For the relations suggested between limit and indefiniteness see particularly Plato *Philebus* 16c–18d, quoted in part at 2.6, and compare the fragments of Philolaus at 1.9ff.

98 On the virtues see chapters 16–17, cf. 126.28–127.7; on bodily beauty see chapter 18, 118.19–28, cf. 127.7–11; on climates and seasons see chapter 19.

99 Compare 14.3ff.

100 The basis of these correspondences is not made clear. Perhaps they reflect again the treatment of the series 1, 2, 4, 8 etc. as being in 'plane' and hence bodiless geometric proportion, while 1, 3, 9, 27 etc. is in 'solid' geometrical proportion. Then the octave, 2:1, involves nothing of the solid: the fourth, 4:3, is based on the insertion of further means between 3 and 9 in the solid sequence; while the fifth, 3:2, is based on the 2 of the plane series but imports the 3 from the solid, see 101.12ff. But a reference to these orders of being in chapter 12 links the divine with the unit (not the plane), aether with the plane, matter with the solid; see n. 116 below. Probably the theory of proportions has temporarily dropped out of sight. The reference to 'them' at the beginning of the sentence is presumably to beings in general.

101 The genera are discussed in Book 1 ch. 9. A similar division of beings is reflected below at 111.3ff.

102 *Chroa*, lit. 'colour', is used sometimes as a name for the chromatic genus, sometimes, as by Aristoxenus, for its various 'shades' or species. Early writers do indeed use it to mean 'surface'. Aristides' distinctions here seem somewhat impressionistic. The fact that the enharmonic is 'undifferentiated' (which might mean that there is only one form of it) relates it to the number 1, but in Aristoxenian theory, as Aristides says in the next paragraph, there are three 'shades' of chromatic and only two of diatonic, the reverse of what would be required (cf. 11 Ptol. *Harm.* 32.18–22). Probably he is thinking only of the relative sizes of their most characteristic intervals. The enharmonic diesis has barely discernible magnitude or 'thickness', like a line; the tone appearing in diatonic is large and substantial; the semitone of chromatic is intermediate.

103 Aristides' account of these Aristoxenian classifications is in Book 1, 17.2ff.

complete *systēma*.[104] If we consider them in relation to mankind, the enharmonic exhibits the essence of the soul, which is unitary and simple,[105] and the chromatic the kind of being which lies between soul and body, and which we call 'nature'.[106] Nature is analogous to the number 3, because of its perfection; and for this reason some people, who speak of what we call 'soul' as 'the mind from without' and give what we call 'nature' the name 'soul', have described the latter as an 'entelechy', on the basis of the perfection it displays.[107] The diatonic displays the perceptible body, which is solid and resistant, just as the diatonic is hard and unyielding;[108] and the constitution it has is similar. For the body – both as a whole and in its parts – is made up of the combination of two and a half...[109] just as the diatonic consists in an arrangement of two tones and a semitone. The chromatic, which consists of nothing but semitones, exhibits the internal uniformity natural to an entelechy, and reveals its perfection in its triplication of the interval of the semitone.[110] The enharmonic, whose sequence is diesis, diesis, ditone, displays in its dieses the way the soul is fitted together [*hērmosmenon*] out of very small parts, and is virtually indivisible and without bulk; while through the ditone it displays the vigorous and united character of the soul's self-movement.

Again, in relation to the universe at large, the enharmonic, being simple and immutable, resembles the efficient cause; the chromatic, being divided into three, a perfect number, implicitly reveals the region which is the cause of the life of bodies; and the diatonic, being divided into two, reveals the divisible and impressionable character of matter.[111]

[104] Again reflecting Aristoxenian, not Pythagorean theory; contrast, for example, 8 Eucl. *Sect. Can.* propositions 9 and 14. [105] See Book II, 66.17–18.

[106] Nature (*physis*) is a directive, purposive principle of change. Natural things, as such, are more than merely inert bodies. But they are not rational or capable of deliberation, hence they are less than souls. See Aristotle *Phys.* Book II, and cf. *SVF* vol. 2, 714, 1,013 (p. 302, lines 36ff.), etc. Cf. also 110.10ff. above.

[107] On the perfection of the number 3 see 100.1, cf. 102.4–5. The general idea of the soul as coming 'from without' is of course common, notably in Plato; cf. also the obscure argument of Aristotle *De Anima* Book III ch. 5. For Stoic versions see, for example, *SVF* vol. 2, 634, 1,038. The expression *nous thyrathen*, 'mind from without', appears in a passage tentatively attributed to Cleanthes at *SVF* vol. 1, p. 118, line 7. The soul was first described as an 'entelechy', i.e., a completeness or actuality, by Aristotle (e.g., *De Anima* 412a 27).

[108] 'Hard' translates *stereon*, rendered as 'solid' at 110.23. With this use of the progression 1, 2, 3, 4 compare also Aristotle's reports about the *De Philosophia* of Plato at *De Anima* 404b.

[109] The text is defective. The form of the sentence suggests that it is the human body (not 'body' generally) that is its subject. Perhaps the sense is that this consists of three main parts (legs, trunk, head), of which the last is roughly half the size of each of the others. A similar analysis could be applied to some of its more prominent parts (upper arm, forearm, hand; thigh, lower leg, foot). Perhaps the text was *tēi duoin schedon kai hēmiseos synthesei*, 'the combination of approximately two and a half (units)'.

[110] There seems to be a gap in the text (after 'entelechy'), but the sense must be roughly as I have given it. The triplication of the semitone occurs in the highest interval of the tetrachord (the usual form of chromatic being semitone, semitone, interval of three semitones). The usage *trihēmitonion* for this interval (e.g., 16.7–8), rather than 'tone and a half', justifies the claim that the chromatic consists of nothing but semitones.

[111] On the contrast between efficient cause and matter see 108.8ff. The region that causes life is the aether, associated with the number 2, not 3, at 109.19ff. (but it is said there

Chapter 12 None of this need conflict with what we said previously about dimensions.[112] Where we are drawing distinctions about magnitude, the region
112 of aether is manifested in two dimensions,[113] that of matter in three. Where we are setting out their capacities [*dynamis*], the region of aether, being perfect, is assigned the number of 3, and that of matter, being imperfect and passive, embraces the number 2.[114] Nor is the explanation of this exchange anomalous [*anharmostos*]. Just as from these numbers there is generated 6, a perfect number, and the first to be the sum of its own elements (which is why it was called 'marriage'),[115] so when these two natures, the aetherial and the material, come together, there is constituted every kind of thing that lives as a body. Again, if we consider the three regions we have described, and set out the ratios which they bear to one another – that of the first to the second and the third, and that of the second to the third – the quantity we have completed will be that of the number 6.[116]

All the numbers to do with music are sacred and of perfect efficacy. Thus the number 9 displays the *harmonia* of the universe[117] (since there are seven planets, the zodiac making eight, and the so-called 'starless sphere' making nine). The number 18 juxtaposes successively three most elegant terms: 16 and 18 are the only plane numbers which have the areas equal to their own circumferences, and thus display the symmetry of the container with that which is contained, that is, of the body with the soul.[118] The number 17 is a mean term between those just mentioned, and displays the mean that lies naturally between them; and it teaches us of the first natural emanation imparted from the moon to things on earth.[119] The number 36, which expresses the diesis, represents the first creation of a human creature.[120]

Chapter 13 Turning to the tables of the *tropoi*, which we also call *tonoi*, if we write each of them out in the three genera mixed together the total number

to be a plane, and planes are implicitly linked with the number 3 at 102.6ff.). More immediately, 3 is the number of nature, and hence life, at 111.5ff.

112 The direct reference is to 110.21–7; see also n. 111 above. From different points of view the numbers 2 and 3 are associated respectively with chromatic and diatonic or vice versa.

113 As being a plane, 109.19ff.

114 The relevant characterisations of these numbers are at 102.3–5.

115 See 102.10–12. It is both the product of 2 and 3, and the sum (and the product) of its own elements, 1, 2, 3.

116 The 'regions' must include that of the divine, beyond the aether; cf. 110.10ff. The ratios, as listed, must be 1:2, 1:3, 2:3.

117 'The number 9', lit. 'the epogdoic number'. Compare the epitritic and hemiolic numbers of 100.1–2. On the astronomical and other significances of the number 9 see 102.16–22. The 'starless sphere' mentioned in the following parenthesis is that of the surrounding aether.

118 A square of 16 square units has a circumference of 16 units; a rectangle with sides 3 and 6 has an area of 18 square units and a circumference of 18 units. The musical significance of the numbers 16, 17, 18 is explained at 95.19ff.

119 The role of the moon in the generation of life is mentioned at 121.4–5. I can find nothing to connect it specifically with the number 17, but its life-giving role may be conceived as a mediation between soul and body.

120 The number 36 is one of those involved in the investigation of the diesis at 96.4ff. It reappears in a cosmological role at 124.28. Its significance for human births depends on its being the square of the 'marriage number', 6; see also chs. 18 and 23.

of notes will be twenty-eight,[121] equal to the number of times the light of the
113 moon appears – for that is the number of times it is seen each month. Next, if they are set out genus by genus, omitting the *systēma synēmmenōn*, which is fundamentally equivalent to the *systēma diezeugmenōn*, they have a total of fifteen notes that are sounded, equal in number to the days in which the moon waxes.[122] Just as the moon, after increasing up to the fifteenth day, begins to wane again, so the voice, after rising as far as fifteen notes, turns back downwards once more, and the number occurring in connection with both subjects is the same.[123] Again, the month goes up to twenty-nine days (sometimes including the 'old and new' day, while if we sing the notes upwards and downwards, sounding the fifteenth (the last in their ascent and the first in their descent) only once, they make up the same number.[124]

Chapter 14 Since there are five tetrachords in each of the *tropoi*,[125] each of them plainly corresponds to one of our organs of sense. The lowest, the tetrachord *hypatōn*, resembles touch; for touch comes first,[126] being possessed even by new-born babies, the coldness of whose surroundings makes them cry: and it is also by nature the heaviest [*barytatē*], since it extends through the whole body. The second, the tetrachord *mesōn*, resembles taste; for we need taste before the remaining senses, in order to live. It is also similar to touch, since taste is the tongue's sense of touch. The third, called the tetrachord *synēmmenōn*, should be aligned with smell; for smell follows on from taste, and the two senses are allied with one another.[127] Many people have successfully used smells, rather than nourishment, as doctors do, to revive those who have fainted. The fourth, which we call the tetrachord *diezeugmenōn*, should be compared with hearing; for hearing is located at a distance from the other senses, and it does not lie all in the same part of the
114 body, as the nostrils do: its organs are separated [*diezygēsan*] from one another, one on the right and one on the opposite side.[128] The last one, the

121 On the *tonoi* see Book I chs. 10–11: the twenty-eight notes are those listed at 7.18ff.

122 The two *systēmata* are alternative series, not parts of the same sequence. See Book I, 8.21–9.9. Aristides is considering the double octave, in any one *tonos* and genus, from *proslambanomenos* through the tetrachords *hypatōn*, *mesōn*, *diezeugmenōn*, *hyperbolaiōn*; this contains fifteen notes.

123 The moon waxes for fourteen days, not fifteen. But Aristides is construing notes in the Aristoxenian manner, as points bounding intervals. Fifteen notes bound fourteen intervals, and fifteen points of time bound fourteen durations, here days. The moon waxes from the initial point of time up to the fifteenth.

124 The expression 'old and new day' commonly refers straightforwardly to the last day of a month. If the sense is comparable here, the point will be that the twenty-ninth point of time in the old month is simultaneously the first of the next; compare Plato, *Cratylus* 409b–c. This would be a relevant explication of the calculations here, but the word 'sometimes' is against it. Possibly the reference is to the extra day occasionally intercalated (as sometimes in the Attic calendar) between one month and the next. But in that case, it is not clear how it is to bear on the moon's 'natural arithmetic', nor how an intercalation could be paralleled in a musical system.

125 Listed in Book I at 15.2–4; cf. also Book I ch. 6.

126 A use of *hypatos* to mean 'first' is mentioned at 8.7–9.

127 The point is that the tetrachord *synēmmenōn* is conjoined with *mesōn*, not disjoined from it as *diezeugmenōn* is; cf. 9.1–2.

128 See the account given of *diezeugmenōn* in Book I, 9.6–8.

tetrachord *hyperbolaiōn*, should be likened to sight. Just as this tetrachord is the sharpest [*oxytaton*] of the *systēmata*, so is sight the sharpest of all the senses:[129] it does not require bodies to be near it, as do the others, but has in itself the power to project itself at its objects.

We also find that the five primary elements are analogous to the tetrachords, earth, being heaviest [*barytaton*], to the tetrachord *hypatōn*, water to the tetrachord *mesōn*, since it is closest to earth, air to the tetrachord *synēmmenōn*. (For air sinks down and is drawn down into the depths of the sea and the caves of the earth to enable creatures there to breathe, since it is essential to them, and without it they would not survive.) Fire is analogous to the tetrachord *diezeugmenōn*, since movement downwards is contrary to its nature and it can be drawn downwards only by an external force, while its natural movement is upwards, away from things on the earth. The aether should be assigned to the tetrachord *hyperbolaiōn*, since aether is the highest.

Further, we can see that each of these elements is analogous to one of our sense organs. Earth, being resistant, is analogous to touch, which apprehends hard and soft things; water to taste, since it is through wetness that it perceives qualities; air to smell, since all pleasant scents are discriminated by breathing in; fire to hearing, since this organ of sense is active in great heat, but is ruined and destroyed by cold, which is why the ears' wind-break has been placed in front of them; and aether to sight, since the activity of this sense comes through the agency of a radiant body.

Chapter 15 Concerning the fifths, of which there are three,[130] we shall once again fit together [*synarmottontes*] like with like, and thus say what is appropriate about each of them. The fifth through *hypatōn* and *mesōn*,[131] considered in relation to the nature of man, represents both taste and touch
115 simultaneously, because their activity goes on among resistant objects: in relation to the universe as a whole it represents earth and water, because of their equal tendency towards the centre. We assign the fifth belonging to the conjunct series to the sense of smell in the one sphere, and to air in the other, for reasons we have given already. The fifth of the disjunct series we assign in the one sphere to both hearing and sight, since the two have a common association with heat, which is never found without light, and in the other to fire and aether, both of which belong to the upper region.[132]

Of the two kinds of octave, the first represents, in man, four of the organs of sense at once, those whose activity involves only external objects and our own faculties: the second represents sight, as being greater, since it does not fulfil its

[129] *Oxys*, 'sharp', the usual Greek term for 'high in pitch', has many of the analogical associations of 'sharp' in English. The sharpness of a knife, of the eyes, of the mind, of shrill sounds, etc., all fall naturally within its scope.

[130] On the vexed issue of the three pentachords see Book I, 15.4–5 with n. 97.

[131] This is presumably the *mesoeidēs* fifth of Book II, 81.21–3, which runs from diatonic *lichanos hypatōn* to *mesē*.

[132] These characterisations draw on the previous chapter in obvious ways. The problem of identifying the latter two fifths is not much eased by this passage, except that the final sentence hints that the third one includes a note or notes in the tetrachord *hyperbolaiōn*, as well as in *diezeugmenōn*; see also n. 142 below.

function through the conjunction of just two things, but needs the help of a third, light.[133] In the universe at large, the first *systēma* represents the material region, which is moved in a straight line, the second that of aether, which is moved in a circle.[134]

Chapter 16 When discussing the human soul, it is entirely reasonable to compare the *systēmata* with the virtues.[135] The *systēma* of *hypatōn* and *mesōn* is to be assigned to moderation, for the activity of this virtue has two aspects.[136] There is one kind of pleasure which is illegitimate: here the state of total abstention from it, and of being undisturbed by it, is what we praise, and we may not unreasonably compare this with the lowest of the *systēmata*. There is another, legitimate kind of pleasure, where praise is given for goodness of proportion [*symmetria*], and we may sensibly classify this with the *systēma mesōn*.[137] The *systēma synēmmenōn* should be associated with justice. The nature of justice links it with moderation, and it exercises its power, both in matters of politics and in private good deeds, by drawing mankind together through sharing.[138] The *systēma diezeugmenōn* should be equated with courage. This virtue is the most effective in keeping us free from vice of every kind, releasing the soul from its fondness for the body.[139] The *systēma*
116 *hyperbolaiōn* is the natural equivalent of wisdom, since the former is the limit of high pitch, and the good of the latter lies in an extreme.[140]

Again, if we liken these things to the three fifths,[141] we shall assign the first of the fifths to two virtues, taking justice and moderation together as constituting the principle of good order for the appetitive part of the soul; the second to courage, since it displays the virtue and the being of the spirited part, and its inclination towards each of the other two natures in turn;[142] the third

133 This recalls Plato *Rep.* 507c–e. The two 'kinds' of octave are simply the lower and the higher of the two-octave system. They differ in that the higher is more complex, including the tetrachord *synēmmenōn* in parallel with *diezeugmenōn*. See further in chapter 17 below.

134 On these movements see 109.18ff. 'Circularity' occurs in the upper octave (not in the lower) in a sense suggested in Book 1 at 16.24–17.2, 29.12–13, where 'circular' melody or progression is that which ascends in conjunction and descends in disjunction, or conversely. (But the technical term for 'circular' in that sense, *peripherēs*, is not used here.)

135 The virtues considered here are those treated as primary by Plato, e.g., *Rep.* 427e ff.

136 This *systēma* may be the whole lower octave, including *proslambanomenos*, but Aristides goes on to reduce it to the conjunction of the two named tetrachords, which together span only a seventh.

137 This complex attitude to pleasures derives from Plato; see *Philebus* 44e ff., particularly 63d–64a. The *systēmata* mentioned here and in the rest of the paragraph are of course the tetrachords.

138 Moderation and justice are distinguished from the other virtues as involving cooperative and harmonious relations, for example, at Plato *Rep.* 403e–432a, 433a–c, 442a–d, 443c ff.

139 'Moral' and not merely 'physical' courage is at issue here; cf. *Rep.* 429a–430b.

140 This imports an Aristotelian touch, reflecting the contrast in his ethical writings between moral virtues, which are dispositions towards a mean between excess and deficiency, and intellectual excellence, which is not. But compare Plato *Rep.* 474b ff.

141 Those discussed above at 114.29ff.

142 Compare *Rep.* 439e–440e. There may be a hint here that this pentachord (*synēmmenōn*) contains notes occurring also in both the pentachords called *mesōn* and *diezeugmenōn* at 15.4–5; cf. n. 132 above.

to wisdom, displaying as it does the essence of rationality. Further, we should compare the duality of the octave to the generic duality of the soul, associating the first with the active and irrational part, the second with the rational, on the basis of their resemblances.[143]

Chapter 17 Actions performed during life fall into two groups, one impelling us towards virtue, the other towards vice. You should notice how music also has implicitly picked out these different patterns of life. The first of the complete *systēmata* resembles, so they say, the whole of our early life, in which we all live in the same way, and all succumb equally to our emotions.[144] The *systēma* which begins from *mesē* represents the alternative ways of life which come after childhood. The conjunct form is shorter, and thereby represents the effortlessness of vice, while the fact that its semitonal conjunction makes it closer and sweeter in sound represents the facile sweetness of vice's nature. Here one changes not at all in adult life, but gains an increase only in wickedness.[145] The disjunct form by its greater length indicates the difficulty of virtue; and its modulation by a tone, and the fact that it is bounded by the power of the voice, indicate both the violence and intensity of the conversion
117 to the better form of life, and the power of virtue.[146] For the being of virtue lies in an extreme, whereas vice of any kind is a sign of an imperfect nature, and is nothing but weakness. This is why some wise men have called the perfection of virtue 'divinity', revealing it to be each man's greatest adornment and preserver; and understanding that the perfection of vice diminishes and destroys every nature and every constitution, they called it 'bestiality', as being unsuitable for a rational creature. Hesiod too speaks somewhere of these two, describing each of the conditions in words which clearly reflect the way a musician might speak of *systēmata*:

'Vice can easily be had in abundance: the way is smooth and lies close by. But the immortal gods have placed sweat between us and virtue. The path to it is long and steep, and rough at the beginning. Yet hard though it is, it is easy once you have reached the summit' (*Works and Days* 287–92).[147]

143 For the generic duality of the soul see, for example, Book II, 54.21–3.

144 The 'complete *systēmata*' are the octaves of the system combining GPS and LPS. Aristides' distinctions in what follows run parallel to those of 115.8ff.

145 The 'conjunct *systēma*' is conceived as ending with the tetrachord *synēmmenōn*, not proceeding to *hyperbolaiōn*; hence it is shorter. (This is the usual treatment, but contrast 10 Nicomachus *Ench.* 259.16–17.) The tetrachord *synēmmenōn* is not only 'closer' to *mesōn* than is *diezeugmenōn*, but movement into it involves no new principle of progression: the conjunction repeats that between *hypatōn* and *mesōn*.

146 On the shift from a disjunct to a conjunct *systēma* (or vice versa) as a form of modulation, see n. 126 to Book I. But in moving from the lower octave to the higher through the disjunctive tone no modulation of *systēma* or *tonos* is involved (on the contrary, it would be involved in progression to the tetrachord *synēmmenōn*, 11 Ptol. *Harm.* Book II ch. 6). By *metabolē* here Aristides probably means only 'shift to a new starting point', i.e., to one not identical with the last note of the previous tetrachord. On modulation see also chapters 26–7 below. On the limits of the voice compare 103.19–21.

147 This passage was a favourite with Greek moralists. Aristides may have recalled Plato's use of it (half quoted, half paraphrased) at *Rep.* 364c–d.

Chapter 18 Let us now show how the procreations of rational creatures are also in sympathy with the musical proportions.[148] We shall find that a pregnancy that comes to fruition after seven monthly cycles corresponds to the harmonic ratios. For if we take the number 6, the first symbol of procreation,[149] and put together the sequence of numbers which exhibit in relation to it the harmonic proportions, the epitritic, the hemiolic and the duple, the numbers in our sequence will be the following: 6, 8, 9, 12. If we add these numbers, they make 35, the number in the course of which, so they say, seven-month children
118 are thoroughly formed:[150] and if we multiply 35 by 6 we make 210, equal to the number of seven-month children's daily cycles.[151] Next, if we add the rhythmical ratios, equal, duple, hemiolic and epitritic, to the same terms, beginning from the unit, the series will be 1, 2, 3, 4. Of these the unit, which has no number preceding it in order, will stand to itself in equal ratio: 2 stands to 1 in duple ratio, 3 to 2 in hemiolic, and 4 to 3 in epitritic. By adding these numbers together we complete the number 10,[152] and by adding this to 35 we shall make 45, the number in the course of which they say that nine-month children are formed.[153] If we multiply 45 by 6, as this is a perfect number, we shall get the number which belongs to nine-month children, 270: for that is the number of daily cycles after which these children are born alive.[154] We can see that eight-month children are sometimes conceived, through their participation in the smallest ratios: but since they do not participate in all of them, they are never born alive.[155]

Again, if we relate the ratios of intervals to what people say about the proportions of bodies,[156] we shall find that those which have a share in beauty are proportional to the concordant intervals, while those of the opposite kind

[148] On cosmic 'sympathy' cf. 105.4–5 and Book II, 89.23 with n. 200. With this whole chapter compare Plato *Rep.* 546a–d, but the calculations involved here are nearer to those found in some of the Hippocratic writings, especially *De Victu* 1.8, *De Septim.* 9, *De Carn.* 19. See also Proclus *In Rempubl.* vol. 2, 26.15–25, 33.9–36.2 (Kroll), where notions attributed to Pythagoras and Pythagoreans closely resemble those used by Aristides. The topic is revisited in chapter 23; cf. also II Ptol. *Harm.* Book III ch. 4.

[149] See 102.12, 112.6.

[150] On the relation between the concordant ratios and the number 35 see 100.8–11. In ancient theory (including the texts cited in n. 148) seven-month and nine-month pregnancies make viable children, whereas eight-month pregnancies do not. For a more cautious and empirical approach see Aristotle *Hist. An.* 584a 35–b25. That Pythagoreans reckoned the formation of a seven-month child to occupy 35 days is also recorded by Proclus (*In Rempubl.* vol. 2, 33.14–16).

[151] Seven months of thirty days each, as in the *De Carn.* and Proclus, and, for example, Plut. *De Anim. Procr.* 1018b.

[152] These are the numbers of the Pythagorean *tetraktys* of the decad.

[153] A different way of reaching the number 45 is given at Proclus *In Rempubl.* vol. 2, 35.2–4.

[154] Nine months of thirty days each; cf. n. 151 above.

[155] Compare n. 150 above. In *De Septim.* and elsewhere the non-viability of eight-month children is related to the 'incompleteness' of even numbers. Aristides' reasoning is not quite clear, but may be as follows. On the analogy of the previous cases, the number in eight months is 240, and the underlying 'formation number' will be found by dividing it by 6. It is therefore 40. This number results from the addition to 35 of some, but not all, of the numbers involved in the rhythmical or concordant ratios. Compare Proclus' rather subtler discussion at *In Rempubl.* vol. 2, 35.4ff.

[156] This paragraph picks up the idea signalled at 110.4–5.

resemble the discordant ones. We should not treat as beautiful a body whose appearance is contemptibly effeminate, but one displaying the readiness of the soul for manly excellence:[157] people who possess this are responsive when summoned to virtue and worthy of being drawn into friendship. This is what the noble Plato means when he says that the objective of music is love of the beautiful (*Republic* 403c).

Chapter 19 Consider now the fact that the ratios of the bodies that constitute the universe are also concordant, in the following way. To fire,
119 because of its affinity with the pyramid, we assign the number 4, which is equal in number to the angles of that figure. To earth, as being a cube, we assign the number 6, equal to its surfaces; to air, as an octahedron, we assign the number 8, because of its surfaces; and to water, as an eikosahedron, we assign the number 12, because of its angles.[158] In both the pair that moves upwards and the pair that moves downwards, we have taken the angles of the active elements, it being by means of their angles that they act, because of their sharpness [*oxytēs*], and the surfaces of the passive ones, it being along their surfaces that they are cut: for air is brought together and dispersed by fire, earth by water.[159]

In view of these facts, it is clear that each of the seasons of the year will also take the number belonging to the element which is like it, spring taking 8, the number of air, since it resembles it in its tenderness, summer taking 4, the number of fire, because of its heat, autumn 6, the number of earth, because of its dryness, and winter 12, the number of water, because of its wetness. Thus spring, as Pythagoras is reported to have said, stands to autumn in the ratio of the fourth, to winter in that of the fifth, and to summer in that of the octave, as is shown in the diagram below. Their proportionality is thus musical, completing the duple ratio in various ways from epitritic and hemiolic, and the other way round.[160]

Chapter 20 There is also, then, a clear paradigm of music in the body of the universe. For the fourth, once again, displays the *tetraktys* of matter, the fifth indicates the additional body, aether, and the octave the melodious movement of the planets, about which we must make a few remarks.[161]

[157] Note the reintroduction of the male–female dichotomy exploited in Book II.

[158] These descriptions are based on Plato *Tim.* 55d–56b.

[159] Compare Plato *Tim.* 56c–57c. With the contrast between active and passive compare 108.8ff., 111.22ff. above.

[160] Plutarch attributes these ratios between the seasons to 'the Chaldeans', i.e., to astronomers rather than to scholars reflecting on the *Timaeus*' identification of the elements with the four regular solids. See *De Anim. Procr.* 1028f with Cherniss' note to the Loeb edition. It is at least plain that Aristides intends the numbers to represent the characters of seasons rather than their relative durations or any other temporal measure. The diagram to which he refers is missing.

[161] A *tetraktys* is any coordinated set of four items. On the aether as a fifth element see particularly Book II ch. 17, Book III, 109.18–21 and ch. 12. On the planets see 102.19–22, 112.15–17. Aristides' 'remarks' about their music occupy chapters 20–2, and some of chapter 23. On the contrast between the body and the soul of the universe see 125.29–31.

Poets, stirred by the breath of music, constantly sing of it, calling it the chorus of the stars; but wise men who pursued truth have hunted it out through demonstrations of the following sort. They say that any body moving at great speed through a homogeneous and readily yielding medium which is made to form waves by its impact, like the circles that spread out in water when a pebble is thrown into it, produces sound of some kind.[162] The air in these regions is simple and well accommodated to our organs, and can make sounds of various kinds (thus a similar impact made by a man and a woman, or by a child and an old man, produces a dissimilar sound): and in just the same way, they say, since the aether in those other regions is simple, and the bodies in it are many and various (as these writers demonstrate) in respect of their sizes and their surfaces, their activities and their emanations, each of them strikes it in accordance with its own special nature and power.[163] Now to us the sounds are imperceptible, since our hearing is not adjusted to it, being made too dull by the great distance and by its contamination with body (just as those of us who are hard of hearing detect neither voices nor even – most remarkably – thunder and other such loud noises); but the better of those who have lived good lives among men are permitted to come close to this experience and to be not wholly deprived of a share in this kind of felicity.[164] Just as our nature makes it hard for us to become spectators of the greater powers,[165] and yet those who have reached the pinnacle of virtue and of the necessary kind of understanding can look unharmed upon the presence of the divine forms, so also it is impossible for men, particularly if they are unworthy, to hear through their own powers the sound of the universe, but men of virtue and understanding are ungrudgingly permitted by the higher powers to share (just occasionally, yet they do share) in this blessed privilege. These sounds progress, they say, in the pattern of organisation which underlies the notes.[166] Some of the notes are

[162] The theory of sound-production implied here has most affinities with that of the Peripatetics: see especially 3.15 Aristotle *De Anima* 419b–421a, 5 *De Audib.* 800a ff., and 1.6 Aristotle *De Caelo* 290b, whose reports are one of Aristides' sources here. For other accounts of celestial harmony see particularly 2.2 Plato *Rep.* 616b ff., 10 Nicomachus *Ench.* ch. 3, 11 Ptol. *Harm.* Book III chs. 8ff. For further references and discussion see Burkert (1972), pp. 350–5.

[163] That the same medium, struck in different ways by qualitatively different agents, produces different sounds, is a major theme of 5 *De Audib.* Aristides' turn of phrase here ('special nature and power') hints at the peculiar character of his own version of the doctrine of cosmic harmony; see n. 172 below.

[164] A commoner explanation of our inability to hear the sounds is that mentioned by Aristotle at 1.6 *De Caelo* 290b. Aristides assimilates it to the decline in our intellectual and moral purity induced by our descent into this world, and it is analogous to the embodied soul's inability, as conceived by Plato, to 'see' the Forms. See particularly chapter 7 above, and the account of the soul's descent in Book II ch. 2; cf. also chapter 27 below.

[165] 'Spectators' translates *epoptai* (cf. Plato *Symp.* 210a); these were initiates to the third level of the mysteries. On the five stages of initiation see Theon Smyrn. 14.18–16.2. On Aristides' conception of music as a 'mystery' see especially Book II ch. 7, Book III chs. 7, 9, 27.

[166] That is, those of ordinary terrestrial music; their 'pattern' is outlined in Book I ch. 6.

male, some female, and some a combination of both:[167] and with each of them they identify the power and activity of a planet.

Chapter 21 We must therefore first define the nature of male and female. Among bodies the male is hard and dry, among souls active and work-loving,
121 while the female is damp and diffuse, static and disinclined to work.[168] It is from the prevalence of these or their equality in a mixture that distinctions between activities arise. Thus the globe of the Moon, which is diffuse and damp, and especially responsible for all bodily generation,[169] emits the note that is sung to the letter epsilon [ĕ], which is female with a slight tinge of the male.[170] For the Moon is mostly female, in that it absorbs emanations from the others, but still has an element of maleness, since it also sends out to the earth, from its own resources, the powers that generate and nourish bodies. This is also made clear by the devotees of its rites and mysteries. They call it 'male-female', though in fact it is more female than male, and they associate with it the horned *aulos*, which has a similar crescent shape, and is very deep-pitched, comparable to *proslambanomenos* in its sounding.[171]

The globe of Mercury is for the most part a source of dryness, because of its nearness, in terms of distance, to the sun, but on the brief occasions when it moves further away from it, it becomes a source of moisture. It enjoys appearing in the daytime, for the most part, and only rarely at night. Its sound is also male-female, but predominantly male, because of its special association with dryness and with daylight appearances. The globe of Venus is bright and sweet and brings joy, like its goddess. It is moist for the most part and loves the night, and the sound it emits is in the highest degree female. The globe of the Sun is drier and burning, and is through and through hot and active: its sound is male. That of Mars is hot and impetuous, but has a delight in appearing among the damps of night: thus it produces a sound which displays both qualities, but is predominantly male. The globe of Jupiter is pleasing in
122 all that it does, and is very similar in almost all respects to that of Venus. It is more relaxed than its neighbours on either side, diminishing the heat of Mars and moderating the cold of Saturn, and having a well blended mixture of both. Like Venus it is fertile among the breaths of day: it brings marriage and sows children when bodies are joined together. Not surprisingly, then, the note it sounds is more female. Lastly, the globe of Saturn, which is harsh, parched and burdensome, sounds a male note.[172]

[167] See Book II, chs. 12–14.

[168] These characterisations were developed in Book II, especially in chapters 8 and 10.

[169] Compare 112.24–6.

[170] For the system of solmisation referred to here see Book II ch., 14. The letter epsilon belongs to the foundational notes of the system, *mesē* and *proslambanomenos*, but the Moon is identified only with the latter. Compare line 14 below and see 122.31–123.4.

[171] The description applies to the Phrygian or *elymos aulos*: see 85.5.

[172] These characterisations imply that the solmisation letter attached to Mercury and Mars is alpha, that of Venus and Jupiter is eta, that of the Sun and Saturn is omega. Those given alpha are the lowest in a tetrachord, those with eta next, those with omega third:

There remains the concord of a fourth upwards from *mesē* in the conjunct series:[173] and we find that the zodiac too has four regions and magnitudes. It will not be unreasonable to assign the first of them to *mesē*, the second to the note that follows it, and so on for the rest.[174] Each of these regions, through its similarity to the planets, naturally shares similarly in their activities too.[175] Those of the planets which occupy smaller or greater regions than each of the magnitudes mentioned may reasonably be supposed to give out sounds which exceed or fall short of those intervals, so that among them is found the constitution of enharmonic melody: for though their bodies are immense, they can sound the smallest of intervals.[176]

Seven other notes still remain.[177] We are prevented from assigning them to the regions of the zodiac, since the activity of the latter is simple. But the planets have two-fold powers, since they exercise one kind of power by night,

see 79.26ff. with n. 131. It becomes clear in the sequel that we have so far worked through seven notes of the lower octave of the system, in order from lowest to highest. Moon is *proslambanomenos*, Mercury is *hypatē hypatōn*, Venus *parhypatē hypatōn*, Sun *lichanos (diatonos) hypatōn*, Mars *hypatē mesōn*, Jupiter *parhypatē mesōn*, Saturn *lichanos (diatonos) mesōn*. The order of planets is similar in most sources, except that Venus usually precedes Mercury, as in 10 Nicomachus *Ench.* ch. 3. Aristides also differs from Nicomachus in making the most distant planet the highest, not the lowest in pitch, and from Ptolemy (who agrees on this point) in other aspects of his system. Though Aristides retains (so far at least) an astronomically orthodox order, his arrangement does not depend on a view about the planets' relative distances or velocities. It is derived wholly from astrological conceptions of the qualities attached to each (cf. Aristotle *Meteorol.* 4.1, Ptol. *Tetr.* Book I chs. 4–23, and 11 *Harm.* Book III ch. 16) and their assimilation to the male–female dichotomy. Nothing is necessarily implied about their spatial relations.

173 That is, the tetrachord *synēmmenōn*.

174 For the four regions of the Zodiac see Ptol. *Tetr.* Book I ch. 18; cf. 11 *Harm.* Book III ch. 9.

175 Each region of the Zodiac is associated with specific planets: the first with the Sun and Jupiter, the second with Venus and Mars, the third with Saturn and Mercury, the fourth with Mars, Venus and Moon.

176 The references to regions and magnitudes (*topoi, megethē*) are best understood, harmonically speaking, in a quasi-Aristoxenian sense. For Aristoxenus, each moveable note has a *topos* within which it moves: *paranētē synēmmenōn*, for instance, like *lichanos*, moves within the region of a tone (see 7 *El. Harm.* 22.24ff.). The magnitude of the interval between one note and the next in a tetrachord depends on the locus of each within its *topos*, which depends in turn on the genus of the system. Each section of the Zodiac here represents a region of pitch, the regions being adjacent and non-overlapping (cf. 7 *El. Harm.* 23.28–33): the magnitude of interval between 'notes' in adjacent regions depends on the position of the governing planets within their sections, perhaps at the time when they are at the maximum distance, in a given passage, from the boundary of the adjacent section. Since the courses of the planets appear irregular, these magnitudes will differ in different phases of the planets' courses, sometimes generating 'enharmonic' magnitudes, sometimes 'chromatic', and so on. Though the four sections of the Zodiac are equal in astronomical space, they must then represent *topoi* of different sizes: the second will span an enharmonic diesis, the third a tone (7 *El. Harm.* 23.24–6), while the first and fourth, representing fixed notes, will have no discernible range of variation (cf. *El. Harm.* 55.3–6). This need be no objection, from Aristides' point of view; cf. n. 172 above. For a different and fuller attempt to correlate planetary movements with differences of genus see 11 Ptol. *Harm.* Book III ch. 11, cf. 10 Nicomachus *Ench.* ch. 3 with n. 17.

177 Those of the tetrachords *diezeugmenōn* and *hyperbolaiōn*.

another by day.[178] Again, then, we shall assign to each of them one of the remaining notes, on the principle of opposition to their daytime powers: to the active or burdensome among them we shall assign the male, or the mixture in which maleness predominates, and with the less active or moister we shall associate the female, or that which is mainly female.[179]

123 *Proslambanomenos* is the only note to be found among the planets which is vocalised through the letter epsilon.[180] This letter corresponds to the nature of the Moon, and men like to call it the symbol of generation, since it is put together, if we consider it as a number, from the first even and the first odd.[181]

Chapter 22 From these points we can see how we shall assign to each of the gods and to the things which resemble them the *systēmata* and *tropoi* which correspond to their own particular notes.[182] Thus we shall allot to the Moon and to the powers that correspond to it the *harmonia* beginning from *proslambanomenos*,[183] to Mercury and the things resembling it the second *systēma*, to the third planet and the things like it the third *systēma*, to the fourth and things resembling it the fourth, and so on.[184] To the zodiac as a whole we shall assign all the species together, but to the magnitudes which form its parts we should assign those corresponding to their respective notes, constructing

178 Compare Ptol. *Tetr.* Book 1 ch. 7.

179 The scheme of oppositions requires that planets previously assigned to female notes become male, and conversely. One would expect those that are intermediate in their daytime powers to remain so by night, while changing in the character that is predominant. But this scheme cannot be followed consistently with the underlying solmisation system: the notes from *paramesē* to *nētē hyperbolaiōn* include no predominantly female intermediates into which those previously allied to the letter alpha could be converted. If the order of planets from lowest to highest remains constant (which is likely, but not certain), Moon becomes *paramesē* (alpha, mainly male), Mercury *tritē diezeugmenōn* (eta, female), Venus *paranētē diezeugmenōn* (omega, male), Sun *nētē diezeugmenōn* (alpha), Mars *tritē hyperbolaiōn* (eta), Jupiter *paranētē hyperbolaiōn* (omega), Saturn *nētē hyperbolaiōn* (alpha). This seems the most probable scheme, but note that Sun and Saturn have only changed from being male to being predominantly male.

180 Compare n. 170 above. The note *mesē*, which also has the letter epsilon, is attached to a sector of the Zodiac (122.9ff.), not to a planet.

181 The letter epsilon serves in Greek for the numeral 5. The reasoning is similar to that which links generation and marriage elsewhere with the number 6 (see, for example, 102.10–12). Their association with 5 is probably older (see Burkert (1972), p. 467 n. 8). On the affinity between epsilon and Moon see 121.3ff.

182 The system of *tropoi* or *tonoi* intended here is plainly one of 'modal' octave species, corresponding to *harmoniai* in the old sense, and not one of key. See Book 1 chs. 10–11 with notes, and ch. 8, particularly 15.9–19. Modal conceptions do appear in chapters 10–11, though confusedly, but the present discussion is to be related primarily to the simpler conceptions of chapter 8, where the system of seven *harmoniai* (corresponding to the seven planets), based in the analysis of the octave, has not been complicated by the introduction of thirteen (or fifteen) double-octave *tonoi* whose *proslambanomenoi* stand a semitone apart. In the present chapter *proslambanomenos* is the name of the lowest note of the GPS, conceived as a fixed framework. It is not a designation for the starting point of each *tonos*.

183 Up to *mesē* in the perfect system. This is structurally identical with the *harmonia* called Hypodorian at 15.15, which is there treated as running upwards from *mesē*.

184 These are apparently the *harmoniai* beginning respectively from *hypatē hypatōn*, *parhypatē hypatōn*, and so on; for their names see 15.11–15.

their concordances downwards.[185] As for the remaining notes, which we have linked with the nocturnal activities of the planets, we shall take them in the order already given, but construct their *harmoniai* downwards. Finally, we shall obviously assign to each the nature of rhythm and of instruments that is appropriate to its *systēma*.[186] Where greater activity is needed, we shall summon up an extreme by means of similarity, and where more muted activity is needed we shall dilute excesses with an admixture of things that are dissimilar.[187]

Chapter 23 There are also ratios that hold between these sounds in the aether. The zodiac is divided into twelve parts, equal in number to the tones of music and to the circumference of a right-angled triangle, since that is the first such triangle that we can construct with sides that are exclusively rational.[188] Right-angled triangles with shorter sides will always have one irrational side, if the square on the hypotenuse is to be equal to those of the sides with the right angle between them. (Hence people also speak of the
124 number 5 as being the first that displays a rational diameter, using the term 'rational diameter' to refer to the diagonal of a rectangle whose right angles lie between sides in epitritic ratio.) Since this sort of triangle is constructed out of 3, 4 and 5, as I said, if we sum the sides arithmetically the total quantity is 12. Further, if we add the number 4 to each of the others in turn, we get first the number belonging to seven-month children, and then the number belonging to nine-month children. In each of these numbers, that which is human is brought to completion, since its composition is from the male and the female, as is expressed by the nature of the numbers which have been put together: whereas if we add 3 and 5, both of which are male, they express the fact that children born at eight months cannot be nourished or

185 The sequences (*harmoniai*) considered in the present passage are octave-structures. Since the double octave system is here conceived as a fixed linear framework, not a cycle, those beginning above *mesē* must be read downwards in order to stay within its compass. The parts of the Zodiac represent the notes of the tetrachord *synēmmenōn* (see 121.9ff.). Aristides' treatment involves an anomaly, since the note an octave below *tritē synēmmenōn* has no designation in the system. (It does exist in the system of *tonoi* and in the Alypian notation; see 23.6ff.) The octave species projected upwards in the diatonic series from this note (semitone; semitone, tone, tone; semitone, tone, tone; semitone) is a monstrosity, since it includes nine notes, as well as a sequence of two semitones, which is impossible in diatonic (7 Aristox. *El. Harm.* 65.8ff.). It appears that Aristides has not thought his scheme out very thoroughly, unless, despite all the evidence, he is thinking after all of pitch-keys and not octave species. (But it seems to me that Mathiesen's construction of the passage in terms of the *tonoi*, pp. 49–50, is quite unworkable.)

186 That is, those planets whose note (and its associated *systēma* or *harmonia*) is female are linked with female rhythms and instruments, and so on. (See Book II chs. 12 and 15 on rhythms, chs. 16, 18, 19 on instruments.)

187 This sentence is not descriptive of planetary music. It returns to the recommendations for musical education and therapy set out in Book II; see particularly 85.21ff.

188 The parts of the Zodiac here are those associated with the twelve familiar 'signs'; cf. 11 Ptol. *Harm.* Book III ch. 9. There are twelve tones (exactly in Aristoxenian theory, approximately in Pythagorean) in the two-octave system. The triangle in question is that with sides of 3, 4 and 5 unit lengths.

live.[189] If, on the other hand, we multiply each of the sides to give a figure with depth – since depth is the essence of body – we shall get a total of 216, roughly equal to the number belonging to seven-month children.[190] Again, if we take the three sides with one another to form a figure with depth, and add this to the previous number, the total is 276, the number belonging to nine-month children.[191] In each case there is an excess of 6, which – for reasons given previously – is the number of marriage.[192]

This figure also admits the four rhythmic ratios. The straight line of three units can be divided in duple ratio, that of four in equal ratio, and that of five in hemiolic. The lines bounding the right angle express the epitritic ratio. It is to this that Plato is referring when he speaks of 'an epitritic foundation linked to a set of five' (*Republic* 546c).[193]

That, then, is why we divide the whole zodiac into twelve. If we multiply 12 by the first perfect number, 3, we get 36, which they also call the 'rulers of the hours': and if we multiply this by another perfect number, 10, since each of
125 them holds power for ten days, we shall make 360, a number equal to the divisions of the whole circle.[194]

Around the centre point there are just four right angles. Each of these is divided into three acute angles by the diameters running from the points on the circumference which separate its twelve segments. Any two of these angles contain a segment of a hexagon, and display equal ratio; three contain that of a square, which stands to that of the hexagon in hemiolic ratio; four contain that of a triangle, standing to one of the others in epitritic and to the other in duple ratio. A group of five such angles stands in none of the harmonic ratios to any of the previous ones, and generates a discordant and uncoordinated chord of the circle. A group of six angles is in concord with all the quantities previously mentioned, and exhibits the diameter.[195] By a 'harmonic' ratio I

[189] On seven and nine month children, and on the non-viability of eight month babies, see chapter 18 above. The sums referred to here are simply 3 + 4, 5 + 4 and 3 + 5, giving the number of months in each case. Odd numbers were male, even numbers female (e.g., Aristotle *Metaph.* 1078b23, Plut. *De E* 388a–c); compare the characterisations of 5 and 6 at 123.1–4 and 102.10–12.

[190] Each number is to be cubed: the sum of the cubes of 3, 4, and 5 is 216. This is close to the number attached to seven-month children, which was 210 (see 117.29–118.1).

[191] That is, we multiply together 3, 4 and 5, giving 60, and add it to 216. The number given to these children was 270 (see 118.14).

[192] See 102.10–12.

[193] The 3 is divided as 2:1, the 4 as 2:2, the 5 as 3:2. The lines bounding the right angle are in the ratio 4:3. Plato's discussion of 'the complete geometrical number governing better and worse births' is evidently much in Aristides' mind, though his own account is much less involved.

[194] On the perfection of the number 3 see 100.17–19; on the number 10 see 118.9–10 with n. 152. 'Ruler of the hour', *hōronomos*, has the astrological sense 'ascendant', referring to the sign of the Zodiac rising at the time of one's birth. The Zodiac was divided into segments of ten degrees each (*dekamoiriai*, see, for example, Ptol. *Almagest* Book II ch. 7), each presided over by one of thirty-six divinities, sometimes called *dekanoi* (e.g., Stobaeus 1.21.9), to which Aristides here refers.

[195] With this discussion compare the similar but more complex treatment of 11 Ptol. *Harm.* Book III ch. 9.

mean one which, through a single large number, expresses potentially, and without division, a smaller one which is concordant with it – for the lower sound is potentially contained in the higher – and by a 'rhythmic' ratio I mean one in which a single number is taken by divisions, with one part assigned to the thesis, one to the arsis.[196]

Chapter 24 Given the concords we have discussed and the capacities that have been assigned to the notes, you may next seek to track down the prognostications attached to births, without straying far from the mark. By studying the contributions of female, male and mixed notes, and of concordant and discordant figures in the composition of the body, in the soul, in characters, in actions, in ways of life and in life itself, you will reach an understanding which is neither unmusical nor far from the truth.[197]

Wise men of ancient times maintained that not only the body but also the soul of the universe is to be understood as constituted through concordant
126 numbers.[198] In the *Timaeus* the noble Plato says something like this: the craftsman who made the soul took a kind of essence intermediate between indivisible and divisible essence, and in the case of the divisible and indivisible varieties of the Same and the Other he put together means with the mean of those essences. When he had made a mixture of these three, he divided up the entire mixture on the basis of the following odd and even numbers: he increased the even numbers in duple ratio, up to the number 8, and the odd ones in triple ratio up to the number 27.[199] Some people argue that Plato says this because it is on numbers that the soul's activities depend, both those of the individual soul in what it does through the arts, and those of the soul of the universe in what it does according to nature. Other more perceptive authorities say that the passage presents the specific character of the soul's essence and power. The fact that the exposition is given in terms of numbers, whose nature lies outside of bodies, expresses the soul's incorporeal origin, while the progressions by ratios and proportions represent its journey towards depth: the progression by twos – 8 being the cube of 2 – represents the bodily depth which we call physical, since it is perishable and divisible: the progression by threes – 27 being the cube of 3 – represents the incorporeal, indivisible and active.[200]

196 Compare the distinction between geometric and arithmetic relations at 101.2–6. But here the point is that harmonic ratios hold between distinct items, one larger, one smaller, whereas rhythmic ratios hold between elements into which a single magnitude or number is divided. On the rhythmic ratios see 33.29ff.

197 The context of this paragraph is astrological prediction, common in later antiquity and espoused by many Stoics.

198 Compare the claim made at 2.14–17. The subsequent passage deals once again with 2.3 Plato *Tim.* 35–6. Of the many ancient authors by whom the passage is expounded, see especially Plut. *De Anim. Procr.* and the commentary of Proclus: cf. ps.-Plut. *De Mus.* 1138c–1139b, and the treatment of 10 Nicomachus *Ench.* ch. 8. The conceptions underlying Aristides' discussion are reviewed in my introduction to his treatise.

199 Plato's divine craftsman appeared previously at 110.2; cf. Book I, 4.5, Book II, 88.11. The two number-sequences are 1, 2, 4, 8 and 1, 3, 9, 27; compare 101.15–23.

200 The text is in some doubt, but these adjectives are certainly applied to a psychic analogue of bodily depth. With this use of the two series of numbers compare particularly chapters 11 and 12 above.

The soul also adopts the body's depth and increases in depth along with it, and often responds in opposition to the depth proper to the soul, since the bodily kind is stronger; and sometimes it turns to better things, which we have decided to relate to the odd, since the odd is indivisible and thus resembles the incorporeal, sometimes to the opposite, whose nature is even, and divisible like that of bodies.[201]

In addition, they have described the greatest of the soul's goods as comparable to the terms we have set out. They say that the four virtues are
127 nothing but resemblances to numbers, making wisdom analogous to the unit, since knowledge of each thing is unitary and simple; courage analogous to the number 2, and comparable to the second position, since it displays impulse and movement from one thing to another; moderation to the number 3, since it is a well blended proportion between deficiency and excess; and justice to the number 4, since it is the first number to display equality, the first composed of equals multiplied equally.[202]

In the case of bodily well being, they make strength analogous to courage, and hence to the number 2, beauty to moderation, since it involves good proportions of parts and of colours, and hence to the number 3, and health to justice, since both involve agreement between their parts. Since they find nothing in bodies comparable to wisdom, they act quite reasonably in using the number 7...

...We find that the wise man puts together his account of the soul on two different principles.[203] He assigns the series of even-numbered positions in duple ratio to the dimension of sensation, and the series of odd numbers in triple ratio to the rational and incorporeal, both being made to depend on the same unit, which provides them with a single first principle.[204] The following diagram will clarify this point. It contains all the proportions, both geometrical and musical, in a double *tetraktys*, which has the odd numbers along a straight line and the even numbers on the circumference.[205] For in bodies the circular

[201] With this paragraph compare Book II's accounts of the human soul and its embodiment, chapters 2, 8, 17.

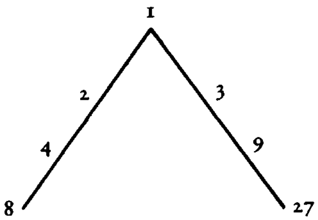
[202] Compare the earlier treatment of the virtues in chapter 16. The equations of the number 1 with wisdom and of 4 with justice, at least, seem to be old; see Burkert (1972), pp. 466–7, with references given there.

[203] Meibom indicates a gap in the text as I have indicated. The Greek, with Winnington-Ingram's small emendation at line 12, can be construed grammatically as it stands: 'Since (or 'while') we find nothing in bodies comparable to wisdom, we find that the wise man acted quite reasonably in putting together his account of the soul, on two different principles, through the number 7'. The wise man is Plato; the divisions of 2.3 *Tim.* 35–6 involve seven basic numbers (1, 2, 3, 4, 8, 9, 27). But the sentence can hardly stand: the connection between its beginning and its end is tenuous to the point of disappearance. Possibly, however, the contents of the gap explained the reference to the number 7, and should be located before it rather than after. The two principles are of course those of the duple and triple progressions.

[204] Plato treats the number 1 as the origin of both progressions. But the associations of the even numbers with sensation etc. are Aristides' own, made earlier in the chapter (126.14ff.)

[205] The missing diagram should bear some relation to one commonly used to set out the number-sequences of *Timaeus* 35–6. It is recorded and discussed at Theon Smyrn.

94.11ff., Plut. *De Anim. Procr.* 1017d ff., 1027d, Proclus *In Tim.* 170–1, 187–8. Its form was that of two diverging lines, like a Greek lambda, on one of which was marked the even series, 2, 4, 8, on the other the odd series 3, 9, 27. Their common origin was the unit, 1.



Proclus attributes to Adrastus a more complex version, in which two further lambdas were placed outside the first, forming a nest of three, corresponding to the three phases of Plato's construction. On the second the numbers were multiplied by 6, and the intervening arithmetic and harmonic means were added. On the third and largest, the *Timaeus* scale was completed by the insertion of terms bounding tones (9:8) and *leimmata* (256:243). Plutarch defends this representation against one in which all the numbers appear on a single straight line; Proclus (*In Tim.* 192) takes the contrary position. But Aristides' diagram seems to have been rather different, since he has the odd numbers on a straight line, the even ones round a 'periphery'. What it was is uncertain, but a related pair of figures in Nicomachus *Intr. Arithm.* II.3.4–4.1 (perhaps also derived from Adrastus) may give a clue. The figures are graphic representations of the fact that in the series of doubles, 2, 4, 8, etc., the first term is the basis for one hemiolic ratio, the second for two, and so on; and that in the series of triples, 3, 9, 27, etc., the first term is the basis for one epitritic ratio, the second for two, and so on, in the following manner.

(a)	1	2	4	8		(b)	1	3	9	27
		3	6	12				4	12	36
			9	18					16	48
				27						64

Figure (a) has the doubles across the top, and the hemiolics constructible in whole numbers in columns below them; (b) works similarly for the triples and epitritics. (Nicomachus adds three more columns to each figure, and the process can be extended indefinitely.)

Now figure (a) contains all the basic *Timaeus* numbers, the even ones across the top, the odd ones on the hypotenuse, creating a deformed version of the lambda diagram. Like the diagram Aristides mentions, all its odd numbers are on a straight line, while even ones (going beyond the basic *Timaeus* sequence) run round a periphery. It displays octave relations in its horizontal lines, and sequences of octave plus fifth on its hypotenuse and the lines parallel to it. The vertical columns contain terms in hemiolic ratio, giving sequences of fifths. Epitritic ratios, giving fourths, are in the lines slanting downwards obliquely from right to left (4:3, 8:6, 12:9). The display is more elegant, however, if figures (a) and (b) are combined.

In (c) the odd numbers are still on a straight line, while the even numbers go around. Hemiolic ratios are in the vertical columns, and epitritic ratios in lines slanting obliquely downwards from right to left. Triangular groups such as 4, 6, 8 or 6, 9, 12 display arithmetic proportion, and ones such as 6, 8, 12 or 9, 12, 18 display harmonic proportion. Both kinds are captured in parallelograms such as 6, 8, 9, 12 or 18, 24, 27, 36. Geometrical proportions are present, of course, in the line of odd numbers and its parallels, and in the horizontal sequences. Possibly, then, it was a figure of this sort that Aristides had in mind, not the simple lambda diagram. But the fit is hardly exact. In

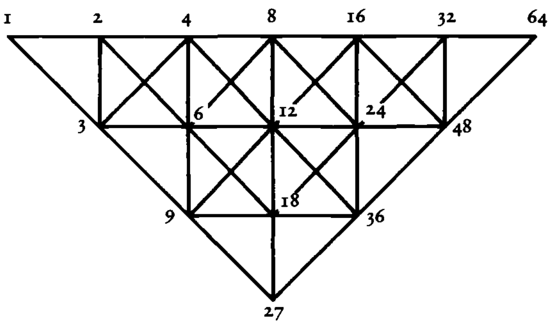
is the more exalted, since it is lighter and more pure, while what follows a straight line is heavier and more material.[206] But in the soul the good is what is straight and undeviating, since that is predominant in equality and homegeneity, while the other is inferior, and through the curvature of the line hints at the changeability of the sensitive part, in that it displays at once both
128 concavity and convexity. Thus the story-tellers of Greece were quite right to associate Pan with the *kalaurops*, since it is entirely reasonable for one who bears the same name as the living universe to use, and to have as his adornment, the instrument which symbolises it.[207]

Of the proportions in the diagram we have given, the arithmetical ones, which are identified by the equality of their excesses, exhibit the fact that the parts of the soul are like one another. The geometrical ones, where the differences relate to the magnitudes, exhibit the fact that the body has bulk. The harmonic proportions, which are made up of both the others, exhibit the fact that the living creature is a combination of the soul and body.[208]

The related issue of the ratio 256:243 has already been discussed.[209] Plato

particular, this is not just a 'double *tetraktys*', which should be simply a pair of sets containing four terms each. For all that, its basis is undeniably a double *tetraktys*, that of 1, 2, 4, 8 and 1, 3, 9, 27. See also n. 207 below.

(c)



206 Compare 109.18ff.

207 Most MSS read *kalabropa*, which would be the accusative of an otherwise unknown noun *kalabrops*. (*Kalabrismos*, a variant of *kolabrismos*, is a Thracian dance possibly associated with pigs, but this seems unhelpful.) One would expect a word related to *kalamos*, 'reed', and the sense 'Pan-pipe' (*syrinx*). Diagrams (a) and (c) in n. 205 above are not unlike the wing-shape of the Hellenistic and Roman *syrinx*. But 'instrument' (*organon*) need not mean 'musical instrument'. *Kalourops* is used by several authors of a shepherd's stick, perhaps a throwing-stick analogous to a boomerang, whose shape could have been similar to the lambda diagrams mentioned in n. 205. (The word *kollorhobon*, which seems to be a related formation, is also given this sense in a number of sources.) Pan has the same name as the universe because that is described as *to pan*, 'the all'; compare Plato, *Cratylus* 408b–c.

208 On these proportions see chapter 5. Here 2, 3, 4 is an arithmetic progression, 2, 4, 8 and 3, 9, 27 geometric ones. The harmonic progressions explained in chapter 5 are not completed in the numbers set out here, but require the insertion of further arithmetic means between terms in the geometric series.

209 The ratio of the *leimma*, the remainder of a 4:3 fourth after two 9:8 tones. See 96.18–28, and 2.3 Plato *Tim.* 36a–b.

constructs the concordant relation which holds between the parts of the soul through the first concord, the epitritic.[210] The ratio relating to this concord signifies the form of animation that consists in length and breadth, which fills up the depth in any distance between things.[211] The two circles express the rotation of the sense-impressions in relation to the ruling principle, and are analogous to the upward and downward movements of the voice.[212] The one based on the *tetraktys* whose factors are even expresses the active aspect of the soul of the universe, which is linked to the body, while the one based on the odd-numbered *tetraktys* represents the theoretical aspect, which is divine and lives apart in company with that which is better.[213] Thus he calls one of them the circle of Sameness, since it expresses the changelessness of the being that is in accordance with understanding, the other the circle of Difference, since it defines the instability of the nature that is impressionable and irrational. So much for this subject.

Chapter 25 Our discussion makes it clear that the first and most natural source of melody is divine possession.[214] When the soul has sunk down towards this world through its abandonment of wisdom, falling into mere ignorance
129 and forgetfulness through the torpor of the body, and becoming filled with confusion and excitement, it is delirious for a while at the moment of birth, and during some periods of its life here it is in what amounts to a lunatic frenzy.[215] Because of this profound ignorance and forgetfulness, which indeed constitutes nothing less than madness, they say that it must be soothed with melody: either the patients must themselves appease the irrational element through imitations of their own (this course is appropriate for those whose characters are savage

[210] This is a very abbreviated (and not quite adequate) way of describing Plato's procedure at 2.3 *Tim.* 35b–36a.

[211] The musical sense of this remark is that the intervals between basic terms in the series are 'filled up' by the insertion of notes at a fourth from each of the boundaries. The metaphysical characterisation of the ratio presumably hangs on the numerological associations of its terms, 4 and 3; see particularly 100.1–3, 17–23, 102.4–9, 110.10–14.

[212] The circles are those described at 2.3 *Tim.* 36b–d, into which the craftsman segments the substance that was harmonically divided. The outer circle (that of the Same) carries the constant movement of the fixed stars; the inner one, in turn divided into seven circles, carries the various movements of the planets. Cf. Plut. *De Anim. Procr.* 1028a ff., and for the analogy with movements of the voice see chapters 21–3 above.

[213] These are connected but not identical with the double *tetraktys* of 127.17ff. That is, the numbers are the same (one containing the odd-numbered series, the other the even), but the figure is different, since both are now circles. Evenness, here as often above, is associated with imperfection, mutability, impressionability, and so on, oddness with their opposites. But there is no warrant in Plato for the assumption that the division of the harmonised series into two strips, which become the circles, separates off the series based on the triple progression from that based on the duple.

[214] The sources of the impulse to melody in the soul are discussed in Book II chs. 4–5. On divine possession see particularly 58.1–2, and ch. 5. That the primary source of melody is divine is a conclusion drawn from the demonstrations, given in the previous chapters of Book III, that the divine order of the universe is itself musical; cf. particularly 105.18–19.

[215] The text of the last clause is doubtful. I follow Borthwick's suggestion *parakopon aluein* in line 4. This sentence and the sequel draw on the accounts of the soul given in Book II chs. 2, 8, 17; cf. Book III ch. 7 (see especially 54.13ff., 86.30ff., 104.24ff.).

and bestial), or they must avert the dreadful affliction through the use of their eyes and ears (this will suit those who are educated and whose nature is more orderly).[216] Hence, they say, there is a degree of reason behind bacchic rites and others of that kind: they serve to cleanse away, with their songs and dances and games, the frenetic excitedness to which foolish folk have become subject through their way of life, or merely through chance.

That all passion is madness is indicated by Homer when he speaks of a woman agitated by desires:

'The wife of Proteus was mad with lust for the man' (*Il.* VI.160): and again when he speaks of a man distraught at the killing of his son:

'Madman, crazed in your wits' (*Il.* XV.128).

Wise men have also indicated this by calling all passion a 'slight seizure':[217] it is also demonstrated by the argument from extremes. If the extremity of every emotion is madness, then plainly other quantities of them will be little madnesses; but the affliction is disguised by the additional presence of a greater or lesser degree of good sense.

130 Whether melody is considered merely in relation to the *systēmata* which are its parts, or is treated as a whole, in relation to all the aspects of composition, one kind proceeds in a straight line, the other through variations in the order of the letters.[218] The first of these symbolises generation, and is to be assigned to earth; the second to water, since it has a share in maleness, and it is through water that nature activates generations in the region of earth; the third to air, since it is female, and expresses the malleability and passivity of that element; the fourth to fire, since it is excessively male, and this element is the most active;[219] and the letter which is attached to all these, the tau, to aether, since it is shaped like a plectrum, and is sacred to the god who, so wise men say, is the plectrum of the universe.[220] Thus the letter tau is attached to all the vowels, when these are associated with notes, just as the aether is attached to the other elements, and transmits to them the power of life.[221] Thus the movement of the elements is the ordering of matter, and the ordering of the soul... is melody.[222]

216 On the contrast of human types see Book II, 63.3–24. But the idea that more cultured persons should look and listen rather than actually performing has not previously been introduced; cf. Aristotle *Pol.* 1339a–b, 1340b–1341b.

217 That is, it is comparable to an epileptic fit.

218 This picks up the theme of the straight and the curved that was elaborated especially at 109.18ff., 127.17ff.; cf. 115.13–16. For the forms of melodic sequence intended see Book I, 16.18ff., 29.8ff. The letters are those of the solmisation of Book II ch. 14; cf. Book III ch. 21.

219 The order of letters is epsilon, alpha, eta, omega.

220 On the use of the letter tau in solmisation see Book II ch. 14. The god referred to is evidently Apollo, whom Aristides adopts as his patron. The plectrum is of course proper to his role as the god of *kithara* and *lyra*, but see also Plut. *De Pyth. Orac.* 402a and Cleanthes at *SVF* vol. I, 499, 502, where the ray of the sun is the plectrum of the cosmos (Apollo is also the sun-god).

221 This is implied by the doctrine of the origin of the soul in the aetherial region, but see also the explicit statements of, for example, 92.5–8, 112.7–9.

222 One or two words in the MSS of line 16 are too distorted for confident restoration. Schäfke's suggestion *hē toutōn* is as likely as any. The sense would be 'and the ordering

In the case of rhythms, whose constitution we identify in their arsis and thesis,[223] the thesis exhibits the generation of particular things, the arsis their destruction. Just as neither of them can create a rhythm without the other, so generation can never be found without destruction. The destruction of one set of things is always the starting point for the generation of others, and generation always bases itself on things that have been destroyed.[224]

Some rhythms are changeless, others undergo modulation.[225]

Chapter 26 Before we close our discussion we shall say a little about modulation.[226] Some complete melodies[227] continue in a way that conforms to the pattern of their initial impulses, while others change into something different as they go along. In the same way, over very long periods in the universe at large, one can see changes in ways of life, alterations in political
131 constitutions, fertile and barren times for animals and plants.[228] Further, if we consider each kind of thing separately, some of them follow in close dependence on the causes of their initial generation, others change their condition and their appearance through modifications made by art or by chance. These principles and ways of life, impulses to action, practical interests and movements from place to place which do not follow the direction initiated by the first causes modify all the distinctive features of a person's birth.[229] Homer gives reliable evidence that this is so. In one place he says:

'No man, I say, has escaped fate' (*Il.* VI.488), but in another:

'Lest you enter the house of Hades contrary to fate' (*Il.* XX.366):

and his poetry speaks constantly of such things. At the beginning of the *Iliad* he says:

'It cast many strong souls into Hades' (*Il.* I.3), as though the destined destruction of the Achaeans was initially determined by the wrath of Achilles. In the *Odyssey*, however, he says:

'Fools, who ate the cattle of the Sun, the child of Hyperion; and he took from them the day of their homecoming' (*Od.* I.8–9), as though they would not have been deprived of their return to their homes if they had not behaved as they did.[230] It is said by the wise, as well, that future events are of two kinds.

of the soul is the melody of the former things' (i.e., of the notes, or perhaps of the vowels attached to them).

[223] Rhythm is discussed in Book I chs. 13ff.: arsis and thesis are introduced at 31.8ff.

[224] A commonplace in Greek cosmological thought from the fifth century B.C. onwards. But Aristides may be thinking particularly of the argument of Plato *Phaedo* 70d–72d.

[225] Rhythmic modulation is defined in Book I at 40.1–7; cf. 34.19ff. and 83.17ff.

[226] The 'modulation' of this chapter is melodic; see Book I ch. 11, but cf. Book II, 83.17ff., on the psychological effects of rhythmic modulation.

[227] Probably not in the technical sense of Book I, 4.20, 5.4, but meaning merely 'taken from start to finish'.

[228] The nature of all these has been shown to depend on musical relations. Hence, radical changes in them must be linked with radical musical shifts, that is, with modulation.

[229] The circumstances of a person's birth are such as to dispose him (astrologically or otherwise) towards a certain destiny. But this result can be deflected by changed circumstances (just as the acorn that 'should' become an oak tree may instead become part of a squirrel).

[230] Note that the 'fate' with which these contrasts are made is not something fixed from the beginning of time, but something flowing 'naturally' from a given set of conditions.

There is that which is necessary and inevitable, which they call 'what will happen', and there is that which can be altered and is not altogether determinate: this they call 'what is probable'.[231] The necessary kind has under its control the things above the Moon, the ambivalent and contingent kind the things in this world. Things which occur universally are inevitable and necessary, while things which occur in particular cases are always capable of being altered.[232] Further, things whose occurrence contributes to the preservation and good order of the universe, and whose failure to occur obstructs it, are necessary: those which neither contribute to the organisation
132 of the universe by their occurrence nor damage its constitution by their absence can, in the event, turn out either way.[233] I have evidence for the truth of this in an ancient oracle delivered from the Pythian tripod,[234] concerning the Persian army which confronted the Greeks at Plataea. It runs:

'There many of the Medes who carry bows will meet sufferings beyond Lachesis and fate, when the due day arrives.'[235] The sense of the first line is plainly that the Medes will die before their fated time. But in the second line, if we take the word 'due' to mean 'in accordance with the destiny that will be' (as Homer says somewhere (*Od.* VIII.511–12), 'It was the city's due to be destroyed when it had taken the great wooden horse inside'), then the discordance of the words is clear.[236] If on the other hand we take it in the sense 'appropriate' (as Homer says in another place (*Il.* III.59), 'Hector, since you have rebuked me duly'), by pursuing what follows concordantly with what the words indicate we shall reach the same conclusion. For what seems appropriate to the divine judgement has thereby been ordained to follow; and when the gods are overseeing it, what follows is necessary; and what is necessary is fated from all time.[237] Thus once again we find the same thing, that the same oracle has described the disaster to the Medes as both in accordance with fate and contrary to it. Can the prophet of the universe really have uttered in so short a space of time statements that so blatantly contradict one another?[238] Far from it. It is rather, I believe, that he is covertly pointing to the dual nature of destiny. By 'beyond Lachesis and fate' he means 'contrary to the first cause of each individual's birth', and by 'when the due day arrives' he means 'in

It can be altered as a consequence of individual choice as well as of new circumstances, but see the opening of chapter 27. Absolute necessities are discussed in what follows.

[231] *Genēsomenon* and *mellon*. The distinction, but not the terminology, corresponds roughly to the Stoic division between what is fated and what is 'up to us', *to kath' heimarmenēn* and *to eph' hēmin* (see the references under *eph' hēmin* at *SVF* vol. 4, p. 53). But the Stoics found it hard to maintain consistently that there are any events or actions not directly subject to fate. See, for example, Cicero *De Fato*, particularly 6.12–8.16, and for other references see Sorabji (1980), ch. 4.

[232] See, for example, Aristotle *De Gen. et Corr.* Book II ch. 11.

[233] The basis of this doctrine is Platonic; see *Rep.* 507a ff., especially 509b, *Tim.* 27d–29d.

[234] That is, the seat of the priestess of the Delphic oracle.

[235] Compare Herodotus IX.43. Lachesis is the name of one of the Fates.

[236] The word translated 'due' is *aisimon*. This and the cognate noun *aisa*, used in the Homeric quotation, have the sense 'destined', 'destiny', but as the sequel suggests, with the colouring 'rightly' or 'duly' destined.

[237] *Ex hapantos*, lit. 'from all': the sense might be simply 'without qualification'.

[238] The prophet is Apollo, to whom the Delphic oracle was dedicated.

accordance with the fate that follows in consequence of the deed that has been done'.[239]

133 *Chapter 27* Now that I have reached this point in the argument, I can see that there exists a satisfactory method of escape even from these problems, in that people can break an unhappy train of events imposed by nature, by introducing other sequences. Just as in music the whole *harmonia* can be changed as the result of a small beginning in a single note, so too in actions a modest initial effort can reshape a whole way of life.[240] But such ways of escape should not be treated as secure or as meriting approval. Those who devise ways of avoiding troubles for the sake of comfort do not entirely escape the confusions of the world of becoming.[241] It is only the divine conversion that comes through philosophy that leads unswervingly and securely to change. It releases the soul from its fellow-feeling with bodies, and makes the man who has knowledge of higher things worthy, through his participation in virtue, of the providence of that which is divine, and resembles himself.[242] Philosophy provides a real escape from the process of becoming.[243] If something unwelcome occurs, it endures it calmly and bravely, and counts nothing as evil or shameful except wickedness and enslavement to vice.[244] All its habits of speech and action are founded in virtue; and virtue has no master, as the prophet mentioned by the noble Plato tells us.[245] So it is as the best partner and servant of this – of philosophy, I mean – that we should practise music and teach it in its entirety, and by comparing as it were the lesser and the greater mysteries we should assign to each its proper value and honour, and join them together in the most proper and legitimate union. Philosophy brings all knowledge to perfection, while music gives preliminary instruction: philosophy is the true and perfect ritual which restores to souls, through recollection, what they lost
134 because of the catastrophe of their birth,[246] while music is an initiation, a gentle

[239] Aristides has drawn distinctions of two main sorts: (a) between what is absolutely necessary and what is contingent (131.20–132.2); (b) between what flows from the initial conditions of a thing's origin and what is due to later events impinging on the thing, through chance or choice. His strategy here is to deny that the 'destiny' referred to in the oracle is that of absolute necessity, which cannot be disrupted. It is that to which the circumstances of one's birth, if left to develop causally on their own, would naturally lead. But their causal development can be deflected by events impinging from outside.

[240] This links the capacity to alter one's 'destiny' (see the previous note) with melodic modulation as discussed in Book I ch. 11.

[241] Since this ability to alter destiny depends on the contingency of all particular chains of causation in the world below the Moon, it is always possible that our efforts to redirect our lives will be frustrated by other events impinging by chance from outside, or by unexpected consequences of our own choices.

[242] On the soul's fellow feeling with bodies see Book II, 54.12–13 and ch. 8; cf. Book III, 115.28–31. The ideas here are drawn from Plato, most notably *Phaedo* 64c–67b. On the ethical project of seeking absolute security from chance see Nussbaum (1986).

[243] Compare, for example, *Phaedo* 67b–e, 114b–c.

[244] This is a commonplace of Platonic dialogues such as *Crito*, *Gorgias*, etc.

[245] At *Rep.* 617e, in the Myth of Er. On the links between philosophy and virtue see especially *Rep.* 484e–487a.

[246] A reference to the Platonic theory of recollection discussed at *Meno* 81a ff., *Phaedo* 73a ff.; cf. *Phaedrus* 249c. See Book II chs. 2, 8, 17.

introduction to the ritual, which offers in advance a little taste of what is brought to complete perfection in philosophy. Music provides the foundations of every field of learning, philosophy the pinnacles.[247]

Here let us bring to an end our discussions of music. If our labours have brought them to a perfect completion, great thanks are due to the god who is the lord of the Muses, who turned our efforts to this task and has brought our endeavours to this end.[248] If we have omitted anything that should have been said, even so no harm is done, as the saying goes. The way has been paved well enough for those who at some future time will be able to present all musical matters complete in a single work.[249]

[247] This thesis is a guiding principle of the educational programme of Plato's *Republic*. The musical education discussed in its second and third books is a necessary preliminary to the intellectual education described in the seventh, which culminates in philosophical dialectic. Aristides' treatment of music and philosophy as 'mysteries' is also derived from Plato, who uses such language frequently, e.g., *Gorg.* 497c, *Euthyd.* 277d–e (which parallels the notion of 'a gentle introduction to the ritual'), *Phaedo* 69c–d, *Symp.* 210a, *Phaedrus* 249c. For Aristides' earlier uses of the conception see n. 165 above. On the relation of music to philosophy compare Book I ch. 1 and especially Book II ch. 3.

[248] Compare the invocations of Book I ch. 3 and Book III ch. 9.

[249] Compare Aristides' complaint against previous musical writers, Book I, 3.12–18.

Bibliography of works by modern authors

This is a selective rather than a comprehensive bibliography. It contains all the modern works referred to in my notes, and those others which I would judge either fundamental to the study of the subject, or particularly useful on special issues. In addition to writings on harmonics and acoustics I have included a small selection of works on related aspects of Greek philosophy and science. Articles in the familiar major encyclopedias, such as *The New Grove* and its predecessors, other musical and classical dictionaries, Pauly's *Real-Encyclopädie*,. etc., are not mentioned here. Editions of the Greek texts translated, and of the principal Greek musicological writings drawn on in my commentary, are listed on pp. vii–viii above under their authors' names. Those which include significant introductions or commentaries reappear below under the names of their editors. The bibliography can be supplemented from the following sources:

Abert, H. 'Literatur zur griechischen Musik 1909–21', *Bursian* 193 (1923)

Fellerer, G. 'Literatur zur griechischen Musik 1921–31', *Bursian* 246 (1935)

Winnington-Ingram, R. P. 'Ancient Greek music 1932–57', *Lustrum* 3 (1958)

Mathiesen, T. J. *A Bibliography of Sources for the Study of Ancient Greek Music*, Hackensack, New Jersey, 1974

There are also good bibliographies in Lippman (1964) and Neubacker (1977); and see the listings in successive numbers of *L'Année Philologique*.

Adam, J. *The Republic of Plato*, 2 vols., Cambridge, 1902

Alexanderson, B. *Textual Remarks on Ptolemy's Harmonica and Porphyry's Commentary*, Göteborg, 1969

Anderson, W. D. 'The importance of Damonian theory in Plato's thought', *Transactions of the American Philological Society* 86 (1955), 88–102

Ethos and Education in Greek Music, Cambridge, Mass., and London, 1966

'What song the Sirens sang', *Research Chronicle* 15 (1979), 1–13

'Musical development in the school of Aristotle', *Research Chronicle* 16 (1980), 78–98

Annas, J. *An Introduction to Plato's Republic*, Oxford, 1981

von Arnim, H. *Stoicorum veterum fragmenta*, Leipzig, 1905–24 (*SVF*)

Barbera, A. 'Arithmetic and geometric division of the tetrachord', *Journal of Music Theory* 21 (1977)

'The consonant eleventh and the expansion of the musical tetractys', *Journal of Music Theory* 28 (1984a), 191–224

'Placing *Sectio Canonis* in historical and philosophical contexts', *Journal of Hellenic Studies* 104 (1984b), 157–61

Barbour, J. M. *Tuning and Temperament*, East Lansing, 1953

'The principles of Greek notation', *Journal of the American Musicological Society* 13 (1960), 1–17

Barker, A. D. 'Music and mathematics: Theophrastus against the number theorists', *Proceedings of the Cambridge Philological Society* n.s. 23 (1977), 1–15

'Music and perception: a study in Aristoxenus', *Journal of Hellenic Studies* 98 (1978), 9–16

'*Symphonoi arithmoi*: a note on *Republic* 531c1–4', *Classical Philology* 73 (1978), 337–42

'*Hoi kaloumenoi harmonikoi*: the predecessors of Aristoxenus', *Proceedings of the Cambridge Philological Society* n.s. 24 (1978), 1–21

'Aristotle on perception and ratios', *Phronesis* 26 (1981a), 248–66

'Methods and aims in the Euclidean *Sectio Canonis*', *Journal of Hellenic Studies* 101 (1981b), 1–16

'Aristides Quintilianus and constructions in early music theory', *Classical Quarterly* 32 (1982), 184–97

'Aristoxenus' theorems and the foundations of harmonic science', *Ancient Philosophy* 4 (1984), 26–64

Greek Musical Writings, vol. 1, *The Musician and his Art*, Cambridge, 1984 (*GMW* vol. 1)

'Theophrastus on pitch and melody', in *Theophrastus of Eresus*, eds. W. Fortenbaugh and others, New Brunswick and Oxford, 1985, 289–324

'Text and sense at *Philebus* 56a', *Classical Quarterly* 37 (1987), 103–9

Barnes, J. 'Aristotle's theory of demonstration', *Phronesis* 14 (1969), 123–52, revised in Barnes, Schofield, Sorabji, vol. 1, pp. 65–87

Aristotle's Posterior Analytics, Oxford, 1975

The Presocratic Philosophers, London, 1979

Barnes, J., M. Schofield and R. Sorabji eds. *Articles on Aristotle*, 4 vols., London, 1975–9

Bataille, A. 'Remarques sur les deux notations mélodiques de l'ancienne musique grecque', *Recherches de papyrologie* 1 (1961), 5–20

Baud-Bovy, S. 'Le dorien était-il un mode pentatonique?', *Revue de Musicologie* 64 (1978), 153–80

Beare, J. I. *Greek Theories of Elementary Cognition*, Oxford, 1906

Bélis, A. 'Les nuances dans le *Traité d'harmonique* d'Aristoxène de Tarente', *Revue des études grecques* 95 (1982), 54–73

'Un nouveau document musical', *Bulletin de correspondance hellénique* 108 (1984), 99–109

Aristoxène de Tarente et Aristote: le Traité d'Harmonique, Paris, 1986

Borthwick, E. K. '*Katalepsis* – a neglected term in Greek music', *Classical Quarterly* n.s. 9 (1959), 23–9

'Some problems in musical terminology', *Classical Quarterly* n.s. 17 (1967), 145–57

'Notes on the Plutarch *De Musica* and the *Cheiron* of Pherecrates', *Hermes* 96 (1968), 60–73

Bowen, A. C. 'The minor sixth in early Greek harmonic science', *American Journal of Philology* 99 (1978), 501–6

'The foundations of early Pythagorean harmonic science: Archytas, fragment 1', *Ancient Philosophy* 2 (1982), 79–104

'Menaechmus versus the Platonists: two theories of science in the early Academy', *Ancient Philosophy* 3 (1983), 12–29

Bower, C. 'Boethius and Nicomachus', *Vivarium* 16 (1978), 1–45

Brancacci, A. 'Aristosseno e lo statuto epistemologico della scienza armonica', in *La scienza ellenistica*, eds. G. Giannantoni and M. Vegetti, Naples, 1984, 154–85

Brumbaugh, R. S. *Plato's Mathematical Imagination*, Bloomington, 1954

'Aristotle as a mathematician', *Review of Metaphysics* 8 (1954–5), 379–93

Bulmer-Thomas, I. 'Plato's Astronomy', *Classical Quarterly* 34 (1984), 107–12
Burkert, W. *Weisheit und Wissenschaft: Studien zu Pythagoras, Philolaus und Platon*, Nürnberg, 1962, translated by E. L. Minar as *Lore and Science in Ancient Pythagoreanism*, Cambridge, Mass., 1972
Chadwick, H. *Boethius: The Consolations of Music, Logic, Theology and Philosophy*, New York, 1981
Chailley, J. 'L'hexatonique grecque d'après Nicomaque', *Revue des études grecques* 69 (1956), 73–100
'Le mythe des modes grecs', *Acta musicologica* 28 (1956), 137–63
L'imbroglio des modes, Paris, 1960
'Nouvelles remarques sur les deux notations musicales grecques', *Recherches de papyrologie* 4 (1967), 201–16
'Nicomaque, Aristote et Terpandre devant la transformation de l'heptacorde grec en octocorde' *Yuval* 1 (1968), 132–54
'La notation archaïque grecque d'après Aristide Quintilien', *Revue des études grecques* 36 (1973), 17–34
La musique grecque antique, Paris, 1979
Cherniss, H. *Aristotle's Criticism of Plato and the Academy*, Baltimore, 1944
The Riddle of the Early Academy, Berkeley, 1945; reprinted New York and London, 1980
'Plato as mathematician', *Review of Metaphysics* 4 (1951), 395–425
Cohen, M. R. and I. E. Drabkin *A Source Book in Greek Science*, Cambridge, Mass., 1949
Commotti, G. 'L'endecacordo di Ione di Chio', *Quaderni urbinati di cultura classica* 13 (1972), 54–61
La musica nella cultura greca e romana, vol. 1, part 1 of *Storia della musica*, Turin, 1979
Cornford, F. M. *Plato's Cosmology: The Timaeus of Plato Translated with a Running Commentary*, London, 1937
Crocker, R. L. 'Pythagorean mathematics and music', *Journal of Aesthetics and Art Criticism* 22 (1963–4), 189–98, 325–35
'Aristoxenus and Greek mathematics', in *Aspects of Medieval and Renaissance Music*, ed. J. La Rue, New York, 1966, 96–110
Crombie, A. C. 'Mathematics, music and medical science', *Organon* 6 (1969)
Dale, A. M. *The Lyric Metres of Greek Drama*, Cambridge, 1968
Denniston, J. D. 'Some recent theories of the Greek modes', *Classical Quarterly* 7 (1913), 83–99
Dicks, D. R. *Greek Astronomy from Thales to Aristotle*, London, 1970
Diels, H., and Kranz, W. *Die Fragmente der Vorskratiker*, 8th edition, Berlin, 1956–9 (DK)
Dillon, J. *The Middle Platonists*, London, 1977
D'Ooge, M. L. *Nicomachus of Gerasa: Introduction to Arithmetic*, New York, 1926
Düring, I. *Die Harmonielehre des Klaudios Ptolemaios*, Göteborg, 1930
Porphyrios Kommentar zur Harmonielehre des Ptolemaios, Göteborg, 1932
Ptolemaios und Porphyrios über die Musik, Göteborg, 1934
'Studies in musical terminology in fifth century literature', *Eranos* 43 (1945), 176–97
'Greek music: its fundamental features and its significance', *Journal of World History* 3 (1956), 302–29
Eitrem, S., L. Amundsen, and R. P. Winnington-Ingram, 'Fragments of unknown Greek tragic texts with musical notation', *Symbolae Osloenses* 31 (1955), 1–87

Festugière, A. J. 'L'âme et la musique d'après Aristide Quintilien', *Transactions of the American Musicological Association* 85 (1954), 55–78

Proclus: Commentaire sur le Timée de Platon, 3 vols., Paris, 1967

Fowler, D. H. *The Mathematics of Plato's Academy*, Oxford, 1987

Gevaert, A. *Histoire et théorie de la musique de l'Antiquité*, 2 vols., Gand, 1875 and 1881, reprinted 1965

Gevaert, A. and J. C. Vollgraff, *Problèmes musicaux d'Aristote*, Gand, 1903

Gombosi, O. J. *Die Tonarten und Stimmungen der antiken Musik*, Copenhagen, 1939

'Key, Mode, Species', *Journal of the American Musicological Society* 4 (1951), 20–6

Gosling, J. C. B. *Plato, Philebus*, Oxford, 1975

Gottschalk, H. B. 'The *De Audibilibus* and Peripatetic acoustics', *Hermes* 96 (1968), 435–60

Guthrie, W. K. C. *A History of Greek Philosophy*, 6 vols., Cambridge, 1962–81

Handschin, J. 'The *Timaeus* scale', *Musica disciplina* 4 (1950), 3–42

Heath, T. L. *A History of Greek Mathematics*, 2 vols., Oxford, 1921, reprinted New York, 1981

Mathematics in Aristotle, Oxford, 1949, reprinted New York, 1980

Henderson, M. I. 'The growth of the Greek *harmoniai*', *Classical Quarterly* 36 (1942), 94–103

'The growth of ancient Greek music', *The Music Review* 4 (1943), 4–13

'Ancient Greek music', in *The New Oxford History of Music*, ed. E. Wellesz, vol. 1, London, 1957, 336–403

Howard, A. A. 'The aulos or tibia', *Harvard Studies in Classical Philology* 4 (1893), 1–60

Huffman, C. A. 'The authenticity of Archytas fr. 1', *Classical Quarterly* 35 (1985), 344–8

Issberner, R. 'Dynamis und Thesis', *Philologus* 55 (1896), 541–60

von Jan, K. 'Die Harmonie der Sphären', *Philologus* 52 (1894), 13–37

Musici Scriptores Graeci, Leipzig, 1895, reprinted Hildesheim, 1962 (*MSG*)

Johnson, C. W. L. 'The motion of the voice in the theory of ancient music', *Transactions of the American Philological Association* 30 (1899), 42–55

Jonker, G. H. *The Harmonics of Manuel Bryennius*, Groningen, 1970

Junge, G. 'Die Sphären-Harmonie und die pythagoreisch-platonische Zahlenlehre', *Classica et medievalia* 9 (1947–8), 183–94

Kirk, G. S., J. E. Raven, and M. Schofield, *The Presocratic Philosophers*, 2nd edition, Cambridge, 1983

Knorr, W. R. *The Evolution of the Euclidean Elements*, Dordrecht, 1975

Kucharski, P. 'La musique et la conception du réel dans le *Philèbe*', *Revue philosophique* 141 (1951), 39–59

'Le *Philèbe* et les *Eléments harmoniques* d'Aristoxène', *Revue philosophique* 149 (1959), 41–72

Kytzler, B. 'Die Weltseele und der musikalische Raum', *Hermes* 87 (1959), 393–414

Laloy, L. 'Anciennes gammes enharmoniques', *Revue de philologie* 23 (1899), 238–48; 24 (1900), 31–43

Aristoxène de Tarente et la musique de l'Antiquité, Paris, 1904

Lexique d'Aristoxène, Paris, 1904

Landels, J. G. 'Shipshape and sambuca-fashion', *Journal of Hellenic Studies* 86 (1966), 69–77

'Music in ancient Greece', *Proceedings of the Classical Association* 77 (1980), 24–39

Lasserre, F. *Plutarque de la musique*, Olten and Lausanne, 1954

Lear, J. 'Aristotle's philosophy of mathematics', *Philosophical Review* 91 (1982), 161–91

Lennox, J. G. 'Aristotle, Galileo and "Mixed Sciences"', in *Reinterpreting Galileo*, ed. W. A. Wallace, Washington, D.C., 1984, 29–51

Letters, R. J. 'The scales of some surviving *auloi*', *Classical Quarterly* n.s. 19 (1969), 266–8

Levin, F. R. 'The hendecachord of Ion of Chios', *Transactions of the American Philological Association* 92 (1961), 295–307

'Synesis in Aristoxenian theory', *Transactions of the American Philological Association* 103 (1972), 211–34

The Harmonics of Nicomachus and the Pythagorean Tradition, American Classical Studies no. 1, Philadelphia, 1975

Lippman, E. A. 'Hellenic conceptions of harmony', *Journal of the American Musicological Society* 16 (1963), 3–35

'The sources and development of the ethical view of music in ancient Greece', *Musical Quarterly* 49 (1963), 188–209

Musical Thought in Ancient Greece, New York, 1964

Lloyd, G. E. R. 'Plato as a natural scientist', *Journal of Hellenic Studies* 88 (1968), 78–92

Aristotle: The Growth and Structure of his Thought, Cambridge, 1968

Early Greek Science: Thales to Aristotle, London, 1970

Greek Science after Aristotle, London, 1973

'Saving the Appearances', *Classical Quarterly* n.s. 28 (1978), 202–22

'Observational error in later Greek science', in *Science and Speculation*, eds. J. Barnes and others, Cambridge, 1982, 128–64

Long, A. A. 'Astrology: arguments pro and contra', in *Science and Speculation*, eds. J. Barnes and others, Cambridge, 1982, 165–92

Long, A. A. and D. N. Sedley, *The Hellenistic Philosophers*, 2 vols., Cambridge, 1987

Lord, C. 'On Damon and music education', *Hermes* 106 (1978), 32–43

Macran, H. S. *The Harmonics of Aristoxenus*, Oxford, 1902

Marenghi, G. *Hosa peri harmonian: problemi musicali*, Florence, 1957

Problemi di fonazione e di acustica, Naples, 1963

Mathiesen, T. J. 'An annotated translation of Euclid's division of the Monochord', *Journal of Music Theory* 19 (1975), 236–58

'Problems of terminology in ancient Greek theory', in *Harmonia: Festival Essays for Pauline Alderman*, ed. B. Kanon, Boston, 1976, 1–17

'Aristides Quintilianus and the harmonics of Manuel Bryennius', *Journal of Music Theory* 27 (1983), 31–49

Aristides Quintilianus: On Music in Three Books, New Haven and London, 1983

Melling, D. J. *Understanding Plato*, Oxford, 1987

Merlan, P. *From Platonism to Neoplatonism*, 3rd edition; The Hague, 1975

Michaelides, S. *The Music of Ancient Greece, an Encyclopedia*, London, 1978

Monro, D. B. *The Modes of Ancient Greek Music*, Oxford, 1894

Mountford, J. F. 'Greek music and its relation to modern times', *Journal of Hellenic Studies* 40 (1920), 13–42

'The musical scales of Plato's *Republic*', *Classical Quarterly* 17 (1923), 125–36

'The Harmonics of Ptolemy and the lacuna in II. 14', *Transactions of the American Philological Association* 57 (1926), 71–95

Mourelatos, A. P. D. 'Plato's "Real Astronomy": *Republic* 527d–531d', in *Science and the Sciences in Plato*, ed. J. P. Anton, Buffalo, N.Y., 1980, 33–73

'Astronomy and kinematics in Plato's project of rationalist explanation', *Studies in the History and Philosophy of Science* 12 (1981), 1–32

Moutsopoulos, E. *La musique dans l'oeuvre de Platon*, Paris, 1959

'Euripide et la philosophie de la musique', *Revue des études grecques* 75 (1962), 396–452
Mueller, I. 'Euclid's *Elements* and the Axiomatic Method', *British Journal for the Philosophy of Science* 20 (1969), 289–309
'Aristotle on geometrical objects', *Archiv für Geschichte der Philosophie* 52 (1970) 157–71; reprinted in Barnes, Schofield, Sorabji, vol. 3, 96–107
'Ascending to problems: astronomy and harmonics in *Republic* VII', in *Science and the Sciences in Plato*, ed. J. P. Anton, Buffalo, N.Y., 1980, 103–21
Najock, D. *Drei anonyme griechische Traktate über die Musik, Ein kommentierte Neuausgabe des Bellermannschen Anonymus*, Kassel, 1972
Neubacker, A. J. *Altgriechische Musik*, Darmstadt, 1977
Neugebauer, O. *The Exact Sciences in Antiquity*, 2nd edition, Providence, 1957; reprinted New York, 1969
A History of Ancient Mathematical Astronomy, New York, Heidelberg and Berlin, 1975
Nussbaum, M. C. *The Fragility of Goodness*, Cambridge, 1986
Owen, G. E. L. '*Tithenai ta phainomena*', in *Aristote et les problèmes de méthode*, ed. S. Mansion, Louvain, 1961, 83–103; reprinted in *Aristotle*, ed. J. M. E. Moravcsik, New York, 1967, 167–90; and in Barnes, Schofield, Sorabji, vol. 1, 113–26
Palisca, C. V. 'Scientific empiricism in musical thought', in *Seventeenth Century Science and the Arts*, ed. H. H. Rhys, Princeton, 1961, 91–137
Humanism in Italian Renaissance Musical Thought, New Haven and London, 1985
Pappalardo, V. 'Vicende della notazione musicale nei testi drammatici greci', *Memorie dell' Istituto Lombardo* 35 (1975), 355–413
Paquette, D. *L'instrument de musique dans la céramique de la Grèce antique*, Paris, 1984
Philip, J. A. *Pythagoras and Early Pythagoreanism*, Toronto, 1966
Phillips, R. C. 'Mean tones, equal-tempered tones, and the harmonic tetrachords of Claudius Ptolemy', *Memoirs and Proceedings of the Manchester Literary and Philosophical Society* 48 (1904)
Pighi, G. B. *Aristoxeni Rhythmica*, Bologna, 1969
Studi di ritmica e metrica, Turin, 1970
Pöhlmann, E. *Denkmäler altgriechischer Musik*, Nürnberg, 1970
Potiron, H. 'Les notations d'Aristide Quintilien et les harmonies dites Platoniciennes', *Revue de musicologie* 47 (1961), 159–76
'La notation grecque au temps d'Aristoxène', *Revue de musicologie* 50 (1964), 222–5
Reese, G. *Music in the Middle Ages*, New York, 1940
Reinach, T. 'Les nouveaux fragments rhythmiques d'Aristoxène', *Revue des études grecques* 11 (1898), 389–418
'La musique des sphères', *Revue des études grecques* 13 (1900), 432–49
La musique grecque, Paris, 1926; reprinted 1979
Reinach, T. and E. d'Eichtal, 'Notes sur les problèmes musicaux attribués à Aristote', *Revue des études grecques* 5 (1892), 22–52
Restani, D. 'Il *Chirone* di Ferecrate e la nuova musica greca', *Rivista Italiana di musicologia* 18 (1983), 139–92
Richter, L. 'Die Aufgaben der Musiklehre nach Aristoxenos und Klaudios Ptolemaios', *Archiv für Musikwissenschaft* 15 (1958), 209–29
Zur Wissenschaftslehre von der Musik bei Platon und Aristoteles, Berlin, 1961
Riethmüller, A. 'Zum Anfang der musikalischen Terminologie', in *International Musicological Society, Report of the Twelfth Congress, Berkeley 1977*, eds. D. Heartz and B. Wade, Kassel, 1981

'Logos und Diastema in der griechischen Musiktheorie', *Archiv für Musikwissenschaft* 42 (1985), 18–36
da Rios, R. *Aristoxeni Elementa Harmonica*, Rome, 1954
Rispoli, G. M. 'Il primo libro del *peri mousikes* di Filodemo', *Ricerche sui papiri ercolanesi*, ed. F. Sbordone, 1, Naples, 1969, 25–287
'Sensazione, esperianza, giudizio tecnico', *Annali della Facoltà di Lettere e Filosofia della Università di Napoli* 24 (1981–2), 105–19
Robbins, F. E. *Ptolemy, Tetrabiblos*, Cambridge, Mass., 1940
Ross, W. D. *Aristotle*, 5th edition, London, 1960
Rowe, C. J. *Plato*, Brighton, 1984
Ruelle, C.-E. 'Etude sur Aristoxène de Tarente et son école', *Revue archéologique* 14 (1857), 413–22, 528–55
Collection des auteurs grecs relatifs à la musique, 6 vols., Paris, 1871–98
Sachs, C. 'Die griechische Instrumentalnotenschrift', *Zeitschrift für Musikwissenschaft* 6 (1923–4), 289–301
'Die griechische Gesangnotenschrift', *Zeitschrift für Musikwissenschaft* 7 (1924–5), 1–5
The History of Musical Instruments, New York, 1940
The Rise of Music in the Ancient World, New York, 1943
Sambursky, S. *The Physical World of the Greeks*, London, 1956
Schäfke, R. *Aristides Quintilianus: von der Musik*, Berlin, 1937
Schlesinger, K. 'The Greek Foundations of the Theory of Music', *Music Standard* n.s. 27, 28 (1926), 27: 23–4, 44–6, 62–3, 96–8, 109–10, 134–6, 162–4, 177–8, 197–8; 28: 31–2, 44–5
'Further notes on Aristoxenus and musical intervals', *Classical Quarterly* 27 (1933), 88–96
The Greek Aulos, London, 1939; reprinted 1970
'The Harmoniai', *Music Review* 5 (1944), 7–39
Scholz, H. 'The ancient axiomatic theory', in Barnes, Schofield, Sorabji, vol. 1, 50–64
Sedley, D. 'The end of the Academy', *Phronesis* 26 (1981), 67–75
Shipton, K. M. 'Heraclitus fr. 10: a musical interpretation', *Phronesis* 30 (1985), 111–30
Shirlaw, M. 'The music and tone systems of ancient Greece', *Music Review* 4 (1943), 14–27
'The music and tone systems of ancient Greece', *Music and Letters* 32 (1951), 131–9
'Claudius Ptolemy as musical theorist', *Music Review* 16 (1955), 181–90
Smyly, J. G. 'Notes on Theon of Smyrna', *Hermathena* 14 (1907), 261–79
Solmsen, F. *Aristotle's System of the Physical World*, Ithaca, N.Y., 1960
Solomon, J. '*Orestes* 344–45: Colometry and Music', *Greek, Roman and Byzantine Studies* 18 (1977), 71–83
'Eklysis and Ekbole', *Symbolae Osloenses* 55 (1980), 111–26
'The diastaltic ethos', *Classical Philology* 76 (1981), 93–100
Sorabji, R. 'Aristotle, mathematics and colour', *Classical Quarterly* 22 (1972), 293–308
Necessity, Cause and Blame, London, 1980
Steinmayer, O. 'Bacchius Geron's introduction to the art of music', *Journal of Music Theory* 29 (1985)
Streich, H. 'Tonarten in mythologischer und psychologischer Sicht', *Eranos-Jahrbuch* 48 (1979)
Strunk, O. *Source Readings in Musical History*, London, 1952; reprinted in five parts (vol. 1, *Antiquity and the Middle Ages*) 1981
Tanner, R. 'La musique antique grecque', *Revue musicale* 248 (1961), 5–72
Tannery, P. 'Sur Theon de Smyrne', *Revue de philologie* 18 (1894), 145–52

'Du rôle de la musique grecque dans la développement de la mathématique pure', *Bibliotheca mathematica* 3 (1902), 161–75
'Sur les intervalles de la musique grecque', *Revue des Études Grecques* 15 (1902), 336–52
'Inauthenticité de la "Division du canon" attribuée à Euclide', *Comptes rendues des séances de l'année 1904 de l'Académie d'Inscriptions et Belles-Lettres* (1904), 439–45
Mémoires scientifiques, Toulouse and Paris, 1912–29
Tarán, L. *Speusippus of Athens: A Critical Study with a Collection of the Related Texts and Commentary*, Leiden, 1981
Thesleff, H. *An Introduction to the Pythagorean Writings of the Hellenistic Period*, Åbo, 1961
The Pythagorean Texts of the Hellenistic Period, Åbo, 1965
Tiby, O. 'Note musicologische al *Timeo* di Platone', *Dioniso* 12 (1949), 33–55
Toomer, G. J. *Ptolemy's Almagest*, London, 1984
Vincent, A. J. H. 'Notices sur trois manuscrits grecs relatifs à la musique, avec une traduction française et des commentaires', in *Notices et extraits des manuscrits de la Bibliothèque du Roi*, vol. 16, part 2, Paris, 1847
Vlastos, G. *Plato's Universe*, Seattle, 1975
'The role of observation in Plato's conception of Astronomy', in *Science and the Sciences in Plato*, ed. J. P. Anton, Buffalo, N.Y., 1980, 1–31
Vogel, M. *Die Enharmonik der Griechen*, 2 vols., Düsseldorf, 1963
van der Waerden, B. L. 'Die Harmonielehre der Pythagoreer', *Hermes* 78 (1943), 163–99
'Grosse Terz, Oktave und Harmonie', *Museum Helveticum* 17 (1960), 111–4
Walker, D. P. *Studies in Musical Science in the Late Renaissance*, Leiden, 1978
Wallis, J. *Claudii Ptolemaei harmonicorum libri tres*, Oxford, 1682
(Ptolemy's *Harmonics* and Porphyry's *Commentary* are also included in his *Operum Mathematicorum*, vol. 3, Oxford, 1699)
Wallis, R. T. *Neoplatonism*, London, 1972
Wasserstein, A. 'Theaetetus and the history of the theory of numbers', *Classical Quarterly* n.s. 8 (1958), 165–79
Wegner, M. *Griechenland*, vol. II.4 of *Musikgeschichte in Bildern*, eds. H. Besseler and M. Schneider, Leipzig, 1963
Wehrli, F. *Aristoxenos*, vol. 2 of *Die Schule des Aristoteles*, Basel, 1945
Weil, E. 'La place de la logique dans la pensée aristotélicienne', *Revue de metaphysique et de morale* 56 (1951), 283–315, translated as 'The Place of Logic in Aristotle's Thought' in Barnes, Schofield, Sorabji, vol. 1, 88–112
Weil, H. and T. Reinach, *Plutarque: de la musique*, Paris, 1900
Westphal, R. G. H. *Griechische Harmonik und Melopöie*, Leipzig, 1863
Aristoxenus von Tarent: Melik und Rhythmik des Classischen Hellenenthums, 2 vols., Leipzig, 1883 and 1893
White, N. P. *Plato on Knowledge and Reality*, Indianapolis, 1976
A Companion to Plato's Republic, Oxford, 1979
Williams, C. F. A. 'The notes mese and hypate in Greek music', *Classical Review* 12 (1898), 98–100
The Aristoxenian Theory of Musical Rhythm, Cambridge, 1911
Winnington-Ingram, R. P. 'The Spondeion Scale', *Classical Quarterly* 22 (1928), 83–91
'Aristoxenus and the intervals of Greek music', *Classical Quarterly* 26 (1932), 195–208
Mode in Ancient Greek Music, Cambridge, 1936; reprinted Amsterdam, 1968

'The pentatonic tuning of the Greek lyre: a theory examined', *Classical Quarterly* n.s. 6 (1956), 169–86

'Die Enharmonik der Griechen', *Die Musikforschung* 18 (1965), 60–4

'The first notational diagram of Aristides Quintilianus', *Philologus* 117 (1973), 243–9

'Two studies in Greek musical notation', *Philologus* 122 (1978), 237–48

Zaminer, F. 'Hypate, Mese und Nete in frühgriechischen Denken', *Archiv für Musikwissenschaft* 41 (1984), 1–26

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